Nonlinear Analysis of Eccentric Bolted Connections

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A simple approximate step-by-step method is proposed for the nonlinear analysis of fastener groups subject to eccentric shear. The procedure replaces the traditional trial and error solution by a series of incremental linear elastic analyses. Rigid plate-piecewise linear bolt model is assumed. Conservative, yet sufficiently accurate results may generally be expected. With this approach it is possible to trace the development of plastification and the migration of the incremental instantaneous center of rotation with increasing loading.

BACKGROUND

Bolted or riveted connections are often called upon to carry eccentric forces. This paper considers the case in which the eccentric load lies in the plane of the fastener group, namely, the fasteners are stressed only in shear. Traditionally, these connections were analyzed by the elastic method (e.g., Ref. 1). However, with the elastic approach, the design load of the whole connection is reached when the design load of the fastener located furthest from the elastic instantaneous center of rotation is reached. However, at this load level, fasteners nearer to the center of rotation are not fully stressed. Therefore, this approach is conservative. This recognition led the AISC,[2] after a series of tests,[3,4] to lower the safety factor by permitting use of a reduced or effective eccentricity in the elastic analysis, a procedure recently removed from the AISC Manual (errata to Ref. 2) because it could lead to unconservative results. At the same time, the plastic approach was applied to these connections,[5-8] in an attempt to improve the agreement between experimental and analytical results. However, early advocates of the plastic method did not give sufficient recognition to the fact that bolts in shear have limited ductility, and therefore it may not be possible to utilize fully the plastic capacity of all the fasteners in the group.[9] The displacement compatibility method proposed by Crawford and Kulak for bearing type connections[10,11] took this effect into account, as well as the actual nonlinear load-displacement relationship of the fastener itself. It will be observed that in practical terms, there is not much difference in the capacities of connections computed by means of the plastic and the compatibility methods.[5,12] Yet, the plastic approach is more appropriate for predicting the limiting slip resistance of friction type connections.[13] A technique to improve the efficiency of computing the ultimate load of eccentric bolted connections has recently been proposed by Brandt.[14]

The limitation of those methods, as far as the design engineer is concerned, is that trial and error procedures are required to obtain the solution. Also, the fact that the equilibrium state at ultimate load, and only this, is the aim of the analysis, results in the absence of information on the development of plastification, changes in load direction on the individual fasteners and the migration of the instantaneous center of rotation with increasing load. Yet, such information is believed to be important for understanding the real behavior of the connection under static loading. Moreover, these methods cannot be used to analyze the response of eccentric connections under dynamic loads.

This paper proposes a direct step-by-step approximate procedure, manual as well as computer-oriented, for the nonlinear analysis of eccentric connections. This procedure replaces the trial and error techniques used for the plastic and the compatibility methods by a series of linear elastic analyses, all having simple explicit solutions. The procedure is applicable to any arbitrary piecewise linear load-displacement relationship of the individual fastener. It can also be adapted to the nonlinear dynamic analysis of eccentric connections.

First, a description of the procedure is given in some detail, followed by a discussion of the approximations involved. Parametric studies are then presented to demonstrate the errors associated with this simplified ap-

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proach are not significant. Two numerical examples illustrating the procedure are given, and results compared with ultimate load values published in the literature.

ANALYSIS

The analysis presented in this section has been developed to approximate the force-displacement history of fasteners in a connection subjected to eccentric shear. This analysis is based on the following assumptions:

1. Due to a load increment, the fastener group rotates about a point called the (instantaneous) incremental center of rotation (IC). In the nonlinear range of the load-displacement relation, the position of this point may change as the load on the connection changes. Observe that this point usually does not coincide with the instantaneous center of rotation as defined in the literature (Refs. 5-8, 10-12), which is the point around which the connection rotates due to the combined effect of all the load increments comprising the applied load.
2. The incremental displacement which occurs at each fastener is proportionate to its distance from the incremental center of rotation, and acts in a direction perpendicular to the radius of rotation of the fastener.
3. Changes in load direction have negligible effect on the response of the fastener at all stages of loading.
4. The connected plates remain rigid during rotation, i.e. rotation is due solely to shear displacements of the fasteners and to local plate deformations around it, or to local slip in slip-resisting connections.
5. The load-displacement relation of a single fastener is assumed to be piecewise linear.
6. The ultimate strength of the group is reached when the ultimate strength of any one fastener is reached. For slip resistant connections, the ultimate strength is reached when all fasteners, or all but one fastener, are "yielding." This statement is clarified later in this section.

Assumption 3, which may appear unwarranted, was motivated by the observation that, in many connections, locations of the elastic and plastic instantaneous centers of rotation are quite close. This suggests only relatively small shifts of the incremental centers with the spreading of plastification in the fastener group, and only minor changes in the direction of the forces acting on the fasteners. This assumption involves two simplifications: algebraic summation of incremental fastener forces may replace their vectorial summation; and additional forces due to changes in direction on already yielded fasteners are ignored. These assumptions have the advantage of simplifying the analysis appreciably, this at the cost of only minor conservative losses in accuracy, as illustrated in the following section. Evidently, the first simplification can easily be dispensed with, if some additional computations are undertaken. On the other hand, the second simplifying assumption avoids the complications associated with applying the theory of yielding with interaction between forces acting on inelastic elements in two perpendicular directions.[15] Observe that changes in load direction have an appreciable effect on the strength and ductility of welded connections.[16,17] Although the problem is beyond the scope of the present paper, its implications are discussed briefly in the concluding section.

Assumption 6 requires some clarification when applied to plastic analysis, since one would expect all fasteners to "yield," unless the (nonincremental) instantaneous center of rotation computed at ultimate load happens to coincide with a fastener. Such situations are quite common when the eccentricity is large and the number of fasteners is uneven. A discussion of some similar singular situations is made by Fazekas.[18] The difficulty, however, is due to the approximations inherent in Assumption 3. In the last stage of the step-by-step analysis, usually only one fastener remains with some unused resisting capacity (unless due to symmetry there are two such fasteners), whereas the other fasteners are assumed to have yielded. Since the torsional rigidity of a single fastener is assumed to be zero, no equilibrium is possible for an additional increment of external load, and the capacity of the group may be considered to have been reached. Alternatively, one may attempt to resolve the problem by recognizing that, in fact, the apparently yielded fasteners have some residual rigidity, and by making a guess as to the location of the rigidity center of the remaining stiffness. The elastic instantaneous center may conveniently serve as a first approximation for this center. It will be seen that with this approximation one is likely to improve the accuracy of the proposed method, but at the loss of the lower bound property of the solution.

Consider the fastener group shown in Fig. 1. The center of gravity of the group or its center of rigidity is located at CG_o, and the load P acts downwards at an eccentricity e_o from CG_o. It is assumed the load-deformation response of a fastener is given by Fig. 2. An initial load increment $\Delta_o P$, as yet to be determined, is applied, assuming the resulting initial force increment $\Delta_o F_i$ acting on the most heavily stressed fastener does not exceed its slip force (Fig. 2a), or the first kink on the force displacement curve (Fig. 2b). Noting the location of the elastic instantaneous center of rotation IC_o is the point of zero displacement, it can easily be shown that IC_o is located at a distance a_o to the left of CG_o, given by:

$$a_o = \frac{K_{\theta o}}{K_{so}\, e_o} = \frac{r_o^2}{e} \qquad (1)$$

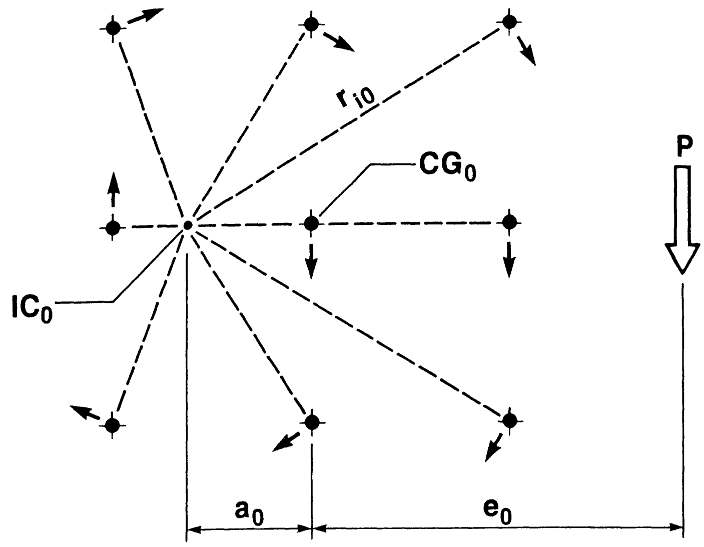

Fig. 1. Eccentrically loaded fastener group

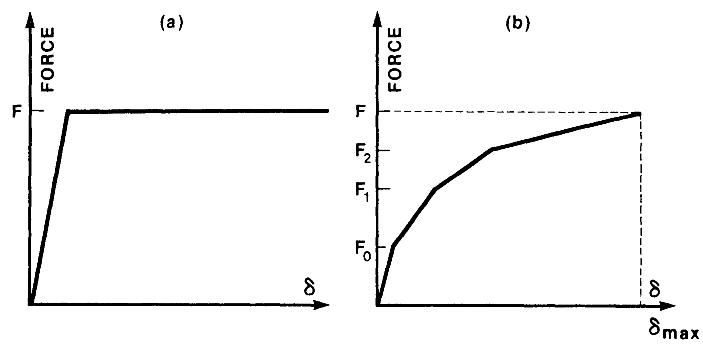

Fig. 2. Load displacement response of a single fastener

in which K_{so} and $K_{\theta o}$ are respectively the shear and torsional rigidities of the system. These are obtained from the individual rigidities K_{io} of the fasteners by means of the following expressions:

$$K_{so} = \sum_{1}^{n} K_{io} \tag{2}$$

$$K_{\theta o} = \sum_{1}^{n} K_{io} \rho_{io}^2 \tag{3}$$

in which n = number of fasteners, and ρ_{io} = the distance of the fastener from CG_o. Note that for loadings inclined to an orthogonal coordinate system with its origin at CG_o (Fig. 6), the coordinates a_{xo} and a_{yo} of IC_o are given by:

$$a_{xo} = -a_o \frac{e_{xo}}{e_o}; \; a_{yo} = -a_o \frac{e_{yo}}{e_o} \tag{1a}$$

Denoting the distance of the fastener to IC_o by r_{io}, the initial force $\Delta_o F_i$ acting on the fastener is given by:

$$\Delta_o F_i = \frac{K_{io} r_{io}}{K_{so} \, a_o} \Delta_o P \tag{4}$$

Equation 4 can easily be verified by noting that, due to the in-plane rigidity of the plate, the displacement $\Delta_o \delta_i$ of a fastener is proportional to the displacement $\Delta_o \delta_{CG}$ of the center of rigidity, or:

$$\Delta_o \delta_i = \frac{r_{io}}{a_o} \Delta_o \delta_{CG} \tag{5}$$

When all fasteners are identical, which is usually the case, Eq. 4 takes the form:

$$\Delta_o F_i = \frac{r_{io}}{n \, a_o} \Delta_o P \tag{4a}$$

For the most heavily stressed fastener j:

$$\Delta_o F_j = F_o \tag{6}$$

and it follows that:

$$\Delta_o P = \frac{K_{so} \, a_o}{K_{jo} \, r_{jo}} F_o = \frac{n \, a_o}{r_{jo}} F_o \tag{7}$$

The preceding relations hold so long as $\Delta_o F_i \leq F_o$. With an additional external load increment $\Delta_1 P$, the effective stiffness of the most heavily loaded fastener is lowered. Therefore, it is necessary to compute new stiffness properties for the group. Since the load-displacement relationship is assumed to be piecewise linear, the computation required to evaluate the effect of $\Delta_1 P$ on all fasteners is similar to the one carried out for the initial stage. The location of the new incremental center IC_1 is given by:

$$a_1 = \frac{K_{\theta 1}}{K_{s1} \, e_1} \tag{8}$$

in which e_1 = the load eccentricity measured from the new center of rigidity CG_1,

$$K_{s1} = K_{j1} + \sum_{i \neq j} K_{io} \tag{9}$$

$$K_{\theta 1} = K_{j1} \, \rho_{j1}^2 + \sum_{i \neq j} K_{io} \, \rho_{io}^2 \tag{10}$$

K_{j1} = the reduced rigidity of fastener j as measured from the secondary slope of the force displacement relationship, and ρ_{j1} the distance of fastener j from CG_1. Again, the incremental force on a fastener is:

$$\Delta_1 F_i = \frac{K_{il}\, r_{i1}}{K_{s1}\, a_1} \Delta_1 P$$

$$(l=0, \text{ for } i \neq j;\ l=1, \text{ for } i=j) \qquad (11)$$

The magnitude of $\Delta_1 P$ can be determined so that the accumulated shear F_{j1} on the most heavily loaded fastener j, computed vectorially, just reaches the second kink F_1 on the load displacement curve (Fig. 2b), or:

$$F_{j1} = \Delta_o \vec{F}_j + \Delta_1 \vec{F}_j \qquad (12)$$

so that the incremental reserve capacity is given by:

$$\Delta_1 F_j = F_1 - F_{j1} \qquad (13)$$

and

$$\Delta_1 P = \frac{K_{s1}\, a_1}{K_{j1}\, r_{j1}} \Delta_1 F_j \qquad (14)$$

However, when the shear on another fastener l reaches F_o at a lower level of external loading, $\Delta_1 P$ in equation 14 should be computed on the basis of:

$$\Delta_1 F_l = F_o - \Delta_o F_l \qquad (15)$$

The repeated vectorial summations indicated in Eq. 12 are rather tedious for hand computations. To simplify matters, it is suggested herein (Assumption 3) to use algebraic summation instead, namely:

$$F_{i1} = \Delta_o \mathrm{F}_i + \Delta_1 F_i \qquad (16)$$

Evidently, this is only an approximation because usually $\Delta_o F_i$ and $\Delta_1 F_i$ are not colinear, since the incremental center of rotation was shifted due to the reduced rigidity of bolt j. Thus, it is assumed that the angle between the two vectors is sufficiently small so that the algebraic sum can be taken as a close approximation of their resultant. The error to be expected from this assumption is discussed in the following section.

In bearing type connections with limited ductility, this step-by-step procedure is continued until the ultimate capacity of the most heavily loaded fastener has been reached, and in slip resistant systems, until all fasteners, or all but one, have yielded (Assumption 6). Note, however, that the contribution to the resisting capacity of the whole group of the remaining strength of the last fastener, which is also the closest to the center of rotation, is very small, even for groups consisting of only three fasteners. Therefore, its effect has little practical significance, a point illustrated in the following section.

ERROR ESTIMATES

The progressive loss of stiffness in a group of fasteners subject to increased loading is accompanied by some shifting of the incremental center. However, it has been observed the shift is likely to be small compared with typical dimensions of the group. For example, graphical presentations showing the proximity of the elastic instantaneous center (IC_o) to its plastic counterpart for two configurations of large irregular fastener groups having each two load eccentricities, were given by Surtees and Pape.[8] Also, relatively large shifts in the incremental center resulting in relatively large angles between two consecutive force vectors in the fasteners

$$\sum_{o}^{j} \Delta_j F_i \text{ and } \Delta_{j+1} F_i$$

do not cause their resultant to differ appreciably from their algebraic sum. This point is illustrated in Fig. 3. It can be seen that when the angle β between the forces F_1 and F_2 is as large as 45°, and the apparent reserve strength ratio $(R\text{-}F_1)/R$ is at maximum (about 45% of capacity), the error is only on the order of 8%. Moreover, the larger errors are likely to occur at fasteners located close to the incremental center, whose contribution to the resisting moment is quite small in any case, so that resulting overall errors are unlikely to be significant. Note that Fig. 3 can be used to improve the accuracy of the analysis if the angle β is computed. Observe that the proximity of the elastic instantaneous center of rotation to its counterpart at ultimate load explains the satisfactory results obtained when the latter center is assumed to be located at the former.[19,20]

The evaluation of errors associated with neglecting the x-y elastic-plastic interaction is more difficult. As an indication, the results of the plastic analysis made by Surtees et al.[7]* for a single column bolt group are compared in Table 1 with the results predicted by the proposed method. It is seen that the agreement is sufficiently close for engineering purposes. The effect of considering the reserve capacity of the central fastener is also shown. Note that the contribution of the latter was computed by considering its full reserve capacity, and assuming it rotates about the initial (elastic) incremental center. Therefore, some overestimation of capacity is to be expected, with the largest errors occurring for three-bolt groups at moderate eccentricities.

Thus it is seen that the conservatism in Assumption 3 is relatively small, suggesting that the benefits to be gained from trying to improve the results may not justify the additional computational effort.

*These are in full agreement with Ref. 6 for all comparable cases.

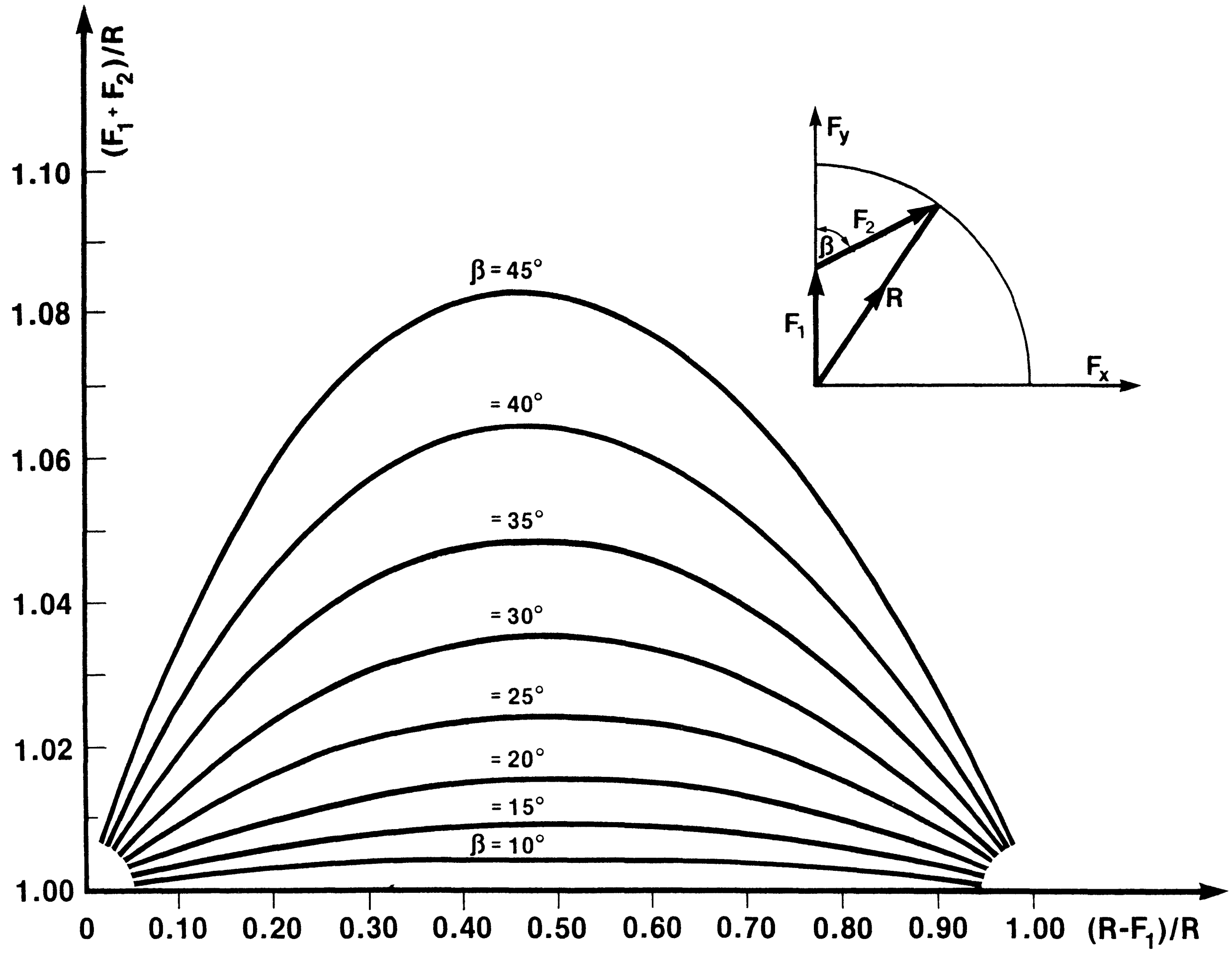

Fig. 3. Errors in algebraic summation of non-colinear vectors

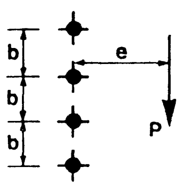

Table 1. Comparison of Results: Proposed Method vs. Plastic Analysis (Trial & Error)

$\frac{e}{(n-1)b}$	n	3		4	5		6	7
0.16	Plastic	0.93		0.92	0.91		0.90	0.90
	Proposed	0.90*	0.93$^{\$}$	0.91	0.89	0.91	0.90	0.88
0.50	Plastic	0.61		0.57	0.55		0.53	0.52
	Proposed	0.55	0.61	0.55	0.52	0.55	0.52	0.52
1.20	Plastic	0.28		0.27	0.25		0.25	0.24
	Proposed	0.27	0.29	0.26	0.24	0.25	0.24	0.23
2.00	Plastic	0.17		0.16	0.15		0.15	0.14
	Proposed	0.16	0.18	0.16	0.15	0.15	0.14	0.14

*Reserve capacity of central fastener ignored
$^{\$}$Reserve capacity of central fastener considered

Example 1

The three-bolt column group shown in Fig. 4 is to be analyzed for the force displacement diagrams given in Figs. 1a and 5. The first is the standard elastic-plastic response curve applicable to slip resistant connections, and the second is a simplified, usually conservative, approximation to the measured response of ASTM A325 bolts in bearing type connections.[10] This particular example was recently analyzed by several investigators,[14,19,20] using a closer fitting exponential response curve,[10] and their results were compared with that given in Table X of the AISC Manual.[2] The initial shear rigidities of the three fasteners, which are all equal, are taken as: $K_{io} = 1.0$.

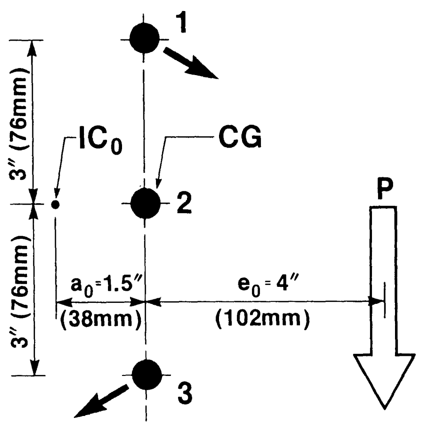

Fig. 4. Example 1: Fastener group geometry and loading

a. Elastic-Plastic Response (Fig. 5)

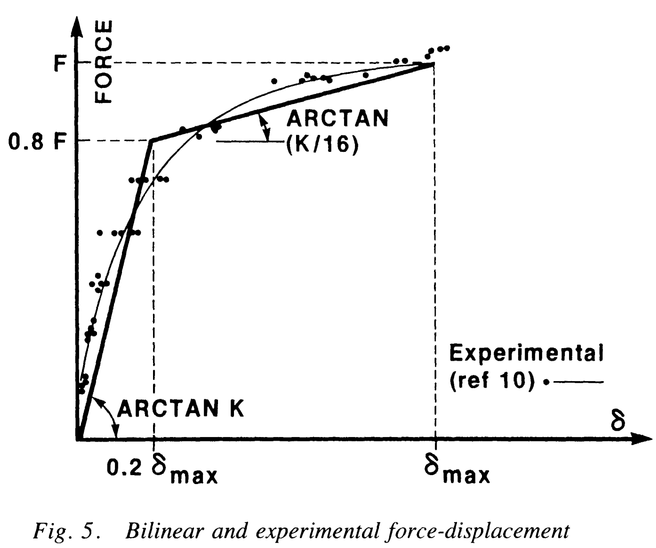

Fig. 5. Bilinear and experimental force-displacement response curves

This is perhaps a trivial example, since for $P \geqslant F$ it can easily be shown that ultimate capacity is given by:

$$P = \frac{1}{1 + (e/b)^2} \{1 + \sqrt{1} + 3\,[1 + (e/b)^2]\}\, F = 1.460F$$

in which $F = F_o =$ the plastic capacity attributed to the fastener, and that the plastic instantaneous center of rotation is located at $a_{pl} = 0.699$. Yet it may be interesting to compare the results with the bilinear analysis which follows, and to illustrate the role played by the central fastener prior to slip.

Step O:

$$K_{so} = 3.0$$
$$K_{\theta o} = 18.0$$
$$e_o = 4.0$$
$$a_o = 18.0/(3 \times 4.0) = 1.5\; e_o + a_o = 5.5$$

Bolts #1 and #3 are located furthest from the incremental center of rotation, therefore, using Eq. 7 with $\Delta_o F_1 = F$:

$$\Delta_o P = \frac{n\, a_o}{r_{1,o}} = 1.342F$$

$$\Delta_o F_2 = \frac{r_{2,o}}{r_{1,o}} = 0.447F\ (<F)$$

Since Bolts #1 and #3 are yielding, the capacity of the group is apparently exhausted, so the incremental approach, which in this case is reduced to the elastic method, is conservative by 8.1%. However, as noted with respect to Table 1, results for larger groups of fasteners are in much better agreement with plastic analysis. Regarding the reserve capacity of Bolt #2, it can only be mobilized through additional rotation about an IC which is closer to CG_o than IC_o, and therefore, the vertical component of the force in Bolts #1 and #3 must be lowered. To avoid these complications, it is tempting to use IC_o instead, as was done for Table 1 (see also Ref. 19). This leads to $P = 1.492F$ which is unconservative by 2.2%.

b. Bilinear Response (Fig. 5)

Step O:

As before, but for 80% of the ultimate load ($F_o = 0.8F$):

$$\Delta_o P = 0.80 \times 1.342F = 1.074F$$
$$\Delta_o F_1 = 0.80F;\ \Delta_o F_2 = 0.358F$$

Step 1:

$$K_{s1} = 1.125,\ K_{\theta 1} = 1.125;\ e_1 = 4.0$$
$$a_1 = 1.125/(1.125 \times 4.0) = 0.250$$
$$\Delta_1 P_1 = \frac{K_{s1}\, a_1}{K_{1,1}\, r_{1,1}}\, \Delta_1 F_1 = 0.299F$$

After this stage, the strength of the connection is assumed to be exhausted. Therefore, adding all the incremental loadings $\Delta_i P$, capacity of the group is given by:

$$P \geq \sum_{o}^{I} \Delta_i P = 1.373\,F$$

As expected, this result is somewhat lower (1.9%) than $P = 1.40F$ given in Table X of the AISC Manual,[2] but it is sufficiently close for engineering purposes. However, it can be improved upon by considering the effect of the angle β between $\Delta_o F_1$ and $\Delta_1 F_1$, and this correction is presented here to illustrate the use of Fig. 3. Since $(F_1 - \Delta_o F_1)/F_1 = 0.20$, and from trigonometry, $\beta = 21.81°$, Fig. 3 gives: $(\Delta_1 F_1 + \Delta_1 F_1)/F_1 \approx 0.012$, i.e. $\Delta_1 F_1 = 0.212F$ (rather than $0.20F$), so that $\Delta_1 P = 0.299F \times 0.212/0.20 = 0.317F$, and $P = 1.391F$ (0.6% lower). Note that Iwankiw's modification[20] of Marsh's single step plastic approximation[19] using the exponential response curve yields $P = 1.435F$ (2.5%, unconservative).

Example 2

The second example is a bolt group with an inclined eccentric load shown in Fig. 6. This group was first analyzed by Brandt[14] and then by Marsh[19] and by Iwankiw,[20] and it illustrates a case not covered by standard tables. As in Ex. 1, the bilinear response curve of Fig. 5 is assumed. The results of the step-by-step analysis are summarized in Table 2, and the progress of stiffness degradation of the connection as well as the locations of the incremental centers are shown in Fig. 7. For illustration, the calculations for the first two steps of analysis are presented below.

Step O:

$$K_{so} = 6.0;\ K_{\theta o} = 10\times 3.0^2 = 90.0$$

$$e_o = 20.0\times 0.8 + 5.0\times 0.6 = 19.0$$

$$a_o = \frac{K_{\theta o}}{K_{so}\, e_o} = \frac{90.0}{6.0 \times 19.0} = 0.790$$

$$a_{xo} = -0.790\times 0.8 = -0.632$$

$$a_{yo} = -0.790\times 0.6 = -0.474$$

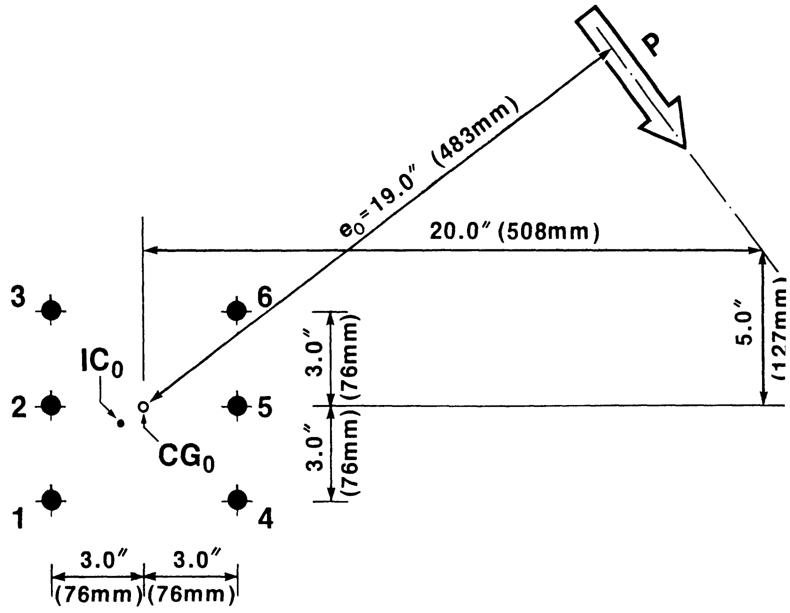

Fig. 6. Example 2: Fastener group geometry and loading

Bolt #6 is located furthest from IC_o, therefore:

$$\Delta_o P = P_o = \frac{K_{so}\, a_o}{K_{1,0}\, r_{6,0}}\ 0.8F = \frac{6.0 \times 0.790}{1.0\times 5.025} = 0.754F$$

in which $5.025 = r_{6,0}$. Since bolt forces are proportional to their distances r_{io} to IC_o, it follows:

$$\Delta_o F_1 = 0.551F$$

$$\Delta_o F_2 = 0.385F$$

Table 2: Sumary of Results for Example 2

Step		0	1	2	3	4	5	6
CG*	−x	0.000	0.556	1.364	0.882	2.500	2.143	0.000
	−y	0.000	0.556	0.000	0.882	1.250	0.000	0.000
e		19.00	19.78	20.09	20.24	21.68	20.71	19.00
K_s		6.000	5.063	4.125	3.188	2.250	1.313	0.375
K_θ		90.00	70.00	48.58	34.41	13.36	8.036	5.625
IC*	−x	0.632	1.150	1.833	1.309	2.719	2.379	0.632
	−y	0.474	0.975	0.352	1.203	1.414	0.177	0.474
F_1		0.551	0.609	0.641	0.683	0.800		
F_2		0.385	0.429	0.442	0.477	0.582	0.800	
F_3		0.669	0.761	0.800				
F_4		0.704	0.800					
F_5		0.583	0.671	0.724	0.800			
F_6		0.800	0.808	0.812	0.818	0.851	0.982	1.000
P/F		0.754	0.828	0.854	0.883	0.928	1.059	1.076

*Measured from CG_o

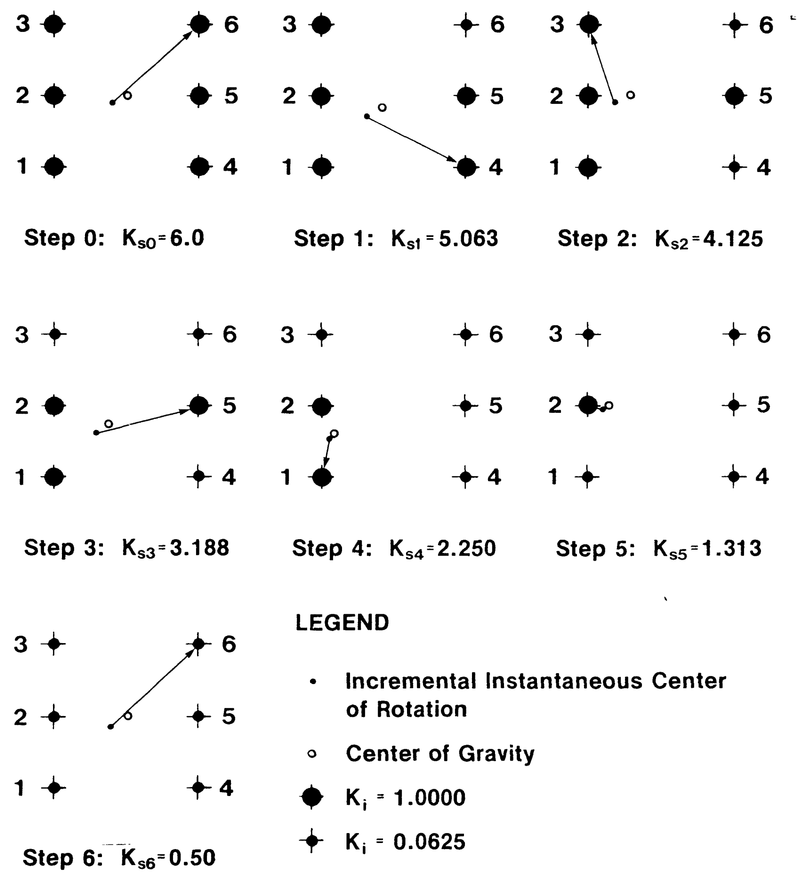

Fig. 7. Example 2: Stiffness degradation and migration of CG and IC with increasing load

$$\Delta_o F_3 = 0.669F$$
$$\Delta_o F_4 = 0.704F$$
$$\Delta_o F_5 = 0.583F$$
$$\Delta_o F_6 = 0.800$$

Step 1:

Now the rigidity of Bolt #6, $K_{6,1} = 0.0625$, rather than 1.0, therefore the coordinates $\bar{x}_1$ and $\bar{y}_1$ of CG_1, as measured from CG_o, and all other properties of the group are modified accordingly:

$$K_{s1} = 5.063$$
$$\bar{x}_1 = -0.556$$
$$\bar{y}_1 = -0.556$$
$$K_{\theta 1} = 5(2.444)^2 + 3.125(3.445)^2 + 2(0.556)^2 = 70.0$$
$$e_1 = 20.556 \times 5.556 \times 0.6 = 19.778$$
$$a_1 = \frac{70.0}{5.063 \times 4.586} = 0.699$$
$$a_{x1} = -0.559$$
$$a_{y1} = -0.419$$

Bolt #4 is now approaching the kink on the response curve:

$$\Delta_1 F_4 = 0.8F - \Delta_o F_4 = (0.800 - 0.704)F = 0.096F$$
$$\Delta_1 P = \frac{5.063 \times 0.699}{1.04 \times 4.586} 0.096F = 0.078F$$
$$P_1 = P_o + \Delta_1 P = 0.828F$$

in which $4.586 = r_{4,1}$. Bolt forces are computed using Eq. 14, noting, however, that $K_{6,1} = 0.0625$ rather than 1.0:

$$\Delta_1 F_1 = 0.058F$$
$$\Delta_1 F_2 = 0.044F$$
$$\Delta_1 F_3 = 0.092F$$
$$\Delta_1 F_4 = 0.096F$$
$$\Delta_1 F_5 = 0.088F$$
$$\Delta_1 F_6 = 0.008F$$

Steps 2-6:

The geometric and stiffness properties of the group and the results of the computations are presented in Table 2 and in Fig. 7.

From Table 2 it is seen that the capacity of connection $P/F = C_u = 1.076$. Brandt's result for this group, assuming an exponential response curve, is $C_u = 1.10$, i.e., the error in the proposed approach is conservative by approximately 2%. This discrepancy results from two sources: the assumed bilinear response curve is slightly more conservative than the exponential curve; and neglecting the effect of changes in force direction in computing the reserve capacities of the bolts. An estimate of the error due to the former can be obtained by means of Iwankiw's approximation.[20] Using IC_o as the instantaneous center of rotation at ultimate load, Iwankiw obtained $C_u = 1.122$ for the exponential response. For the bilinear response, Iwankiw's approach yields $C_u \leq 1.115$. The difference between the two results, namely 0.007, which accounts for one third of the discrepancy, may be attributed to the conservatism built into the bilinear response curve. In automatic computation the remaining error can be practically eliminated by incorporating in the computer program vector addition of incremental bolt forces.

It may be of some interest to note that if slip resistant, rather than bearing, connection is assumed, the step-by-step analysis yields $C_u = 1.144$. In this value, however, the contribution of the reserve capacity of Bolt #2 is considered based on rotation about IC_o. Marsh's upper bound solution[19] for this case is: $C_u = 1.172$. The exact value lies in between.

CONCLUSION

A simple step-by-step method is described for the nonlinear analysis of fastener groups subject to eccentric shear. Using the rigid plate piecewise linear bolt model, the proposed procedure replaces the traditional trial and error search solution by a series of incremental linear elastic analyses, all having simple explicit solutions. This simplification follows from the conservative assumption that changes in load direction during progressive slip or stiffness degradation have negligible effect on the response. The approach yields results which are sufficiently accurate for engineering purposes, yet are always conservative. The proposed approach has the advantage of tracing the development of plastification, changes in load direction, and shifts in the incremental instantaneous center of rotation, and thus helps the designer to gain insight into the actual nonlinear behavior of the connection under increasing loads. It should not be overlooked, however, that for asymmetric configurations and loadings, the number of steps is approximately equal to the number of fasteners. Thus for the larger groups, hand computations become tedious, and a computer solution should be sought. This, of course, is also the case when trial and error procedures are applied.

The incremental approach has been applied in this paper to bolted connections, and it may be of some interest to discuss its applicability to eccentric welded connections. Whereas changes in the force orientation on the individual bolt with increasing external loading do not affect the force deformation relationship of the bolt, the implication of force orientation changes on eccentric welded connections are, at least theoretically, quite important. The state of the art approach[16,17] assumes that for a given external load level, the force-displacement relationship of a unit weld run is *predetermined* by its orientation as computed from the location of the instantaneous center of rotation at that load level (usually ultimate load). In fact, with increasing external loading, the force orientation on any weld run is continuously changing. Therefore, the force-displacement relationships in present use, which were derived by applying an increasing load on a weld with a *given orientation*, cannot be applicable. The reported agreement between experimental and computed results[16,17] merely reflects the known fact that the ultimate capacity of eccentric connections is not particularly sensitive to small variations in the governing constitutive laws. At present, the application of the proposed approach to welded connections suffers from the same limitations as the state of the art methods. Yet, when more is learned about the effect of load direction history on weld runs, it is believed the need for a historical analysis for eccentric welded connections will become more apparent.

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NOMENCLATURE

a	= distance from incremental center to center of rigidity
CG	= center of rigidity (gravity) of fastener group
e	= eccentricity of applied load from center of rigidity
F_i	= fastener force
IC	= incremental center
K_i	= shear rigidity of a fastener
	= 1 for elastic behavior
	= tangent slope for inelastic behavior
K_s	= shear rigidity of fastener group
K_θ	= torsional rigidity of fastener group with respect to CG
n	= number of fasteners in group
P	= applied force
r_i	= distance from the incremental center to a fastener
δ_i	= shear displacement of a fastener
ρ_i	= distance from the center of rigidity to a fastener

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