Concrete Slab Stresses in Partial Composite Beams and Girders

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Although calculation of compressive flexural stresses in the concrete slab of composite steel beams and girders is a routine procedure, engineers are discovering certain ambiguities in such an analysis when the *partial* composite concept is used.

The problem presents itself when applying the rules in Section of 1.11.2.2 of the AISC Specification.[1] This section states, in part, "The *actual* section modulus of the transformed composite section shall be used in calculating the concrete flexural stress. . . ." The intent of the Specification is to determine the actual stress distribution corresponding to the actual number of shear connectors in the partial composite design approach. This paper describes a suggested procedure to determine concrete slab stresses based on assumed elastic behavior.

The basic justification for the partial composite theory is the fact that shear connectors are inadequate to maintain strain compatibility at the interface of the concrete slab and the top of the steel beam. This results in a limited amount of horizontal slip, which would reduce the concrete compressive strains, and their corresponding stresses.[2] Figure 1 shows the relative flexural strain

PARTIAL COMPOSITE
THEORETICAL ELASTIC BEHAVIOR

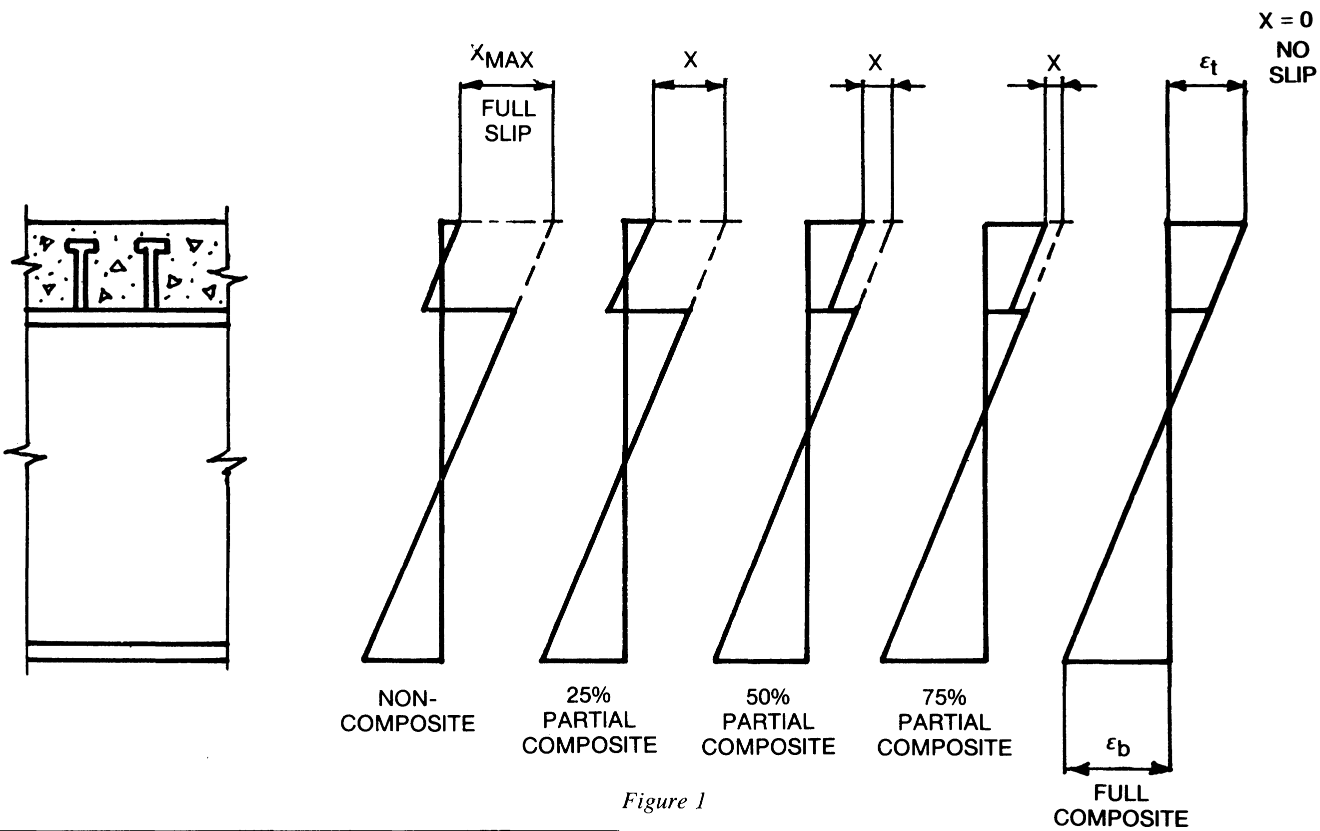

Figure 1

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distributions just described for a typical composite flexural member with shear connection conditions ranging from zero to full composite action. As the percentage of shear connection is reduced, the relative horizontal slip is increased, and the concrete strains correspondingly decrease approaching zero at the non-composite limit.

For *full* composite properties, the calculation of the maximum concrete flexural stress results in a value equal to the value for 100% shear connection. This is a reasonably accurate value where the amount of shear connection is above 80% or so, but, as fewer connections are used, it becomes overly conservative for mid-range partial composite designs. The following suggested procedure gives more reasonable results.

SUGGESTED PROCEDURE

A method for calculating concrete flexural stresses that would approximate the actual stress under partial composite conditions including a design example is developed here. The procedure is based on the following assumption:

1. The location of the neutral axis from the bottom of the steel of the partial composite beam is equal to

$$y_{eff} = \frac{I_{eff}}{S_{eff}} \qquad (1)$$

where

I_{eff} is determined from AISC Formula 1.11-6

S_{eff} is determined from AISC Formula 1.11-1

2. The effective width of the concrete slab (b_{eff}) varies from the full composite value (b/n) to zero for the non-composite condition as described by the relationship from the definition of neutral axis;

$$b_{eff} = \frac{A_s}{t}\left(\frac{y_{eff} - d/2}{d + t/2 - y_{eff}}\right) \qquad (2)$$

where the quantities are as shown in Fig. 2.

3. The second modulus for the top of the equivalent transformed steel section is

$$S_{t\text{-}eff} = \frac{I_{eff}}{(t + d - y_{eff})} \qquad (3)$$

and the concrete stress at that point is

$$f_c = \frac{M}{S_{t\text{-}eff}}\left(\frac{b_{eff}}{b}\right) \qquad (4)$$

As I_{eff} and S_{eff} approach I_s and S_s, y_{eff} (Eq. 1) approaches d/2, b_{eff} (Eq. 2) approaches zero, $S_{t\text{-}eff}$ (Eq. 3) approaches $S_t = I_s/(t + d/2)$, and f_c (Eq. 4) approaches zero - the non-composite case. As I_{eff} and S_{eff} approach I_{tr} and S_{tr} (full composite), $y_{eff} = I_{tr}/S_{tr} = y_b$, $b_{eff} = b/n$, and $f_c = M/(nS_t)$.

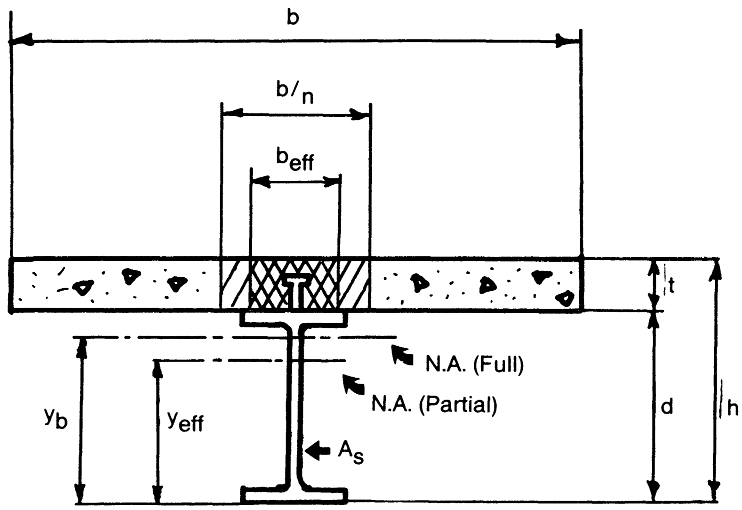

Figure 2

DESIGN EXAMPLE

Fully composite beam
L = 40 ft and S = 10 ft

Material
Concrete:
w = 110 pcf, f'_c = 3,000 psi
n = 14 (for deflections)
n = 9 (for strength)

Steel:
F_y = 50 ksi

Loading
Live load = 120 psf
Ceiling = 5 psf
Partitions = 20 psf
Mechanical = 5 psf
Design requirements:
flat soffit,
t = 4-½ in. and unshored construction

Solution

4-½ in. slab	= 0.042 ksf
steel (assumed)	= 0.008
W_D	= 0.0050 ksf

M_D = 1/8 × 0.05 × 10 × 40² = 100 kip-ft

Live load	= 0.120 ksf
Partitions	= 0.020
Mech & Ceil	= 0.010
W_L	= 0.150 ksf

M_L = 1/8 × 0.15 × 10 × 40² = 300 kip-ft

$$S_{tr}(\text{req'd}) = \frac{(100 + 300)12}{0.66(50)} = 145.5 \text{ in.}^3$$

$$S_s(\text{req'd}) = \frac{12(100)}{33} = 36.4 \text{ in.}^3$$

Effective slab width

$b = L/4 = 1/4 \times 40 \times 12 = 120$ in.
$b = \text{Spacing} = 10 \times 12 = 120$ in.
$b = 16t + b_f = 16 \times 4.5 + 7.0 = 79$ in. (governs)

Select composite section (Manual p. 2-108)

for 4-½ in. slab, select W24×55;
$S_{tr} = 169 \text{ in.}^3 > 145.5$

Steel Properties	**Composite Properties**
$A_s = 16.2 \text{ in}^2$	$A_c = 79.0 \times 4.5 = 355.5 \text{ in}^2$
$d = 23.57$ in	$h = 23.57 + 4.5 = 28.07$ in
$I_s = 1350 \text{ in}^4$	
$S_s = 114 \text{ in}^3$	

(See Fig. 2) $y_b (A_s + A_c/n) = \Sigma\, \bar{y}_i A_i$

for n = 14 (deflections)

$$y'_b \left(16.2 + \frac{355.5}{14}\right) = \left(\frac{355.5}{14}\right)\left(23.57 + \frac{4.5}{2}\right) + 16.2\left(\frac{23.57}{2}\right)$$

$$y'_b = \frac{655.6 + 190.9}{41.59} = 20.35 \text{ in}$$

$$I'_{tr} = 1350 + 16.2\left(20.35 - \frac{23.57}{2}\right)^2$$

$$\frac{1}{12}\left(\frac{79}{14}\right) 4.5^3 + \frac{355.5}{14}\left(23.57 + \frac{4.5}{2} - 20.35\right)^2$$

$$I'_{tr} = 1350 + 1188 + 42.9 + 759.8 = 3341 \text{ in}^4$$

Calculate composite flexural strength

for n = 9 (strength)

$$y_b \left(16.2 + \frac{355.5}{9}\right) = \frac{355.5}{9}\left(23.57 + \frac{4.5}{2}\right) + 16.2\left(\frac{23.57}{2}\right)$$

$$\text{steel:} \quad y_b = \frac{1020 + 190.9}{55.7} = 21.74 \text{ in}$$

$$I_{tr} = 1350 + 16.2\left(21.74 - \frac{23.57}{2}\right)^2 + \frac{1}{12}\left(\frac{79}{9}\right) 4.5^3 + \frac{355.5}{9}\left(23.57 + \frac{4.5}{2} - 21.74\right)^2$$

$$I_{tr} = 1350 + 1605 + 66.7 + 658 = 3679 \text{ in}^4$$

$$S_{tr} = \frac{I_{tr}}{y_b} = 169.2 \text{ in}^2 \text{ (agrees with Manual Table)}$$

concrete stress:

$$S_t = \frac{I_{tr}}{(h - y_b)} = 582 \text{ in.}^3$$

$$f_c = \frac{300 \times 12}{9 \times 582} = 0.69 \text{ ksi} < 0.45 \times 3 = 1.35 \textbf{ o.k.}$$

Check Formula 1.11-2 for unshored construction

$$\left[1.35 + 0.35\,\frac{300}{100}\right] \times 114 = 274 \text{ in.}^3 > 169 \text{ in.}^3 \quad \textbf{o.k.}$$

Check deflections (note: use n = 14 for deflections)

recall $I'_{tr} = 3341 \text{ in.}^4$

$$\Delta_L = \frac{300 \times 40^2}{160 \times 3341} = 0.90 \text{ in.} < L/360 \quad \textbf{o.k.}$$

$$\Delta_D = \frac{100 \times 40^2}{160 \times 1350} = 0.74 \text{ in.} < 1.50 \quad \textbf{o.k.}$$

Determine shear studs

Use 3/4 in. × 3 in. studs

$q = 11.5 \times 0.83$ (reduce for l.w.c.) $= 9.5$ kips

$$V_h \text{ (con.)} = \frac{0.85 \times 3 \times 355.5}{2} = 453 \text{ kips}$$

$$V_h \text{ (steel)} = \frac{16.2 \times 50}{2} = 405 \text{ kips (governs)}$$

$$N = \frac{405}{9.5} = 42.6$$

Use: 43 studs/beam half

Redo for partial composite action

Use $S_{eff} = S_{tr(req'd)} = 145.5 \text{ in.}^3$

$$\text{(Formula 1.11-1)} \quad \frac{V'_h}{V_h} = \left(\frac{S_{eff} - S_s}{S_{tr} - S_s}\right) = 0.328 > 0.25 \quad \textbf{o.k.}$$

$$V'_h = 0.328 \times 405 = 133 \text{ kips}$$

$$N = \frac{133}{9.5} = 14.0$$

Use: 14 Studs/beam half

Check concrete stress for partial composite action

S_{eff} from Formula 1.11-1

$$= 114 + \sqrt{\frac{133}{405}}(169 - 114) = 145.5 \text{ in.}^3$$

I_{eff} from Formula 1.11-6

$$= 1350 + \sqrt{\frac{133}{405}}(3684 - 1350) = 2687 \text{ in.}^4$$

$$y_{eff} = \frac{I_{eff}}{S_{eff}} = \frac{2687}{145.5} = 18.47 \text{ in.}$$

$$b_{eff} = \frac{16.2}{4.5}\left(\frac{18.47 - 11.785}{23.57 + 2.25 - 18.47}\right) = 3.27 \text{ in.}$$

$$S_{t\text{-}eff} = \frac{2687}{28.07 - 18.47} = 280 \text{ in.}^3$$

$$f_c = \frac{300 \times 12}{280}\left(\frac{3.27}{79}\right)$$
$$= 0.53 \text{ ksi} < 0.69 \text{ ksi for full composite}$$

REFERENCE

1. *American Institute of Steel Construction* Specification for the Design, Fabrication and Erection of Structural Steel for Buildings *Chicago, Ill.*, November 1978.