Discussion

Column Stability under Elastic Support

Paper presented by T. R. HIGGINS (April, 1965, Issue)

Discussion by **ALFRED ZWEIG**

THE ARTICLE by Mr. T. R. Higgins in the April issue of the AISC ENGINEERING JOURNAL is a most valuable contribution to this rather intricate question. It permits the solution of certain problems which could not be solved with the nomographs and tables of the present Code, as demonstrated in the examples in Mr. Higgins' article.

Introduction of required lateral stiffness as the criterion for investigating the stability of a frame, in lieu of using the nomographs presented in the Commentary, is an interesting and novel example of a "rational method", permitted by Section 1.8.3 of the 1963 AISC Specification.

Another example, this one taken from the writer's practice, serves to emphasize the significance of Mr. Higgins' method and shows the savings which can be materialized by its application. Take the part of a large one-story industrial plant consisting of eight bays at 50 ft-0 in. in each direction and separated from the adjacent buildings by expansion joints, where no vertical bracing or masonry walls will be permitted. The columns form rigid frames with the 7 ft-6 in. deep roof trusses having a moment of inertia of say 25,000 in.^{4} which, in relation to the columns, can for all practical purposes be considered of infinite rigidity. The column bases are assumed to be hinged and because of the lack of a vertical bracing system the overall stability of the columns depends upon the bending stiffness of the frame, which means—by definition of the Code—that the columns are subject to sidesway. Assuming the height to the bottom of the trusses as 20 ft-0 in. and the typical column load to be 156 kips, and assuming further that wind loading does not govern, the nomographs and table in the Commentary on the 1963 Code would lead to the following column analysis:

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Bottom hinged: $G = 10$

Top fully fixed in rigid trusses: $G = 0$

From Figure Cl.8.3 of the Commentary, $K = 1.65$

For $KL = 1.65 \times 20 = 33$ ft-0 in. and, for a 12 WF 53 column with $r_y = 2.48$, $KL/r_y = 1.60$ and $F_a = 5.83$ ksi for A36 steel. Therefore the allowable load on a 12 WF 53 column would be $15.59 \times 5.83 = 91$ kips, which is inadequate. A 12 WF 65 column would be required.

Mr. Higgins' method can be used with great advantage in this case by turning alternate columns 90 degrees with respect to their weak and strong axes. In this way, considering two adjacent columns, there will be always one column resisting lateral movements with the strong axis and one column with the weak axis.

Assuming that the weak-axis column is no more effective in resisting frame sidesway than a pin-ended column would be, the strong-axis column would have to provide lateral support for the full frame load, using Mr. Higgins' analysis.

$$\Sigma P = 2 \times 156 = 312 \text{ kips}$$

$$L/r_x = 240/5.23 = 46$$

$$F_a = 18.70 \text{ ksi}$$

$$f_a = 156/15.59 = 10.0 \text{ ksi}$$

$$f_a/F_a = 0.53$$

From Chart A of Mr. Higgins' paper,

$$\frac{\psi}{\tau} = 1.09$$

$$\gamma_B = \frac{2 \times 25{,}000}{50 \times 12} = 83$$

$$A = \frac{312 \times 240}{47{,}000} = 1.59$$

$$\text{Req'd } I_c = \frac{1.09 \times 1.59 \times 240 \times 83}{83 - 1.59} \qquad \text{(Eq. 6)}$$

$$= 423 \text{ in.}^4 < 426.2 \text{ in.}^4 \quad \textbf{OK}$$

Note that Equation (6) can also be written as

$$\text{Req'd } I_c = \frac{\psi}{\tau}\left[\frac{AL_c}{1 - (A/\gamma_B)}\right]$$

and that, in the given example,

$$A/\gamma_B = 1.59/83 = 0.019$$

In this case, therefore, had the trusses been considered as infinitely rigid, the error in computing the required I_c would have been less than 2 percent.

It is interesting to note in passing that the AISC Code in force prior to 1963 permitted the use of a 10BP42 column for the above-described case, a size which was used at numerous occasions in the writer's practice for similar conditions. One explanation of the satisfactory behavior of these columns in the past might be found in the great help they receive from the actual restraint at their base, which is not reflected in the figures of this example.

This comparison shows that, whereas the existing Code would seem to require an increase of well over 50 percent in the weight of this typical interior factory column as compared to the previous Code provision, the design method suggested by Mr. Higgins reduces this difference to slightly more than 25 percent.

It also shows that neither in the previous Code, nor in the present one using the nomograph, did it make any difference in the buckling analysis of the described system if adjacent columns were turned alternately 90 degrees, whereas the Higgins method accounts in a rational way for this fact and reaps the logical and economic benefit for it. This turning of the columns, parenthetically stated, is used for similar reasons in all earthquake regions to provide for equal stiffness in both directions.

In accepting the Higgins' method these questions arise:

1. What figure should be used for the value of C_m if combined axial and bending stresses are to be considered? Is it the value prescribed in Section 1.6.1 for frames subject to joint translation, that is, $C_m = 0.85$, or is it rather the one specified for frames braced against joint translation?

2. The case of a frame with fixed column bases should also be treated in the same manner as Mr. Higgins has done for the hinged column base. This would offer further economies to the practical designer. While it is not very hard to develop such formulas neglecting the influence of the stability factors ψ and τ, it becomes a rather involved procedure if these factors are to be considered. That they cannot be disregarded is apparent from the above-cited example.

Summarizing, it can be stated that the concept of an elastic support as outlined by Mr. Higgins is of great benefit to the economic design of steel columns.

Discussion by **T. R. HIGGINS**

In presenting the frame stability analysis, it was the author's hope that it would stimulate further study of the subject. Mr. Zweig's suggested application is a step in that direction and is of real practical value. It points to possible economies, heretofore not readily apparent, in a type of framing frequently used.

An error in setting type* in the formula for equivalent axial load on page 47 of the April, 1965, article, which should have read:

$$P_0 = P + \frac{MB_xF_a}{(1 - f_a/F'_e)F_b}$$

may have prompted Mr. Zweig's question concerning the value of C_m where combined axial and bending stress is considered. The procedure suggested in the article requires two separate analyses, one for working stress and one for frame stability. Under the former, the requirements of interaction formulas 7(a) and 7(b), given in Section 1.6 of the AISC Specification, must be satisfied, basing the value of F_a on the actual unbraced length in the plane of bending. Since the top of the columns are free to translate, C_m is taken as 0.85. The stability check is performed using the formula given above for equivalent P_0, again basing F_a on the actual unbraced length. In Mr. Zweig's example, since the weak-axis column is assumed to depend upon the strong-axis column for lateral support, all resistance to wind moment would be assigned to the latter.

While further economy might be possible using an analysis based upon fixed column bases, Mr. Zweig has correctly pointed to the difficulties. Expressions similar to Equations (6) and (7) in the original article, for this case, and for the case of combined wind and gravity loading, become too involved for practical use. Likewise, a rigorous solution based upon the assumption of beams rigidly connected at their far end yields expressions too complicated for ready application. Hence, at some sacrifice of economy, a simpler procedure, admittedly providing lower bound solutions, seemed warranted.

It should be noted that the procedures suggested are limited in application and merely constitute a first step in an area of design where improved general methods of analysis could lead to more efficient designs than are now possible.

* *The last line under Example 1 in Mr. Higgins' article was also set incorrectly. It should have stated: "10 WF 39 is adequate", as indicated in the line above, reading "212 in.^{4} $\cong$ 209.5 in.^{4}" — Ed.*

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