

Finite Element Modeling of Girders with Large Web Openings

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INTRODUCTION

In finite element modeling, the objective is to simulate the actual behavior of a loaded structure so that the resulting structural stress predictions are sufficient to make design decisions. Very often it is not economically viable to generate a single mathematical model able to accurately reflect all the phenomena related to the behavior of a particular structure. In such situations, it is necessary to consider multiple models, some or all of which are reasonable compromises of accuracy vs. cost. The final choice of the model depends on engineering judgment, and differing situations demand different considerations. The primary purpose of this paper is to discuss some of the trade-offs involved for one particular type of structure.

The structure considered here can be described as a multi-celled girder (Fig. 1), which contains manholes placed in vertical plates called bulkheads. These access openings need to be provided for construction as well as for inspection. This type of structure is encountered in large mining and earthmoving equipment. The behavior of plate shear walls in buildings, and diaphragms in box girders that contain manholes, is similar to the bulkheads analyzed here.

The primary interest here lay with the bulkheads containing the manholes. Many of these manholes exist in a given structure, hence an accurate, but economical model is sought to enable the designer to determine the extent of stress concentration for each configuration.

INITIAL MODELING ASSUMPTIONS

The multi-celled girder is so complex that modeling the complete structure is impractical, particularly during the

early design phase. A better approach is to model the complete structure using a representation of the web that is much too crude to permit evaluation of local stresses. However, the crude model can give a good estimate of the total shear load carried by each bulkhead. A second model can then be used to evaluate the local stresses

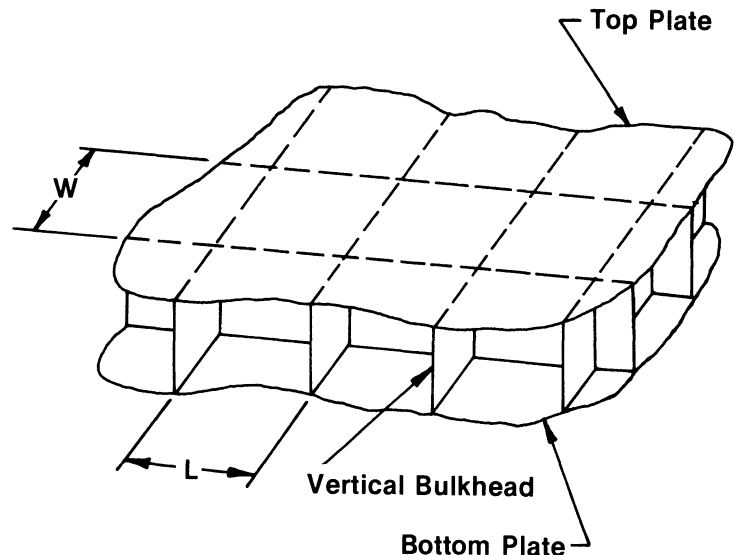


Fig. 1. Section of a multi-celled girder

which occur because of the presence of the manhole. The primary concern here is with the latter model.

A single bulkhead, with its associated flanges, was considered. To isolate a single bulkhead, dividing planes are passed through the structure parallel to the bulkhead to be isolated, midway between it and the adjacent bulkheads on either side. Similar dividing planes are passed through the center of the transverse bulkheads at each end of the isolated bulkhead (see Fig. 2).

The top and bottom plates (now flanges) are assumed to be equal in thickness so that three planes of symmetry exist. Symmetry is used to reduce the model to that shown in Fig. 3, which permits a tremendous reduction in both total degrees of freedom and band width of the finite

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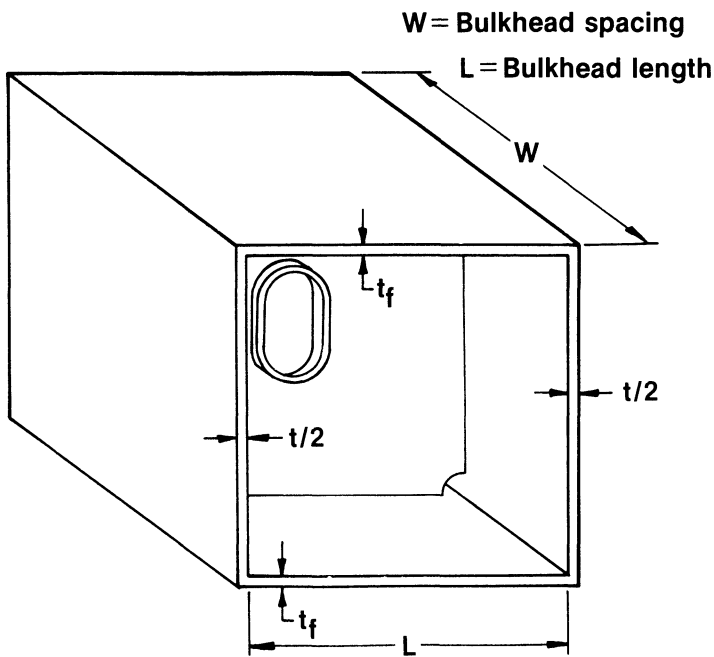


Fig. 2. Isolation of single bulkhead for modeling purposes

element models used. Using the coordinate system shown in Fig. 3, the planes of structural symmetry are: x - y plane at $z = 0$, x - z plane at $y = 0$ and y - z plane at $x = 0$. In addition the following planes, which form the boundaries of the reduced model, are also assigned the roles of symmetry planes: y - z plane at $x = \pm w/2$, x - y plane at $z = \pm (L + t)/2$. See Fig. 3 for definitions of L , t and w . The loads are such that some of the planes above become planes of antisymmetry (first and second). This produces the need for displacement boundary conditions as described in Table 1.

The load is assumed to be shear only applied at $z = (L + t)/2$ in the y direction and was taken so that the average shear on the web was 10 ksi (shear load/web area). This shear load was applied uniformly along the

Table 1. Boundary Conditions Due to Symmetry of the Structure

Edge	Degrees of Freedom					
	δ_x	δ_y	δ_z	θ_x	θ_y	θ_z
a-b	1	0	1	0	1	1
b-c	1	0	1	1	1	0
c-d	1	0	1	1	1	1
d-e	1	0	0	0	1	1
e-f	1	1	0	0	0	1
f-g	1	1	0	0	1	1
g-a	1	0	0	0	1	1
h-b	1	0	1	1	1	1
h-f	1	0	0	0	1	1
h-d	0	0	1	1	1	0

1 = fixed
0 = free

bulkhead's outer edge (edge b - h of Fig. 3). Because of the relatively large stiffness of the cross bulkhead, the precise distribution of the load was not important.

The reason only shear loading was considered is that it has been found to be of greater significance to local stresses in and around the manhole reinforcing ring than the girder moment. The primary cause is that the bulkhead serves as the girder web which, as is well known, resists the major portion of the shear, but only a small fraction of the imposed moment. However, moment loading could be included, but there is no reason to believe the conclusions regarding the modeling options would be changed.

Because fatigue has been observed as the primary design criterion, elastic behavior is assumed. The desired end result is the determination of stress concentration.

FINITE ELEMENT MODELS

A typical finite element program, SAP IV, which is similar to many other such programs, was used (Ref. 1). At least four distinct models can be formulated using SAP. The most obvious and most difficult would be to use three-dimensional solid elements. However, because of the large number of degrees of freedom required, this model would demand a large amount of computer time, which makes each run very expensive. The high cost of such an analysis is not justified, since significant ap-

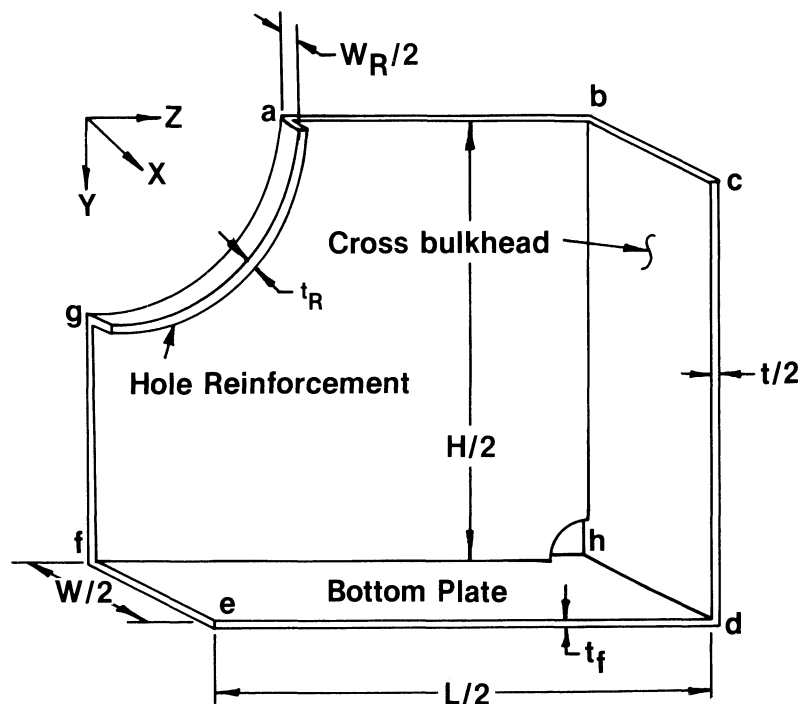


Fig. 3. Model of isolated bulkhead after application of symmetry conditions

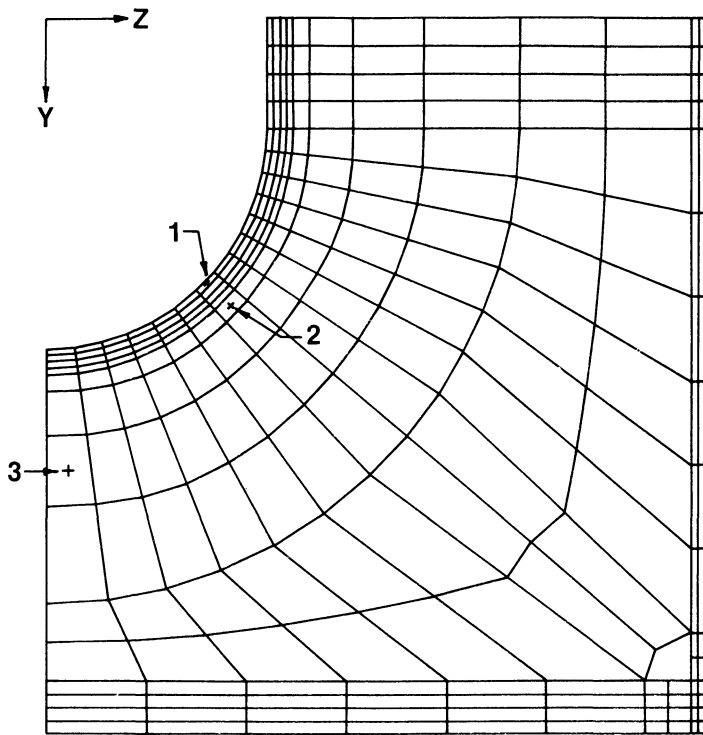


Fig. 4. Finite element idealization (plane stress model) of isolated bulkhead

proximation has already been introduced in moving from the three-dimensional multi-celled structure to a single panel. Therefore, the full three-dimensional solid element model was not considered appropriate.

The primary interest of this work involves the shear resisting characteristics of the web. Some shear is resisted by the flanges, and for greatest accuracy this additional shear capacity should be taken into account. Some of the primary differences in the following three models are associated with the ability of the model to accurately account for the shear resisting capacity of the flange.

The simplest model (Model I) is a two-dimensional one (Fig. 4) with plane stress elements (referred to as element Type 4 in SAP). The model is made up of quadrilateral elements so the element size around the edge of the hole is smallest where stress gradients were expected to be maximum. The hole reinforcing is, of course, made up of thicker elements than those representing the web but are not nearly as thick as those elements which represent the bottom plate (flange). Since the assumption of zero stress perpendicular to the plane of the web corresponds to the one in a simple beam, the above model should be at least as good as simple beam theory for a prismatic beam with similar flanges. However, the two-dimensional model is not able to account for the shear lag phenomenon (discussed later) because, just as in elementary beam theory, the plane stress model does not

recognize any variation of displacement or stress in a direction orthogonal to the web plane.

Therefore, a three-dimensional model was considered also, which is capable of reflecting shear lag behavior in the flange (Fig. 5). Two separate models using two different types of elements were utilized: plane stress membrane elements (Model II) and plate elements (Model III). A fourth model will be discussed later. The plane stress membrane element and the plate element are referred to in SAP as Type 3 and Type 6 respectively. Membrane elements have fewer degrees of freedom than plate elements and permit a significant reduction of the required computer time, and therefore the cost of each run. However, contrary to the two-dimensional model, the flange elements in the three-dimensional model with membrane elements cannot contribute to the shear carrying capacity at all, since they sustain only stresses in the plane of the flange. This situation is improved in the case of plate elements, where the flange model is able to sustain some vertical shear. With either type of element, an additional row of web elements is required to "bridge the gap" between the edge of the web and the center of the flange. This at least partially offsets the loss of shear resistance by the flange elements, but distorts the flange portion of the model by adding area which must be compensated for by a reduction in flange width.

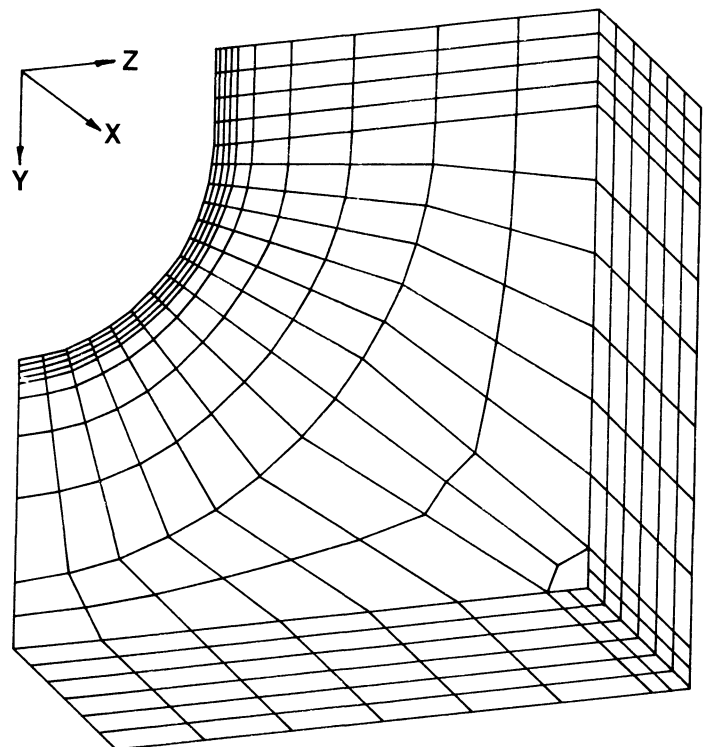


Fig. 5. Finite element idealization of three-dimension model of isolated bulkhead

Table 2. Computer Cost Comparison of Different Finite Element Idealizations of Bulkhead

Model	Number of Elements	Number of Nodes	Number of DOF	Band Width	Computer Cost*
I	244	282	520	61	\$ 5.44
II	300	338	609	69	7.55
III	300	338	749	94	18.43
IV	300	338	748	94	13.80

*Total computer cost (Univac 1100/80) including CPU time, I/O-time and time-core charge. Due to differing price structures, these costs are useful only for comparison between models.

The multiplicity of choices of a finite element model of the isolated bulkhead and its associated flanges raises the question of which model to use. As is typically the case, the answer involves both solution cost and solution quality.

The easiest comparison is one of direct computer solution cost. This is shown in Table 2 and, as expected, the two-dimensional plane stress solution is least costly. Note that the finite element mesh for the web is essentially identical for all models. Probably of more importance is the input preparation time where it was the author's experience that an even greater advantage lay with the two-dimensional model. This was in spite of the use of a mesh generator and band width optimizer. For example, the application of the correct boundary conditions to the finite element model gave the authors much more trouble with the three dimensional models than the two-dimensional.

However, the shear lag behavior of short-wide flange beams produces serious doubts about accuracy. Therefore, the question of accuracy of the predicted stresses must be addressed carefully.

COMPARISON OF THE BULKHEAD STRESSES

Since the state of stress in an element is complicated, it is convenient to perform the comparison using stress intensity, defined as $(\sigma_2 - \sigma_1)$, $(\sigma_2 - \sigma_3)$ or $(\sigma_3 - \sigma_1)$ where one of the three principal stresses is always zero for the situation here. The maximum value of the stress intensity was calculated at the center of three elements (see Fig. 4):

- Location 1 at the hole edge (45° off vertical), where the tensile/compressive stress in the reinforcing ring reaches the highest value,
- Location 2 at the reinforcing weld, and
- Location 3 below the hole, where a state of almost pure shear occurs.

Examination of Table 3 reveals that Model I (two-dimensional plane stress model) predicts stresses about four to six percent less than the three-dimensional models, Models II and III, which produce very similar results. If

Table 3. Comparison of Stresses in the Two-dimensional and Three-dimensional Models. $L = 50$ in., $W = 25$ in., $H = 50$ in., $t = t_R = 1$ in., $W_R = 6$ in., $t_f = 2$ in., Manhole Opening Dimensions (clear distances): End Radius = 8.33 in., Total Depth = 25 in.

Location Number	Model	Principal Stresses		Maximum Stress Intensity
		σ_1	σ_2	
1	I	-0.45	-24.1	24.1
	II	-0.48	-25.6	25.6
	III	-0.52	-25.4	25.4
	IV	-0.48	-25.4	25.4
2	I	-8.5	-15.7	15.7
	II	-9.0	-16.5	16.5
	III	-8.8	-16.3	16.3
	IV	-8.9	-16.3	16.3
3	I	18.0	-19.6	37.6
	II	19.0	-20.6	39.6
	III	18.8	-20.4	39.2
	IV	18.7	-20.4	39.1

Model I 2-D plane stress, Type 4 elements

Model II 3-D membrane, Type 3 elements

Model III 3-D plate, Type 6 elements

Model IV 3-D Type 3 web elements, Type 6 flange elements

one assumes that the Model II/III results are essentially correct and the difference in the results represents error in the Model I representation, then the accuracy of the Model I results is satisfactory, considering the degree of approximation already introduced and the approximate nature of the allowable stresses (particularly allowable ranges fatigue stress). If one could accept this last statement, then the only remaining step is to recognize that the Model I formulation is significantly more economical in both input preparation time and computer cost and is the most reasonable way to proceed.

However, such a procedure is unacceptable without further study. If a six percent error occurs in one example, then in a different configuration the error might be larger. Further, one cannot be sure that the Model II/III results are accurate. Therefore, it is necessary to study the models further to try to understand why the differences exist and attempt to determine if greater differences could occur. First, consider the results from Models II and III. They are very close, with only slightly higher stresses produced in the Model II results. This indicates the ability of the flange to resist shear out away from the web is insignificant in this problem. There is also another factor which must be taken into account in this comparison. The membrane formulation used in the SAP Type 6 plate elements is different from that used for the SAP Type 3 elements (assembly of four constant stress triangles versus a single isoparametric quadrilateral). Thus the Type 6 elements are stiffer as membrane elements than the Type 3, which could modify the stress results in a way such as to place the conclusion above in doubt.

To investigate this effect, another run was made in which membrane, Type 3, elements were used for the web and plate, Type 6, for the flange (Model IV). The resulting stresses from this model (Table 3) are much closer to those for Model III than Model II, which indicates the difference in the element formulations is insignificant in this application.

Now consider the difference between the results of Model I and those of Models II and III. Because Models II and III are closer to agreement does not imply these results are closer to the correct solution than those of Model I. However, one good reason to suspect the results of Models II and III are better is that these models are capable of reflecting the shear lag behavior of the short, wide-flanged beam. Shear lag behavior has the effect of reducing the bending stresses in the outer portion of the flange relative to the value predicted by simple beam theory.

It is important to observe the nature of this particular problem is such that the total internal resisting moment and total resisting shear at any given section is fixed by statics. Therefore, the net result of the occurrence of shear lag is not a change in the total moment or shear at a section but rather results in a redistribution of the stresses which integrate to the total moment or shear. The influence of shear lag on the stress concentration in the web will be discussed later after enumeration of some other possible causes of differences between the Model I results and those of Models II/III.

Unfortunately, the move to the three-dimensional formulation (Models II and III) introduces other approximations. Due to the finite width of the flange, there is a "gap" between the edge of the web and the middle surface of the flange. It is possible (though not with SAP IV) to "tie" the edge of the web to the flange middle surface by the use of constraint equations (rigid links) which would not contribute area to resist bending nor shear stresses. This would clearly reduce the shear resisting capability of the flange. The modeling procedure used herein for Models II and III was to utilize another row of elements (same thickness as web) between the web edge and the flange middle surface. This introduces some shear resisting capability. But is it a realistic model? Another row of web elements could have been added above the flange middle surface extending to the outer surface of the flange.

It can be seen that, in adding the ability to include the effect of shear lag, other errors/approximations are added. Consider first the shear lag effect. As mentioned above, shear lag produces a reduction in the bending stress in the flange, with the greatest reduction occurring at the outer edge of the flange (furthest removed from the web). This is somewhat equivalent to an imaginary flange that is fully stressed in accordance with beam theory, but which is not as wide. Hence the effect of shear lag on the result can be estimated by considering the effect of

a reduction in width of the flange on the shear stresses in a simple prismatic beam.

An evaluation of the relative magnitudes of shear stress shows that for a prismatic beam section, but with no hole, similar to the section under consideration, a halving of the flange width produces an increase in the maximum shear stress (at the neutral axis) of about five percent. However, the increase in total shear resisted by the web is less than one percent. Of course, the "beam" considered in this paper is not prismatic but contains a large hole in the web. Nevertheless these results suggest that the reduced effective width of the flange could be the major source of the stress difference in the models.

Consider another approximation associated with both Models II and III. These models concentrate all the flange area at the flange midsurface with a slight extension to the web depth (between web edge and flange middle surface). Considering a simple prismatic beam with zero thickness but finite area flanges, it is seen that all shear must be resisted by the web. However, the web is slightly deeper, hence has more area. Computation of the shear stresses using simple beam theory shows the shear stress at the neutral axis is unchanged (to three significant figures) relative to a cross-section with the true flange thickness. In addition, the shear stress in the web at the location of the true edge of the web (inside edge of flange) is also about the same for both the zero thickness flange and the finite thickness flange. This suggests this particular modeling error of Models II and III is insignificant.

Finally there is a difference between Models I and II/III which is associated with the stress condition in the flange transverse to the bulkhead span. Model I, by definition, assumes zero stress in that direction (a plane stress approximation). However, the boundary conditions (Table 1) place the flange in a state of plane strain which is imposed in Models II/III. To evaluate this effect, Model II was rerun with the plane strain condition ($\delta_x = 0$) at the flange edge (edge *e-d* of Fig. 3) removed. The differences in the resulting stresses were very small, as indicated in Table 4. Hence, this difference can be disregarded.

Table 4. Comparison of Stress Results in Model II with and without Imposition of Plane Strain Conditions on Flange

Location Number	Model	Principal Stresses		Maximum Stress Intensity
		σ_1	σ_2	
1	II	-0.48	-25.6	25.6
	II*	-0.48	-25.7	25.7
2	II	-9.0	-16.5	16.5
	II*	-9.0	-16.5	16.5
3	II	19.0	-20.6	39.6
	II*	19.1	-20.7	39.8

*Model II with δ_x boundary condition removed on flange edges.

Some other minor problems plagued this investigation, which are mentioned only for the sake of completeness. One must be careful that the predicted stresses compared are for the same physical point in the models. Since automatic mesh generators were used, it was necessary to be sure the elements where stresses were compared had identical nodal coordinates. Also, it was necessary to insure the shear load was applied at the same distance from the hole center for all models, due to the moment stresses generated. The moment induced stresses are not large in this problem, but comparisons involving nearly equal numbers can be distorted in puzzling ways by reasonably small changes in this stress. Finally, one must be careful about the use of "incompatible modes", or non-conforming deformations between elements, when using the SAP finite element program. Since the Type 6 element does not have this capability, the incompatible modes were not used in any model. However, evaluation using them in Models I or II showed a change in stress (in the elements checked) of less than one percent.

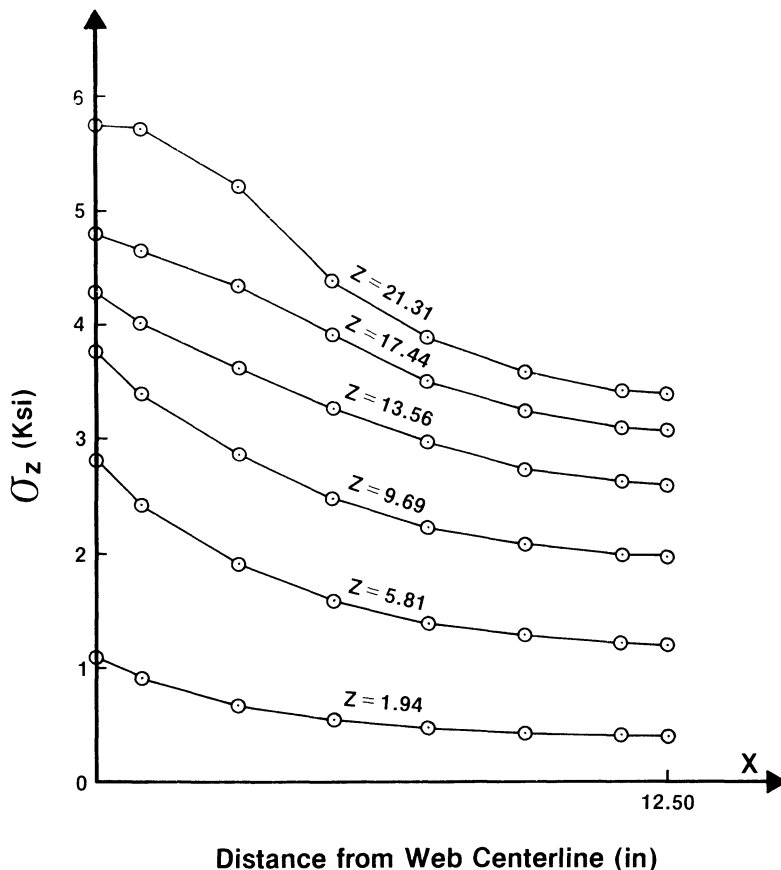


Fig. 6. Variation of bending stress across flange at different stations along span of section of multi-celled girder with manhole

SHEAR LAG

In the section above, it was suggested the major cause of differences in predicted stresses between Models I and II/III was the inability of Model I to account for shear lag in the flanges. This phenomenon, referred to as "shear lag" (or "diffusion"), is due to the shear deformations of the flange in the plane of the flange. The resulting warping of the cross-section affects the longitudinal displacements in such a way that the longitudinal strains in the flange away from the web are reduced. The greater the distance from the web, the greater the reduction. Since the bending stresses in the flange depend, through Hooke's Law, on strains, this brings about a reduction of the flange stresses. For an in-depth discussion of the shear lag effect, the reader is referred to Refs. 2 to 11.

The shear lag effect on the problem was investigated by examination of the results of the three-dimensional models described earlier. From the results of Models II or III, the change in the bending stresses in the flange (Fig. 6), is significant. Hence, an attempt was made to determine the effective width of the flange for this problem.

Several authors have determined the effective width on the basis of equal forces in the actual flange and the transformed one (Fig. 7).

$$2t \int_{x=0}^{W/2} \sigma dx = W_{eff} t \sigma_0$$

where: σ_0 = the maximum value of bending stress in flange

σ = value of bending stress at any point in flange

W = actual width

W_{eff} = effective width

t = flange thickness

x = distance from center of web

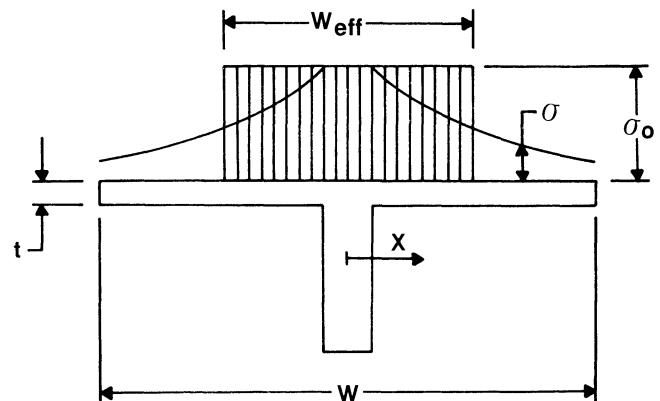


Fig. 7. Definition of terms used in equivalent width determination

For the work here, a slightly different version of the above concept was adopted which lent itself better to the models under consideration. The effective width was defined as the width which, if assigned to the two-dimensional model flange, produced the same total flange force as exists in the three-dimensional model flange. Unfortunately, there is still one variable left to fix which is the location, along the span of the panel, to make the evaluation. In this case, a location about one third the distance from directly under the hole to the panel edge was chosen. This location was chosen since it was directly under the edge of the hole, hence in the depth transition zone. However, investigation showed that moving the evaluation location to another point did not change the results significantly.

Figure 8 shows the results of this investigation. For each of several flange widths, the total flange force as predicted by the two-dimensional model was calculated and plotted vs. the flange width. As would be expected, the larger widths resulted in larger flange forces. The total flange force for the three-dimensional model (Model III) was calculated to be 131.5 kips. The intersection of this value with the previous curve gives an effective width of 18.4 in. Hence it can be seen that the effective width is about 74% of the actual width.

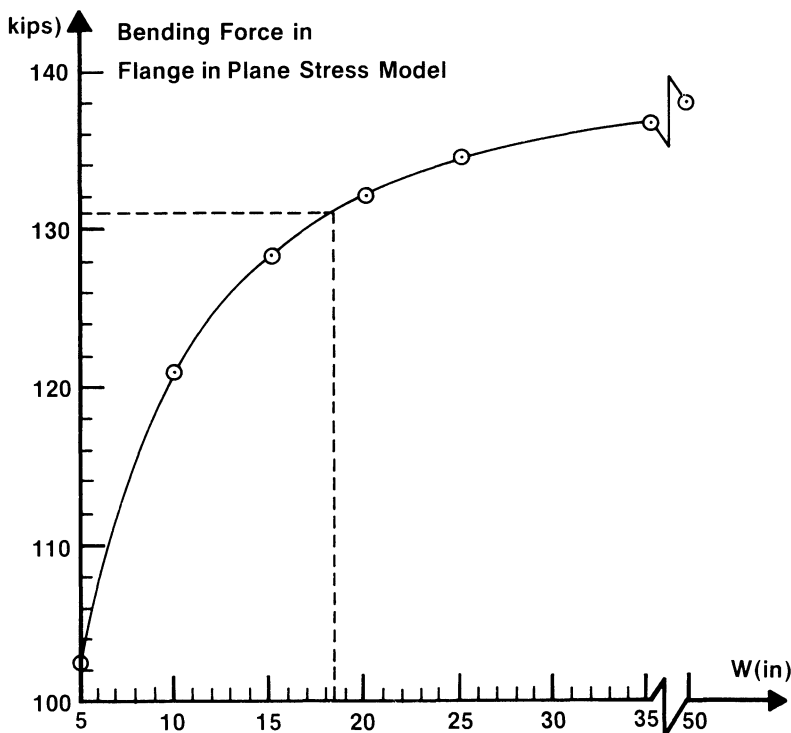


Fig. 8. Variation of bending force in flange as function of flange width

Table 5. Comparison of Stress Results Between Finite Element Models Using Actual and Equivalent Widths for Flange

Location Number	Model	Principal Stresses		Maximum Stress Intensity
		σ_1	σ_2	
1	I	-.45	-24.1	24.1
	Ia	-.46	-24.6	24.6
2	I	-8.5	-15.7	15.7
	Ia	-8.6	-16.0	16.0
3	I	18.0	-19.6	37.6
	Ia	18.2	-19.9	38.1

Notes: Model I: same as before - plane stress

Model Ia: Model I with flange width adjusted for equivalent width

CORRECTED SOLUTION

The discussion above suggested that the major portion of the difference between the two-dimensional and three-dimensional stress results was due to shear lag effects. To check this, another two-dimensional solution was made using the reduced width found above for the flange and the results are compared with the three-dimensional Model III results in Table 5. The results are closer than before, although the "correction" made up less than half of the previous difference. Therefore, one must conclude the shear lag error is only partially responsible for the difference. It is not clear precisely what causes the remaining differences, but they could just as well lay with the three-dimensional models as with the simpler plane stress model.

GENERAL APPLICATION OF RESULTS

If the error associated with the two-dimensional model using the true dimensions is felt to be unacceptable, then a correction can be applied by using an effective width for the flange in place of the actual one. Clearly, this is an easy correction to make to the plane stress model. However, it places the burden on the designer to determine the effective width.

Many investigators have studied the shear lag problem for a prismatic beam. In most of these results the effective width is expressed as a function of the beam length, and in some the flange thickness also affects the result. The work in Ref. 11 provides an estimate of the effective width for a simple beam (similarly loaded) of 31% of the span length independent of the flange thickness. This would give an effective width of 15.5 in., which is reasonably close to the result above. Other effective width estimates for various loading conditions are given in Refs. 12 to 15. Unfortunately, further investigation showed the effective width as determined here varied with the flange thickness (Table 6) which indicates a stronger dependence on flange thickness than might be hoped for. The size of the hole in the web, as well as other geometric

Table 6. Change in Effective Width Due to Change in Flange Thickness

Flange Thickness (in.)	Effective Width at Flange (in.)
1	21.1
2	18.4
3	15.0
4	10.8

variables such as the amount of hole reinforcing, most likely also affect the effective width. Therefore, it appears an accurate determination of the effective width is quite difficult. Fortunately, the stress concentration in the web is not strongly dependent on the effective width. Thus, a reasonable choice of the effective width for this type of configuration would seem to be about one third of the span length.

SUMMARY

This paper has addressed the problem of determining an accurate but economic finite element model which will permit determination of the stress concentration in the vicinity of a reinforced hole present in a multi-celled girder web. It was pointed out that simulation of the entire structure in sufficient detail to adequately predict the stress concentration is impractical. Instead, cruder models of the entire structure are used after which the resulting shear carried by each bulkhead can be imposed on a smaller model with much greater detail. However, several alternatives exist regarding the type of finite element model to use. Three different finite element models were compared with regard to economy and accuracy. The primary errors associated with the use of those models were discussed, and it was found that the difference between results of the various models considered was not great, and no significant reasons for those differences other than shear lag could be found. Thus, it follows that a reasonable estimate for the effective width used in conjunction with the two-dimensional model should be sufficient for most design applications. The authors believe

an effective width of about one third of the panel span would provide acceptable results for configurations similar to the one here.

REFERENCES

1. *Bathe, K. J., E. L. Wilson and F. E. Peterson* SAP IV, A Structural Analysis Program for Static and Dynamic Response of Linear Systems *EERC Report No. 73-11, Rev. April 1974.*
2. *Hildebrand, F. B. and E. Reissner* Least Work Analysis of the Problem of Shear Lag in Box Beams *NACA Technical Notes No. 893, 1943.*
3. *Hadji-Argriris, J.* Diffusion of Symmetrical Loads into Stiffened Parallel Panels with Constant Area Edge Members *Reports and Memoranda No. 2038, 1944.*
4. *Reissner, E.* Analysis of Shear Lag in the Box Beams by the Principle of Minimum Potential Energy *Quarterly of Applied Mathematics, Vol. 4, 1946, p. 268.*
5. *Kuhn, P. and P. T. Chiarito* Shear Lag in Box Beams, Methods of Analysis and Experimental Investigations *NACA Report No. 739.*
6. *Kuhn, G. P.* Stress Analysis of Beams with Shear Deformation of the Flanges *NACA Report No. 608, p. 669.*
7. *Kuhn, P.* Stress in Aircraft and Shell Structures *McGraw-Hill Book Co., 1956, p. 101.*
8. *Niles, A. S. and J. S. Newell* Airplane Structures *John Wiley & Sons, Vol. 2, 1943, p. 218.*
9. *Kristek, V.* Theory of Box Girders *John Wiley & Sons, 1979, p. 120.*
10. *Oden, J. T. and E. A. Ripperger* Mechanics of Elastic Structures *McGraw-Hill Book Co., 1981, p. 128.*
11. *Timoshenko, S. P.* Theory of Elasticity *McGraw-Hill Book Co., 1970, p. 267.*
12. *Schnadel, G.* Die mittragende Breite in Kastenrägern und im Doppelboden *Werft und Reederei, Vol. 9, 1928, p. 92.*
13. *Reissner, E.* Über die Berechnung von Plattenbalken *Der Stahlbau, Vol. 7, 1934, p. 206.*
14. *Chwalla, E.* Die Formeln zur Berechnung der 'voll mittragenden Breite' dünner Gurt- und Rippenplatten *Der Stahlbau, Vol. 9, 1936, p. 73.*
15. *Beschkine, L.* Determination de la Largeur Utile des Tables de Compression des Poutres *International Association for Bridge and Structural Engineering, Memoires, Vol. 5, 1937-38, p. 65.*