Analysis of Knee-braced Portal Frames for Vertical Loading: Part 2–Columns of Unequal Stiffness

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In our previous paper, "The Analysis of Knee-Braced Portal Frames for Vertical Loading,"[1] the example presented had columns of equal stiffness, resulting in a simple computation for sway shear force. The sway shear force was equally divided between the two columns to find the knee-brace loads. This paper investigates the sway analysis of frames of unequal column stiffness. For continuity, the equation and figure numbers are continuous from Part 1.[1]

The first step in determining sway shear for the frame is described in the previous paper. Frame sway is first prevented by introducing a temporary horizontal restraint at the beam level. For a one bay portal frame, the initial knee-brace loads due to vertical loading are calculated using Eq. 7 of Ref. 1, and the restraint reactions determined. In the second step, the temporary restraint is removed and the frame allowed to sway due to the sway load, giving the sway correction. The sum of the two steps gives the required result.

SWAY CORRECTION

The number of equations to be solved is the same as the number of knee braces in the frame. One of these equations derives from equating frame base shears to the sway load. For the frame shown in Fig. 9 the equation is:

$$P_1\cos\theta_1 \frac{a_1}{L_1} + P_2\cos\theta_2 \frac{a_2}{L_2} = H \quad \text{or}$$

$$CP_1 + DP_2 = H \tag{9}$$

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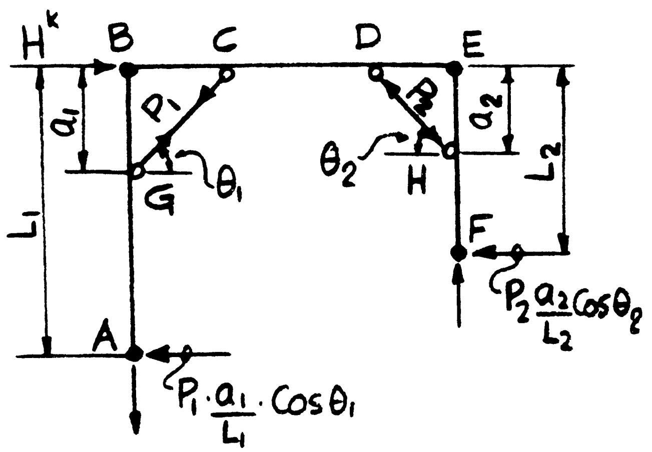

Figure 9

The second equation is obtained by equating horizontal deflections d_B and d_E. The horizontal deflection comprises of two parts. In the case of Col. AB, the first part is the displacement of GB relative to base A due to base shear of

$$P_1 \cos\theta_1 \frac{a_1}{L_1} \text{ given by } P_1 \cos\theta_1 \frac{a_1}{L_1} d_{aa}$$

The second part of the horizontal deflection is given by $d_G \frac{L_1}{a_1}$ where d_G is the horizontal movement of G considering vertical deflection of beam at C and axial deformation of knee brace. For the direction of sway shown, P_1 is tensile and P_2 is compressive. Referring to Fig. 10,

$$d_G = (P_1 d_{brc1} + d_C \sin\theta_1) \,/\, \cos\theta_1$$

$$\text{and } d_C = P_1 \sin\theta_1 d_{cc} - P_2 \sin\theta_2 d_{cd}$$

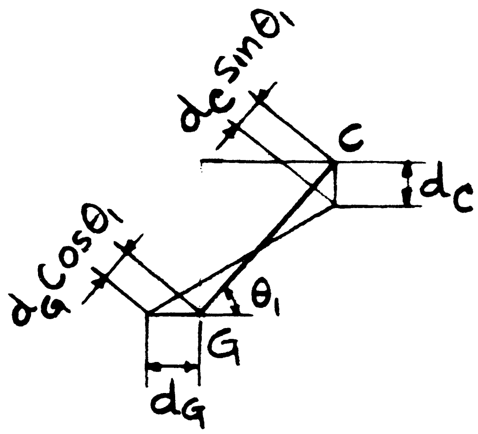

Figure 10

Substituting the value of d_C in the equation for d_G, we get

$$d_G = (P_1 d_{brc1} + P_1 \sin^2\theta_1 d_{cc} - P_2 \sin\theta_1 \sin\theta_2 d_{cd})/\cos\theta_1$$

Now the total lateral deflection of B is given by:

$$d_B = P_1 \cos\theta_1 \frac{a_1}{L_1} d_{aa} + d_G \frac{L_1}{a_1}$$

$$= P_1 \cos\theta_1 \frac{a_1}{L_1} d_{aa} + (P_1 d_{brc1} + P_1 \sin^2\theta_1 d_{cc} - P_2 \sin\theta_1 \sin\theta_2 d_{cd}) \frac{L_1}{a_1} / \cos\theta_1 \qquad (10)$$

Similarly, lateral deflection of E is given by,

$$d_E = P_2 \cos\theta_2 \frac{a_2}{L_2} d_{ff} + (P_2 d_{brc2} + P_2 \sin^2\theta_2 d_{dd} - P_1 sin\theta_1 sin\theta_2 d_{cd})\ (L_2/a_2)/\cos\theta_2 \qquad (11)$$

Equating d_B and d_E and rearranging we get,

$$AP_1 - BP_2 = 0 \qquad (12)$$

where coefficients A and B are as follows,

$$A = \frac{a_1}{L_1} d_{aa} \cos\theta_1 + \frac{L_1 d_{brc1}}{a_1 \cos\theta_1} + \frac{L_1 \sin^2\theta_1}{a_1 \cos\theta_1} d_{cc} + \frac{L_2 \sin\theta_1 \sin\theta_2}{a_2 \cos\theta_2} d_{cd} \text{ and}$$

$$B = \frac{a_2}{L_2} d_{ff} \cos\theta_2 + \frac{L_2 d_{brc2}}{a_2 \cos\theta_2} + \frac{L_2 \sin^2\theta_2}{a_2 \cos\theta_2} d_{dd} + \frac{L_1 \sin\theta_1 \sin\theta_2}{a_1 \cos\theta_1} d_{cd}$$

Solving Eqs. 9 and 12, the knee-brace loads are,

$$P_1 = HB/(AD + BC) \text{ and } P_2 = HA/(AD + BC)$$

Following is an example of a single bay knee-braced portal frame with columns of unequal stiffness. Referring to Fig. 11,

Col. AB: $I = 750$ in.^{4}

$L_1 = 20$ ft

$a_1 = 8$ ft

$d_{gg} = \dfrac{353.89}{E}$ in./kip

$d_{aa} = \dfrac{2{,}211.84}{E}$ in./kip

Col. EF: $I = 500$ in.^{4}

$L_2 = 8$ ft

$a_2 = 4$ ft

$d_{hh} = \dfrac{36.86}{E}$ in./kip

$d_{ff} = \dfrac{147.46}{E}$ in./kip

Beam BE: $I = 1{,}000$ in.^{4}

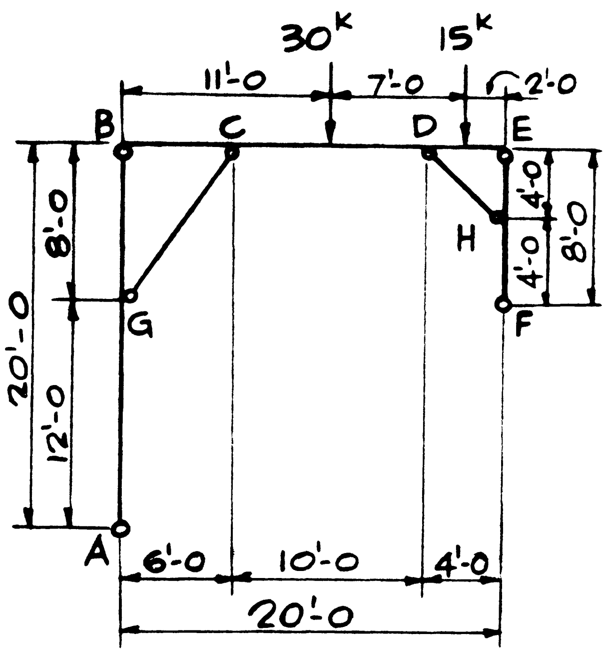

Figure 11

$$l = 20 \text{ ft}$$

$$d_{cc} = \frac{203.21}{E} \text{ in./kip}$$

$$d_{cd} = \frac{120.27}{E} \text{ in./kip}$$

$$d_{dd} = \frac{117.97}{E} \text{ in./kip}$$

Due to vertical loading on beam,

$$d_{C1} = \frac{7{,}534.9}{E} \text{ in.}$$

$$d_{D1} = \frac{5{,}966.78}{E} \text{ in.}$$

Knee brace 1: $L = 10$ ft

$$A = 2.5 \text{ sq.in.}$$

$$\sin\theta_1 = 0.8$$

$$\cos\theta_1 = 0.6$$

$$d_{brc1} = \frac{48}{E} \text{ in./kip}$$

Knee brace 2: $L = 5.656$ ft

$$A = 2.5 \text{ sq.in.}$$

$$\sin\theta_2 = \cos\theta_2 = 0.707$$

$$d_{brc2} = \frac{27.15}{E} \text{ in./kip}$$

a. Sway Prevented Case

Using Eq. 7 of Ref. 1,

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} M_{11} \; M_{12} \\ M_{21} \; M_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} 7{,}534.9 \times 0.8 \\ 5{,}966.78 \times 0.707 \end{bmatrix}$$

where

$$M_{11} = 0.64 \times 203.21 + 0.36 \times 353.89 + 48$$

$$M_{12} = 0.8 \times 0.707 \times 120.27$$

$$M_{21} = 0.8 \times 0.707 \times 120.47$$

$$M_{22} = 0.5 \times 117.965 + 0.5 \times 36.86 + 27.15$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 305.46 & 68.04 \\ 68.04 & 104.56 \end{bmatrix}^{-1} \begin{bmatrix} 6{,}027.95 \\ 4{,}219.15 \end{bmatrix} = \begin{bmatrix} 12.568^k \\ 32.174^k \end{bmatrix}$$

Sway shear $= 12.568 \times 0.6 \times 0.4 - 32.174 \times 0.707 \times 0.5 = -8.359^k$

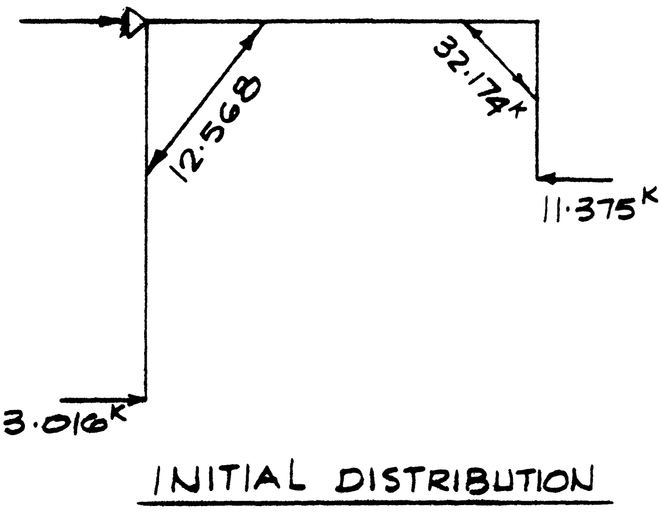

Figure 12

Initial distribution is shown in Fig. 12.

b. Sway Correction

Coef. A $= (8/20) \times 0.6 \times 2{,}211.84 + (20/8) \times 0.8 \times (0.8/0.6) \times 203.213 + (8/4) \times 0.8 \times 1 \times 120.27 + [(20 \times 48)/(8 \times 0.6)] = 530.84 + 541.9 + 192.43 + 200.0 = 1{,}465.17$

Coef. B $= (4/8) \times 0.707 \times 147.46 + (8/4) \times 0.707 \times 1 \times 117.97 + (20/8) \times 0.707 \times (0.8/0.6) \times 120.27 + [(8 \times 27.15)/(4 \times 1)] \times 0.707 = 52.13 + 166.83 + 283.48 + 76.8 = 579.24$

Coef. C $= (8/20) \times 0.6 = 0.24$

Coef. D $= (4/8) \times 0.707 = 0.35355$

AD + BC $= 1{,}465.17 \times 0.35355 + 579.24 \times 0.24 = 657.035$

$$P_1 = \frac{579.24}{657.035} \times 8.359^k = 7.369^k$$

$$P_2 = \frac{1{,}465.17}{657.035} \times 8.359^k = 18.64^k$$

These values are shown in Fig. 13.

The resulting knee-brace loads, together with column and beam moments, are shown in Figs. 14 and 15. The above problem was also solved using a computer, with the results within 2% of the values calculated above.

This paper, and the previous one, have looked at the analysis of a one bay portal frame. The same reasoning can be extended to cover multibay frames with knee braces. The number of equations to be solved for each of the cases of sway prevented and sway correction would depend on the number of knee braces totally present.

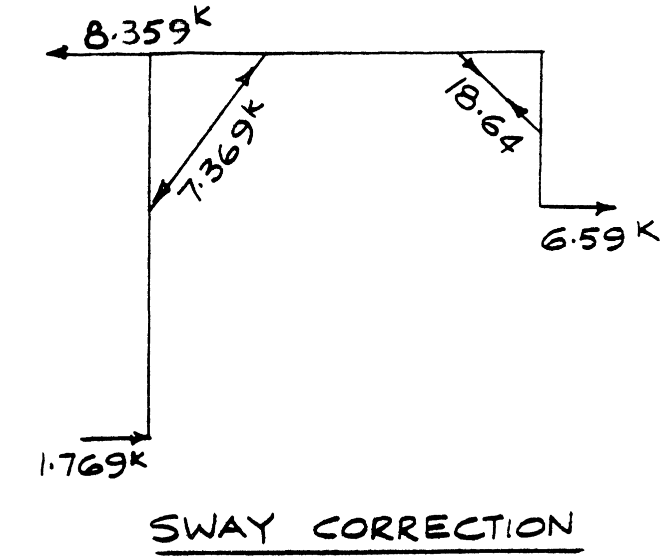

Figure 13

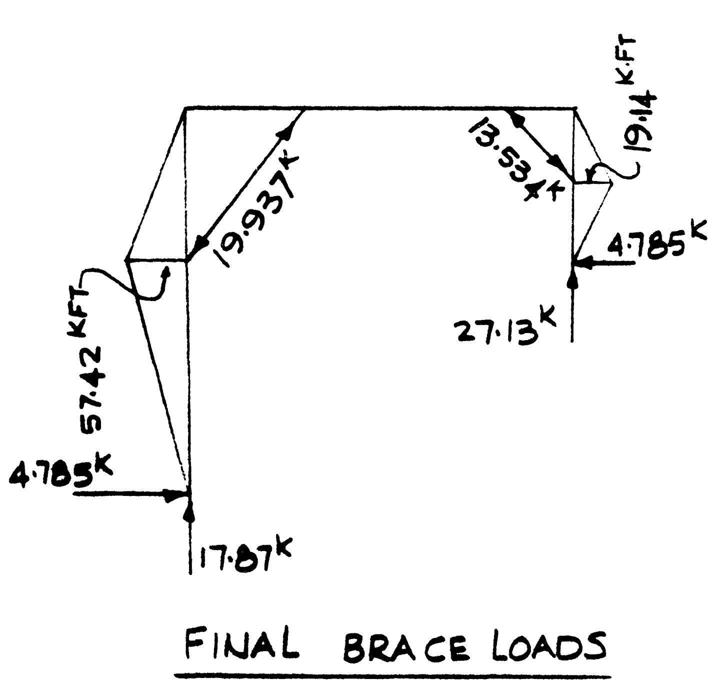

Figure 14

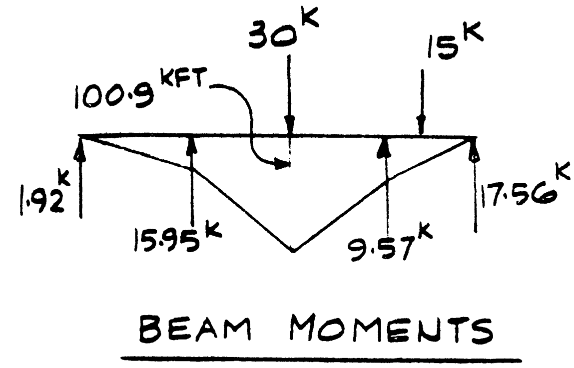

Figure 15

NOMENCLATURE

d_{aa} = cantilever deflection of base A due to unit load at A on the Col. AB supported at G and B
d_{cc} = deflection in beam at C due to a unit load at C
d_{cd} = deflection in beam at C due to a unit load at D
d_{dd} = deflection in beam at D due to a unit load at D
d_{gg} = deflection in Col. AB at G due to unit load at G
d_{hh} = deflection in Col. EF at H due to unit load at H
d_{ff} = cantilever deflection of Base F due to a unit load at F on the Col. EF supported at H and E
d_{brc1} = shortening of Brace GC due to a unit load along GC
d_{brc2} = shortening of Brace DH due to a unit load along DH
d_{C1} = deflection in beam at C due to external loading
d_{D1} = deflection in beam at D due to external loading
P_1 = force in Brace GC
P_2 = force in Brace DH

REFERENCE

1. *Vilas H.K. and J.O. Surtees* Analysis of Knee-Braced Portal Frames for Vertical Loading *AISC Engineering Journal, 4th Qtr., 1983, Chicago, Ill.*