Lateral Stiffness of Core/Outrigger Systems

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It is common to increase the lateral stiffness of braced cores by placing horizontal trusses or outriggers from the core to the exterior columns. Outriggers, in effect, apply moments which reduce the rotation of the core at outrigger locations. The smaller rotations at those points in turn reduce the overall sway of the structure.

How effective are such outriggers? What are the primary parameters which control their effectiveness? Where along the height are they most effective? For such systems, where should an increment of material be added to affect the largest increment in lateral stiffness? Structural engineers need quantitative answers to such questions to make design decisions.

Taranath,[1] McNabb and Muvdi[2,3] and others have analyzed such systems and have provided insights useful for design. McNabb and Muvdi[3] analyzed the case of multiple outriggers and concluded that the gain in stiffness decreases as more outriggers are added. Herein the case of outriggers at only one location is analyzed. The work presented generalizes the results of the references cited by explicitly considering flexible outriggers, nonprismatic elements and a triangularly distributed lateral load.

The system model analyzed is shown in Tables 1 and 2. The outriggers are prismatic with a moment of inertia, I_G; they can be located at any height, bH, from the base. The exterior columns are pin ended; their area increases *linearly* from the outrigger location to the base. The vertical core is nonprismatic, with a moment of inertia which increases *linearly* from the top to the base. Two load models are used; one is a uniform load, the other is a triangular load. The model is statically indeterminate to the second degree, however it is antisymmetric, therefore only one redundant needs to be determined.

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ANALYTICAL RESULTS

A flexibility approach was used to solve for the redundants and to obtain the horizontal deflection at the top. All results are given by Equations 1–8 in Tables 1 and 2. For a uniform load and nonprismatic elements Equations 1 and 2 give the force in the columns and the deflection at the top normalized by the deflection of the system without outriggers. Equations 1 and 2 yield an indeterminate form (i.e., 0/0) when the members become prismatic, therefore limits must be taken which result in Equations 3 and 4. Corresponding comments apply to Equations 5–8 for the case of a triangular lateral load.*

Equations 1–8 indicate the effectiveness of the outriggers is controlled by five parameters:

$I_0/H/I_G/D$ = Ratio of flexural stiffness of core to flexural stiffness of outrigger

I_0/A_0D^2 = Ratio of moment of inertia of core to an effective moment of inertia of the exterior columns (this parameter is also used in Ref. 2)

I_1/I_0 = Ratio of maximum moment of inertia of core to its minimum value

A_1/A_0 = Ratio of maximum area of column to its minimum value

BEHAVIOR AT EXTREME PARAMETER VALUES

It is useful to examine Equations 4 and 8 (i.e., the case of prismatic elements, $I_1/I_0 = A_1/A_0 = 1.0$) with $(I_0/H)/(I_G/D) = I_0/A_0D^2 = 0.0$. The latter extreme parameter values imply infinitely rigid outriggers and columns, which in turn imply that the point at which the outrigger is attached is fixed against rotation. For outriggers at the top, i.e.

* *To evaluate Equations 1, 2, 5, and 6 when* I_1/I_0 *and* A_1/A_0 *are smaller than 1.1, use double precision in computer calculations.*

Table 1. Column Forces and Deflection Ratios for a Uniform Lateral Load

For $I_1/I_0 > 1$ and $A_1/A_0 > 1$

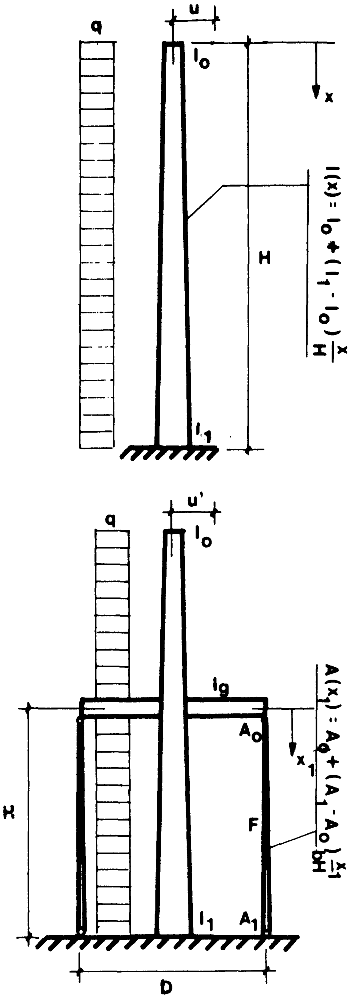


Fig. 1. Analytical model with a uniform lateral load

$$F = \pm \frac{qH^2}{4D} \cdot \frac{c^2(1-(1-b)^2) - 2cb - 2Ln\left(1 - \frac{bc}{c+1}\right)}{c^2\left[\frac{2c(I_0/A_0D^2)}{((A_0/A_0)-1)} Ln\frac{A_1}{A_0} + \frac{c(I_0/H)}{12(I_G/D)} - Ln\left(1-\frac{bc}{c+1}\right)\right]} \qquad (1)$$

$$\frac{u'}{u} = 1 - \frac{\left[3cb + 3Ln\left(1-\frac{bc}{c+1}\right)\right]\left[c^2(1-(1-b)^2) - 2cb - 2Ln\left(1-\frac{bc}{c+1}\right)\right]}{\left[\frac{2c(I_0/A_0D^2)}{((A_1/A_0)-1)} Ln(A_1/A_0) + \frac{c(I_0/H)}{12\,(I_G/D)} - Ln\left(1-\frac{bc}{c+1}\right)\right][2c^3 - 3c^2 + 6c - 6Ln(c+1)]} \qquad (2)$$

in which $c = \frac{I_1}{I_0} - 1$

For $I_1/I_0 = A_1/A_0 = 1.0$

$$F = \pm \frac{qH^2}{4D} \cdot \frac{1-(1-b)^3}{\frac{3b}{2} + \frac{(I_0/H)}{8(I_G/D)} + 3b(I_0/A_0D^2)} \qquad (3)$$

$$\frac{u'}{u} = 1 - \frac{(1-(1-b)^2)(1-(1-b)^3)}{\frac{3b}{2} + \frac{(I_0/H)}{8(I_G/D)} + 3b(I_0/A_0D^2)} \qquad (4)$$

$$I_1/I_0 = A_1/A_0 = 1.0$$
$$\frac{I_0/H}{I_G/D} = I_0/A_0D^2 = 0.0$$
$$b = 1$$

the deflection ratios become:

$$u'/u = 1/3 \qquad (4)$$
$$u'''/u'' = 7/22 \qquad (8)$$

The position b, which yields minimum deflection ratios is found by differentiation. Letting $b_1 = 1 - b$, then

$$\frac{d(u'/u)}{db_1} = 0 = 1 - 3b_1^2 - 4b_1^3 \text{ solving,}$$

$$b_1 \simeq 0.455 \text{ or } b = 0.545$$

$$\frac{d(u'''/u'')}{db_1} = 0 = 3 - 12b_1^2 - 12b_1^3 + 5b_1^4 \text{ solving,}$$

$$b_1 \simeq 0.430 \text{ or } b = 0.570$$

The optimum b_1 for the case of a uniform load was previously given in Refs. 1 and 2.

The corresponding deflection ratios for the optimum position of the outrigger are:

Table 2. Column Forces and Deflection Ratios for a Triangular Lateral Load

For $I_1/I_0 > 1$ and $A_1/A_0 > 1.0$

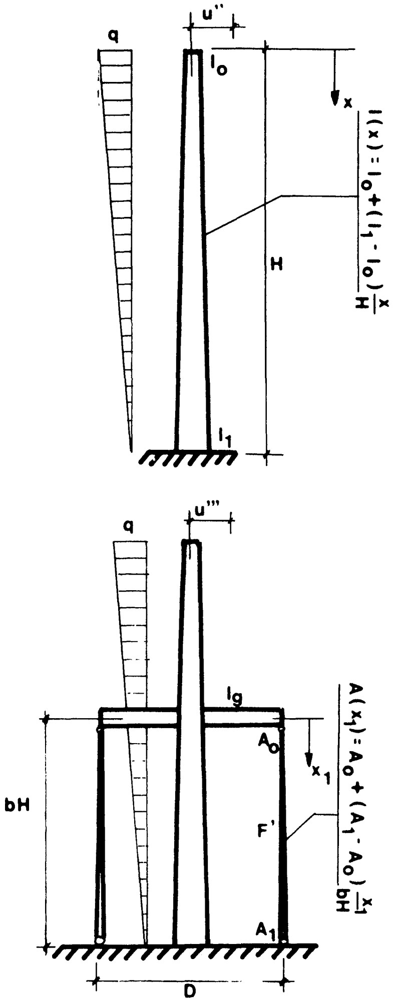


Fig. 2. Analytical model with a triangular lateral load

$$F' = \pm \frac{qH^2}{4D} \cdot \frac{\left[c^2(1-(1-b)^2) - 2cb - 2Ln\left(1-\frac{bc}{c+1}\right)\right]\left(c+\frac{1}{3}\right) - \frac{2c^3}{9}(1-(1-b)^3)}{c^3\left[\frac{2c(I_0/A_0D^2)}{((A_1/A_0)-1)}Ln(A_1/A_0) + \frac{c(I_0/H)}{12(I_G/D)} - Ln\left(1-\frac{bc}{c+1}\right)\right]} \qquad (5$$

$$\frac{u'''}{u''} = 1 - \frac{\left[3cb + 3Ln\left(1-\frac{bc}{c+1}\right)\right] \cdot \left[\left(c^2(1-(1-b)^2) - 2cb - 2Ln\left(1-\frac{bc}{c+1}\right)\right)\left(c+\frac{1}{3}\right) - \frac{2c^3}{9}(1-(1\cdots\right.}{\left[\frac{2c(I_0/A_0D^2)}{((A_1/A_0)-1)}Ln\,(A_1/A_0) + \frac{c(I_0/H)}{12(I_G/D)} - Ln\left(1-\frac{bc}{c+1}\right)\right]\left[(2c^3 - 3c^2 + 6c - 6Ln(c+1))\left(c+\cdots\right.\right.} \qquad (6$$

in which $c = \frac{I_1}{I_0} - 1$

For $I_1/I_0 = A_1/A_0 = 1.0$

$$F' = \pm \frac{qH^2}{4D} \cdot \frac{\frac{3}{4} - (1-b)^3 + \frac{(1-b)^4}{4}}{\frac{3}{2}b + \frac{(I_0/H)}{8(I_G/D)} + 3b(I_0/A_0D^2)} \qquad (7$$

$$\frac{u'''}{u''} = 1 - \frac{\frac{15}{11}(1-(1-b)^2)\left(\frac{3}{4} - (1-b)^3 + \frac{(1-b)^4}{4}\right)}{\frac{3}{2}b + \frac{(I_0/H)}{8(I_G/D)} + 3b(I_0/A_0D^2)} \qquad (8$$

$$u'/u = 0.121 \qquad (4)$$

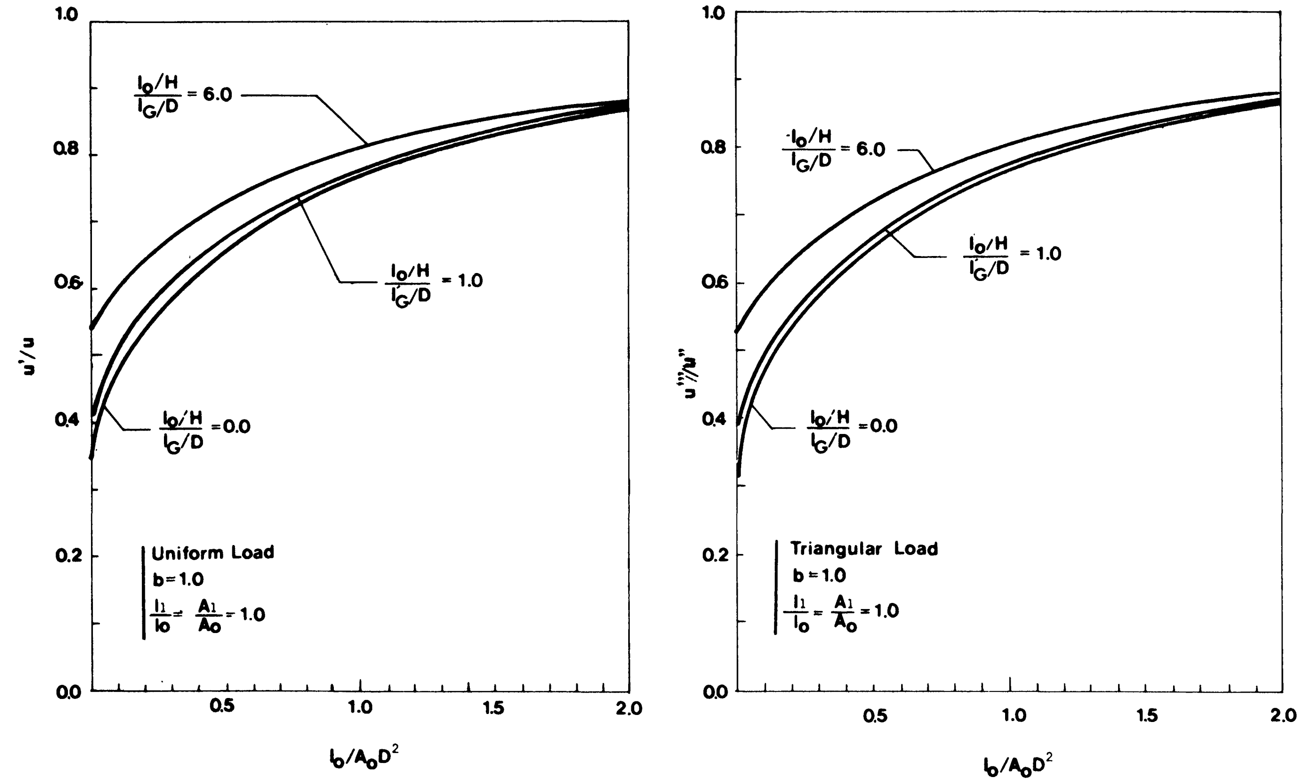
$$u'''/u'' = 0.117 \qquad (8)$$

For the parameter extremes considered, the best position for the outriggers and its effectiveness are not significantly different for the two load distributions.

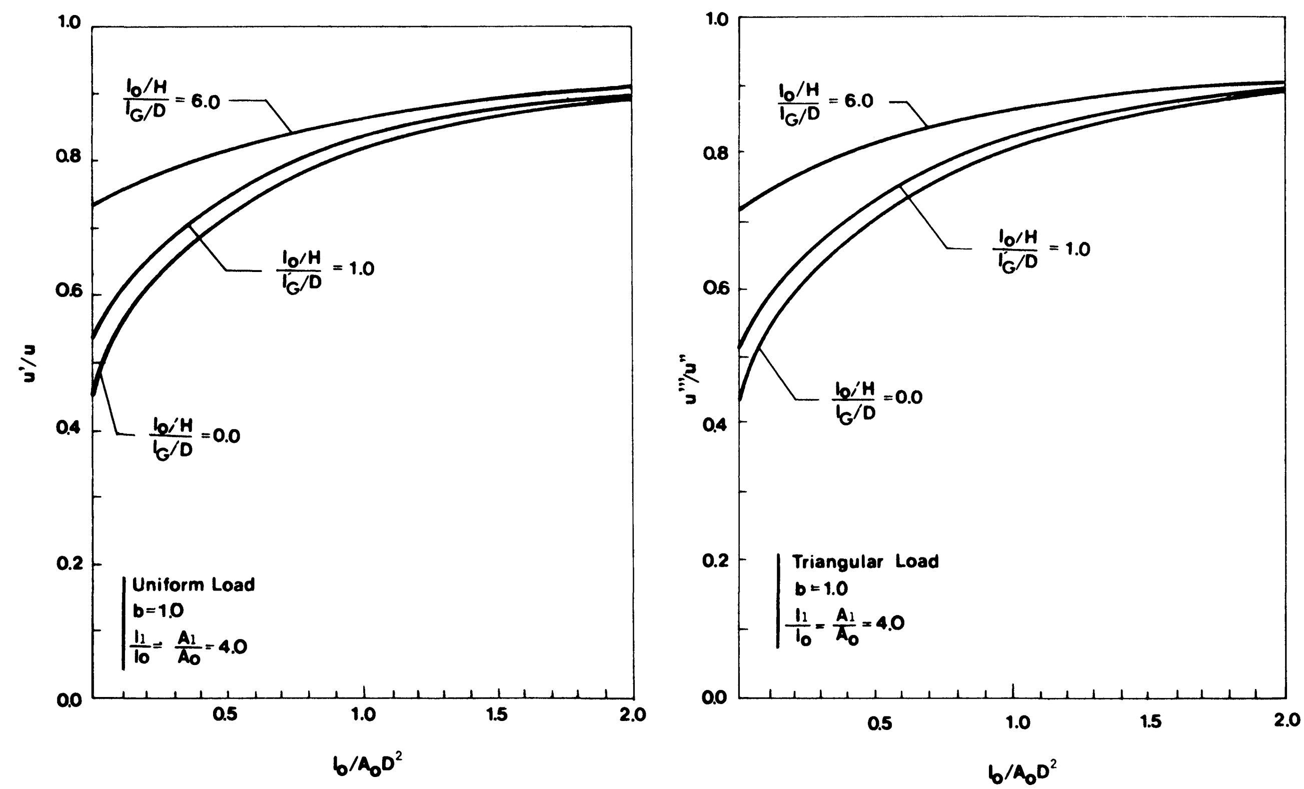
BEHAVIOR AT OTHER PARAMETER VALUES

The preceding discussion was for the limiting cases of prismatic elements and infinitely stiff columns and outriggers. However, the analytical solutions given in Tables 1 and 2 are correct for any values of the five controllin parameters. To illustrate behavior at other paramete values, Figs. 3–5 show displacement ratios as functions o I_0/A_0D^2 for a set of $(I_0/H)/(I_G/D)$ values. The fol lowing observations can be made:

1. Deflection ratios, and hence the effectiveness of th outriggers, are very close for the two loading condi tions.
2. Deflection ratios vary sharply when I_0/A_0D^2 i smaller than about 0.5 and $(I_0/H)/(I_G/D)$ is als small. This sensitivity is greater for $b = 0.6$ and 0.4

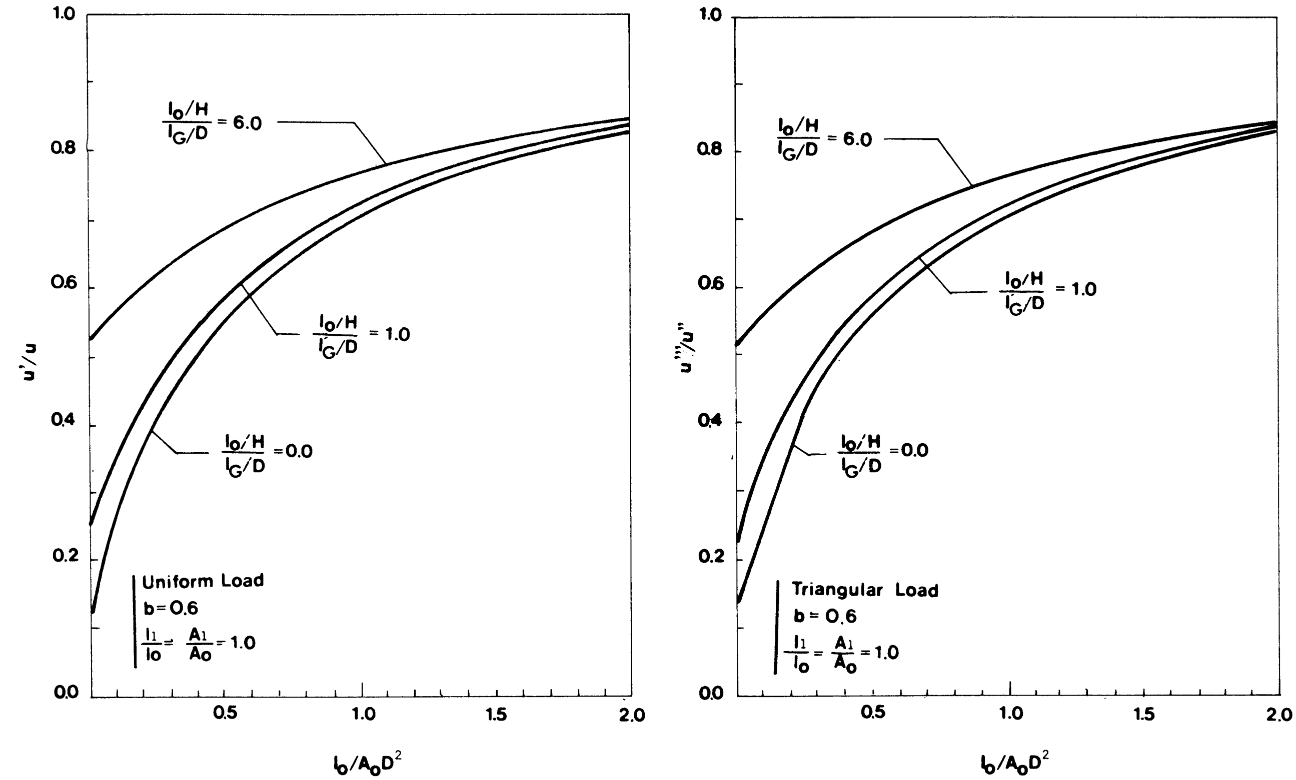


(*a*)



(*b*)

Fig. 3. Deflection ratios for b = *1.0 (outrigger at top) (a) Prismatic elements (b) Nonprismatic elements*



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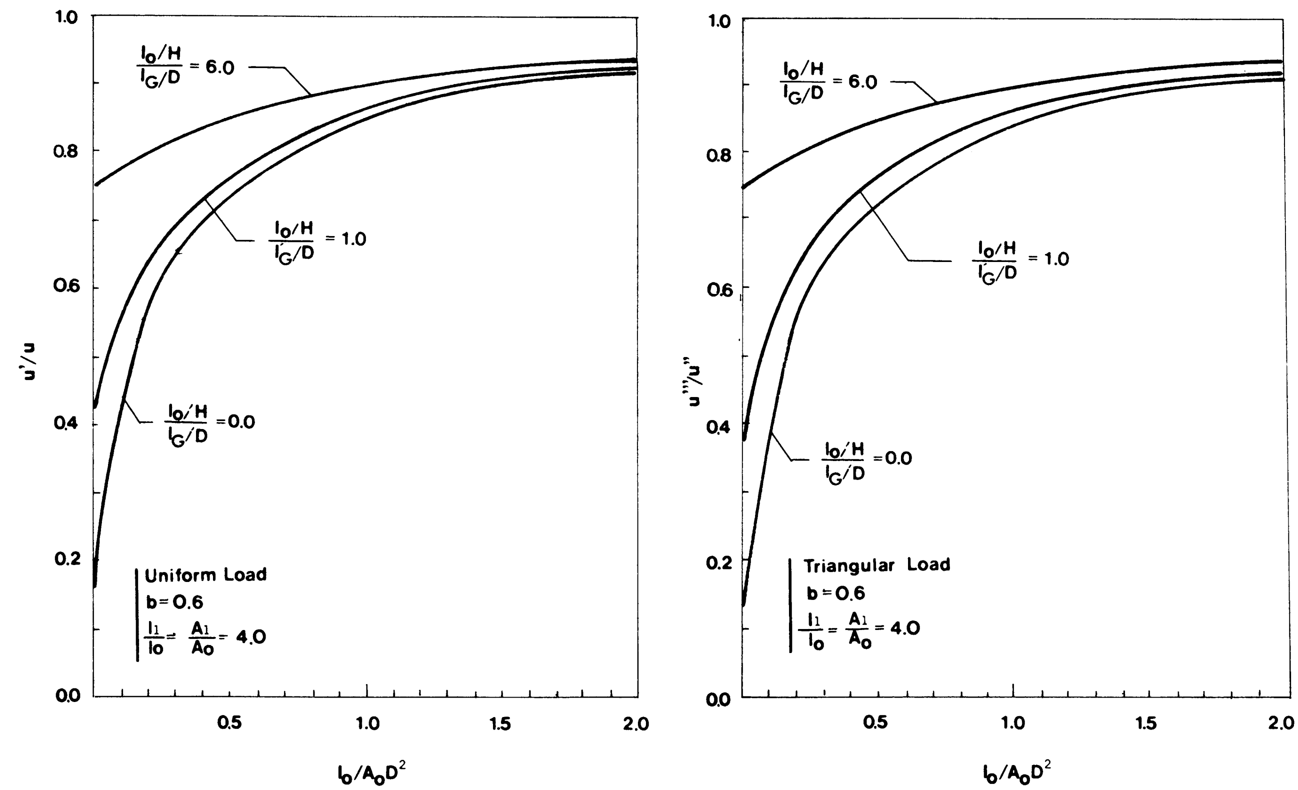
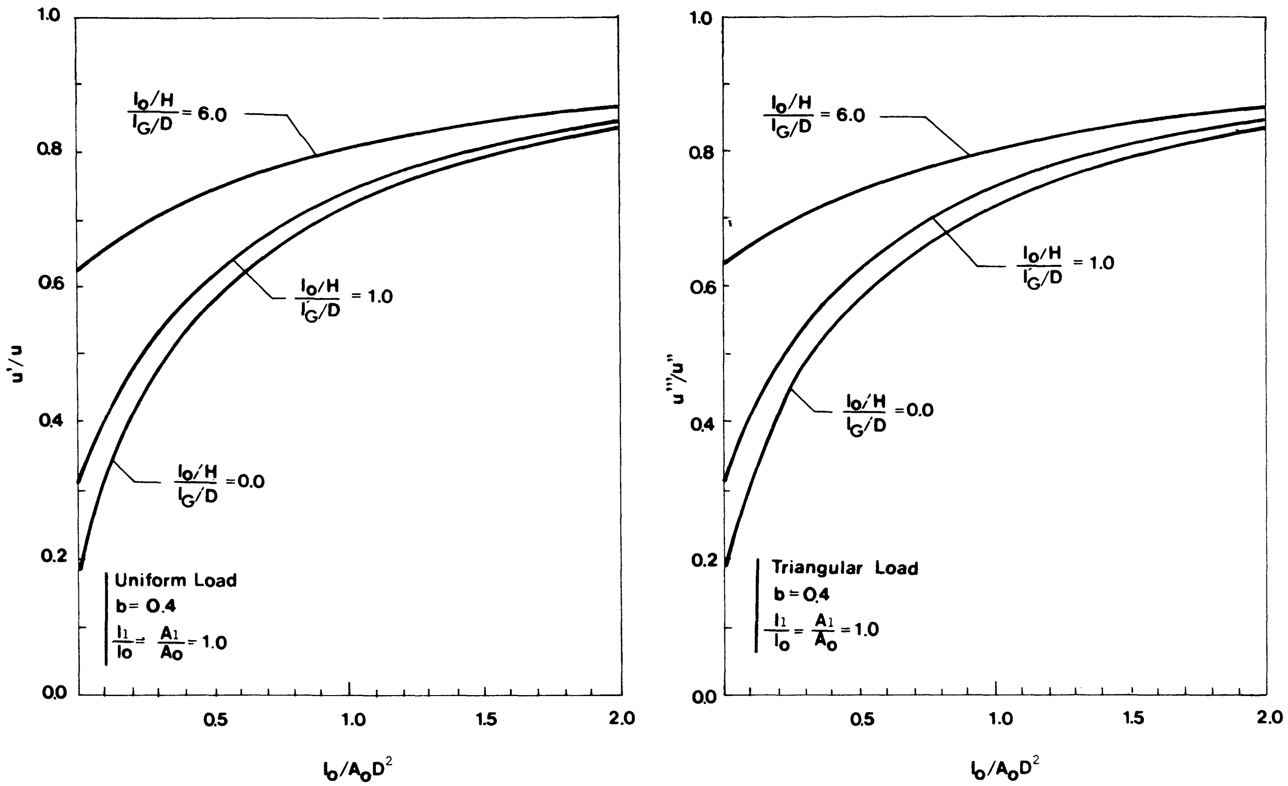
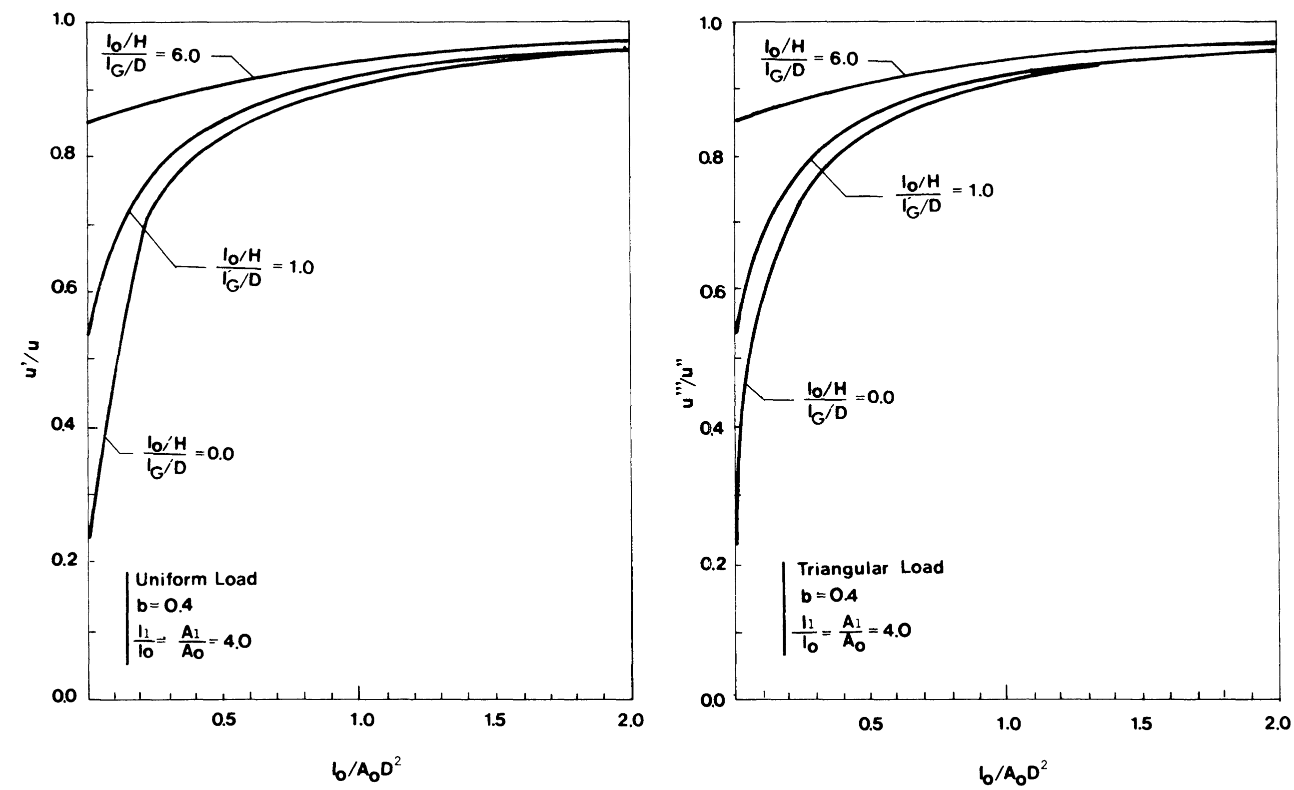


(*b*)

Fig. 4. Deflection ratios for b = *0.6* (*a*) *Prismatic elements* (*b*) *Nonprismatic elements*



(a)



(b)

Fig. 5. Deflection ratios for b = *0.4 (a) Prismatic elements (b) Nonprismatic elements*

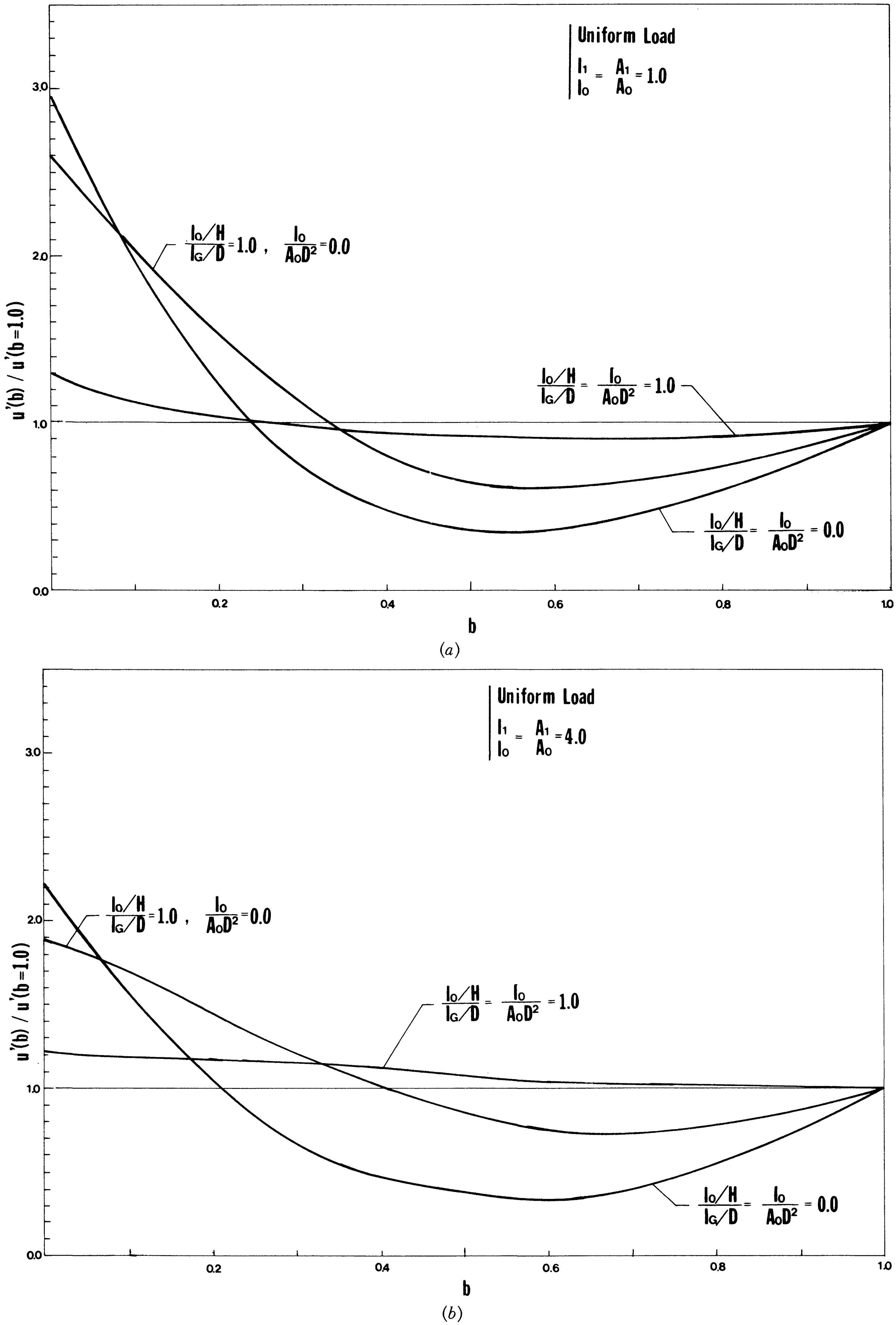
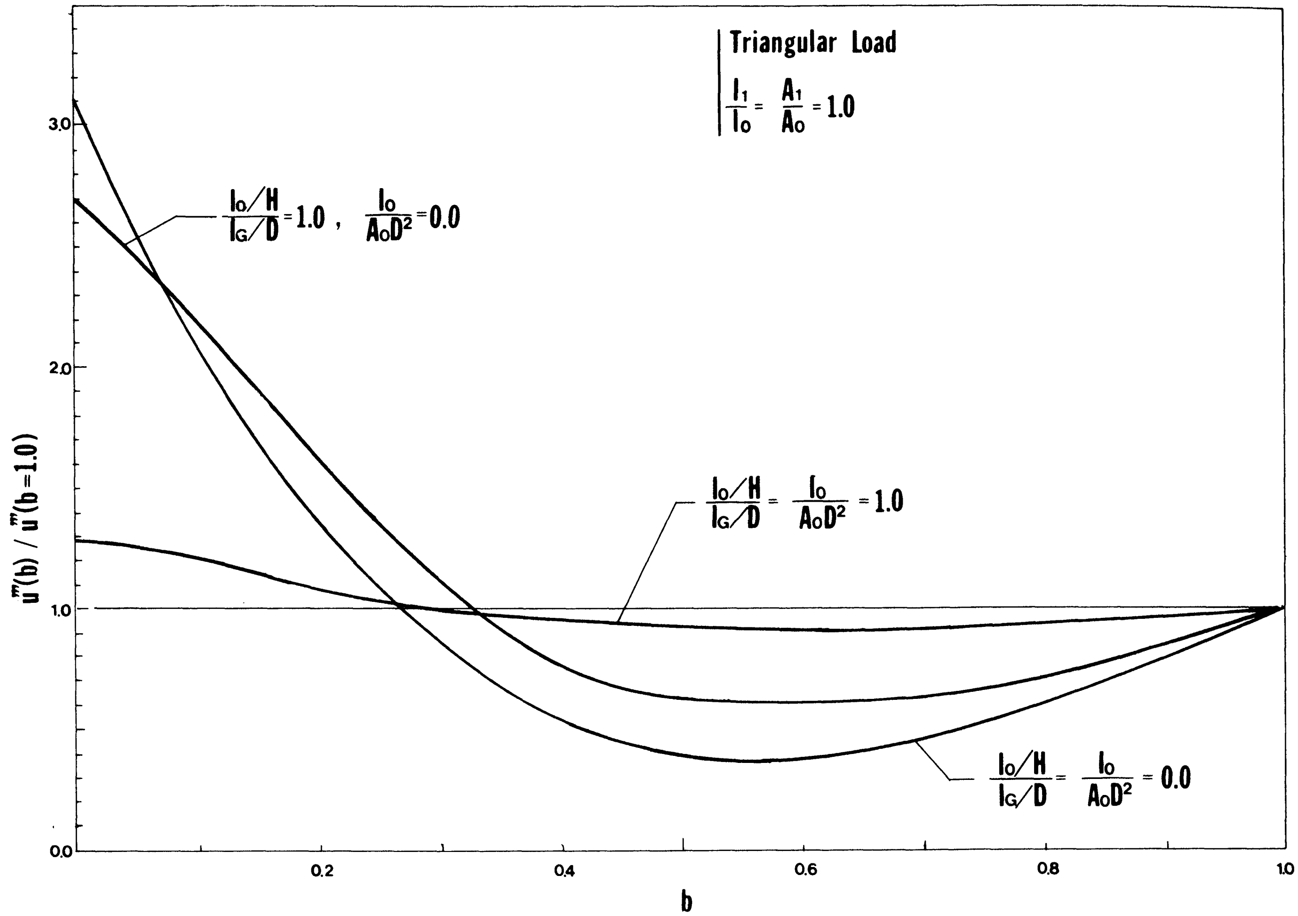
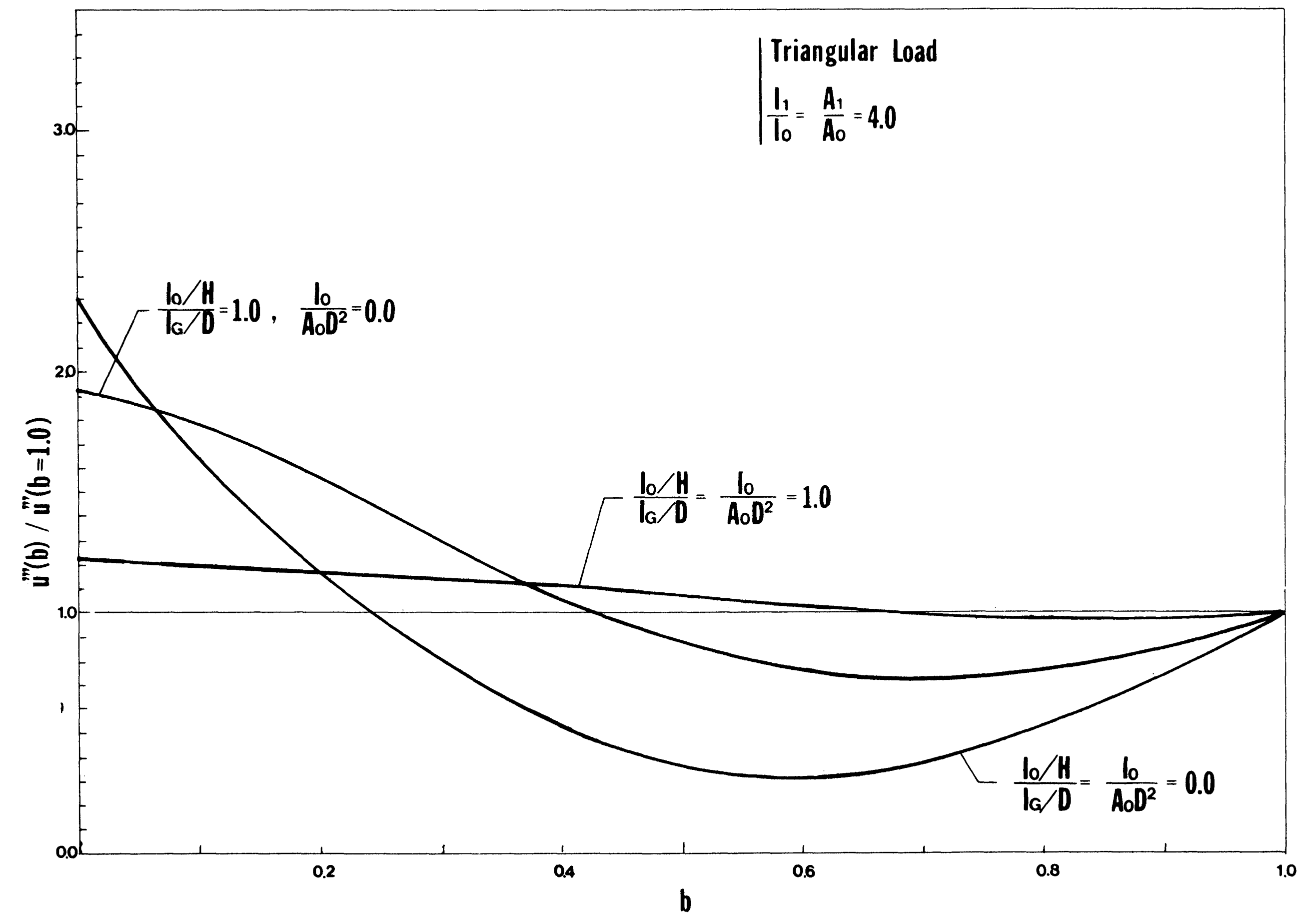


Fig. 6. Effectiveness of alternate positions of outrigger—uniform load (a) Prismatic elements (b) Nonprismatic elements



(*a*)



(*b*)

Fig. 7. Effectiveness of alternate positions of outrigger—triangular load (*a*) *Prismatic elements* (*b*) *Nonprismatic elements*

i.e., at positions where outriggers are more effective. Of course, as the outriggers become more flexible (i.e., as the ratio $(I_0/H)/(I_G/D)$ increases), they become ineffective irrespective of the stiffness of the columns.

3. Higher deflection ratios are obtained for the cases of nonprismatic cores simply because the stiffer cores carry more of the lateral load.
4. Deflection ratios are smallest for $b \simeq 0.55$. This is consistent with the optimum positions derived for the extreme parameter values.

Figure 6a, b and 7a, b also show the relative effectiveness of alternate outrigger positions along the height. Note that the horizontal displacements in these figures are normalized by the horizontal displacement from the outrigger at the top, i.e., $b = 1.0$. The ordinate at $b = 0$ is simply

$$\frac{u'(b=0)}{u'(b=1)} = \frac{u}{u'(b=1)} = \frac{1}{u'(b=1)/u}.$$

Figures 6 and 7 show:

1. The relative effectiveness of alternate outrigger positions is essentially the same for both uniform and triangular loads.
2. The optimum position shifts toward the top for the cases of nonprismatic cores and columns.

SUMMARY

The analytical results presented in Tables 1 and 2 for the core/outrigger model shown in Fig. 1 allow rapid calculation of the effectiveness of preliminary designs. The results show the main parameters which control the behavior of such systems. Effects of the flexibility of the outriggers and of nonprismatic elements are explicitly included. The results can be useful to designers of high-rise structures.

NOMENCLATURE

I_0 = Smallest moment of inertia of vertical core
I_1 = Largest moment of inertia of vertical core
H = Height of structure
D = Width of structure
I_G = Moment of inertia of outrigger
A_0 = Smallest area of exterior column
A_1 = Largest area of exterior column
q = Magnitude of distributed lateral load
u = Top deflection of core without outrigger foı uniformly distributed lateral load
u^1 = Top deflection of core with outrigger for a uı formly distributed lateral load
u^{11} = Top deflection of core without outrigger foı triangularly distributed lateral load
u^{111} = Top deflection of core with outrigger for a t angularly distributed lateral load
bH = Position of outrigger along the height of ı structure
F = Axial force in exterior column for a uniforn distributed lateral load
F^1 = Axial force in exterior column for a triangula distributed lateral load
b = Position of outrigger along height

REFERENCES

1. *Taranath, B. S.* Optimum Belt Truss Locations for Hi Rise Structures *AISC Engineering Journal, Vol. 11, 1, First Quarter, 1974.*
2. *McNabb, J. W. and B. B. Muvdi* Drift Reduction Fact for Belted High-Rise Structures *AISC Engineering Jourr Vol. 12, No. 3, Third Quarter, 1975.*
3. *McNabb, J. W. and B. B. Muvdi* Discussion of Drift I duction Factors for Belted High-Rise Structures *AI Engineering Journal, Vol. 14, No. 1, First Quarter 1977*