Web Design Under Compressive Edge Loads

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Webs of rolled and built-up beams and girders can be subjected to local in-plane compressive patch loads. Examples are, wheel loads, loads from purlins and roller loads during construction. For practical and/or economic reasons, transverse stiffeners are to be minimized or avoided except at critical sections. It is, therefore, necessary to check the unstiffened web under the edge compressive loading to insure no localized failure will occur.

The type of loading under consideration is shown in Fig. 1. The length of the loaded patch "c" can vary between being so small as to be assumed concentrated to so large it can be extended over the entire length of the web panel. The later case will be referred to as distributed-edge loading. The localized stresses due to edge loading can be combined with global stresses of bending and/or shear.

During the past 50 years, tests have been performed by several investigators to study the web behavior under compressive edge loads. These loads are mostly of the type shown in Fig. 1. However, the compression of the web over a support bearing block, as in Fig. 2, was also investigated. Extensive analytical and experimental studies of the elastic buckling and ultimate strength of webs loaded, shown in Fig. 1, were carried out during the last 20 years.

The purpose of this report is to summarize the available analytical and experimental studies and develop recommendations for the design of unstiffened webs under compressive edge loads.

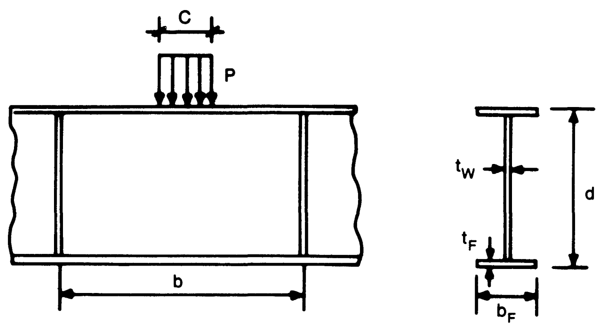

Fig. 1. Patch loading and girder dimensions

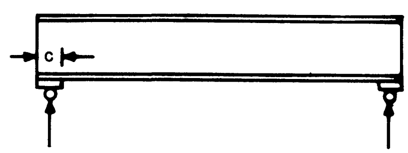

Fig. 2. Web-bearing strength over a support

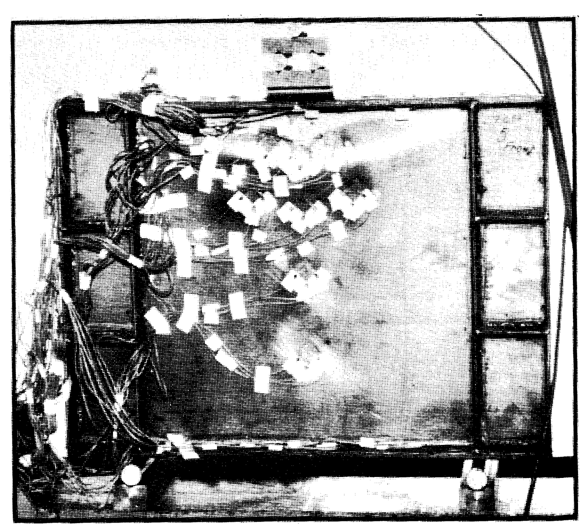

Fig. 3. Wrinkling, or folding, of web plate, Elgaaly[30]

FAILURE MODES

Local membrane stresses in the web under the load can reach the yield stress of the web material. The localized membrane yielding may not necessarily constitute failure, but eventually will induce web crippling, a localized wrinkling or folding of the web plate, shown in Fig. 3. During testing of thick webs, the girders sustained higher loads than those which caused membrane yielding and

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failed in the crippling mode. Inspection of the load deflection curves obtained from these tests reveal a change in slope at about the yield load, which is due to a significant membrane yielding of the web.

In thin webs, crippling can occur prior to yielding. Rigorous analytical and numerical solutions for elastic buckling of thin web panels with assumed idealized conditions are available. Web buckling, however, is not synonymous with failure due to the post-buckling reserve of strength possessed by restrained thin panels. Little or no correlation between the theoretical buckling loads and the experimental failure loads can be established. Furthermore, during testing, it is very difficult, if not impossible, to define the buckling load. This is due to the initial out-of-plane crookedness of the web panel, unavoidable even under laboratory conditions.

ELASTIC BUCKLING—ANALYTICAL AND EXPERIMENTAL STUDIES

At the beginning of this century, Sommerfield[1] and Timoshenko[2] were the first to obtain approximate solutions for the buckling of a plate subjected to equal and opposite concentrated forces applied at the midpoints of the longitudinal edges, as in Fig. 4a. About 30 years later, Leggett[3] presented a more rigorous solution to the same problem. Recently, Khan and Walker[4] obtained solutions for this problem with the applied forces distributed over a finite length "c," as shown in Fig. 4b.

Girkman[5] was the first investigator to study the problem of buckling of a rectangular plate subjected to discrete edge loading at the middle of one longitudinal edge and supported at its two vertical edges. Zeltin[6] provided a more detailed study of the problem using energy methods. Zeltin assumed the plate to be simply supported (out-of-plane) along all four edges, and the applied load to be supported by shear stresses distributed parabolically along the two vertical edges, as in Fig. 5a. Using a finite difference solution, White and Cottingham[7] examined the buckling of a plate (with simply supported and clamped boundaries) when loaded and supported in-plane, as shown in Fig. 5b. Rockey and Bagchi[8] solved similar problems using the finite element method.

The presence of either in-plane shear or moment will reduce the applied edge load necessary to buckle the panel. These effects were studied and presented by Rockey, Elgaaly, and Bagchi.[9] Most recently, in a series of papers, Khan and Walker[4]; Khan and Johns[10]; and Khan, Johns and Hayman,[11] examined the buckling of plates under localized edge loading, analytically using the energy method and experimentally employing the Southwell plot technique. In their studies, Khan and his associates did include the effects of localized edge loading combined with in-plane bending and shear. In all the studies, the buckling load is given by:

$$f_{cr} = \frac{P_{cr}}{bt} = k\left(\frac{\pi^2 D}{d^2 t}\right) \qquad (1)$$

where:

P_{cr} = critical edge load which will cause buckling
b, d, and t = width, depth and thickness of the plate, respectively
k = a non-dimensional buckling coefficient
D = flexural rigidity of the plate $[= Et^3/12(1 - \mu^2)]$
E and μ = modulus of elasticity and Poisson's ratio, respectively.

The nondimensional buckling coefficient "k" is a function of the relative length of the loaded patch ($\beta = c/b$), the aspect ratio of the panel ($\alpha = b/d$), and the boundary conditions. Variations of k with respect to β and α for simply supported boundaries, as presented by Rockey, Elgaaly, and Bagchi in Ref. 9, are shown in Fig. 6.

The results of combined in-plane bending or shear with edge loading are presented in the form of interaction curves. The curves for simply supported square plate, presented

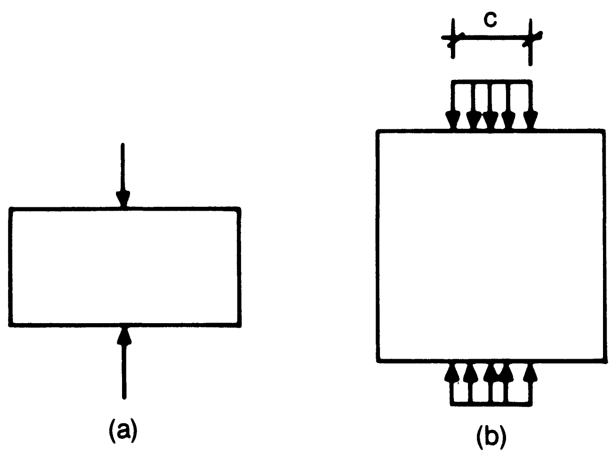

Fig. 4. Plates subjected to equal and opposite forces

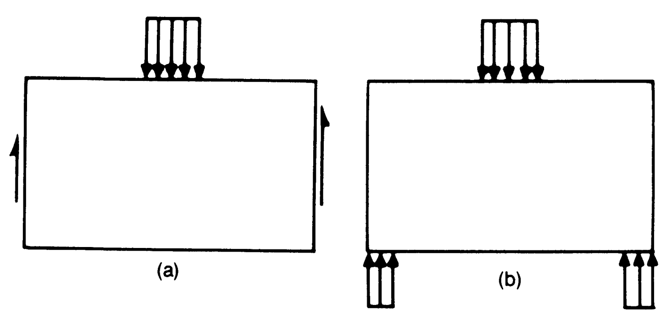

Fig. 5. Support plates subject to discrete edge loading

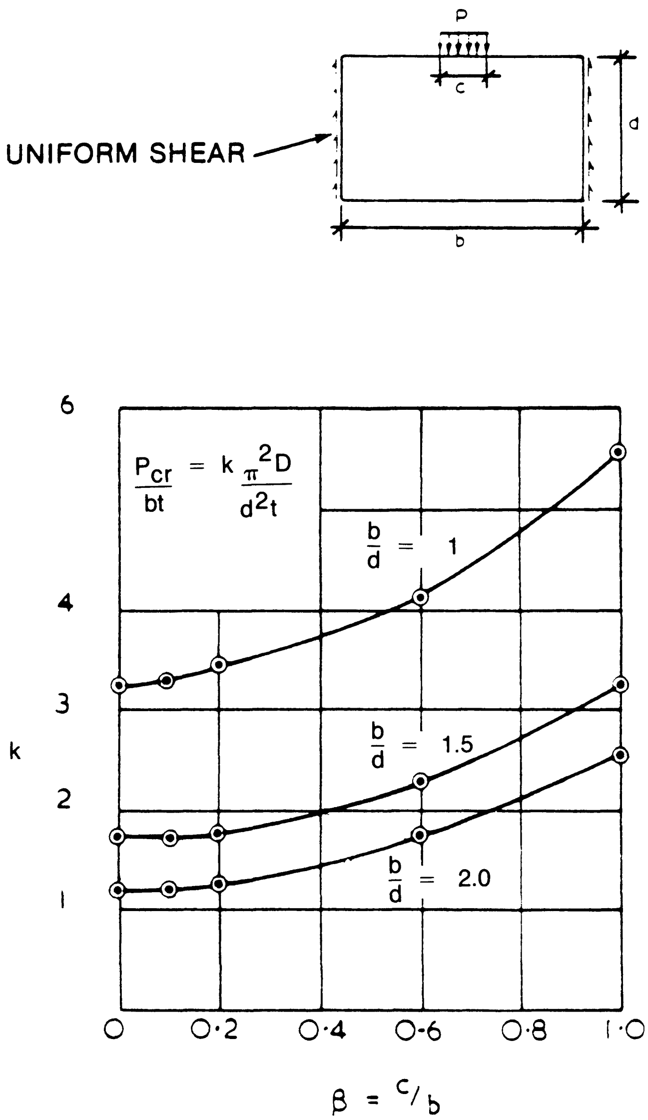

Fig. 6. Variation of buckling coefficient K *and relative length of the loaded patch* β[9]

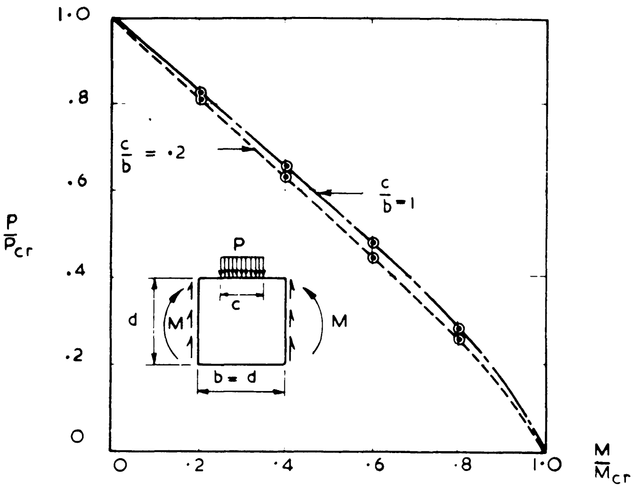

Fig. 7a. Interaction curves for edge loading and moment[9]

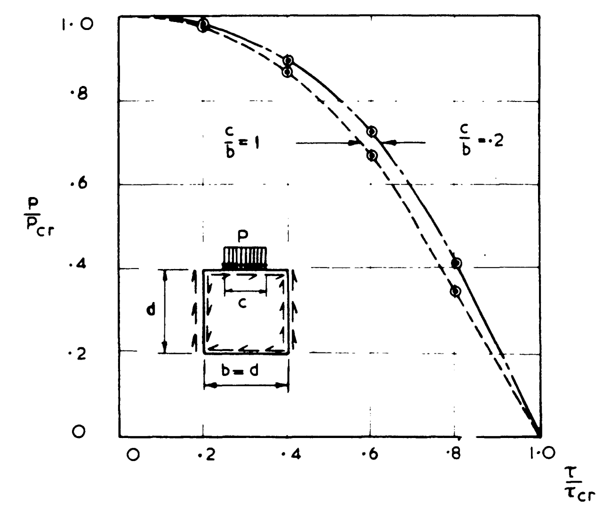

Fig. 7b. Interaction curves for edge loading and shear[9]

by Rockey, Elgaaly, and Bagchi in Ref. 9, are illustrated in Fig. 7.

As mentioned earlier, web buckling is not synonymous with failure, and little or no correlation exists between the elastic buckling load and the web failure load. For thin webs, the failure load is several times the elastic buckling load; and, in the case of thick webs, the web will yield locally before it buckles. In addition, the idealized boundary conditions assumed in all the analytical solutions of the buckling problem seldom exist in practice. The elastic buckling studies are summarized in this report for the sake of completeness; and, since some specifications still use the buckling load with a low safety factor, to establish design rules.

ULTIMATE STRENGTH—EXPERIMENTAL STUDIES

Ketchum and Draffin,[12] in the course of studies on the strength of light I-beams in the early 30's, tested the beams for compression of the web over the support bearing block, as in Fig. 8. They have tested 66 specimens, which are 6-, 10-, and 12-in. deep, with web slenderness ratio d/t_w of approximately 50, 60, and 70, respectively. The flange-to-web thickness t_f/t_w is about 1.3 in all specimens, and the yield stress of the material varies between 38 and 43 ksi. The ratio between the width of the bearing block and the depth of the beam c/d varies between 0.08 and 0.50. In eight specimens, the distance between the outer edge of the bearing block and that of the beam was not equal to zero,

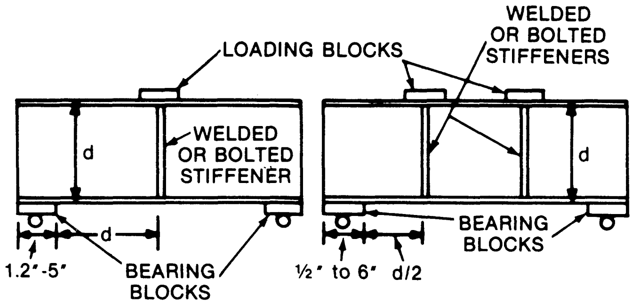

Fig. 8. Specimens used to test for bearing strength of web[12]

but varied between 0.75 and 3.25 in. Ketchum and Draffin concluded the failure of the beams was generally due to buckling of the web rather than yield failure of the material at the junction of the web and the flange. They suggested the ultimate load, which can be carried by the web, is equal to the buckling load of a fixed-ended column with a length that equals the depth of the beam; and a cross-section with an equivalent width of the web that equals the width of the bearing block, plus one quarter the depth of the beam.

Lyse and Godfrey[13] tested six rolled I-beams, 22-in. deep weighing 58 lbs/ft, under central or end patch bearing load. The slenderness ratio of the web d/t_w was about 60, and the length of the bearing block varied between 7 and 11 in. for central bearing and 3.5 and 5.5 in. for end bearing. In four out of the six tests, the yield point load was reached when the bearing stress in the vicinity of the bearing block, based on a bearing length equal to the length of the bearing block plus twice the thickness of the flange measured to the root of the web, reached the yield stress of the web material F_y.

In the remaining two tests, the yield point load was reached before the bearing stress, calculated as described above, reached F_y. In one case, the bearing stress was 0.82 F_y, and in the other it was 0.86 F_y. The authors stated that the results from these two beams are doubtful, due to thin bearing blocks. In all six tests, the beams continued to carry loads beyond the yield-point load until an ultimate capacity was reached, at which time the web crippled over or under the bearing block. The ratio between the ultimate load and yield point load varies between 1.1 and 1.4

Based on results of tests on welded plate girders (Refs. 14 and 15), Granholm concluded the web thickness is the most important parameter affecting the ultimate capacity of the web under concentrated edge load. He came to the conclusion that the ultimate capacity P_u is directly proportional to the square of the web thickness t_w^2 (P_u = constant $\times\ t_w^2$). The slenderness ratio of the web for the girders tested varied between 150 and 350, and they had relatively thin flanges.

The work started by Granholm was carried on under the leadership of Bergfelt. A great number of girders were tested to increase the knowledge of the parameters that influence the ultimate load-carrying capacity of the web under edge loading.[15-22] In these tests, it was demonstrated that failure involved a combination of yielding and buckling of the web, and that an increase in the ultimate carrying capacity of the web under edge loading with an increase in the flange to web thickness ratio t_f/t_w, can be obtained.

The influence of the length of the loaded patch c was examined, and an increase in the ultimate capacity up to a maximum of 30% was found possible with an increase in the c/t_f ratio. Furthermore, Bergfelt considered the influence of the web yield stress F_y and, inspired by von Karman's approximation for failure of plates in the post-buckling domain, was able to non-dimensionalize the Granholm formula by replacing Granholm constant by another constant multiplied by $(EF_y)^{0.5}$.

Several other researchers carried out experimental investigations into the effects of parameters that can influence the ultimate strength of the web under partial edge loading. Among those researchers are Skaloud, Novak, Drdacky and Novotny in Czechoslovakia[23-25]; and Rockey, Elgaaly, Bagchi, and Roberts[9,26-33] in Great Britain. The majority of the available test data has been summarized by Roberts,[33] shown in *Appendix 1.* Most of the tests were carried out on short-span girders with web thickness less than 0.2 in. The tested girders have a web slenderness ratio d/t_w between 75 and 505; flange width to thickness ratio b_f/t_f between 3 and 49; web panel aspect ratio b/d between 0.75 and 14; width of patch load to girder depth ratio c/d between 0 (concentrated load) and 0.33; flange to web thickness ratio t_f/t_w between 1 and 12; and the yield stress of the web material varies between 28 and 51 ksi.

All test results indicate the ultimate capacity (crippling load) is almost independent of the web slenderness ratio and the flange width to thickness ratio. The collapse load, however, is more or less directly proportional to the square of the web thickness, and is influenced to a lesser extent by the length of the patch load, the flange stiffness and the web material yield stress.

Bossert and Ostapenko[34] carried out a series of tests on three slender, long-span girders with vertical web stiffeners similar to the girder shown in Fig. 9. The girders were loaded by a uniformly distributed edge load over one panel at a time, and by additional loads acting through vertical web stiffeners to vary the magnitude of the coexistent

Fig. 9. Details of girders tested[34]

bending and/or shear stress in the loaded panel. The ultimate loads were found to be from three to four times the theoretical elastic buckling loads. Failure of the test panels was indicated by the formation of two bands of yielding across the test panel. The first band was almost circular in shape and formed at the points of maximum deflection on the concave side of the plate. The second band of yielding formed directly under the compression flange on the other side of the plate. This is identical to the web-crippling collapse mode observed in all tests under patch loading.

The influence of coexistent global bending stresses in the panel subjected to the patch load was examined by some of the aforementioned researchers. Very limited research work was carried out to study the effect of the presence of global shear stresses on the ultimate capacity of the web under patch loading. The reduction of the ultimate capacity under patch loading due to the presence of global bending or shear stresses will be discussed in a later section of this report.

ANALYTICAL PREDICTION OF ULTIMATE STRENGTH

Rigorous analytical solutions to calculate the ultimate capacity of the web under direct compressive loads do not exist, due to the complex nature of the problem. Simplified analytical solutions based on assumed collapse mechanisms deduced from experimental observations have been developed recently. It should be noted, however, that strict adherence to these solutions is not theoretically justifiable and that these solutions have been reduced to simple closed forms.

Two simplified analytical solutions will be discussed briefly in this report. The first solution is by Bergfelt.[17,22] At small loads, the flange behaves as a beam on an elastic foundation consisting of the web. At increasing load, a plastic hinge forms in the flange just under the load. Then the web stresses start yielding below the hinge, then the yielding region extends. The negative bending moments in the flange increase, and the failure starts when a plastic hinge forms at the location of the maximum negative moments on each side of the load.

If the flange is very stiff, there might be a hinge at each end of the flange and no hinge forms under the load. Considering the end stage, and noting there is no shear force at the hinges, Bergfelt derives a simple equation for the ultimate load. The equation derived by Bergfelt, when compared with test results, was found not to be in agreement for $t_f/t_w > 2$. The reason, as explained by Bergfelt, seems to be that for higher t_f/t_w values, web crippling under the load occurs before web yielding. Bergfelt extended his solution to cover ratios of t_f/t_w between 2 and 5, assuming web crippling rather than yielding.

Roberts and Rockey[29,31,33] developed a mechanism solution for predicting the collapse load of plate girders subjected to direct loading. Experimental results have shown that collapse occurs by formation of plastic hinges in the loaded flange accompanied by yield lines in the web.

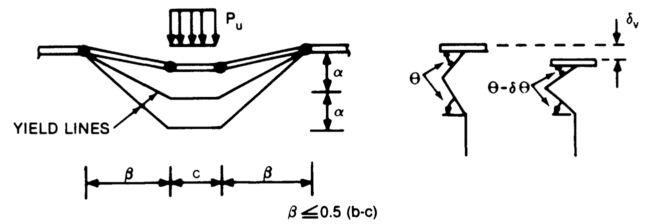

Fig. 10. Collapse mechanism (Roberts & Rockey)

Based on this experimental observation, Roberts and Rockey developed the collapse mechanism shown in Fig. 10. Equating external and internal work, minimizing the ultimate load P_u with respect to β, and calculating the deflection of the flange just prior to collapse (using elastic theory and assuming that the moment in the flange varies linearly between the plastic hinges) and equating it to the web deformation; Roberts and Rockey derived an expression for P_u. To simplify their solution and to rectify an anomaly in their derivation, they have assumed $\alpha = 25\ t_w$ (by observation from test results), $\beta = 0.5\ (b-c)$, the yield stress of the web equal to that of the flange, and replaced a complex function of the girder dimensions and material properties by $3c/d$, for a lower bound solution.

For stockey webs, assuming that failure may be initiated by membrane yielding of the web, Roberts and Rockey, by considering an alternative failure mechanism, derived another formula for the ultimate load "P_u." Robert and Chong,[35] using modified versions of the mechanisms for patch loading, derived formulae for the failure load under distributed edge loading assuming web crippling or membrane yielding failure mode.

The semi-empirical formulae derived by Bergfelt, and by Roberts, Rockey and Chong, will be discussed in the next section of this report.

FORMULAE TO PREDICT THE ULTIMATE STRENGTH

For a concentrated load ($c = 0$) acting directly on the web through the flange, Granholm[14] proposed the following equation to determine the ultimate capacity of the web.

$$P_u = 1{,}209\ t_w^2, \qquad (2)$$

where:

P_u is the ultimate load in kips,
and
t_w is the web thickness in inches.

Further tests by Bergfelt to study effects from other parameters coupled with analytical studies, as discussed earlier in this report, resulted in several semi-empirical formulae to predict the ultimate capacity of the web under patch loading. These formulae predict the Bergfelt test results to a reasonable degree of accuracy, and are applicable to girders within the range of parameters covered by the tests. Bergfelt formulae can be summarized as follows:

$$P_u = k_1 F_{yw} t_w^2 \qquad (3a)$$

and

$$P_u = k_2 (E F_{yw})^{0.5} t_w^2 \qquad (3b)$$

where:

$k_1 = 13\eta(t_i/t_w)f(c)$
$t_i = t_f(b_f/25t_f)^{0.25}$
$f(c) = \gamma/(1 - e^{-\gamma}\cos\gamma) \leq 1.3$
$\gamma = c/13.4t_f$
η = 0.55, 0.65, 0.85, and 1.00 for t_i/t_w = 0.5, 1.0, 1.5, and 2.0, respectively
$k_2 = 0.6(1 + 0.4t_i/t_w)$, or
$= 0.77(t_i/t_w)^{0.5}$, or
$= 1.3(t_i/t_w)^{0.6}(F_{yw}/E)^{0.1}$, or
$= 0.68(t_i/t_w)^{0.6}$

Equation (3a) is based on failure initiated by web yielding and will control for t_i/t_w less than about 2, while Eq. 3b is based on failure initiated by web crippling and will control for t_i/t_w between approximately 2 and 5. It has been recommended that P_u is calculated from 3a and 3b and the smaller value to be used.

The formulae for k_2 give almost identical results as shown in the following table.

	$\frac{t_i}{t_w} = 2$	$\frac{t_i}{t_w} = 3$	$\frac{t_i}{t_w} = 4$	$\frac{t_i}{t_w} = 5$
$k_2 = 0.6(1 + 0.4t_i/t_w)$:	1.08	1.32	1.56	1.80
$k_2 = 0.77(t_i/t_w)^{0.5}$:	1.09	1.33	1.54	1.72
$k_2 = 1.3(t_i/t_w)^{0.6}(F_{yw}/E)^{0.1}$, F_{yw} = 36 ksi:	1.01	1.29	1.53	1.75
$k_2 = 1.3(t_i/t_w)^{0.6}(F_{yw}/E)^{0.1}$, F_{yw} = 50 ksi:	1.04	1.33	1.58	1.81
$k_2 = 0.68(t_i/t_w)^{0.6}$:	1.03	1.31	1.56	1.79

Herzog[36,37] analyzed the results of the tests reported in Refs. 15, 17, and 38, and developed the following empirical formula:

$$P_u = 1{,}430t_w^2[1.2 + 1.25(I_c d/I_w t_w)(1 + c/d)^2(0.85 + b/100d)] \qquad (4)$$

where:

I_c = moment of inertia of compression flange about weak axis
I_w = moment of inertia of web about strong axis

The mean value of the ratio of the test loads to predicted loads by Eq. 4 is 1.001, with a standard deviation of 0.141.

Rockey, Elgaaly, and Bagchi[9] analyzed the results of tests on cold-formed trough section beams ($t_f = t_w$), and developed the following empirical formula which predicts the test results to a reasonable degree of accuracy:

$$P_u = 26.2t_w^2[4.5 + 6.4c/b]f(c/b,b/d) \qquad (5)$$

In Eq. 5, $f(c/b,b/d)$ is the elastic buckling coefficient "k" defined in Eq. 1 of this report multiplied by the panel aspect ratio "b/d" (can be determined from Ref. 9).

Equations 4 and 5 are not a function of the yield stress of the web material; it should be noted, however, that these equations are empirical equations derived to fit test results of mild steel members.

Dubas and Gehri[39] proposed the following empirical formula:

$$P_u = 0.75t_w^2(EFy_w t_f/t_w)^{0.5} \qquad (6)$$

Skaloud and Drdacky[24] incorporated the following empirical formula in the Czechoslovak Code of Practice:

$$P_u = 0.55t_w^2(0.9 + 1.5c/d)(EFy_w t_f/t_w)^{0.5} \qquad (7)$$

Finally, Roberts[33] developed the following semi-empirical formulae:

$$P_u = 2(b_f + t_f^2 F_{yf} Fy_w t_w)^{0.5} + F_{yw} t_w c \qquad (8a)$$

$$P_u = 0.5t_w^2(EF_{yw}t_f/t_w)^{0.5}[1 + (3c/d)(t_w/t_f)^{1.5} \qquad (8b)$$

Equation 8a is based on failure by initiation of yielding and 8b is based on failure by initiation of crippling. It is recommended that P_u be calculated using 8a and 8b, and the smaller of the two values be used.

If the ultimate load P_u calculated from 8a is denoted by P_{uy}, and that calculated from 8b is denoted by P_{uc}; the ratio P_{uc}/P_{uy} is plotted versus c/d in Fig. 11 for t_f/t_w = 1, 1.5,

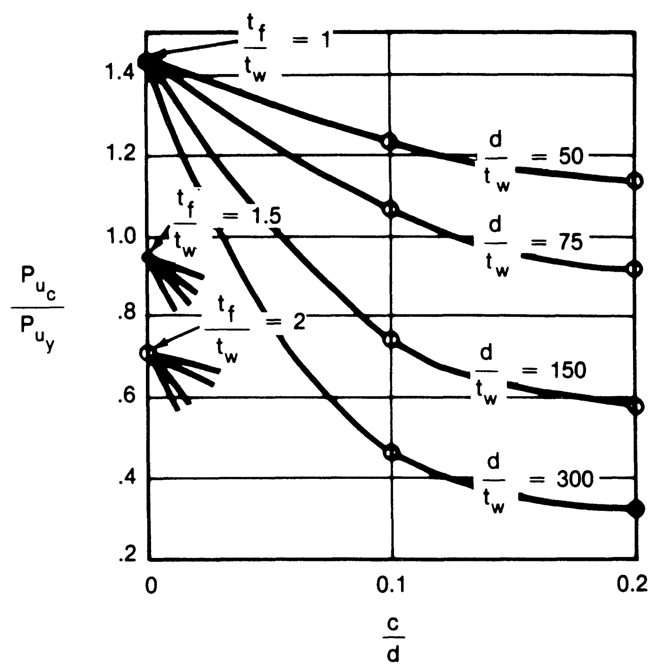

Fig. 11. Ratio of ultimate load initiated by crippling P_{uc} *to that initiated by yielding* P_{uy} *(Eqs. 8a, 8b)*

and 2; and d/t_w = 50, 75, 150, and 300 for the case where $b_f = 25t_f$ and $F_{yf} = F_{yw}$ = 36 ksi. As can be noted from the figure, in most practical cases, failure will be initiated by crippling. Only for thin flanges and stockey webs subjected to almost concentrated loads failure will be initiated by yielding. The same conclusions can be deduced by examining Eqs. 3a and 3b developed by Bergfelt.

For girders with failure initiated by yielding, the girders sustained significantly higher loads than P_{uy} and eventually failed in the crippling mode at loads above P_{uc}. The table below shows the results of tests on two girders by Roberts,[40] which confirm this conclusion.

c/d	b/d	d/t_w	b_f/t_f	t_f/t_w	t_w	P_{test}/P_{uc}	P_{uc}/P_{uy}
0	1	50	15	1	0.39 in.	1.22	3.3
0.2	1,	50	15	1	0.39 in.	1.14	1.57

It appears, therefore, that P_{uc} provides satisfactory prediction of the collapse load. However, at loads above P_{uy}, significant membrane yielding of the web may occur. Tests by Roberts[41] on rolled I-beams having web slenderness ratio as low as 20, compressed by two equal and opposite concentrated forces, confirm this conclusion. For girders subjected to uniformly distributed edge loading, Roberts and Chong[35] developed equations similar to 8a and 8b to predict the ultimate capacity of the girder. The predicted loads using Roberts and Chong equations were compared with the test results of Bossert and Ostapenko.[34] The ratio between the test load and the predicted load varies between 1.06 and 1.44.

EDGE LOADING COMBINED WITH GLOBAL BENDING OR SHEAR STRESSES

Little research work has been done to investigate the effect of coexistent global bending stresses on the crippling load of plate girders, even though this combined loading is often encountered in practice. Herzog[37] suggested multiplying P_u by $[1 - (f_b/F_y)^2]^{1/8}$ to allow for the effect of the global bending stress f_b on the collapse load P_u. Herzog's reduction factor was based on very limited number of test results.[17] Elgaaly and Rockey,[26] based on tests on 20 trough-section beams of 15-ft span, found the coexistent moment did not significantly reduce the patch loading (c/d = 0.2 for all tests) until it exceeded 50% of the pure bending ultimate strength of the panel, as shown in Fig. 12. A conservative approximation of the interaction effect illustrated in Fig. 12 is to multiply P_u by $[1 - (f_b/F_y)^3]^{1/3}$ to allow for the effect of the global bending stress f_b on the collapse load P_u. Skaloud[24] suggested a conservative reduction factor of $[1 - (f_b/F_y)^2]^{1/2}$, which has been incorporated in the Czechoslovak Code of Practice. The reduction factors discussed above are plotted in Fig. 13 for comparison; the most conservative approach is recommended for use until more test data are available.

The interaction between patch loading (at the center of the web panel with c/d = 0.2) and coexistent global shear has been examined by Elgaaly.[27] Tests were carried out on

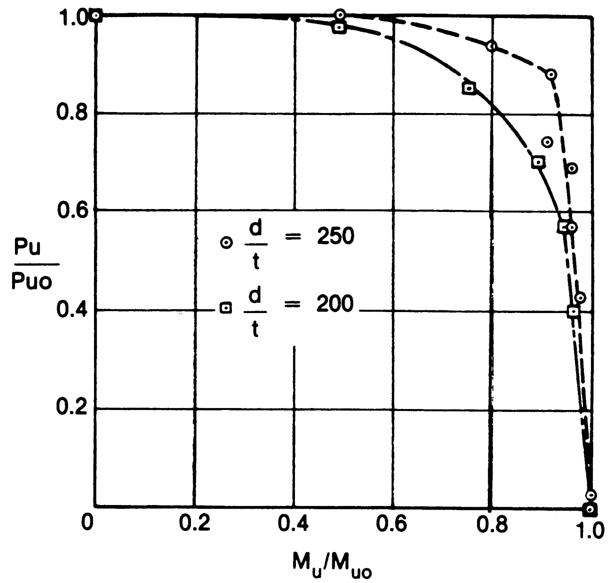

Fig. 12. Relation between P_u/P_{uo} *and* M_u/M_{uo}[26]

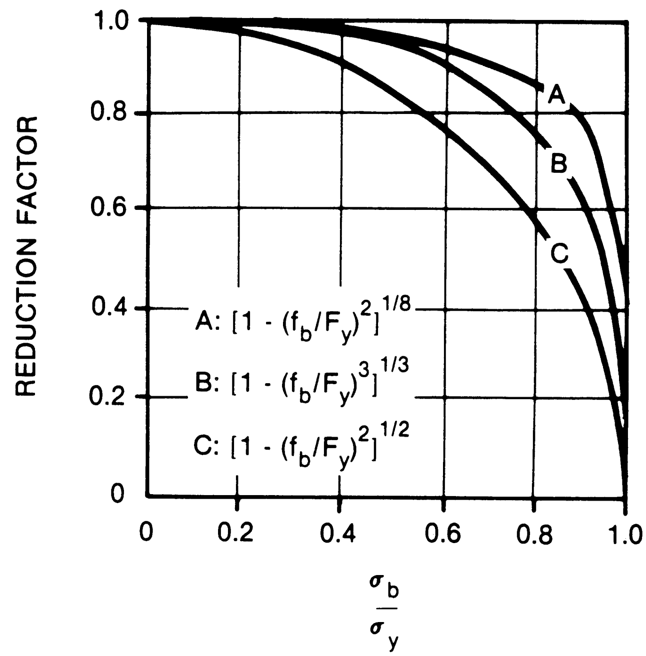

Fig. 13. Reduction in crippling load due to presence of global bending

18 trough-section specimens, the results shown in Fig. 14.

DESIGN RECOMMENDATIONS

The American Institute of Steel Construction (AISC) Specification[42] excludes crippling failure of welded plate

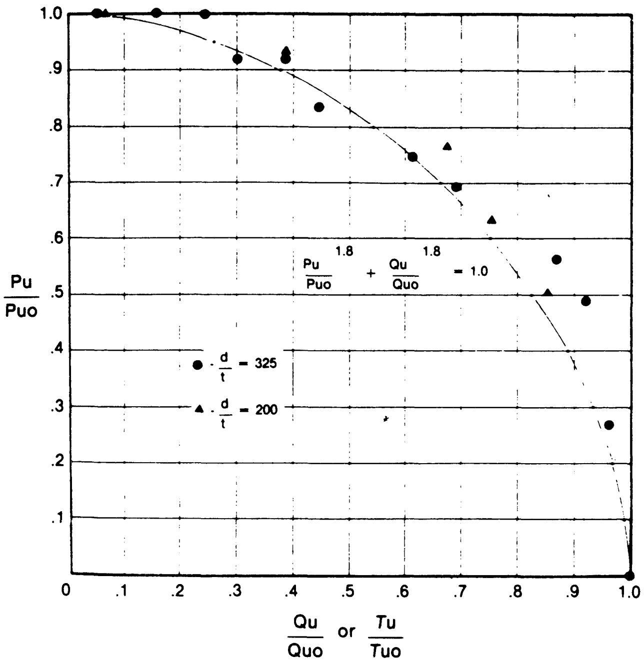

Fig. 14. Relation between P_u/P_{uo} *and* Q_u/Q_{uo}[27]

girder webs under direct in-plane compressive loads, and does not allow the compressive stress at the web toe of the fillets, resulting from direct loads not supported by bearing stiffeners to exceed 0.75 × the web yield stress. The governing formula is:

$$P/[t_w(c + 2k)] \leq 0.75F_y \qquad (9)$$

where:

k = distance from outer face of flange to web toe of fillet

Furthermore, according to the AISC Specification, the web shall be designed so that the compressive stresses from loads, bearing directly on or through a flange plate, upon the compression edge of the web plate and not supported by bearing stiffeners shall not exceed:

$$[5.5 + 4/(b/d)^2]10{,}000/(d/t)^2\text{ksi} \qquad (10a)$$

when the flange is restrained against rotation, and

$$[2 + 4/(b/d)^2]10{,}000/(d/t)^2\text{ksi} \qquad (10b)$$

when the flange is not so restrained

Stresses shall be computed by dividing the total direct loads by the product of the web thickness and the girder depth or the length of panel in which the load is placed, whichever is the lesser.

Results from 122 tests, *Appendix 1*, are compared with the AISC requirements for yielding (Eq. 9) and buckling, assuming that the flange does not restrain the web from rotation (Eq. 10b), in Figs. 15 and 16, respectively. As noted from Fig. 15, unless the parameter $(d/t_w)(c/t_f)$ is less than 900, Eq. 9 will overestimate the capacity of the web. This condition can materialize for loads of length c more than $2t_f$, $3t_f$, $4.5t_f$, $9t_f$, and $18t_f$ for d/t_w = 450, 300, 200, 100, and 50, respectively. A governing formula for yield, based on the test results, as shown in Fig. 15, can be written as follows:

$$P_u/[t_w(c + 2k)] \leq RF_y \qquad (11)$$

where:

$$R = 1.7 - 1.2 \times 10^{-3}(d/t_w)(c/t_f), \quad \text{for } (d/t_w)(c/t_f) \leq 1{,}000$$

and

$$R = 0.525 - 0.025 \times 10^{-3}(d/t_w)(c/t_f), \quad \text{for } (d/t_w)(c/t_f) \geq 1{,}000$$

The buckling requirements (Eq. 10b), on the other hand, are conservative and the web can carry more than five times the estimated capacity based on Eq. 10b, if the parameter $(d/t_w)(t_f/t_w)$ is more than 1,000 and at least 1.5 times the estimated capacity based on Eq. 10b, if the parameter $[(d/t_w)(t_f/t_w)]$ is 200 or less. A minimum factor of safety FS in Eq. 10b against failure can be calculated as follows:

$$\text{FS} = 1.0 + 0.003(d/t_w)(t_f/t_w) \qquad (12)$$

The recommendation in this report is to use formula, Eq. 13 (see Fig. 17), to calculate the ultimate capacity P_u of the web under patch direct loading, namely:

$$P_u = 0.5[EF_{yw}t_f/t_w]^{0.5}t_w^2 \qquad (13)$$

Equation 13 is compared with the semi-empirical formulae recommended by Bergfelt, Dubas and Gehri, Skaloud, and Roberts, which have been discussed earlier in this report, Fig. 18. The AISC proposed load and resistance factor design (LRFD) specification for structural steel buildings,[43] gives the following formula for P_u:

$$P_u = 108(F_y)^{0.5}t_w^2 \qquad (14)$$

A comparison between the proposed formula, Eq. 13, and the LRFD Specification formula, Eq. 14, is given in the following table.

	t_f/t_w	1	2	3	4	5
$P_u/t_w^2(F_y)^{0.5}$,	Proposed:	85	121	148	170	191
$P_u/t_w^2(F_y)^{0.5}$,	LRFD Spec:	108	108	108	108	108

As discussed earlier in this report, for girders where failure can be initiated by yielding, the girders will sustain higher loads than those estimated by Eq. 13. Therefore, the recommendation in this report is that Eq. 13 provides satisfactory estimate for the collapse load for all practical dimensions of girders and beams subjected to compressive patch loading directly applied to the web through the flange. The AISC proposed LRFD specification to exclude local web yielding recommends that the ultimate capacity P_u shall not exceed the value calculated as follows:

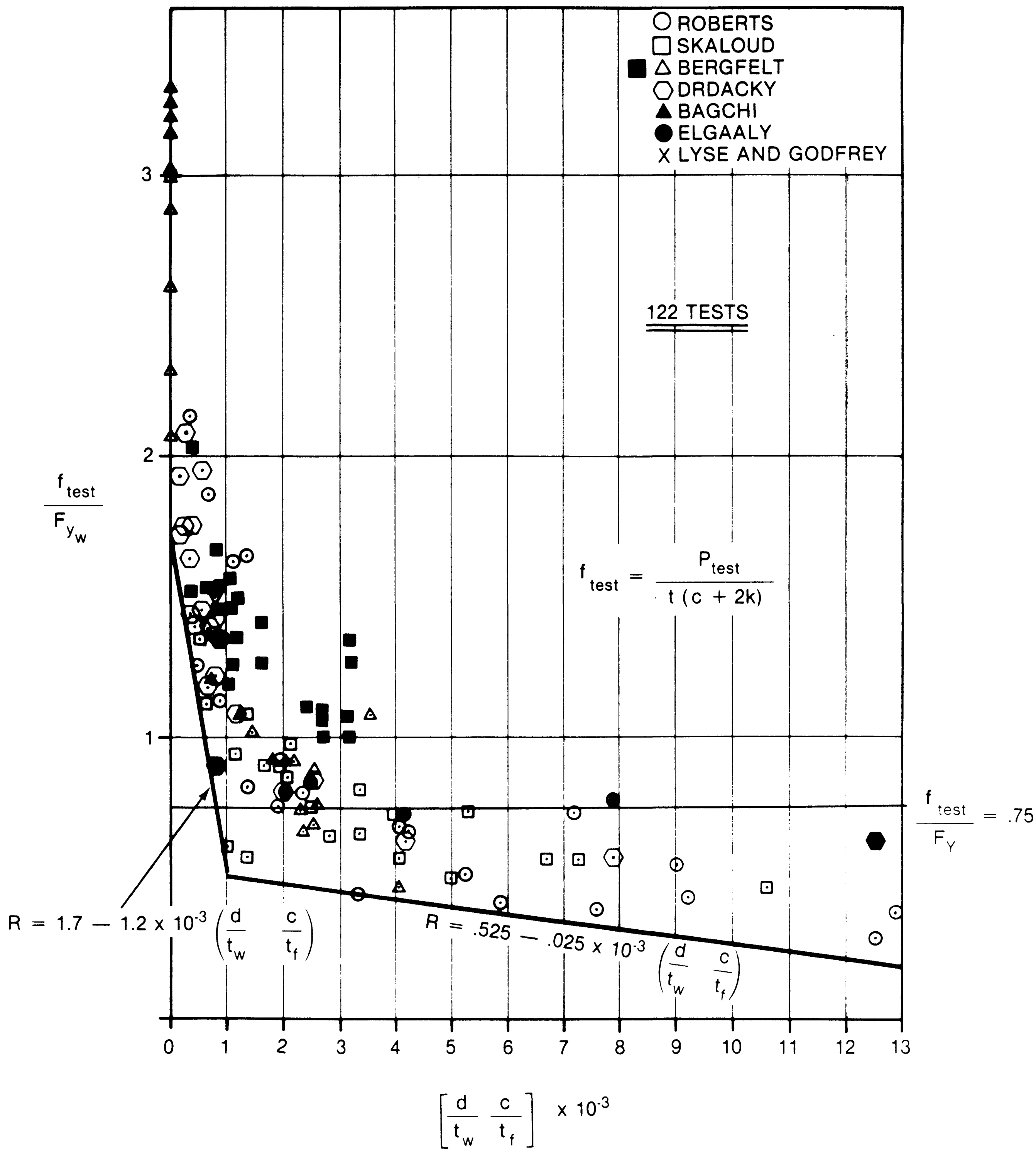

Fig. 15. Test results and AISC Specification equation[9]

$$P_u = (0.5c + 5k)F_{yw}t_w \qquad (15)$$

There is no correlation between Eq. 15 and available test results. To control local vertical deflection of the flange for the very special cases discussed earlier in this report, it is suggested that P_u calculated from Eq. 13 shall not exceed P_u calculated from Eq. 8a developed by Roberts.

Since Eq. 13 is a semi-empirical formula based partially

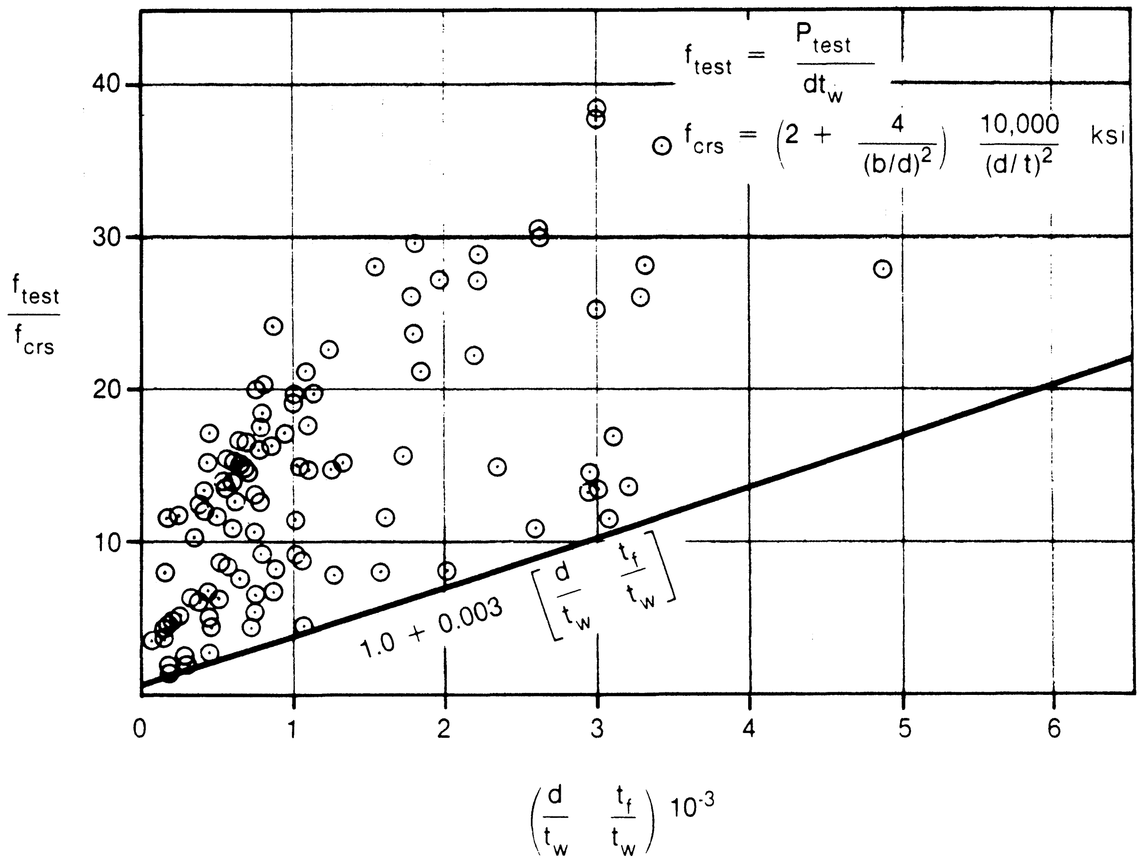

Fig. 16. Test results and AISC Specification Eq. 9b

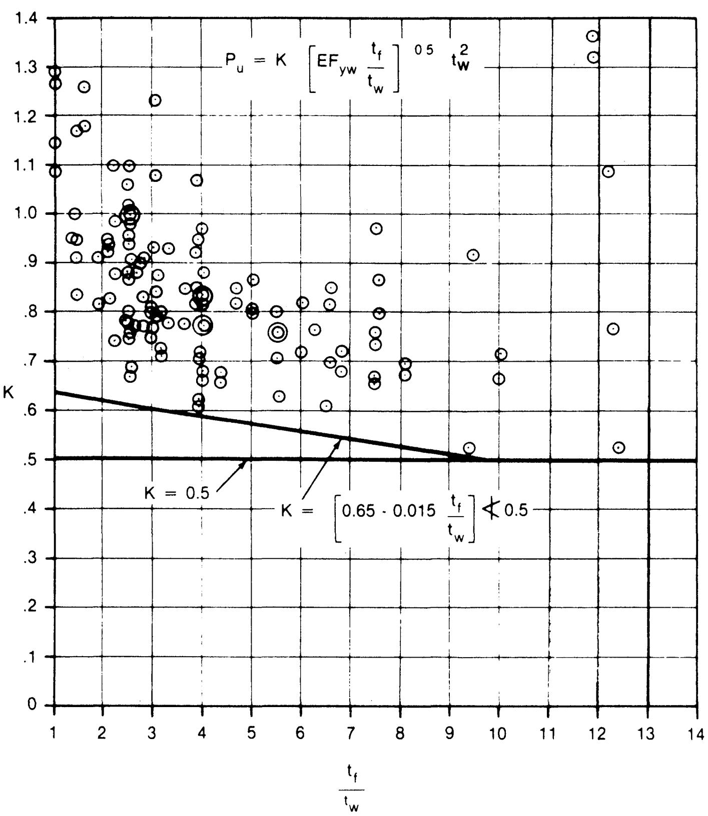

Fig. 17. Proposed design formula and test results

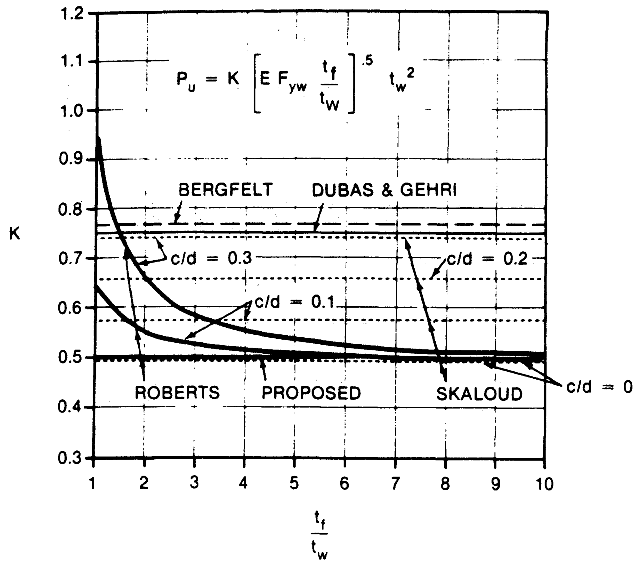

Fig. 18. Various semi-empirical formulae and proposed design formula

on test results, and since in these tests c/d was less than $1/3$, it is recommended to limit its application to cases where $c/d \leq 1/3$. For distributed edge loading, refer to the work by Roberts and Chong.[35]

For end reactions it is always recommended to have end bearing stiffeners. For direct loading on the compression flange near the end of the girder or for end reactions without bearing stiffeners, the AISC Specification controls the compressive stress at the web toe of the fillets as follows:

$$P/[t_w(c + k)] \leq 0.75F_{yw} \tag{16}$$

Results from 69 tests, given in *Appendix 2*, are compared with the AISC requirements (Eq. 16) in Fig. 19. As noted from the figure, unless the parameter $(d/t_w)(c/t_f)$ is less than 600, Eq. 16 will overestimate the capacity of the web.

The recommendation in this report is to use the following formula to calculate the ultimate capacity P_u:

$$P_u = 0.25[EF_{yw}t_f/t_w]^{0.5}t_w^2 \tag{17}$$

Equation 17 is compared with the test results given in *Appendix 2* in Fig. 20.

The proposed AISC LRFD Specification for Structural Steel Buildings gives the following formulae:

$$P_u = 54(F_y)^{0.5}t_w^2 \tag{18a}$$

or

$$P_u = (0.5c + 2.5k)F_{yw}t_w \tag{18b}$$

whichever is less.

The previous discussion relative to yielding versus crippling is applicable for end loads or reactions. It has to be noted that the load given by Eqs. 17 and 18a is one half the load given by Eqs. 13 and 14, respectively.

SUMMARY AND CONCLUSIONS

The studies on crippling of rolled beams and plate girders' webs, subjected to patch or distributed edge loading, have been summarized in this report. Solutions are available for the elastic critical buckling loads of idealized web panels which show little or no correlation to crippling loads determined from tests. Semi-empirical formulae are available to predict the crippling load with an accuracy that is acceptable for practical purposes. Formulae to predict the crippling load are recommended in this report. Semi-empirical formulae to predict loads above which membrane yielding of the web becomes pronounced are also available. The effects of coexistent global bending moment or shearing force were discussed. Reduction of P_u due to coexistent global moment has been established by the way of a formula for a reduction factor. A conservative approach is recommended in this report.

The requirements to prevent crippling given in the AISC building specification, as well as the proposed AISC LRFD specification, have to be reexamined in the light of the information in this report.

NOMENCLATURE

- b = width of web panel
- b_f = width of flange
- c = length of patch load
- d = depth of girder or plate panel
- E = Young's modulus of elasticity
- f_b = global bending stress
- f_t = direct stress from test
- F_{crc} = web panel buckling stress with flanges providing restraint against rotation
- F_{crs} = web panel buckling stress with flanges providing no restraint against rotation
- F_y = yield stress
- f_{yp} = flange yield stress
- f_{yw} = web yield stress
- k = critical buckling coefficient, or
  = distance from outer face of flange to web toe of fillet
- K = constant
- M = global bending moment
- M_{cr} = global bending moment which will cause buckling of web
- M_u = ultimate global bending capacity of web
- M_{uo} = panel's ultimate capacity under global bending only
- P = direct load on web
- P_{cr} = direct load which will cause buckling of the web
- P_u = ultimate direct load that can be carried by the girder

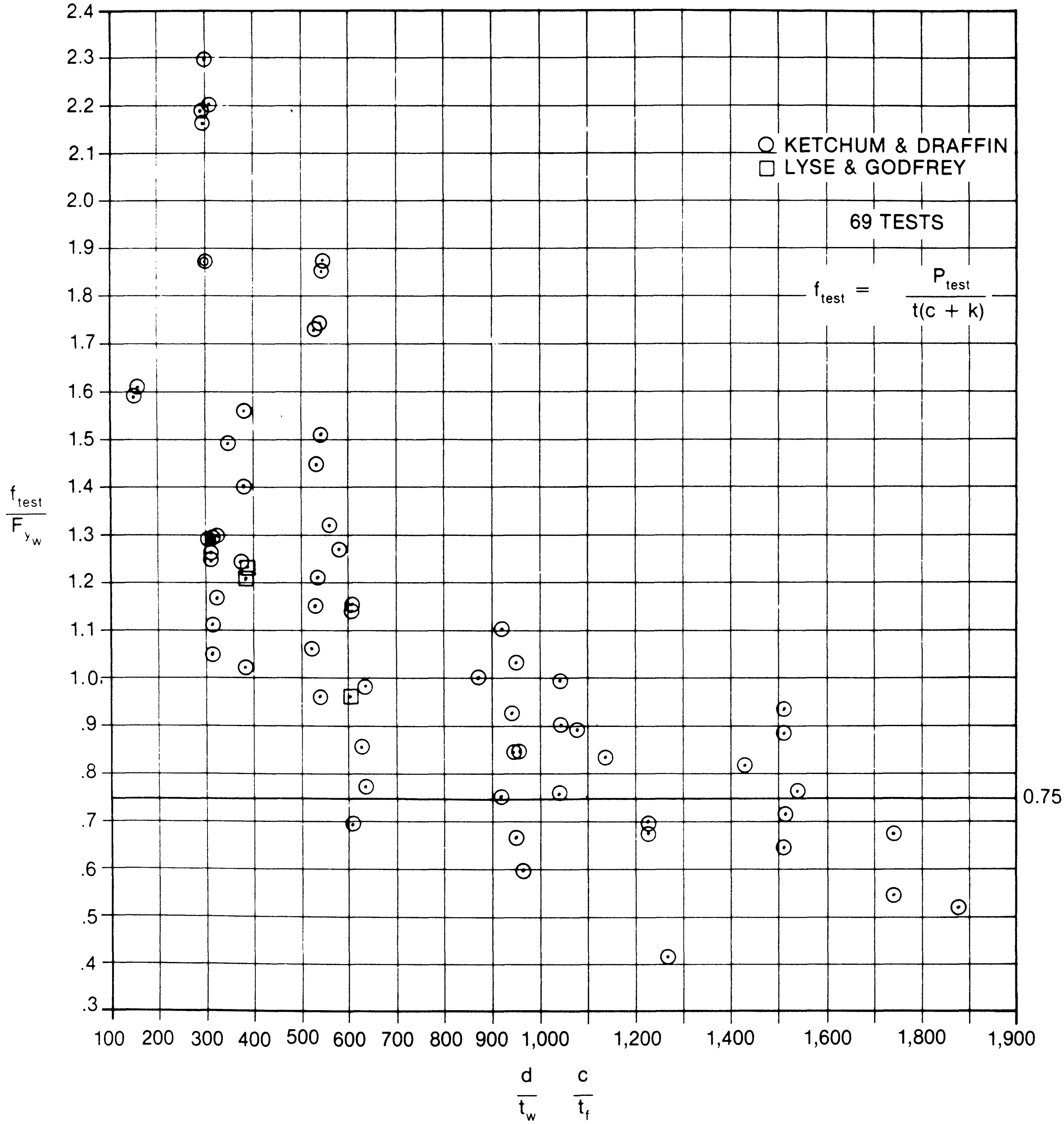

Fig. 19. Test results and specification Eq. 16

P_{uc} = ultimate direct load, failure initiated by crippling
P_{uo} = ultimate direct load in absence of global shear or bending
P_{uy} = ultimate direct load, failure initiated by yielding
P_t = ultimate direct load from test
Q_u = global shear ultimate capacity
Q_{uo} = panel's ultimate capacity under global shear only
t = thickness of plate
t_F = thickness of flange

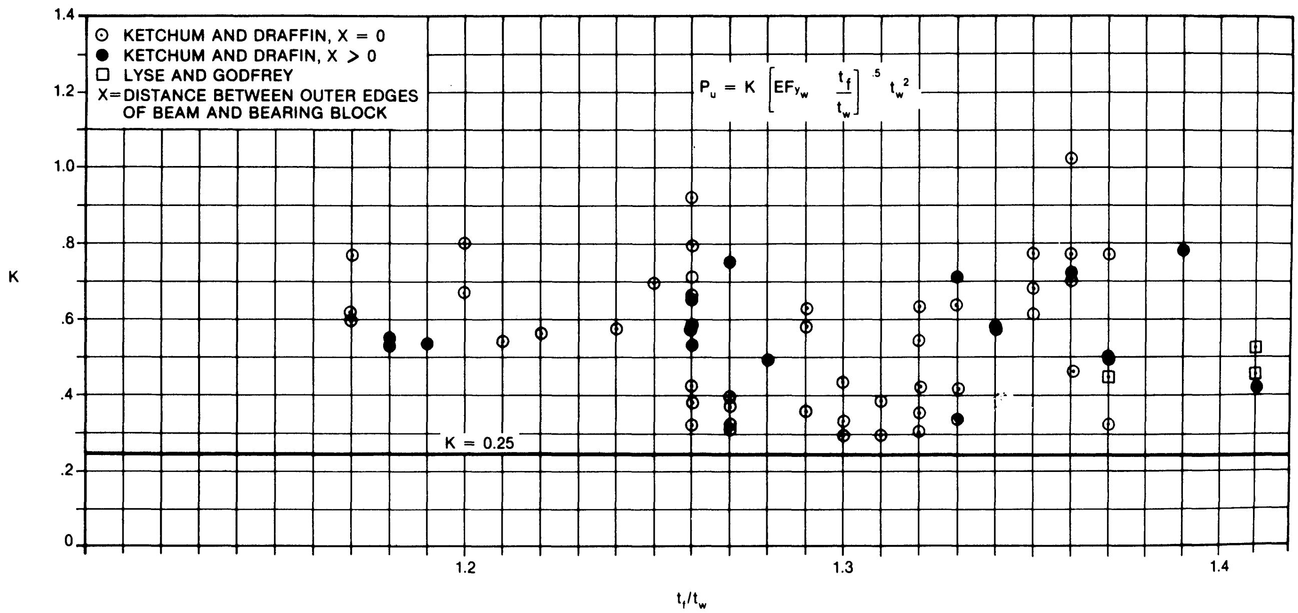

Fig. 20. Proposed design formula and test results

t_w = thickness of web
α = panel's aspect ratio, = b/d, or
α = depth of first web yield line from flange and spacing between the two yield lines under the load
β = relative length of loaded patch, = c/b, or
β = distance between positive and negative moments hinges in the flange
τ = shear stress
τ_{cr} = critical buckling shear stress
τ_u = ultimate shear stress
τ_{uo} = ultimate shear stress for panels subjected to global shear only

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APPENDIX 1

Results of Tests on Simply Supported Welded Girders and Rolled Beams Subjected to Patch Loading on the Top Flange at Mid-Span

	Test No.	$\frac{c}{b}$	$\frac{b}{d}$	$\frac{d}{t_w}$	$\frac{b_f}{t_f}$	$\frac{t_f}{t_w}$	t_w	F_{yw}	K	$\frac{f_t}{F_{yw}}$	$\frac{f_t}{F_{crs}}$	$\frac{f_t}{F_{crc}}$
Roberts 32	1	0.083	2.40	253	48.9	3.08	.039	28.0	0.84	0.65	12.5	5.5
	2	0.083	2.40	253	22.1	6.82	.039	28.0	0.72	0.76	15.9	6.9
	3	0.083	2.40	253	12.7	11.9	.039	28.0	1.32	1.63	38.6	16.8
	4	0.083	2.40	118	48.9	1.44	.083	32.5	0.91	1.04	4.7	2.0
	5	0.083	2.40	118	22.1	3.18	.083	32.5	0.80	1.13	6.0	2.6
	6	0.083	2.40	118	12.7	5.54	.083	32.5	0.76	1.26	7.6	3.3
	7	0.083	2.40	82	48.9	1.00	.120	32.0	1.29	1.65	3.8	1.7
	8	0.083	2.40	82	22.1	2.21	.120	32.0	1.10	1.88	4.8	2.1
	9	0.083	2.40	82	12.7	2.85	.120	32.0	1.07	2.15	6.1	2.7
	10	0.083	4.03	505	48.9	3.08	.039	27.8	0.79	0.24	28.1	11.0
	11	0.083	4.03	505	22.1	6.82	.039	27.8	0.68	0.29	36.0	14.1
	12	0.083	4.03	505	12.7	11.90	.039	27.8	1.37	0.73	95.7	37.4
	13	0.083	4.03	236	48.9	1.44	.083	32.5	0.95	0.39	11.7	4.6
	14	0.083	4.03	236	22.1	3.18	.083	32.5	0.71	0.41	13.0	5.1
	15	0.083	4.03	236	12.7	5.54	.083	32.5	0.63	0.46	15.1	5.9
	16	0.083	4.03	236	7.4	9.46	.083	32.5	0.92	0.81	29.0	11.3
	17	0.083	4.03	164	48.9	1.00	.120	32.0	1.15	0.56	8.1	3.2
	18	0.083	4.03	164	22.1	2.21	.120	32.0	0.99	0.69	10.3	4.0
	19	0.083	4.03	164	12.7	3.85	.120	32.0	0.92	0.80	12.7	5.0
	20	0.083	4.03	164	7.4	6.58	.120	32.0	0.82	0.87	14.9	5.8
	21	0.083	4.03	246	48.9	1.44	.083	32.5	0.84	0.23	15.4	6.0
	22	0.083	4.03	246	22.1	3.18	.083	32.5	0.72	0.29	19.8	7.7
	23	0.083	4.03	246	12.7	5.54	.083	32.5	0.76	0.39	27.3	10.6
	24	0.083	4.03	246	48.9	1.00	.120	32.0	1.09	0.37	11.5	4.5
	25	0.083	4.03	246	22.1	2.21	.120	32.0	0.88	0.43	13.8	5.4
	26	0.083	4.03	246	12.7	3.85	.120	32.0	0.82	0.51	17.1	6.7
Novak 23	27	0.100	1.00	400	29.1	2.2	.098	43.2	0.74	0.56	8.2	5.2
	28	0.100	1.00	400	19.8	4.0	.098	43.4	0.77	0.73	11.5	7.3
	29	0.100	1.00	400	12.3	6.5	.098	36.4	0.61	0.74	10.7	6.8
	30	0.100	1.00	400	9.9	8.1	.098	36.8	0.70	0.90	13.7	8.7
	31	0.100	1.00	400	8.2	12.2	.098	41.9	1.09	1.41	28.0	17.7
	32	0.050	2.00	333	25.4	2.1	.118	42.0	0.83	0.74	14.6	6.8
	33	0.050	2.00	333	20.0	3.3	.118	43.1	0.78	0.81	17.6	8.1
	34	0.050	2.00	333	12.1	5.5	.118	44.7	0.71	0.86	21.1	9.7
	35	0.050	2.00	333	10.1	6.6	.118	43.5	0.70	0.90	22.4	10.4
	36	0.050	2.00	333	8.3	10.0	.118	43.4	0.67	0.94	26.3	12.2
	37	0.100	2.00	333	25.4	2.1	.118	42.0	0.94	0.47	16.6	7.7
	38	0.100	2.00	333	20.0	3.3	.118	43.1	0.93	0.56	21.1	9.7
	39	0.100	2.00	333	12.1	5.5	.118	44.7	0.80	0.58	23.7	11.0
	40	0.100	2.00	333	10.1	6.6	.118	43.5	0.85	0.66	27.2	12.6
	41	0.100	2.00	333	8.3	10.0	.118	43.4	0.72	0.64	28.3	13.1
	42	0.100	1.00	250	8.4	3.0	.079	35.2	0.77	0.98	5.6	3.5
	43	0.100	1.00	250	2.8	8.1	.079	35.2	0.67	1.12	8.1	5.1
	44	0.100	1.00	250	2.0	12.30	.079	35.2	0.77	1.35	11.4	7.2
	45	0.100	2.00	250	10.1	2.50	.079	40.6	0.75	0.50	10.7	4.9
	46	0.100	2.00	250	2.8	9.40	.079	40.6	0.53	0.57	14.8	6.9
	47	0.100	2.00	250	2.4	12.40	.079	40.6	0.53	0.61	17.0	7.8

APPENDIX 1 (*continued*)

Source	Test No.	$\frac{c}{b}$	$\frac{b}{d}$	$\frac{d}{t_w}$	$\frac{b_f}{t_f}$	$\frac{t_f}{t_w}$	t_w	F_{yw}	K	$\frac{f_t}{F_{yw}}$	$\frac{f_t}{F_{crs}}$	$\frac{f_t}{F_{crc}}$
Bergfelt & Hovik 15-17	48	0.000	3.43	215	24.6	1.87	.128	47.3	0.82	3.21	12.0	4.8
	49	0.042	3.43	215	24.6	1.87	.128	47.3	0.91	1.08	13.4	5.4
	50	0.000	3.43	215	23.5	2.61	.128	47.3	0.77	3.02	13.3	5.3
	51	0.042	3.43	215	23.5	2.61	.128	47.3	0.88	0.86	15.3	6.1
	52	0.000	3.43	215	24.8	3.10	.128	47.3	0.80	3.15	15.2	6.1
	53	0.042	3.43	215	24.8	3.10	.128	47.3	0.88	0.92	16.8	6.7
	54	0.000	3.43	215	21.0	3.65	.128	47.3	0.78	3.01	16.0	6.4
	55	0.042	3.43	215	21.0	3.65	.128	47.3	0.85	0.94	17.6	7.0
	56	0.000	3.43	215	19.6	4.69	.128	47.3	0.82	3.07	19.1	7.7
	57	0.042	3.43	215	19.6	4.69	.128	47.3	0.85	1.01	19.8	7.9
	58	0.000	8.00	150	16.7	3.00	.079	42.6	1.09	3.51	15.2	5.6
	59	0.042	8.00	150	16.7	3.00	.079	42.6	1.23	0.87	17.3	6.4
	60	0.000	6.00	200	12.5	4.00	.079	42.6	0.88	2.89	18.5	7.0
	61	0.042	6.00	200	12.5	4.00	.079	42.6	0.97	0.76	20.3	7.6
	62	0.000	4.80	250	10.0	5.00	.079	42.6	0.80	2.60	22.8	8.8
	63	0.042	4.80	250	10.0	5.00	.079	42.6	0.81	0.70	14.6	8.9
	64	0.000	4.00	300	8.3	6.00	.079	42.6	0.72	2.31	26.1	10.6
	65	0.042	4.00	300	8.3	6.00	.079	42.6	0.82	0.74	29.7	12.1
	66	0.000	3.43	350	6.7	7.50	.079	42.6	0.66	2.07	30.3	12.1
	67	0.042	3.43	350	6.7	7.50	.079	42.6	0.67	0.66	30.7	12.3
	68	0.000	14.00	206	25.0	2.94	.134	40.6	0.75	3.27	14.2	5.2
	69	0.010	14.00	206	25.0	2.94	.134	40.6	0.80	0.93	15.1	5.5
	70	0.020	14.00	206	25.0	2.94	.134	40.6	0.81	0.47	15.4	5.6
Drdacky & Novotny 25	71	0.100	1.00	76	4.9	2.51	.156	41.3	0.69	1.76	1.5	1.0
	72	0.100	1.00	75	5.1	2.48	.158	39.1	0.79	2.08	1.7	1.1
	73	0.100	1.00	75	3.1	3.90	.158	40.7	0.71	1.93	1.9	1.2
	74	0.100	1.00	113	4.9	2.52	.156	47.3	0.67	1.46	2.1	1.3
	75	0.100	1.00	114	3.1	3.99	.156	36.1	0.68	1.64	2.6	1.7
	76	0.100	1.00	168	5.1	2.80	.141	37.3	0.91	1.50	4.4	2.8
	77	0.100	1.00	165	4.9	2.78	.143	40.9	0.90	1.42	4.9	2.8
	78	0.100	1.00	164	3.1	4.36	.145	44.4	0.68	1.23	4.4	2.8
	79	0.150	1.00	76	4.9	2.52	.156	41.3	0.80	1.64	1.7	1.1
	80	0.200	1.00	75	5.1	2.48	.158	39.1	0.78	1.40	1.6	1.0
	81	0.100	1.00	75	3.1	3.97	.158	40.7	0.62	1.72	1.7	1.1
	82	0.133	1.00	113	4.9	2.52	.156	37.3	0.76	1.40	2.4	1.5
	83	0.100	1.00	114	3.1	3.99	.156	36.1	0.72	1.75	2.8	1.8
	84	0.050	1.00	168	5.1	2.80	.141	37.3	0.77	1.96	3.8	2.4
	85	0.085	1.00	165	4.9	2.78	.143	40.9	0.83	1.66	4.2	2.6
	86	0.100	1.00	164	3.1	4.36	.145	44.4	0.66	1.19	4.3	2.7
Bagchi & Rockey 28	87	0.039	2.00	195	12.0	3.91	.128	36.3	0.61	1.21	20.0	9.2
	88	0.058	1.36	195	12.0	3.91	.128	36.3	0.84	1.67	20.0	10.8
	89	0.114	1.04	195	12.0	3.91	.128	36.3	0.95	1.49	16.6	10.3
Elgaaly 30	90	0.200	1.00	250	50.0	1.00	.079	31.9	1.27	0.64	5.1	3.2
	91	0.200	1.00	250	31.3	1.60	.079	32.5	1.26	0.78	6.4	4.1
	92	0.200	1.00	250	16.7	3.00	.079	34.5	0.93	0.73	6.7	4.3
	93	0.200	1.00	250	10.0	5.00	.079	32.6	0.87	0.85	7.9	5.0
	94	0.200	1.00	250	8.0	6.25	.079	33.1	0.77	0.81	7.9	5.0

APPENDIX 1 (*continued*)

	Test No.	$\frac{c}{b}$	$\frac{b}{d}$	$\frac{d}{t_w}$	$\frac{b_f}{t_f}$	$\frac{t_f}{t_w}$	t_w	F_{yw}	K	$\frac{f_t}{F_{yw}}$	$\frac{f_t}{F_{crs}}$	$\frac{f_t}{F_{crc}}$
Bergfelt 22	95	0.050	1.00	390	19.4	7.56	.081	30.5	0.80	1.18	13.4	8.4
	96	0.050	1.00	400	23.5	2.55	.079	30.5	0.91	1.01	9.1	5.8
	97	0.050	1.00	390	19.4	7.56	.081	30.5	0.87	1.29	14.7	9.3
	98	0.050	1.00	400	23.5	2.55	.079	30.5	0.98	1.08	9.8	6.2
	99	0.016	3.13	400	20.0	7.50	.079	44.7	0.74	1.20	38.1	15.6
	100	0.033	1.50	400	20.0	7.50	.079	44.7	0.76	1.24	25.1	13.0
	101	0.067	0.75	400	20.0	7.50	.079	44.7	0.97	1.57	13.3	9.6
	102	0.016	3.13	267	20.8	4.00	.118	35.5	0.67	1.43	15.0	6.1
	103	0.033	1.50	267	20.8	4.00	.118	35.5	0.68	1.46	9.7	5.0
	104	0.067	0.75	267	20.8	4.00	.118	35.5	0.77	1.65	4.6	3.3
	105	0.018	3.24	340	24.0	2.50	.079	51.3	0.88	1.01	24.3	9.8
	106	0.039	1.50	340	24.0	2.50	.079	51.3	0.94	1.07	16.3	8.5
	107	0.078	0.75	340	24.0	2.50	.079	51.3	0.96	1.10	6.9	5.0
	108	0.050	1.00	400	24.0	2.50	.079	41.3	1.00	1.27	11.5	7.3
	109	0.050	1.33	300	24.0	2.50	.079	41.3	0.87	1.11	10.7	5.9
	110	0.050	2.00	200	24.0	2.50	.079	41.3	1.00	1.27	11.5	5.3
	111	0.050	2.67	150	24.0	2.50	.079	41.3	1.02	1.29	12.3	4.7
	112	0.100	1.00	200	24.0	2.50	.079	41.3	1.10	1.41	6.4	4.0
	113	0.100	1.33	150	24.0	2.50	.079	41.3	1.06	1.36	6.4	3.5
	114	0.050	1.00	267	20.8	4.00	.118	47.6	0.83	1.54	8.7	5.5
	115	0.050	1.33	200	20.8	4.00	.118	47.6	0.83	1.53	9.1	5.0
	116	0.050	2.00	133	20.8	4.00	.118	47.6	0.82	1.52	8.5	3.9
	117	0.066	2.00	194	26.2	1.56	.077	25.8	1.18	1.34	8.2	3.8
	118	0.066	2.00	127	12.8	2.09	.118	35.5	0.93	1.45	5.8	2.7
	119	0.066	2.00	77	10.0	2.02	.195	42.3	0.95	2.03	3.8	1.8
Lyse & Godfrey 13	120	c/d = 0.32		61	15.2	1.41	.397*	49.0	1.00	1.36	4.75#	1.73#
	121	c/d = 0.50		61	15.2	1.41	.397*	50.0	1.18	1.09	5.63#	2.05#
	122	c/d = 0.32		61	15.2	1.37	.407*	44.7	0.90	1.24	4.35#	1.58#

* - measured near flange
\# - F_{crs} and F_{crc} are based on b/d = ∞

APPENDIX 2

Results of Tests on Simply Supported Rolled Beams Subjected to Loading at the Bottom Flange from Supporting Pads

	Test No.	$\frac{c}{d}$	$\frac{d}{t_{w1}}$	t_{w1}	t_{w2}	t_f*	$\frac{t_f}{t_{w2}}$	Fy_w	K	$\frac{f_t}{F_{yw}}$
Ketchum & Draffin [12]	1	0.320	47	.127	.133	.171	1.29	43.0	.63	1.15
	2	0.320	46	.132	.133	.171	1.29	43.0	.58	1.06
	3	0.200	46	.130	.136	.171	1.26	43.0	.42	1.17
	4	0.200	52	.116	.125	.171	1.37	43.0	.49	1.30
	5	0.100	62	.161	.172	.206	1.41	38.3	.42	1.87
	6	0.100	61	.163	.172	.206	1.41	38.3	.42	1.87
	7	0.120	59	.169	.169	.206	1.22	38.3	.42	1.49
	8	0.120	65	.153	.152	.206	1.36	38.3	.46	1.56
	9	0.175	63	.159	.163	.206	1.26	38.3	.60	1.51
	10	0.175	62	.161	.164	.206	1.26	38.3	.58	1.45
	11	0.100	61	.163	.173	.206	1.19	38.3	.53	2.19
	12	0.100	62	.161	.164	.206	1.26	38.3	.53	2.16
	13	0.192	59	.170	.170	.206	1.21	38.3	.54	1.27
	14	0.192	60	.167	.169	.206	1.22	38.3	.56	1.32
	15	0.296	66	.152	.153	.206	1.35	38.3	.68	1.03
	16	0.296	65	.154	.153	.206	1.35	38.3	.61	0.92
	17	0.100	61	.163	.175	.206	1.18	38.3	.55	2.30
	18	0.100	62	.161	.175	.206	1.18	38.3	.53	2.20
	19	0.350	61	.164	.150	.206	1.37	38.3	.77	0.99
	20	0.350	61	.164	.151	.206	1.36	38.3	.70	0.90
	21	0.175	63	.160	.151	.206	1.36	38.3	.72	1.74
	22	0.175	64	.157	.162	.206	1.27	38.3	.75	1.87
	23	0.500	62	.161	.153	.206	1.35	38.3	.77	0.71
	24	0.500	62	.161	.164	.206	1.26	38.3	.65	0.64
	25	0.175	62	.162	.155	.206	1.33	38.3	.71	1.73
	26	0.175	64	.157	.148	.206	1.39	38.3	.78	1.85
	27	0.100	71	.170	.173	.225	1.30	43.1	.29	1.02
	28	0.100	71	.168	.169	.225	1.33	43.1	.41	1.40
	29	0.100	70	.171	.175	.225	1.29	43.1	.36	1.24
	30	0.160	71	.169	.176	.225	1.28	43.1	.49	1.15
	31	0.160	71	.170	.176	.225	1.28	43.1	.49	1.14
	32	0.250	72	.167	.172	.225	1.31	43.1	.38	0.59
	33	0.250	71	.168	.173	.225	1.30	43.1	.43	0.66
	34	0.250	71	.169	.170	.225	1.32	43.1	.54	0.84
	35	0.083	53	.114	.130	.171	1.32	43.0	.30	1.61
	36	0.083	52	.115	.131	.171	1.31	43.0	.29	1.59
	37	0.167	53	.113	.132	.171	1.30	43.0	.33	1.05
	38	0.167	53	.114	.130	.171	1.32	43.0	.35	1.11
	39	0.333	54	.112	.128	.171	1.34	43.0	.57	0.98
	40	.333	52	.116	.135	.171	1.27	43.0	.39	0.69
	41	.500	54	.112	.137	.171	1.25	43.0	.69	0.84
	42	.500	52	.116	.130	.171	1.32	43.0	.63	0.75
	43	.100	63	.160	.165	.206	1.25	38.3	.32	1.29
	44	.100	63	.159	.150	.206	1.37	38.3	.32	1.25
	45	.175	63	.159	.150	.206	1.37	38.3	.50	1.21
	46	.175	63	.159	.164	.206	1.26	38.3	.38	0.96
	47	.300	63	.158	.164	.206	1.26	38.3	.71	1.10
	48	.350	63	.159	.164	.206	1.26	38.3	.66	0.89

APPENDIX 2 *(continued)*

	Test No.	$\frac{c}{d}$	$\frac{d}{t_{w1}}$	t_{w1}	t_{w2}	t_f*	$\frac{t_f}{t_{w2}}$	Fy_w	K	$\frac{f_t}{F_{yw}}$
	49	.350	61	.163	.154	.206	1.34	38.3	.58	0.75
	50	.400	63	.158	.163	.206	1.26	38.3	.57	0.67
	51	.500	62	.161	.164	.206	1.26	38.3	.92	0.88
	52	.500	62	.161	.151	.206	1.36	38.3	1.02	0.93
	53	.500	63	.158	.151	.206	1.36	38.3	.77	0.71
	54	.500	63	.160	.164	.206	1.26	38.3	.79	0.76
	55	.083	70	.171	.177	.225	1.27	43.1	.32	1.29
	56	.083	70	.171	.177	.225	1.27	43.1	.31	1.26
	57	.167	70	.172	.177	.225	1.27	43.1	.37	0.85
	58	.167	71	.170	.169	.225	1.33	43.1	.34	0.77
	59	.250	65	.186	.193	.225	1.17	43.1	.60	1.00
	60	.333	64	.187	.187	.225	1.20	43.1	.67	0.83
	61	.333	69	.175	.181	.225	1.24	43.1	.57	0.69
	62	.333	71	.170	.169	.225	1.33	43.1	.34	0.41
	63	.417	64	.187	.188	.225	1.20	43.1	.80	0.81
	64	.500	65	.185	.193	.225	1.17	43.1	.77	0.67
	65	.500	70	.171	.169	.225	1.33	43.1	.63	0.51
	66	.500	65	.186	.193	.225	1.17	43.1	.62	0.54
Lyse & Godfrey [13]	67	.160	61	.360	.397	.559	1.41	49.0	.45	1.23
	68	.251	61	.360	.397	.559	1.41	49.0	.52	0.96
	69	.160	61	.360	.407	.559	1.37	49.0	.44	1.22

t_{w1} and t_{w2} = Thickness of web near mid-depth and near flange, respectively
* Average thickness of flange