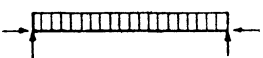
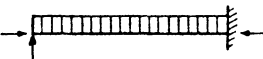
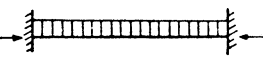
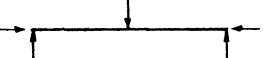
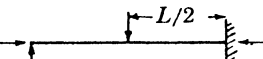
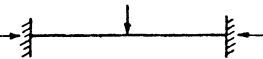


# Note on Beam-Column Moment Amplification Factor

NESTOR R. IWANKIW

This short note has been prepared to clarify and explain the theoretical derivation of the expressions shown in Table C1.6.1 of the AISC Specification Commentary.

**Table C1.6.1.**

Case	$\psi$	$C_m$
	0	1.0
	-0.4	$1 - 0.4 \frac{f_u}{F'_e}$
	-0.4	$1 - 0.4 \frac{f_u}{F'_e}$
	-0.2	$1 - 0.2 \frac{f_u}{F'_e}$
	-0.3	$1 - 0.3 \frac{f_u}{F'_e}$
	-0.2	$1 - 0.2 \frac{f_u}{F'_e}$

Nestor R. Iwankiw is Assistant Director of Engineering, American Institute of Steel Construction, Chicago, Illinois.

This table shows suggested bending moment amplification factor coefficients,  $\psi$  and  $C_m$ , for braced transversely loaded beam-columns which are to be used in conjunction with the Specification stability interaction Formula 1.6-1a. This  $C_m$  moment reduction factor may be written as

$$C_m = 1 + \psi \frac{f_a}{F'_e} \quad (1)$$

where

$f_a$  = computed axial stress  
 $F'_e$  = Euler buckling stress, divided by factor of safety

As the Commentary states,  $\psi$  for single curvature simply supported members may be approximated by

$$\psi = \frac{\pi^2 \delta_o EI}{M_o L^2} \quad (2)$$

where

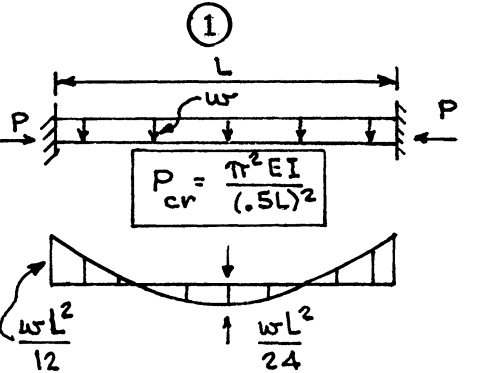
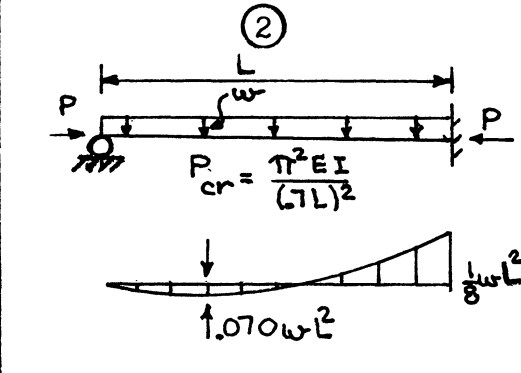


$\delta_o$  = maximum deflection due to transverse loading  
 $M_o$  = maximum moment between supports due to transverse loading

In the previous 7th Edition of the AISC Manual, Eq. 2 was erroneously used to also determine  $\psi$  values for the four indeterminate end restrained cases.

Prof. J. A. Yura, University of Texas at Austin, calculated the correct values for amplified moments on the basis of exact theoretical solutions presented by Timoshenko and Gere. He then developed approximate expressions for  $\psi$  and  $C_m$  that are contained in the current Manual edition. A brief summary of this derivation is presented in Table 1 (see next page). The two simply supported cases shown in Commentary Table C1.6.1 were not affected by the changes.

It is hoped this will resolve some of the common inquiries relative to  $\psi$  and Table C1.6.1.

Table 1. Derivation of  $\psi$  Factors for End-Restrained Members

CASE																																																																															
<p>EXACT SOLUTION with P (1)</p> <p> <math display="block">u = \frac{L}{2} \sqrt{\frac{P}{EI}}</math>                     OR  <math display="block">u = \frac{\pi}{2K} \sqrt{\alpha}</math> </p> <p> <math display="block">M_{\xi} = \frac{wL^2}{24} \frac{6(u - \sin u)}{u^2 \sin u}</math> <math display="block">M_{END} = \frac{wL^2}{12} \frac{3(\tan u - u)}{u^2 \tan u}</math> </p> <p>Amplified Moments <math>\times \frac{wL^2}{24}</math></p>	 <p> <math display="block">M_{\xi} = \frac{wL^2}{24} \frac{6(u - \sin u)}{u^2 \sin u}</math> <math display="block">M_{END} = \frac{wL^2}{12} \frac{3(\tan u - u)}{u^2 \tan u}</math> </p> <p>Amplified Moments <math>\times \frac{wL^2}{24}</math></p> <table border="1" data-bbox="371 1032 859 1585"> <thead> <tr> <th><math>\alpha = P/P_{cr}</math></th> <th><math>M_{\xi}</math></th> <th><math>M_{END}^*</math></th> <th><math>M_{END}</math> (APPROX.)</th> </tr> </thead> <tbody> <tr><td>0.0</td><td>1.000</td><td>2.000</td><td>2.00</td></tr> <tr><td>0.1</td><td>1.129</td><td>2.146</td><td>2.14</td></tr> <tr><td>0.2</td><td>1.291</td><td>2.324</td><td>2.30</td></tr> <tr><td>0.3</td><td>1.501</td><td>2.554</td><td>2.52</td></tr> <tr><td>0.4</td><td>1.782</td><td>2.854</td><td>2.80</td></tr> <tr><td>0.5</td><td>2.179</td><td>3.272</td><td>3.22</td></tr> <tr><td>0.6</td><td>2.778</td><td>3.892</td><td>3.83</td></tr> <tr><td>0.7</td><td>3.782</td><td>4.922</td><td>4.85</td></tr> <tr><td>0.8</td><td>5.798</td><td>6.960</td><td>6.88</td></tr> <tr><td>0.9</td><td>11.87</td><td>13.08</td><td>12.98</td></tr> </tbody> </table> <p>*always max.</p>	$\alpha = P/P_{cr}$	$M_{\xi}$	$M_{END}^*$	$M_{END}$ (APPROX.)	0.0	1.000	2.000	2.00	0.1	1.129	2.146	2.14	0.2	1.291	2.324	2.30	0.3	1.501	2.554	2.52	0.4	1.782	2.854	2.80	0.5	2.179	3.272	3.22	0.6	2.778	3.892	3.83	0.7	3.782	4.922	4.85	0.8	5.798	6.960	6.88	0.9	11.87	13.08	12.98	 <p> <math display="block">M_{\xi}</math> - not calculated, obvious it cannot control  <math display="block">M_{END} = \frac{wL^2}{8} \frac{(\tan u - u)}{\frac{1}{2} u^2 \left( \frac{1}{2u} - \frac{1}{\tan 2u} \right)}</math> </p> <p>Amplified Moments <math>\times \frac{wL^2}{8}</math></p> <table border="1" data-bbox="974 1032 1495 1585"> <thead> <tr> <th><math>M_{\xi}</math></th> <th><math>M_{END}^*</math></th> <th><math>M_{END}</math> (APPROX.)</th> </tr> </thead> <tbody> <tr><td>Does not control</td><td>1.000</td><td>1.00</td></tr> <tr><td></td><td>1.074</td><td>1.07</td></tr> <tr><td></td><td>1.166</td><td>1.15</td></tr> <tr><td></td><td>1.282</td><td>1.26</td></tr> <tr><td></td><td>1.434</td><td>1.40</td></tr> <tr><td></td><td>1.645</td><td>1.61</td></tr> <tr><td></td><td>1.959</td><td>1.92</td></tr> <tr><td></td><td>2.475</td><td>2.42</td></tr> <tr><td></td><td>3.493</td><td>3.44</td></tr> <tr><td></td><td>6.481</td><td>6.49</td></tr> </tbody> </table> <p>*always max.</p>	$M_{\xi}$	$M_{END}^*$	$M_{END}$ (APPROX.)	Does not control	1.000	1.00		1.074	1.07		1.166	1.15		1.282	1.26		1.434	1.40		1.645	1.61		1.959	1.92		2.475	2.42		3.493	3.44		6.481	6.49
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<p>APPROXIMATE SOLUTION</p> <p> <math display="block">M_{END} = \frac{wL^2}{12} \left( \frac{1 - 0.39 P/P_{cr}}{1 - P/P_{cr}} \right)</math> <math display="block">K = 0.5</math> <math display="block">\therefore \psi = -0.4</math> </p>	<p> <math display="block">M_{END} = \frac{wL^2}{8} \left( \frac{1 - 0.39 P/P_{cr}}{1 - P/P_{cr}} \right)</math> <math display="block">K = 0.7</math> <math display="block">\therefore \psi = -0.4</math> </p>																																																																														

(1) Timoshenko & Gere, Theory of Elastic Stability, Chapter 1

CASE

M DIAGRAM without P

EXACT SOLUTION with P

$$\mu = \frac{L}{2} \sqrt{\frac{P}{EI}}$$

OR

$$\mu = \frac{\pi}{2K} \sqrt{\alpha}$$

$$\alpha = P/P_{cr}$$

$$M_{END} = M_{\xi} = \frac{1}{8} WL \frac{2(1 - \cos \mu)}{\mu \sin \mu}$$

Amplified Moments  $\times WL/8$

$M_{\xi}$	$M_{END}^*$	$M_{END}$ (APPROX.)
0.0	1.000	1.00
0.1	1.091	1.09
0.2	1.205	1.20
0.3	1.351	1.35
0.4	1.543	1.54
0.5	1.817	1.81
0.6	2.224	2.21
0.7	2.901	2.89
0.8	4.254	4.24
0.9	8.308	8.29

\*equal to  $M_{\xi}$

$$M_{END} = M_{\xi} = \frac{WL}{8} \left( \frac{1 - 0.19 P/P_{cr}}{1 - P/P_{cr}} \right)$$

$K = 0.5$   
 $\therefore \psi = -0.2$

CASE

M DIAGRAM without P

EXACT SOLUTION with P

$$M_{\xi} = \frac{5WL}{32} \left[ 1.6 \frac{\tan \mu}{\mu} - 0.6 \sec \mu \frac{\lambda(\mu)}{\psi(\mu)} \right]$$

$$M_{END} = \frac{3}{16} WL \frac{\lambda(\mu)}{\psi(\mu)}$$

WHERE:  $\lambda(\mu) = 2(1 - \cos \mu) / \mu^2 \cos \mu$   
 $\psi(\mu) = \frac{3}{2\mu} (\frac{1}{2\mu} - \tan 2\mu)$

Amplified Moments  $\times \frac{3}{16} WL$

$M_{\xi}$	$M_{END}^*$	$M_{END}$ (APPROX.)
.833	1.00	1.00
.900	1.083	1.08
.982	1.186	1.17
1.086	1.317	1.30
1.222	1.488	1.47
1.412	1.725	1.70
1.908	2.076	2.05
2.152	2.653	2.63
3.059	3.790	3.80
5.722	7.122	7.30

\*always max.

$$M_{END} = \frac{3}{16} WL \left( \frac{1 - 0.30 P/P_{cr}}{1 - P/P_{cr}} \right)$$

$K = 0.7$   
 $\therefore \psi = -0.3$