Design Aid for Deflection of Simple Beams Under Concentrated Loads

M. MAX MAZZARELLA, SR.

INTRODUCTION

The AISC *Manual of Steel Construction*, and a number of other publications, provide beam diagrams and formulas for various loading conditions. These diagrams can be found in the Manual, beginning on page 2-114.

With the aid of the appropriate formulas in the Manual, the expected elastic deflection of a beam, subjected to various loading conditions, can be determined at any point along its length. However, the Manual does not provide deflection formulas for loading conditions with unsymmetrically placed concentrated loads. Presented here is a method to compute the expected deflection of a beam subjected to any number of unequal concentrated loads, regardless of their location along the beam.

DEFLECTION COEFFICIENTS

Consider a simple beam subjected to two unequal concentrated loads P_1 and P_2 (Fig. 1). Maxwell's theory of reciprocal deflections states that deflection at B due to the load P at A is equal to the deflection at A due to the load P at B.

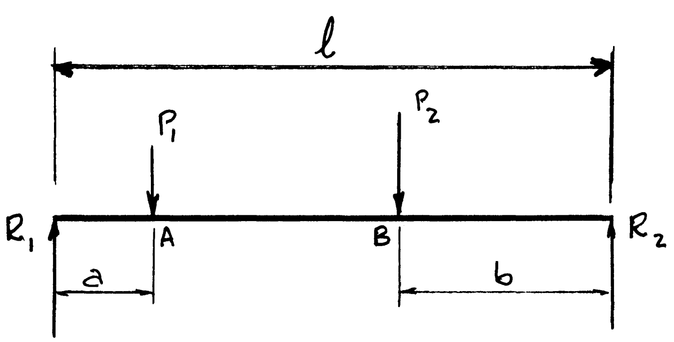


Figure 1

M. Max Mazzarella, Sr. is Vice President and Chief Engineer, Structural Engineering Services, Inc. Mobile, Alabama.

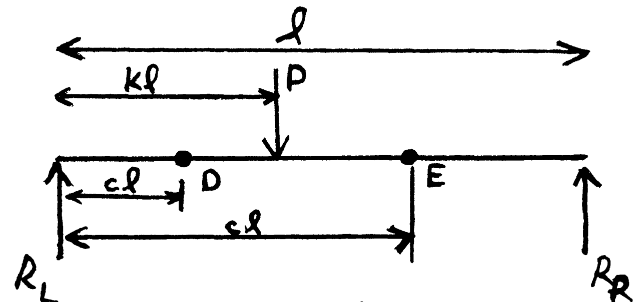


Figure 2

Expanding this in Fig. 2, the deflection at any point D along a simple beam caused by a concentrated load P can be written as:

$$\Delta_D = \frac{Pl^3}{6EI} c[k(2-k) - c^2](1-k) \quad (\text{for } c < k) \qquad (\text{Eq. 1})$$

According to Maxwell's theorem the deflection at E is obtained by interchanging the coefficients c and k so that:

$$\Delta_E = \frac{Pl^3}{6EI} k[c(2-c) - k^2](1-c) \quad (\text{for } c > k) \qquad (\text{Eq. 2})$$

Letting N represent the expression involving c and k in Eqs. 1 and 2, the general formula for deflection can be written as

$$\Delta_c = \frac{Pl^3}{6EI}(N)$$

Finally, using the principle of superposition, the total deflection at any point caused by any number of unequal concentrated loads unsymmetrically placed is obtained from:

$$\Delta_c = \frac{[(P_1N_1) + (P_2N_2) \ldots + P_nN_n]\,(l)^3}{6EI}$$

or

$$\Delta_c = \frac{(\Sigma PN)\, l^3}{6EI} \qquad (\text{Eq. 3})$$

Table 1. Values of N

c \ k	0.05	0.10	0.125	0.15	0.20	0.25	0.30	0.333	0.35	0.375	0.40	0.45	0.5	
0.05	0.0045	0.0084	0.0101	0.0117	0.0143	0.0163	0.0178	0.0184	0.0187	0.0190	0.0191	0.0191	0.0187	0.95
0.10	0.0084	0.0162	0.0196	0.0227	0.0280	0.0321	0.0350	0.0364	0.0369	0.0375	0.0378	0.0378	0.0370	0.90
0.15	0.0117	0.0227	0.0278	0.0325	0.0405	0.0467	0.0512	0.0533	0.0541	0.0550	0.0556	0.0557	0.0546	0.85
0.20	0.0143	0.0280	0.0344	0.0405	0.0512	0.0596	0.0658	0.0687	0.0699	0.0712	0.0720	0.0723	0.0710	0.80
0.25	0.0163	0.0320	0.0396	0.0467	0.0596	0.0703	0.0783	0.0822	0.0837	0.0855	0.0866	0.0873	0.0859	0.75
0.30	0.0178	0.0350	0.0433	0.0512	0.0658	0.0783	0.0882	0.0931	0.0951	0.0974	0.0990	0.0999	0.0990	0.70
0.35	0.0187	0.0369	0.0457	0.0541	0.0699	0.0837	0.0951	0.1011	0.1035	0.1065	0.1087	0.1107	0.1098	0.65
0.40	0.0191	0.0378	0.0468	0.0556	0.0720	0.0866	0.0990	0.1058	0.1087	0.1124	0.1152	0.1183	0.1180	0.60
0.45	0.0191	0.0378	0.0469	0.0557	0.0723	0.0873	0.1002	0.1075	0.1107	0.1149	0.1183	0.1225	0.1232	0.55
0.50	0.0187	0.0370	0.0459	0.0546	0.0710	0.0859	0.0990	0.1065	0.1098	0.1143	0.1180	0.1232	0.1250	0.50
0.55	0.0179	0.0354	0.0440	0.0523	0.0682	0.0827	0.0955	0.1030	0.1063	0.1108	0.1148	0.1205	0.1232	0.45
0.60	0.0168	0.0332	0.0412	0.0491	0.0640	0.0778	0.0900	0.0972	0.1005	0.1049	0.1088	0.1148	0.1180	0.40
0.65	0.0153	0.0304	0.0377	0.0449	0.0586	0.0713	0.0827	0.0894	0.0925	0.0967	0.1005	0.1063	0.1098	0.35
0.70	0.0136	0.0270	0.0335	0.0399	0.0522	0.0636	0.0738	0.0799	0.0827	0.0866	0.0900	0.0955	0.0990	0.30
0.75	0.0117	0.0232	0.0288	0.0343	0.0449	0.0547	0.0636	0.0689	0.0713	0.0747	0.0778	0.0827	0.0859	0.25
0.80	0.0096	0.0190	0.0236	0.0281	0.0368	0.0449	0.0522	0.0566	0.0586	0.0615	0.0640	0.0682	0.0710	0.20
0.85	0.0073	0.0145	0.0180	0.0215	0.0281	0.0343	0.0399	0.0433	0.0449	0.0471	0.0491	0.0523	0.0546	0.15
0.90	0.0049	0.0098	0.0122	0.0145	0.0190	0.0232	0.0270	0.0293	0.0304	0.0319	0.0332	0.0354	0.0370	0.10
0.95	0.0025	0.0049	0.0061	0.0073	0.0096	0.0117	0.0136	0.0148	0.0153	0.0168	0.0168	0.0179	0.0187	0.05
	0.95	0.90	0.875	0.85	0.80	0.75	0.70	0.667	0.65	0.625	0.60	0.55	0.5	k \ c

Table 1 shows values of N for any value of c and k where

c = any point on the beam where deflection is desired, expressed as a fraction of the total span

k = distance from left support to each concentrated load on the span, expressed as a fraction of total span

COROLLARY

When the load applied on the beam consists of a uniform load partially distributed on the span (Case 4 in AISC Manual), Table 1 can be used by breaking up the uniform load into short, equal sections which may be replaced by the equivalent total load on that section, considered as applied at its center of gravity (Fig. 3). Even though this procedure yields approximate results, it is an acceptable solution since it will result in deflection values greater than those from a distributed load.

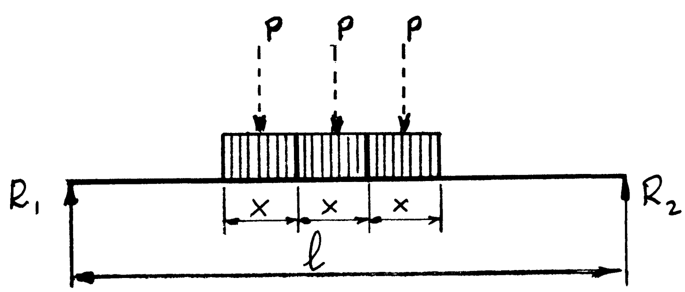
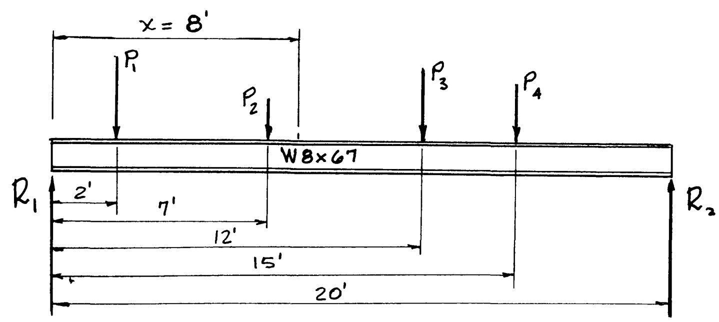


Figure 3

Example



Given:

P_1 = 7,000 lb. P_3 = 6,000 lb.
P_2 = 2,000 lb. P_4 = 5,000 lb.
I = 272 in.^{4} E = 29,000,000 psi

Required:

Deflection at cl = 8 ft-0 in.

Solution: (obtaining values of N from Table 1):

i	c	k_i	N_i	P_iN_i		
1	0.4	0.1	0.0378	7,000(0.0378)	=	264.6
2	0.4	0.35	0.1087	2,000(0.1087)	=	217.4
3	0.4	0.6	0.1088	6,000(0.1088)	=	652.8
4	0.4	0.75	0.0778	5,000(0.0778)	=	389.0
				ΣPN	=	1,523.8

From Equation 3:

$$\Delta_{cl=8} = \frac{(\Sigma PN)l^3}{6EI}$$

$$= \frac{(1{,}523.8)\,(20)^3\,(1{,}728)}{6 \times 29{,}000{,}000 \times 272} = 0.445 \text{ in.}$$