Adjusting the Beam-line Method for Positive-moment Yielding

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The beam-line method[1] is a convenient tool for the analysis of nonlinear building connections[2] and inelastic rotations over the interior supports of highway bridge girders.[3] However, the method presently presumes that a bending member remains elastic in positive bending. While this presumption may be acceptable at low load levels, residual stresses in rolled beams and welded girders combined with applied stresses are known to create inelastic behavior at moderate load levels and above.

The purpose of this paper is to demonstrate how positive-moment yielding may be accounted for in the beam-line method. As a result, the method may then be applied throughout the elastic range, the inelastic range and up to mechanism formation. An example problem will illustrate the adjusted method, and then some potential applications will be discussed.

EXAMPLE PROBLEM

The structure to be analyzed is a two-span continuous prismatic beam subjected to two simultaneously applied concentrated loads (Fig. 1). The loads are positioned to create a critical negative moment over the interior support while the beam is totally elastic.

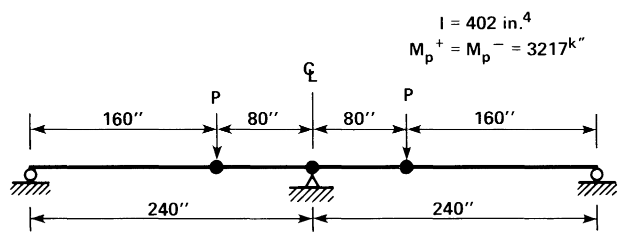

Figure 1

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The beam cross section and lateral bracing are assumed to produce the same moment-rotation behavior as that of Specimen 188-3-6 from Ref. 4. This beam was proportioned to have the maximum web and flange slenderness limits permitted by the AISC Specification[5] for plastic design at a yield point of $F_y = 50$ ksi. Because the effective plastic moment[3] for this beam was about 97% of the full plastic moment (M_p), it was assumed for this example they were equal.

The solid circles in Fig. 1 are the locations where all inelastic behavior is assumed to occur; the remainder of the structure is assumed always to be elastic. The ultimate load (P_u) for this structure obtained from conventional plastic analysis, which assumes elastic perfectly plastic hinges, is 100.5 kips. Taking advantage of symmetry, this value is computed by assuming M_p at the plastic hinges, taking moments about one load to determine the end reaction, and moments about the interior support to determine P_u. The required inelastic rotation (θ_{req}^-) corresponding to P_u for the hinge over the interior support is 17.2 mrads (1 mrad = 1 radian/1,000). θ_{req}^- is computed by applying the portion of P above that required to form a hinge at the interior support on a simple beam, calculating the end slope, and multiplying by two because of symmetry. For elastic-perfectly plastic hinges, the inelastic rotation under each load, $\theta_{req}^+ = 0$ for this example because the hinges under the loads are the last to simultaneously form. The superscripts refer to the sense of rotation.

Elastic Beam in Positive Bending

The beam-line method graphically determines the equilibrium position of a beam, assuming elastic behavior in positive bending. Two curves are plotted: the beam line, which is computed, and the negative moment (M^-) vs. negative rotation (θ^-) curve, which is experimentally determined. For a connection, the M^- vs. θ^- curve may contain both elastic and inelastic components, whereas for the sample problem, θ^- is totally inelastic and therefore is referred to as θ_p^-, the inelastic rotation.

The two curves for the example problem are plotted in Fig. 2. The experimental M^- vs. θ_p^- curve for the given section is based on Ref. 4, but has been broken into linear sections. The limit of the elastic region was set at $M^-/M_p^- = 0.5$. Although the beam-line method can handle a non-linear moment-rotation curve, making it stepwise linear has the advantage of allowing a clear definition of the elastic region; another advantage will be discussed later.

A beam line may be constructed for any value of P. The ordinate value ($M^- = M_{be}^-$), when $\theta_p^- = 0$, is computed from an assumed elastic structure that has full continuity at the interior support. The abscissa value ($\theta_p^- = \theta_{be}^-$), when $M^- = 0$, is computed from an assumed elastic structure that has a free hinge at the interior support. The subscript e in M_{be}^- and θ_{be}^- indicates the beam is elastic in positive bending. All ordinate values are normalized by M_p^- or the effective plastic moment, as appropriate; for this example, they are equal. The slope of the beam line is the change in M^- per change in θ_p^- for elastic behavior, and is thus a straight line.

Two beam lines are shown in Fig. 2. The lower is at the limit of ideal elastic behavior—when $\theta_p^- = 0$. The upper beam line is at P_u, the ultimate load from conventional plastic design. θ_{req}^- can be read from the intersection of the beam line and the elastic-perfectly plastic horizontal line and is the same value as previously mentioned. From Fig. 2, the actual M^- at P_u can be seen to be about 1.05 M_p at the intersection of the beam line and the experimental curve. In passing, it should be noted that a distinction is made between M_p^- and M_p^+ because, although they are equal in this example, that is not necessarily true.

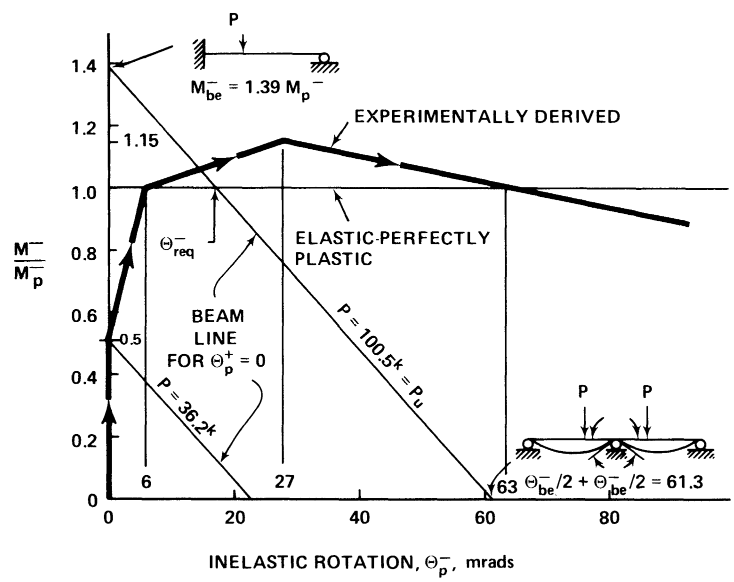

Figure 2

Inelastic Beam in Positive Bending

As mentioned, the value of $M^- = 1.05M_p^-$ at $P = 100.5$ kips presumes the beam is elastic in positive bending. M^+ under the load $P = 100.5$ kips is computed from the statically determinate beam in Fig. 3 as $M^+ = 0.967\ M_p^+$.

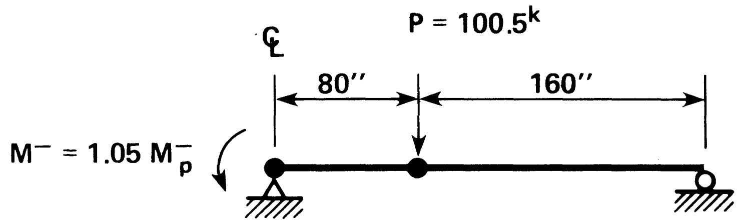

Figure 3

For this example, since the beam is assumed prismatic, the same M vs. θ_p curve applies in positive and negative bending. Thus, from Fig. 2, $\theta_p^+ = 5.5$ mrads at $M^+ = 0.967\ M_p^+$. The assumption of an elastic beam in positive bending at this value of P is therefore incorrect, because $0.967\ M_p^+ > 0.5\ M_p^+$.

We thus reach the main objective of this paper: how to modify Fig. 2 for θ_p^+. The procedure to be followed is similar to moment distribution, where all joints are locked but one and, in an iterative fashion, the moments are gradually redistributed until convergence. We redistribute the effect of θ_p by iteratively assuming that the inelastic rotation of the opposite sign is locked in and therefore behaves elastically. The end result is to recompute the location of the beam line.

The abscissa value, when $M^- = 0$, is equal to θ_{be}^- plus the change caused by θ_p^+, θ_{bp}^-. The subscript p indicates that the beam is inelastic in positive bending. This can be computed from Fig. 4.

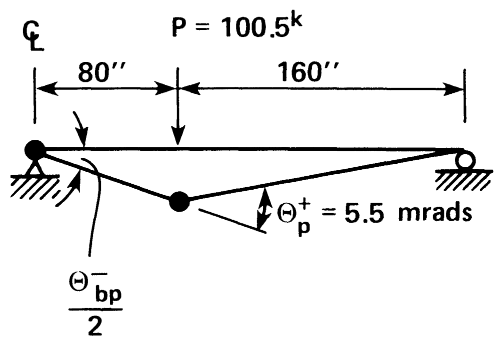

Figure 4

Since θ_p^+ occurs in both spans, $\theta_{bp}^-/2$ is shown in Fig. 4. From geometry, $\theta_{bp}^- = 7.4$ mrads is computed. Thus, $\theta_{be}^- + \theta_{bp}^- = 61.3 + 7.4 = 68.7$ mrads.

The ordinate value, when $\theta_p^- = 0$, is equal to M_{be}^- plus the change caused by θ_p^+, M_{bp}^-. The subscript p again indicates that the beam is inelastic in positive bending. This can be computed from Fig. 5.

The effect of θ_p^+, when $\theta_p^- = 0$, is that part of the load (P_1) is resisted as a cantilever. The remainder, $P - P_1$, is resisted as an elastic beam. The effect of part of the load being carried as a cantilever is to increase the negative moment. P_1 may be computed from the geometry in Fig. 5 and the conjugate-beam method. Knowing P_1, M_{bp}^- is easily computed so that $M_{be}^- + M_{bp}^- = 1.55\ M_p^-$ (see Appendix A for full derivation).

The beam line adjusted for $\theta_p^+ = 5.5$ mrads is plotted in Fig. 6. The adjusted beam line is parallel to the original beam line, a fact that can be used to advantage in further computations because only the adjusted abscissa need be computed. Indeed, upon reflection it could have been reasoned the lines are parallel because the slope of the line is the ratio of the change in negative moment to change in negative rotation; it does not matter what changes that rotation—i.e., θ^+ or θ^-.

Continuing the process, we read a new value of M^- from Fig. 6, which is increased because of θ_p^+. This process is repeated until convergence. For $P = 100.5$ kips, the adjusted beam line in Fig. 6 is essentially converged, and positive and negative rotations are shown by circles. For comparison, hinge rotations for elastic-perfectly plastic hinges at the same load are also plotted in Fig. 6 as squares. It can be noted at $P = 100.5$ kips that, for this structure, section and loading, conventional plastic design underestimates both rotations and M^-, and overestimates M^+.

This process is repeated for increasing values of P, and certain levels are plotted in Fig. 7. Once both inelastic rotations are past the point of maximum moment, that is, on the unloading branch, the process diverges, indicating an equilibrium position is not possible for such load levels and collapse of the structure is imminent. The computed ultimate load is 110.5 kips, 10% above the value computed for the elastic-perfectly plastic assumption. At this load, M^+ reaches its maximum value and M^- is decreasing.

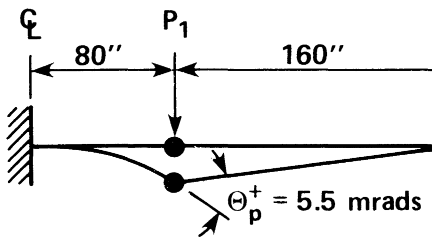

Figure 5

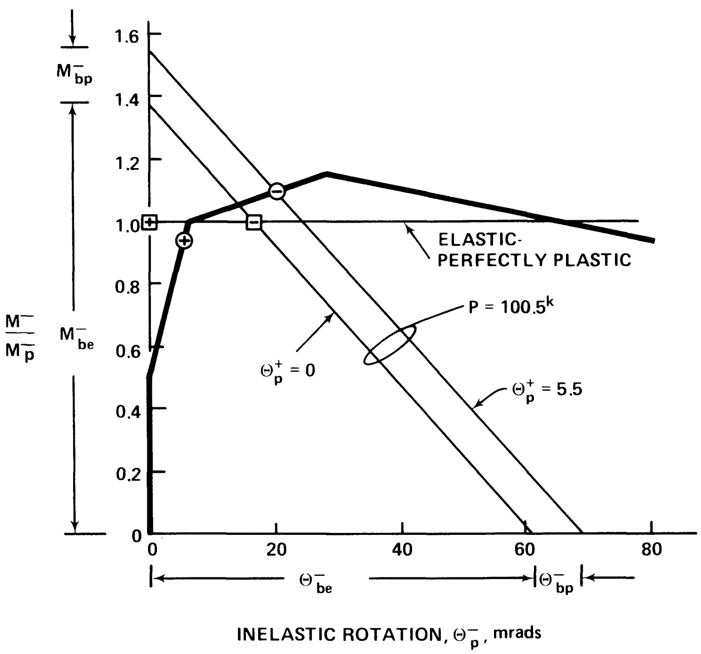

Figure 6

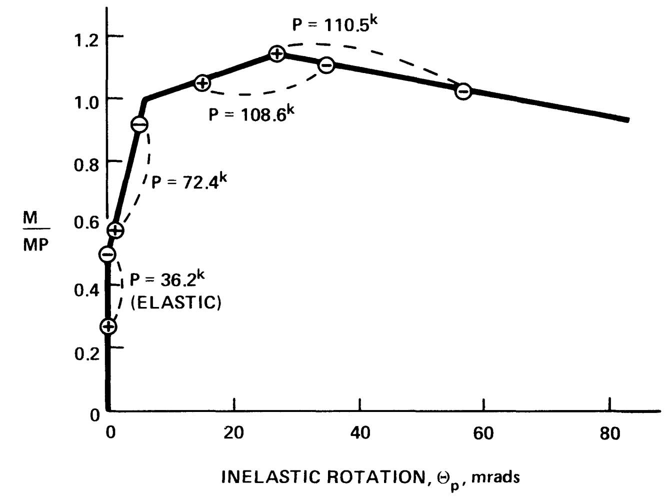

Figure 7

POTENTIAL APPLICATIONS

Elastic analysis requires three conditions: continuity, moments below the yield moment and equilibrium. Conventional plastic analysis has similar requirements: mechanism formation, moment not above M_p and equilibrium. The method described herein generally provides a logical link between these two limit states. And since this is the region where realistic loads occur, the method should have application. Although this method may not be suitable for routine design, it may have application in setting the required parameters for plastic design. The present AISC compactness rules are intended to permit a plastic hinge to reach strain hardening,[6] which may be too conservative for certain structures.

Figure 7 demonstrates stable equilibrium can exist when one of the hinges is on the unloading branch of the moment vs. inelastic-rotation curve. This will be useful in demonstrating that effective plastic moment[3] provides the same stability as conventional plastic design. Based on this one example, it cannot be concluded any particular structure has an equilibrium position on its unloading branch—especially when the branch is steep. However, the method can evaluate such cases.

The modified beam-line method described is useful if a structure is reasonably simple and if the support rotation remains greater than the rotation in positive bending. If these situations are not met, a similar procedure can be employed: instead of the beam-line method, the finite element method can be used. An incremental approach would follow the loading, and each linear zone of the M vs. θ_p curve would be a separate linear finite-element analysis. The points on Fig. 7 were independently verified by a series of piecewise linear analyses using the finite-element program MSC/NASTRAN.[7]

Bridges

Proposed limit states[3] would permit a controlled amount of yielding at an interior support at the overload level. To ensure plastic behavior in positive bending, the corresponding stress is limited to $0.95F_y$. The procedure described will permit any inelastic behavior in positive bending at that stress level to be evaluated.

The same proposed limit states would permit plastic design of composite beams at maximum load level. Barnard and Johnson[8] realized that under certain conditions, which they termed "strain-softening hinges," conventional plastic design may overestimate the collapse load. The procedure described will permit such conditions to be evaluated when the M^+ vs. θ_p^+ curve of a composite beam is known.

Buildings

Certain flexible connections, when used with a relatively stiff beam, may not be able to reach their maximum strength concomitantly with the beam. The proposed method would permit the adequacy of combinations of connections and beams to be evaluated.

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NOMENCLATURE

E	=	Young's modulus = 29,000 ksi
F_y	=	Yield strength, ksi
I	=	Moment of inertia = 402 in.^{4}
M	=	Resisting moment
M_{be} M_{bp}	=	Beam-line ordinate value with elastic and inelastic beam in positive bending
M_p	=	Full plastic moment
P	=	Applied load
P_u	=	Ultimate load for elastic-perfectly plastic hinges
P_1	=	Portion of P resisted as a cantilever
θ_p	=	Inelastic rotation
θ_{be}, θ_{bp}	=	Beam-line abscissa value with elastic and inelastic beam in positive bending
θ_{req}	=	Required inelastic rotation for elastic-perfectly plastic hinges.

Superscripts "+" and "−" refer to positive and negative bending.