Analysis of Knee-Braced Portal Frames for Vertical Loading

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Knee-braced portal frames such as those shown in Figs. 1 and 2 are encountered from time to time in the design of buildings. The analysis of these bents for the vertical loading is easily done if a computer and necessary software are accessible to form and solve the stiffness matrix for the frame. If they are not accessible, then one is in a quandary, because the simultaneous deflections in the beam and the columns have to be accounted for in the solution.

This paper looks into the longhand solution of such portal frames. Equations are developed and solutions given which are easy to apply. What is needed are the unit load and beam load deflections at the knee-brace locations. And, for given frame dimensions, these are calculated only once. As the member sizes are altered, one need only to change the matrix coefficients, which can be accomplished in little time. With two or three trials, economical selection of the members can be made.

In the following paragraphs, the equations for bent in Fig. 1 will be developed, followed by an example. In the end, the equations for bent in Fig. 2 will be given.

Axial deformations in beam and columns are neglected. Referring to Fig. 3, for small angle changes the vertical components of the knee braces are $P_1 \sin \theta_1$ and $P_2 \sin \theta_2$ while horizontal components are $P_1 \cos \theta_1$ and $P_2 \cos \theta_2$. Consider the loading on the beam as shown in Fig. 4.

Net downward deflection at $C = d_C = d_{C1} - d_{C2}$

$$d_C = d_{C1} - (P_1 \sin \theta_1 d_{cc} + P_2 \sin \theta_2 d_{cd}) \quad (1)$$

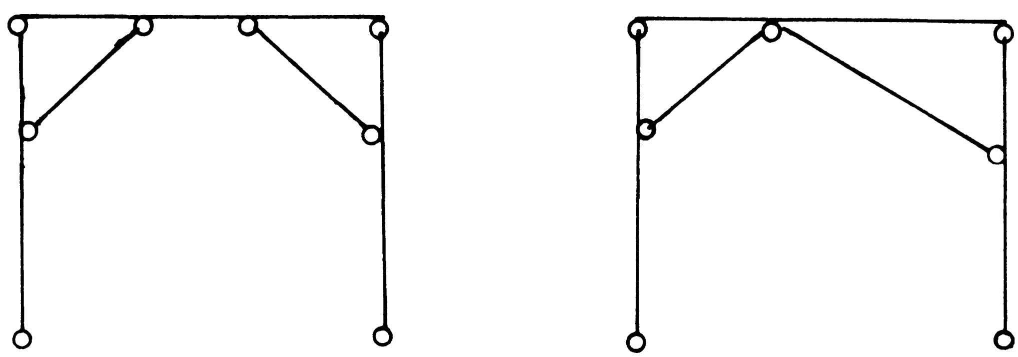
Similarly, net downward deflection at D =

$$d_D = d_{D1} - (P_1 \sin \theta_1 d_{cd} + P_2 \sin \theta_2 d_{dd}) \quad (2)$$

Horizontal deflection d_G in column $AB = P_1 \cos \theta_1 d_{gg}$ (3)

The deflection d_H in column $EF = P_2 \cos \theta_2 d_{hh}$ (4)

Referring now to Fig. 5, the beam and column deflections at the ends of the knee brace can be written in terms of brace shortening. Shortening in brace $GC = P_1 d_{\text{brace } 1}$



(1) (2)

Figs. 1 and 2. Knee-braced portal frames commonly encountered

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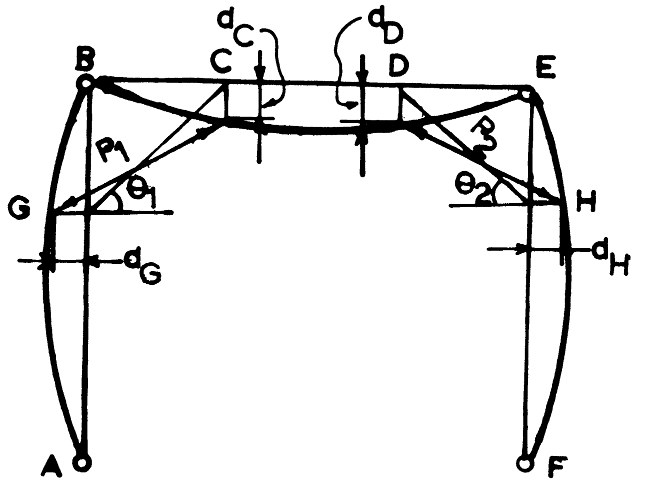

Fig. 3. Deflected configuration of portal under beam vertical loading

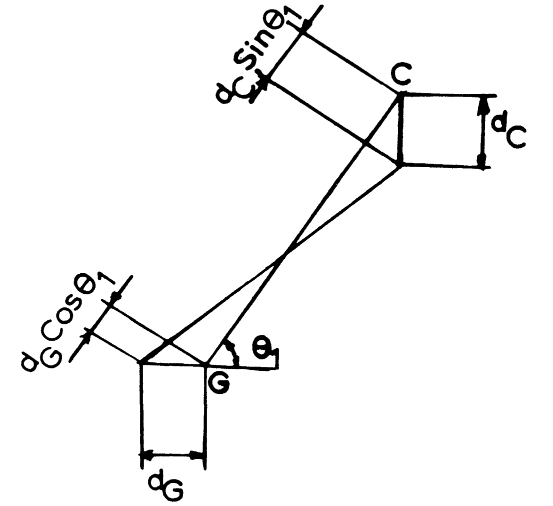

Fig. 5. Brace shortening in terms of beam and column deflection

$= d_C \sin\theta_1 - d_G \cos\theta_1$. Substituting the value of d_C and d_G from Eqs. 1 and 3, we get: $P_1 d_{\text{brace }1} = (d_{C1} - P_1 \times \sin\theta_1 d_{cc} - P_2 \sin\theta_2 d_{cd}) \sin\theta_1 - P_1 \cos\theta_1 d_{gg} \cos\theta_1$.

Rearranging,

$$P_1(\sin^2\theta_1 d_{cc} + \cos^2\theta_1 d_{gg} + d_{\text{brace }1}) + P_2 \sin\theta_1 \sin\theta_2\, d_{cd} = d_{C1} \sin\theta_1 \quad (5)$$

Similarly, for brace DH,

$$P_1 \sin\theta_1 \sin\theta_2 d_{cd} + P_2(\sin^2\theta_2 d_{dd} + \cos^2\theta_2 d_{hh} + d_{\text{brace }2}) = d_{D1} \sin\theta_2 \quad (6)$$

The two equations, 5 and 6, when arranged in matrix form and solved give:

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} (\sin^2\theta_1 d_{cc} + \cos^2\theta_1 d_{gg} + d_{brc\ 1}) & (\sin\theta_1 \sin\theta_2 d_{cd}) \\ (\sin\theta_1 \sin\theta_2 d_{cd}) & (\sin^2\theta_2 d_{dd} + \cos^2\theta_2 d_{hh} + d_{brc\ 2}) \end{bmatrix}^{-1} \times \begin{bmatrix} d_{C1} \sin\theta_1 \\ d_{D1} \sin\theta_2 \end{bmatrix} \quad (7$$

The solution above is analogous to the initial momer distribution for a pin-based portal with unsymmetrica beam loading needing a sway correction. Similarly, a swa correction is applied to the above solution if required. Th method is worked out in the following example.

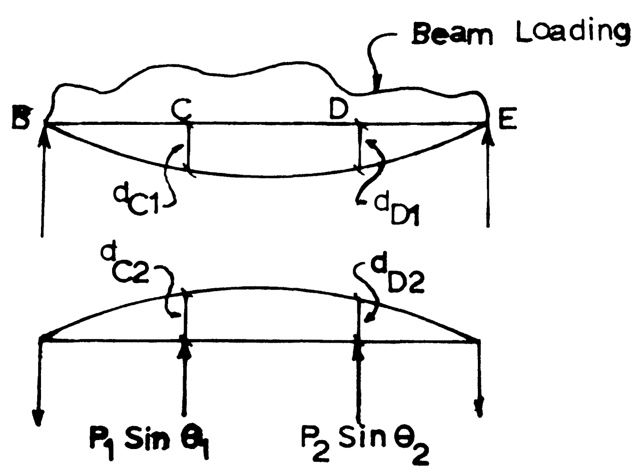

Fig. 4. Loads and deflections in the portal beam

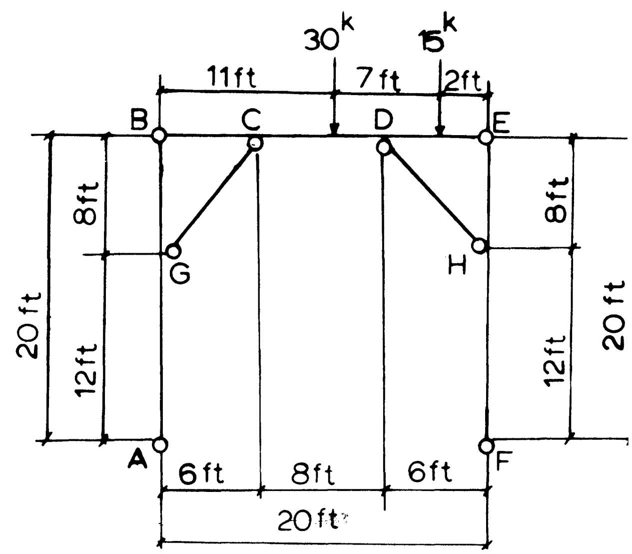

Fig. 6. Example: portal frame dimensions and loading

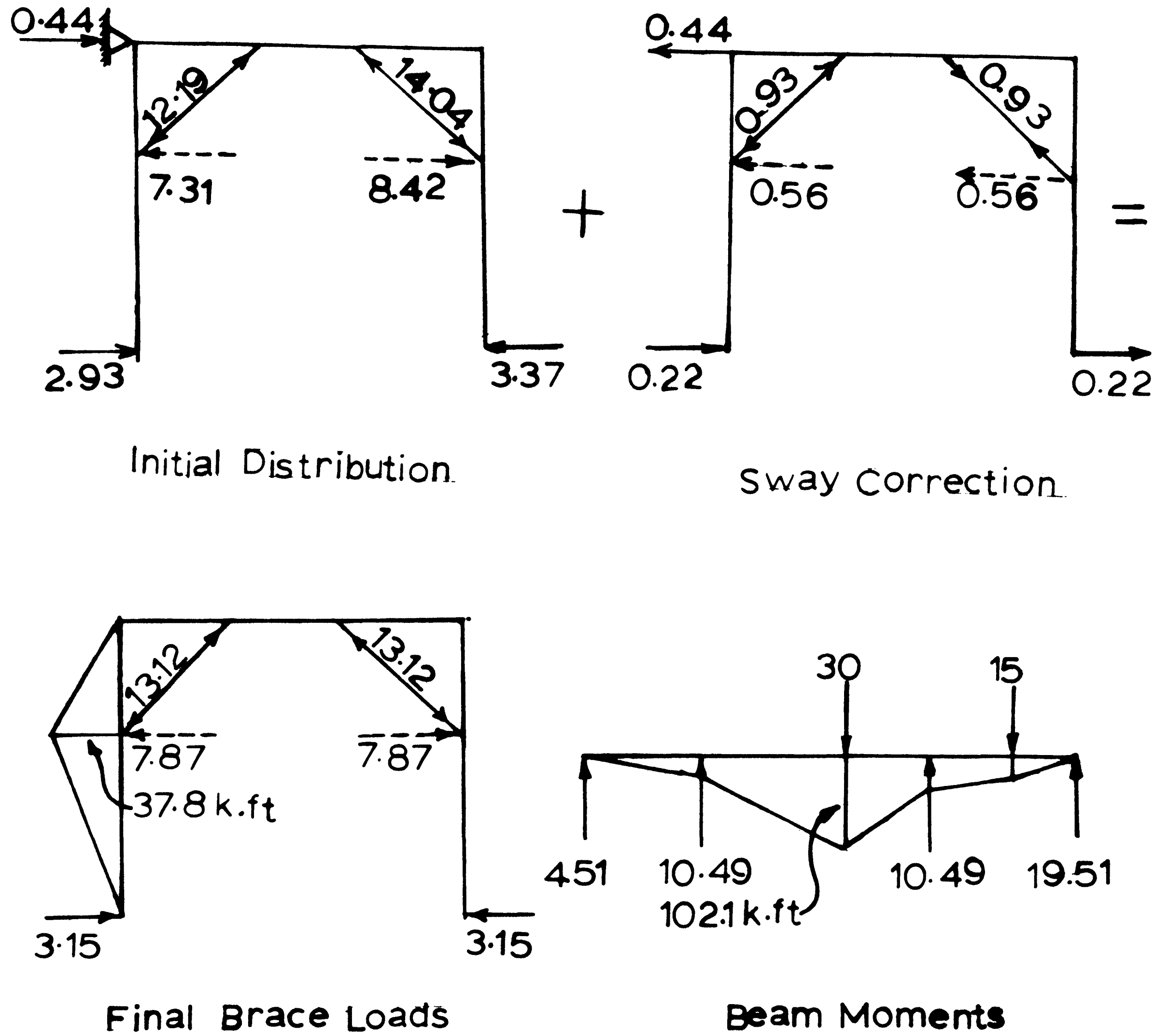

Fig. 7. Example: initial shears, sway correction, final brace loads, beam and column moments

For the frame in Fig. 6, the following information is given:

Moment of inertia of beam = 1000.0 in.^{4}
Moment of inertia of each column = 500.0 in.^{4}
Area of each knee brace = 2.5 sq. in.

$$\sin\theta_1 = \sin\theta_2 = 0.8, \cos\theta_1 = \cos\theta_2 = 0.6$$

$$\sin^2\theta_1 = \sin^2\theta_2 = 0.64, \cos^2\theta_1 = \cos^2\theta_2 = 0.36$$

$$d_{gg} = d_{hh} = \frac{153.6 \times 1728}{500E} = \frac{530.84}{E} \text{ in./}k$$

$$d_{brc\ 1} = d_{brc\ 2} = \frac{10 \times 12}{2.5E} = \frac{48}{E} \text{ in./}k$$

$$d_{cc} = d_{dd} = \frac{117.6 \times 1728}{1000E} = \frac{203.21}{E} \text{ in./}k$$

$$d_{cd} = \frac{98.4 \times 1728}{1000E} = \frac{170.04}{E} \text{ in./}k$$

$$d_{C1} = \frac{4360.5 \times 1728}{1000E} = \frac{7534.9}{E} \text{ in.}$$

$$d_{D1} = \frac{4709.5 \times 1728}{1000E} = \frac{8138.02}{E} \text{ in.}$$

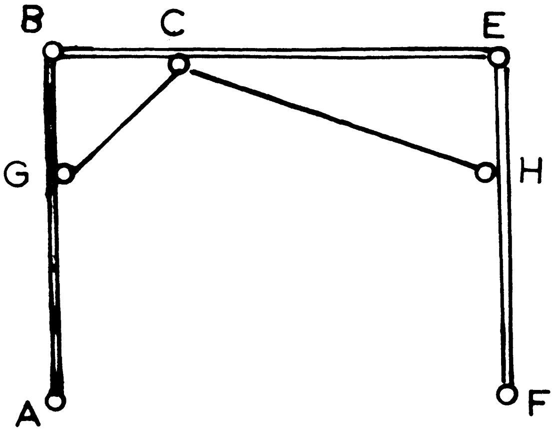

Figure 8.

Substituting the above values in Eq. 7.

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} (203.21 \times 0.64 + 530.84 \times 0.36 + 48)(170.04 \times 0.64) \\ (170.04 \times 0.64)(203.21 \times 0.64 + 530.84 \times 0.36 + 48) \end{bmatrix}^{-1} \times \begin{bmatrix} 7534.9 \times 0.8 \\ 8138.02 \times 0.8 \end{bmatrix}$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 369.15 & 108.83 \\ 108.83 & 369.15 \end{bmatrix}^{-1} \times \begin{bmatrix} 6027.96 \\ 6510.41 \end{bmatrix} = \begin{bmatrix} 12.19^k \\ 14.04^k \end{bmatrix}$$

Initial frame shears and sway corrections with final beam and column moments are given in Fig. 7. This problem was also solved using a computer and the results were within 1% of the values obtained above.

The two equations necessary to solve the knee brace loads in the frame of Fig. 8 formulated in the matrix form are:

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} (\sin^2\theta_1 d_{cc} + \cos^2\theta_1 d_{gg} + d_{brc\ 1})(\sin\theta_1 \sin\theta_2 d_{cc}) \\ (\sin\theta_1 \sin\theta_2 d_{cc})(\sin^2\theta_2 d_{cc} + \cos^2\theta_2 d_{hh} + d_{brc\ 2}) \end{bmatrix}^{-1} \times \begin{bmatrix} d_{C1} \sin\theta_1 \\ d_{C1} \sin\theta_2 \end{bmatrix} \quad (8)$$

Here, too, sway correction may have to be applied depending upon the presence of the unbalanced shear on the frame.

NOMENCLATURE

d_{cc} = Deflection in beam at C due to a unit load at C.
d_{cd} = Deflection in beam at C due to a unit load at D.
d_{dd} = Deflection in beam at D due to a unit load at D.
d_{gg} = Deflection in column AB at G due to a unit load at G.
d_{hh} = Deflection in column EF at H due to a unit load at H.
$d_{brc\ 1}$ = Shortening of brace GC due to a unit load along GC.
$d_{brc\ 2}$ = Shortening of brace DH due to a unit load along DH.
d_{C1} = Deflection in beam at C due to external loading.
d_{D1} = Deflection in beam at D due to external loading.
P_1 = Force in brace 1.
P_2 = Force in brace 2.

REFERENCES

Wang, C. K. Statically Indeterminate Structures.