Simplifications in the Solution of Column Interaction Problems

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THIS PAPER outlines time saving procedures that may be employed by structural engineers to design columns subjected to combined axial and bending stress. Two procedures are described, each of which involves pre-calculated coefficients for insertion in the present column tables of the AISC Manual. A discussion of these tables and their use has already been published.[1]

Method A provides a short-cut procedure for selecting a column shape from the Manual tables with a reasonable degree of accuracy. It is a rapid method for preliminary design, sufficiently accurate for many commonly encountered design problems.

Method B is a modification of the existing interaction Equations (6), (7a) and (7b) which appear on page 3–10 of the AISC Manual, which are derived directly from formulas in the AISC Specification.

METHOD A

This method utilizes three factors which may be calculated for each group of columns having similar properties. These factors, in their respective order, are equal to the expressions

$$\frac{12\,F_a B_x}{F_{bx}}, \quad \frac{12\,F_a B_y}{F_{by}} \quad \text{and} \quad \frac{F_a}{0.6\,F_y}$$

Because the bending factors (B_x and B_y) and the radius of gyration (r_y) are nearly constant for any group of compact or non-compact column shapes of the same nominal size, there is little error introduced in applying these factors to such groups. For these groups of shapes, F_a is nearly constant over a range of effective lengths of 5 ft. Therefore, when these factors are substituted in interaction Equations (6), (7a) and (7b), simplified expressions result, which are applicable within each group over a 5 ft range of effective length. These give the designer a close approximation of the equivalent axial load for selection of a proper column size.

Figure 1 shows a typical page of column load tables in the Manual, with the above factors inserted at 5 ft increments of length. The heavy line added to the table indicates the effective length L_c above which the allowable bending stress F_b may be taken at $0.66\,F_y$. For all lengths below this line, F_b shall not exceed $0.60\,F_y$. Likewise, the dashed line indicates when the effective length L_u has been reached. Below this line F_b may be determined by Formulas (4) and (5) of the AISC Specification. However, in column design this last case seldom becomes a problem.

Direct interpolation to determine factors for any column length is possible, provided the L_c or L_u line (see Fig. 1) does not separate the factors under consideration. Such refinement is not warranted for preliminary design purposes.

Figure 2 shows an alternate method of listing the factors on an insert sheet.

Example A illustrates how these factors are employed to determine a proper column shape subject to combined axial and bending stress.

EXAMPLE A

Assume the same design conditions as shown in Example 6, pages 3–10 and 3–11 of the AISC Manual:

Given:
$P = 600$ kips
$M_x = 190$ kip-ft
$KL = 18$ ft
$M_y = 0$
Use A36 steel
Sidesway assumed uninhibited
$\therefore C_{mx} = 0.85$ (Spec. Sect. 1.6.1)

Solution: Neglecting wind:
From Table I select 14 WF 119, good for 618 kips > 600 kips o.k.

Including wind:
$P = 600 \times 0.75 = 450$ kips
$M_x = 190 \times 0.75 = 142.5$ kips

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1. New Manual Makes Steel Column Design Easy, *Engineering News-Record, September 5, 1963.*

ASTM A 36
F_y = 36 ksi

COLUMNS
WF shapes

14 I

TABLE I
Allowable concentric loads in kips

Nominal Depth and Width		14 × 16			14 × 14½						
Weight per Foot		**158**	**150**	**142**	**136**	**127**	**119**	**111**	**†103**	**†95**	**†87**
Effective lengths in feet KL with respect to least radius of gyration	6	963	913	867	826	771	722	674	625	577	527
	7	954	905	859	818	764	716	667	618	571	522
	8	946	897	851	810	756	708	661	612	565	517
	9	936	888	843	801	748	701	653	605	559	511
	10	927	879	834	792	739	693	646	599	552	505
		(1.85 •	4.92 •	0.91)	(1.82 •	5.16 •	0.90)		(2.00 •	5.76 •	0.90)
	11	917	869	825	783	731	684	638	591	546	499
	12	906	859	815	773	721	676	630	584	539	493
	13	895	849	805	763	712	667	622	576	532	486
	14	884	838	795	752	702	658	613	568	524	479
	15	873	827	785	742	692	648	604	560	516	472
		(1.73 •	4.56 •	0.85)	(1.71 •	4.80 •	0.84)		(1.87 •	5.28 •	0.84)
	16	861	816	774	730	682	638	595	551	508	465
	17	849	804	763	719	671	628	585	542	500	457
	18	836	793	751	707	660	618	576	533	492	449
	19	823	780	740	695	648	607	566	524	483	441
	20	810	768	728	683	637	596	555	514	474	433
		(1.75 •	4.68 •	0.79)	(1.72 •	4.93 •	0.78)		(1.73 •	4.93 •	0.78)
	21	796	755	715	670	625	585	545	504	465	425
	22	783	742	703	657	613	573	534	494	456	416
	23	768	728	690	643	600	562	523	484	446	408
	24	754	714	676	630	587	550	511	473	436	399
	25	739	700	663	616	574	537	500	462	426	389
		(1.60 •	4.20 •	0.72)	(1.54 •	4.33 •	0.70)		(1.55 •	4.45 •	0.70)
	26	724	686	649	602	561	525	488	451	416	380
	27	708	671	635	587	547	512	476	440	406	370
	28	693	656	621	572	533	499	463	429	395	361
	29	677	641	606	557	519	485	451	417	384	351
	30	660	625	591	541	504	472	438	405	373	340
		(1.43 •	3.72 •	0.65)	(1.37 •	3.84 •	0.62)		(1.36 •	3.84 •	0.61)
	32	626	593	560	509	474	443	411	380	350	319
	34	591	559	528	476	443	414	384	355	326	297
	36	555	525	495	442	411	383	355	328	301	275
	38	517	489	461	406	377	352	325	300	276	251
	40	478	451	425	368	342	319	294	271	249	227
		(1.02 •	2.76 •	0.47)	(0.91 •	2.64 •	0.41)				

Properties

	158	150	142	136	127	119	111	†103	†95	†87
Area A (in.^{2})	46.47	44.08	41.85	39.98	37.33	34.99	32.65	30.26	27.94	25.56
Ratio r_x/r_y	1.60	1.60	1.59	1.67	1.67	1.67	1.67	1.67	1.66	1.66
r_y (in.)	4.00	3.99	3.97	3.77	3.76	3.75	3.73	3.72	3.71	3.70
L_c (ft.)	16.8	16.8	16.8	16.0	15.9	15.9	15.8	...	...	...
L_u (ft.)	56.0	53.5	50.8	48.3	45.6	42.9	40.2	37.9	35.0	32.5
B_x Bending factors	.183	.184	.185	.185	.185	.185	.185	.185	.186	.185
B_y Bending factors	.485	.487	.491	.519	.520	.521	.525	.525	.529	.530
a_x Multiply values by 10^6	283.6	266.5	249.1	237.2	220.1	204.3	188.8	173.9	158.5	144.0
a_y Multiply values by 10^6	110.8	104.6	98.3	84.7	78.6	73.3	67.7	62.4	57.3	52.1

Loads below heavy line are for main members with Kl/r ratios between 120 and 200.
† Non-compact section; see discussion under Allowable Loads on Columns, General Notes.

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Fig. 1. Factors inserted in parentheses are

$$\left(\frac{12\,Fa}{F_{bx}} \cdot \frac{12\,F_aB_y}{F_{by}} \cdot \frac{Fa}{0.6\,F_Y}\right) \textit{respectively}$$

	14 × 16			14 × 14½					
	158	150	142	136 127 119 111			103	95	87
	$\frac{12 F_a B_x}{F_b}$	$\frac{12 F_a B_y}{F_b}$	$\frac{F_a}{22}$	$\frac{12 F_a B_x}{F_b}$	$\frac{12 F_a B_y}{F_b}$	$\frac{F_a}{22}$	$\frac{12 F_a B_x}{F_b}$	$\frac{12 F_a B_y}{F_b}$	$\frac{F_a}{22}$
10	1.85	4.92	0.91	1.82	5.16	0.90	2.00	5.76	0.90
15	1.73	4.56	0.85	1.71	4.80	0.84	1.87	5.28	0.84
20	1.75	4.68	0.79	1.72	4.93	0.78	1.73	4.93	0.78
25	1.60	4.20	0.72	1.54	4.33	0.70	1.55	4.45	0.70
30	1.43	3.72	0.65	1.37	3.84	0.62	1.36	3.84	0.61
40	1.02	2.76	0.47	0.91	2.64	0.41			

Fig. 2. Sample arrangement of factors for insert sheet

Try: 14 WF 119 with $KL = 18$ ft
Select factors as shown in Fig. 1 or Fig. 2 under column group which includes 14 WF 119.

$$\frac{12\,F_a\,B_x}{F_b} \simeq 1.72\;;\;\frac{F_a}{22} \simeq 0.78$$

Check: By Equations (7a) and (7b) respectively,

$$\frac{a_x}{a_x - P(Kl)^2} = \frac{204.3}{204.3 - 21.0} = 1.11$$

$$P + P' = 450 + (1.72 \times 142.5 \times 0.85 \times 1.11) \simeq 681 \text{ kips}$$

By Equation (7b),

$$P + P' = (0.78 \times 450) + (1.72 \times 142.5) \simeq 596 \text{ kips}$$

Equation (7a) governs; therefore, enter column Table I, find 14 WF 136 with allowable axial load good for 707 kips.

Use: 14 WF 136

The method outlined will lead the designer to a very accurate approximation of the desired column section; however, for final design purposes the column selected should be checked by a more accurate method.

METHOD B

The design methods using the interaction equations given on page 3–10 of the AISC Manual provide an accurate check. Although these methods, using Equations (6), (7a) and (7b) present a simplified, direct and logical approach to a column interaction problem, they can be further simplified.

In using these interaction equations, it is necessary to consult Tables 1 in the Specification Appendix in order to determine F_a. The necessity of referring to these tables can be eliminated by adding to the column property section new bending factors which are designated m_x and m_y.

These new bending factors can be calculated for each column shape and inserted at the bottom of the column load tables. The derivation of m_x and m_y follows.

If bending occurs about the X–X axis, the maximum allowable bending moment for a given shape may be expressed as

$$M_{cx} \text{ (in kip-in.)} = F_{bx}\,S_x$$

If the section is compact, $F_{bx} = 24$ ksi, for A36 steel. In this case L_c must equal or exceed the effective length (KL).

$$M_{cx} \text{ (in kip-ft)} = \frac{F_{bx}\,S_x}{12}$$

The factor m_x is defined as the reciprocal of $M_{cx} = 1/M_{cx} = 12/(F_{bx}\,S_x)$.

If P_a, in kips, represents the tabulated value given in the column load tables for a particular column with a given effective length (KL), in feet, F_a then equals P_a/A.

The ratio F_a/F_{bx} may be expressed as

$$(P_a S_x)/(A M_{cx}).$$

The bending factor B_x, with respect to the X–X axis, is equal to A/S_x.

With respect to the major axis, Equation (6), as it appears in the Manual, is written as

$$P + P' = P + [B_x M_x\;(F_a/F_{bx})]$$

For a particular shape, each of the terms in this equation, except F_a, are either calculated or may be found by reference to a single page in the column load tables. The allowable axial stress (F_a) must be selected from Tables 1 of the Specification Appendix for a calculated slenderness ratio. In order to eliminate this unnecessary step the foregoing terms may be inserted into

Equation (6); the equation may then be expressed as:

$$P + P' = P + \left[\frac{A}{S_x} M_x \times \frac{P_a S_x}{A\, M_{cx}}\right]$$

$$= P + \left[\frac{M_x}{M_{cx}}\, P_a\right]$$

where M_x and M_{cx} are expressed in kip-in.

$$P + P' = P + (m_x M_x P_a)$$

where M_x is expressed in kip-ft

Note that if the effective length exceeds L_c but is less than L_u, the bending component of the modified equation must be multiplied by the ratio 24/22 for A36 steel.

By inserting into the column load tables the m_x and m_y bending components for each section listed, further reference to other portions of the Manual is not necessary.

Equation (7a), with respect to bending about the major axis, is presently expressed as:

$$P + P' = P + \left[B_x M_x C_{mx} \left(\frac{F_a}{F_{bx}}\right)\left(\frac{a_x}{a_x - P(Kl)^2}\right)\right]$$

where M_x is expressed in kip-in.

By substitution, Equation (7a) may be modified:

$$P + P' = P + \left[m_x\, M_x\, P_a\, C_{mx} \left(\frac{a_x}{a_x - P(Kl)^2}\right)\right]$$

where M_x is expressed in kip-ft.

Also, Equation (7b) which now reads:

$$P + P' = P\left(\frac{F_a}{0.6\, F_y}\right) + \left[B_x\, M_x \left(\frac{F_a}{F_{bx}}\right)\right]$$

may be expressed for A36 steel as:

$$P + P' = \frac{P_a P}{22\text{A}} + P_a\, m_x\, M_x$$

$$P + P' = P_a\left[\frac{P}{22\text{A}} + m_x\, M_x\right]$$

Note that all terms in the foregoing modified equations are directly calculated or are presently given in the column load tables except the m_x value. This factor, with respect to both the X − X and Y − Y axis, can easily be determined for each column shape listed and inserted at the bottom of each respective page of the Manual.

Example B illustrates how the modified equations may be employed in solving a column interaction problem.

EXAMPLE B

Given: Assume the same design conditions as outlined in Example A. Select a trial section which satisfies axial loads only:

Solution:

Try: 14 WF 119; $KL = 18$ ft (see Example A)

From Table I:

$P_a = 618$ kips; $A = 34.99$ in.^{2}; $L_c = 15.9$ ft;

$a_x = 204.3$; $^*m_x = 0.00264$

$L_c < KL = 15.9 \text{ ft} < 18 \text{ ft}$

$\therefore\ F_b = 22$ ksi and $m_x = 24/22 \times 0.00264 = 0.00288$

Check: By Modified Equation (7a),

$$\frac{204.3}{204.3 - 21.0} = 1.11$$

$$P + P' = 450 + (0.00288 \times 142.5 \times 618 \times 0.85 \times 1.11) = 690 \text{ kips}$$

By Modified Equation (7b),

$$P + P' = 618\left[\frac{450}{22 \times 34.99} + (0.00288 \times 142.5)\right] = 618 \times 0.996 = 616 \text{ kips}$$

Equation (7a) governs; enter column Table I and select 14 WF 136 with an allowable axial load of 707 kips.

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* *To be precalculated and inserted in the tables for each shape. Solve* m_x *for* 14 WF 119 (*A36 steel, compact section*):

$M_{cx} = 24\, S_x = 24 \times 189.4 = 4550$ *kip-in.;*
$m_x = 12/4550 = 0.00264$