End Restraint and Column Design Using LRFD

E. M. LUI AND W. F. CHEN

The ultimate strength of a structural member is the maximum resistance it can offer to resist the applied force before failure. Design based on the ultimate strength of the structure or its component parts is referred to as limit state approach to design. A valid limit state approach to analysis and design of any structural member requires the consideration of all important factors that are imperative to determining the behavior of the member throughout the entire range of loading up to the maximum capacity of the member. In the case of columns, the three main factors that need to be considered are (1) residual stresses; (2) initial geometrical imperfections; and (3) end restraints. The purpose of this paper is to summarize the results of some recent research on in-plane behavior of imperfect columns with modest end-restraints. The incorporation of end restraint in the design of columns using the Load and Resistance Factor Design (LRFD) format is then proposed. Simple design examples will be given to illustrate how the effective length concept can be used for the design of imperfect columns with small end restraints.

NOMENCLATURE

D_n = Dead load
E = Young's modulus
E_n = Earthquake load
F_y = Yield stress
I_c = Moment of inertia of column
I_g = Moment of inertia of girder
K = Effective length factor
K_{el} = Elastic effective length factor
L = Length of column (= L_c)
L_g = Length of girder
L_n = Live load
M = Moment at the connection
M_T = Transition moment
P_{max} = Maximum load
P_y = Yield load
Q_n = Load effect
r = Radius of gyration
R = Restraining factor
R_k = Rotation stiffness of connection
$\overline{R}_k$ = Relative rotation stiffness of connection
R_n = Nominal resistance
S_n = Snow load
W_n = Wind load
α = Coefficient of end restraint
$\overline{\alpha}$ = Relative coefficient of end restraint
β = Safety index
δ_{im} = Initial mid-height deflection
δ_m = Mid-height deflection
η = Curve-fitting parameter
γ = Load factor
λ = Non-dimensional effective slenderness ratio
λ_o = Non-dimensional slenderness ratio
ϕ = Resistance factor
θ = Rotation of connection

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END RESTRAINT ON COLUMN STABILITY

The Column Problem—It is generally agreed that residual stresses and initial crookedness have a deleterious effect on the load-carrying capacity of columns. Research on the effect of residual stresses on column strength of wide-flange cross sections in the early 1950's has led to the development of the CRC column strength curve. The CRC curve is based on the tangent modulus concept and thus recognized for the first time the role of residual stress as a primary factor in determining the strength of centrally loaded pin-ended columns. Although initial crookedness was not considered, the CRC curve satisfactorily represented the average test strength of a number of small and medium-size hot-rolled wide-flange shapes of mild structural steel, ASTM A7, F_y = 33 ksi that were in common use at that time. The CRC curve, divided by a variable factor of safety in the plastic range and a constant factor of safety in the elastic range, becomes the allowable stress column design curve that is contained in the present AISC Specification.[22]

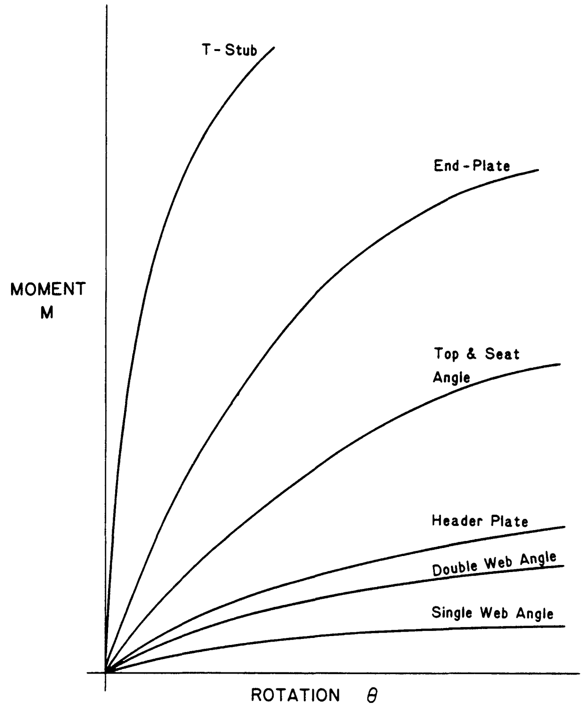


Fig. 1. Typical M-θ curves

Realizing that perfectly straight columns are rarely encountered in real life, researchers incorporated initial crookedness as well as residual stress into the analysis of centrally loaded pin-ended columns based on a computer model developed. The result is the development of SSRC multiple column curves.[3,14] The SSRC curves are based on the stability limit load of pin-ended columns and take into account the influence of actual measured residual stress distributions and assumed initial out-of-straightness at mid-height equal to $0.001L$ (L = length of the column). As a result, the basic strength of pin-ended H-columns can be assessed quite accurately; both in-plane[6] and in-space[7,8] behavior of these columns have been formulated and proposed[14] for general use.

However, as pointed out by Chen,[9] columns in an actual framework are connected to other structural members and so their ends are restrained. It is believed that the behavior and strength of columns in actual building frames will be affected significantly by the presence of these unavoidable end restraints and must therefore be included in the determination of their load-carrying capacity. To this end, SSRC Task Group 23, entitled "Effect of End Restraint on Initially Crooked Columns," was established in 1979; its aim is to study the combined effect of residual stresses, initial out-of-straightness and end restraints on column strength.

The End Restraint Modeling—Under the current AISC Specification,[22] the effective length concept makes reference only to columns in continuous frames classified as Type 1 construction. Columns in structures designated as simple framing (Type 2 construction) are designed as if their ends are pinned. However, experimental investigations of actual joint behavior conducted at various times during the past five decades have shown that typical simple connections do possess a certain amount of rotational rigidity. The importance of end-restraint was realized over 50 years ago when research workers[17,23,27] measured the relationship between the end-moment and the relative rotation of the beam to column at the connection. Typical moment-rotation (M-θ) curves for some commonly used beam-to-column connections are shown in Fig. 1. The closer the curve is to the horizontal axis, the more flexible is the connection. The slope of the M-θ curve is a measure of the stiffness of the connection. It can be seen from Fig. 1 that connection stiffness decreases as rotational deformation increases and that most connections exhibit non-linear M-θ behavior almost over the entire range of loading.

The rigidities of the various types of connections are determined by a number of factors[15] and their M-θ curves do not follow any simple mathematical form. Before any analytical solution is attempted, methods of modeling these M-θ curves have to be developed. Numerous methods have been used over the years. The simplest model assumed a linear M-θ relationship.[17] This linear model utilized the initial tangent stiffness factor R_k, shown in Fig. 2. Thus, the behavior of the connection is assumed to be linear over the entire range of loading. As seen in Fig. 2, the R_k value becomes less representative as the moment increases.

To overcome this, Romstad and Subramanian[18] used a bilinear model in which the initial slope of the M-θ line was replaced by a shallower line at a certain transition moment, M_T (see Fig. 2). Sommer[21] and Frye and Morris[11] curve-fit experimental M-θ data with a polynomial function. However, this polynomial curve-fitting technique was criticized by Jones, Kirby and Nethercot[13] in that it may produce unsatisfactory M-θ descriptions. Since the nature of any polynomial function is to peak and trough within the

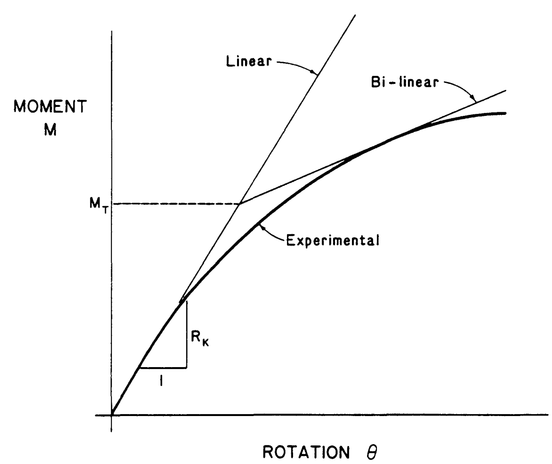


Fig. 2. Linear M-θ models

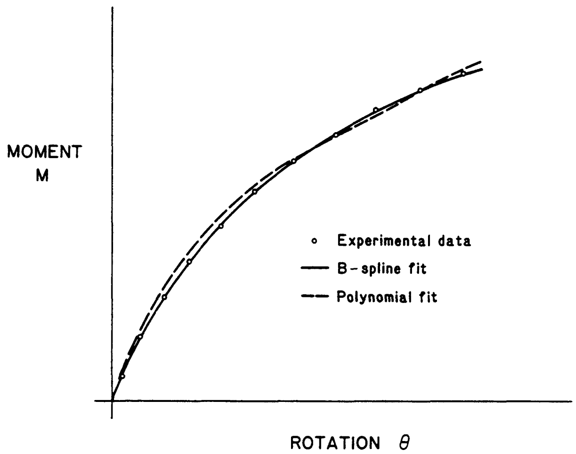


Fig. 3. B-spline and polynomial curve fit models

range of the function, connection stiffness which corresponds to the slope (first derivative) of the function may be negative. This incorrect negative stiffness can lead to difficulties in the analytical procedure. Jones *et al.*[13] therefore utilized a more sophisticated cubic B-spline curve-fit to experimental data to describe its M-θ behavior. This method requires the division of the range of connection rotations into a finite number of smaller ranges. Within each range a cubic function is fitted in turn with first and second derivative continuities maintained between adjacent ranges. This method has been found[13] to produce accurate curve descriptions of experimental M-θ data (Fig. 3).

The Analysis of Columns—With proper end-restraint modeling, the analysis of the combined effect of residual stresses, initial crookedness and end restraints can proceed. The load-deflection approach has to be used in the analysis in which the full load-deflection behavior is traced up to the ultimate load, including the post-buckling unloading branch. The simpler and more straightforward eigenvalue approach cannot be used, because it neglects the effect of geometrical imperfections which have been known to play an important role in the buckling strength of columns.

The governing differential equations describing the behavior of these imperfect end-restrained columns are highly non-linear, and closed form solutions are impossible to obtain. Recourse to numerical methods is necessary in order to obtain approximate solutions. Several numerical techniques are used by researchers to approach this problem. These include finite element, finite difference and tangent stiffness methods. Jones *et al.*[13] employed the finite element method with the cubic B-spline M-θ model. Vinnakota[25] used a finite-difference approach with the bi-linear M-θ model. Sugimoto and Chen[24] used the tangent stiffness method with the bi-linear M-θ model.

The load-deflection curves computed using these different approaches were shown in Fig. 4 in non-dimensional form, in which the load is normalized by the yield load and the mid-height deflection is normalized by the initial mid-height deflection. Also shown in Fig. 4 is the experimental end-restrained load-deflection curve tested by Bergquist[2] and the corresponding pin-ended load-deflection curve. The column is a hot-rolled W10x29, of slenderness ratio, $L/r_y = 189.7$. W10x21 beams are attached to the weak-axis, using web cleats fastened with A325 bolts. Despite different models and techniques used, the computed load-deflection curves showed similar behavior and agreed well with the test data.

Two important observations are obtained from these curves: (1) for the same deflection, the column with end restraint can carry more load than the corresponding pin-ended column; and (2) for the same load, the mid-height deflection of the end-restrained column is considerably less than that of the hinge-ended column. Since the peak load of the end-restrained column is higher than that of the pin-ended counterpart, ultimate strength of the former is greater than the latter. Recall that ultimate strength is the basis of the limit state design; the increase in load-carrying capacity due to end restraint cannot be ignored if the structure is designed to the limit of its usefulness.

The Effects of End Restraint—The amount of increase of maximum load due to end restraint depends on a number of factors. Some of the important ones are:

1. The rotational stiffness of the connection
2. The magnitude of the initial out-of-straightness
3. The axis of bending and slenderness ratio of the column
4. The magnitude and distribution of residual stresses over the cross section of the column

These factors were studied by a number of researchers.[2,13,16,20,24,25] The general conclusion is that maximum load-carrying capacity increases as the rotational stiffness of the connection increases, except at very low slenderness when yielding is the primary cause of failure. The increase

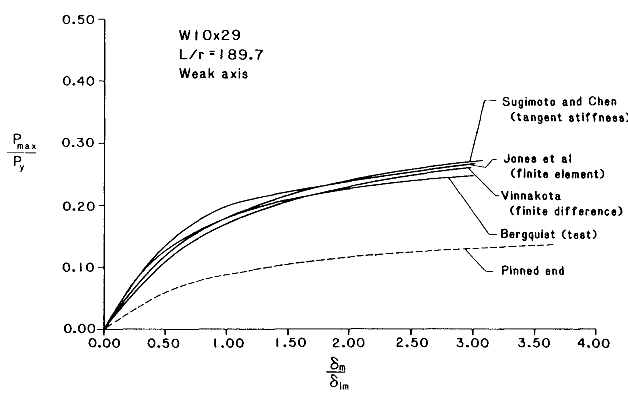


Fig. 4. Comparison of analytical and experimental load-deflection curves

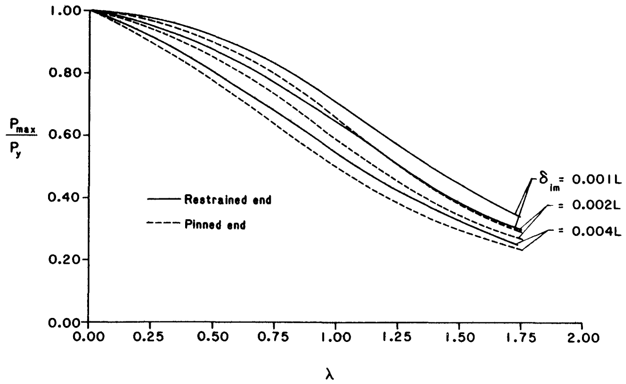


Fig. 5. Calculated column curves (W12x65, strong axis)

in load-carrying capacity is more noticeable at high slenderness ratio. An increase in the magnitude of initial crookedness has a detrimental effect on the strength-increasing effect from end restraint. The strength-increasing effect due to end restraint is more pronounced in weak-axis bending than strong-axis bending. Residual stress has little or no effect on column strength at high slenderness ratio, when compared with initial crookedness and end restraint.

Figure 5 shows a set of six column curves, which are plots of the non-dimensional load (P_{max}/P_y) versus the non-dimensional slenderness ratio $\lambda = (1/\pi)\sqrt{(F_y/E)(KL/r)}$ for three different conditions of initial out-of-straightness: $\delta_{im} = 0.001L$, $0.002L$ and $0.004L$. The column is a hot-rolled W12x65 shape connected to two W12x30 beams by riveted double-web angles. The three solid lines correspond to the end-restrained column and the three dotted lines correspond to their pin-ended counterpart. These curves demonstrate the increase of maximum load for each initial imperfection; but at high λ, the increase becomes less as the initial imperfection increases. For example, at λ = 1.5, the increases in strength are 16%, 12% and 8.5% for $\delta_{im} = 0.001L$, $0.002L$ and $0.004L$, respectively. Since the maximum allowable initial crookedness under the current specification is $0.001L$, discussion from this point on will focus on an initial imperfection of $0.001L$.

To investigate the magnitude of end fixity on column strength, four different beam-to-column connections were used in a recent study.[16] The result is shown in Fig. 6, in which the ratio P_{max} (end-restrained)/P_{max} (pin-ended) is plotted against the non-dimensional slenderness, λ. It can be seen that as the end fixity increases, the percentage gained in strength also increases. This increase is not significant at low λ values when yielding controls the failure, but becomes more noticeable at higher λ values when stability is more crucial. The rate of increase slows down and becomes almost constant after reaching some value of λ. This is due to the limited capacity in end-restrained moment from the connection. The above observations were also reported by Shen and Lu.[20]

In Fig. 7, the percentage increase in strength due to end restraint is plotted against the restraining factor R, expressed as a multiple of EI_c/L_c. The column is a W8x31 shape of A36 steel with $\delta_{im} = 0.001$ bent about the weak axis. Note the increase in strength as λ increases. For example, at $R = 2EI_c/L_c$, the increase is only 10% at λ = 0.5, but jumps up to 50% at λ = 1.5.

In Fig. 8, in which P_{max}/P_y is plotted against R, it can be seen that the variation of P_{max}/P_y over the range of R becomes less significant as λ decreases. For example, at λ = 1.5, the variation is from 0.35 to 0.54 at $R = 0$ to $2EI_c/L_c$; but at λ = 0.5, the variation is from 0.87 to 0.95 at $R = 0$ to $2EI_c/L_c$.

It is now obvious that residual stresses and initial crookedness have a destabilizing effect on columns, whereas end restraint can provide stabilizing effect. The combined

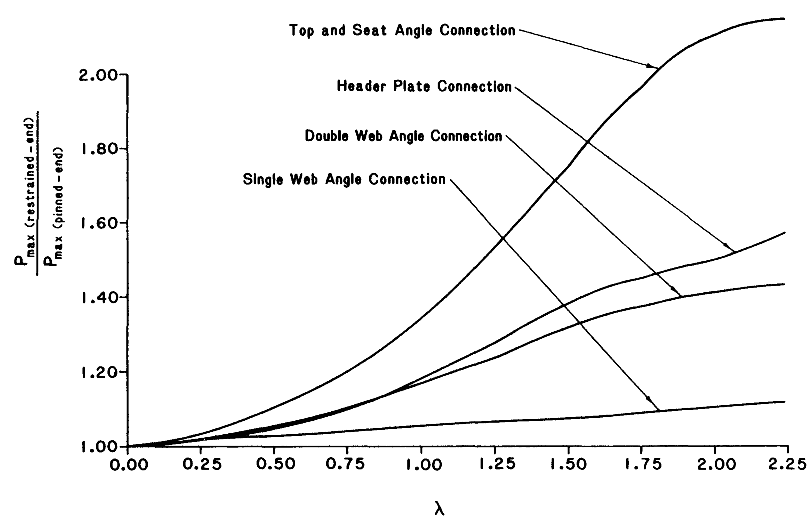


Fig. 6. Comparison of the maximum strength of restrained-end columns with pinned-end columns

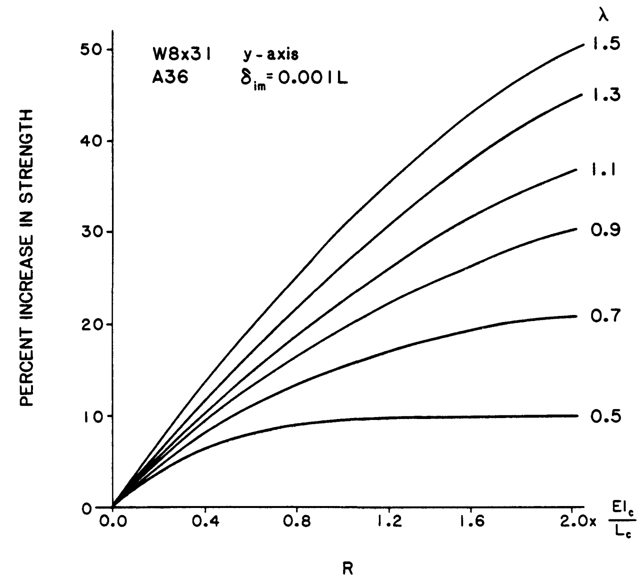


Fig. 7. Percent increase in column strength due to end restraint

effect of these factors was studied systematically by Shen and Lu.[20] Figure 9 shows a plot of the percentage reduction in strength due to crookedness versus λ at four different R values. Three observations are noted:

1. For each value of R, there exists a value of λ at which strength reduction is a maximum.
2. As R increases, the λ corresponding to the maximum strength reduction also increases.
3. As R increases, the maximum strength reduction decreases (see dotted line).

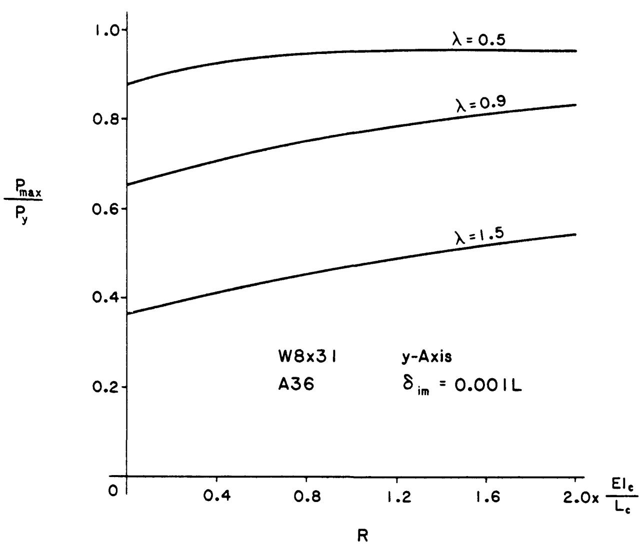


Fig. 8. Influence of λ and R on column strength

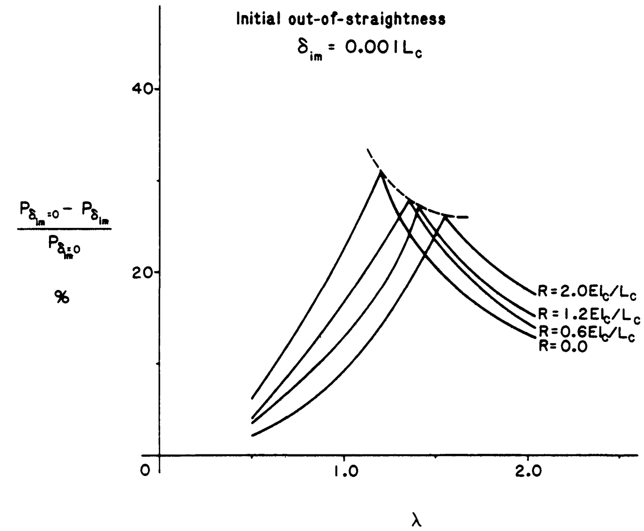


Fig. 9. Influence of initial crookedness and end restraint on the percent reduction of column strength

Observation 3 suggests that the strengthening effect of end restraint can be used to offset to some extent the weakening effect of the crookedness, but the amount of fixity required is highly dependent on λ.

The counteracting effect of end restraint on initial crookedness and its dependence on λ was also revealed by Chapuis and Galambos[5] in a study of restrained crooked aluminum columns. A three parameter law was used to describe the stress-strain relations for the aluminum alloys and the Column Deflection Curve (CDC) approach was used for integrating the beam-column differential equation.

Another interesting observation on end-restrained columns is that the increase in strength is the largest for the weakest column. Because of this, the band width of the set of column curves generated with consideration of end restraints will be reduced. This reduction of band width implies that the scatter of column strength due to different manufacturing method (rolled vs. welded), different grade of steel (A36 vs. A514) and size (light vs. heavy) will be less significant.

EFFECTIVE LENGTH FACTOR

The effective length factor, K, is a multiplier to convert the actual unbraced length of a column with any boundary conditions at its ends to an equivalent hinged-hinged column. This factor can be determined graphically as shown in Fig. 10.

A horizontal line is drawn from the ordinate at point a to the end-restrained column curve at point c with length λ_{ac}. This line will cut the pin-ended curve at point b with length λ_{ab}. The K factor is obtained as the ratio of the two values of λ:

$$K = \lambda_{ab}/\lambda_{ac} \tag{1}$$

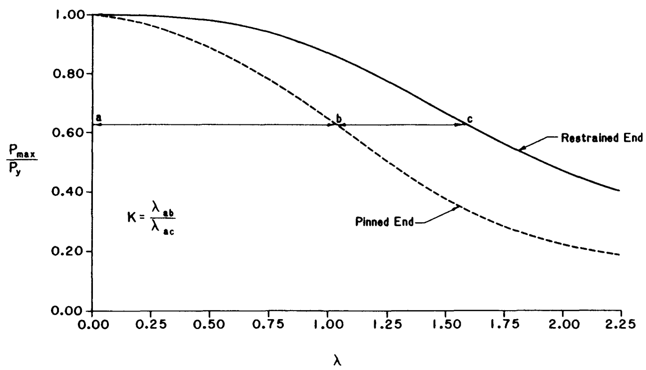


Fig. 10. Determination of effective length factor, K

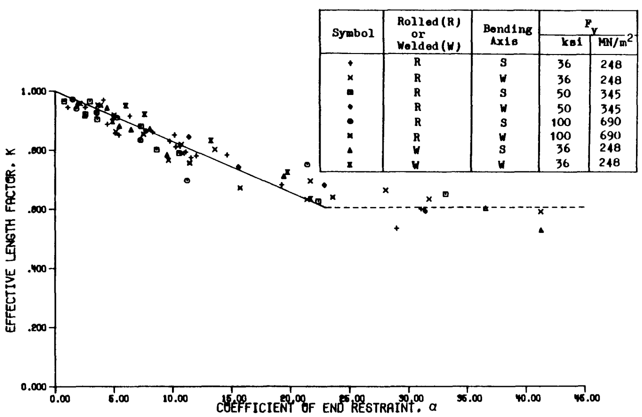


Fig. 11. Relationship between K and α for 83 column curves

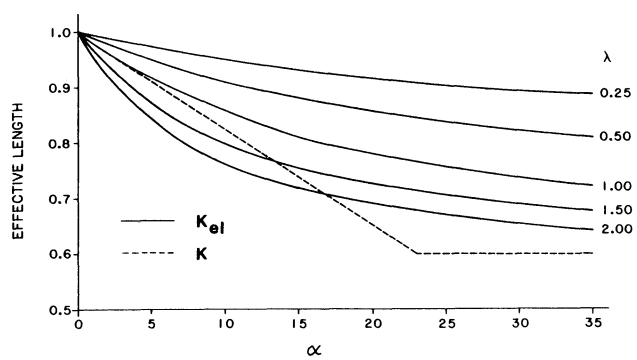


Fig. 12. Comparison of K_{el} and K

Upon investigations of 83 end-restrained column curves,[16] it was found that the values of K for each curve with a known value of end restraint do not vary significantly over the load level. This observation was also indicated by Jones *et al.*[13] and Sugimoto and Chen.[24] As a result of this, an expression for K and the magnitude of end restraint can be established. The magnitude of end restraint is expressed here in a non-dimensional form, α, defined as

$$\alpha = R_k/M_{pc} \tag{2}$$

where

R_k = rotational stiffness of the connection (Fig. 1)
M_{pc} = plastic moment of the column

The relationship between K and α is plotted in Fig. 11. It can be seen that there is an inverse relation between K and α for a certain range of α. For simplicity, a linear relation is assumed. The formula expressing this relationship is

$$K = 1.000 - 0.017\alpha \text{ (for } 0 \leq \alpha \leq 23) \tag{3a}$$

$$K = 0.600 \text{ (for } \alpha > 23) \tag{3b}$$

Equations (3a) and (3b) can be used to calculate the effective length of a column with equal end restraints at its ends provided that α can be estimated.

Before proceeding any further, it is of interest to consider the effective length concept in the context of an elastic stability approach. To determine this elastic effective length factor, designated as K_{el}, an eigenvalue approach assuming an ideal column restrained at both ends with linear springs having rotational stiffness R_k gives:

$$\frac{R_k L_c}{EI_c} = \frac{\dfrac{\pi}{K_{el}} \sin \dfrac{\pi}{K_{el}}}{\cos \dfrac{\pi}{K_{el}} - 1} \tag{4}$$

where

I_c = moment of inertia of the column.

Figure 12 shows the plot of K_{el} vs. α at various λ for the 83 columns. The relation between K and α as described in Eqs. (3a) and (3b) is also plotted in the figure (dotted line). It can be seen that K_{el} gives a conservative estimate of the effective length of a crooked column at low λ and high α values, but it is not conservative at high λ or low α. If K_{el} is used in place of K in determining the effective length of end-restrained columns, a cutoff has to be made to ensure safety, since K_{el} is not always conservative.

EFFECT OF BEAM FLEXIBILITY

The discussion so far has been focused on columns with ends restrained by beam-to-column connections framed to beams having infinite stiffness. In an actual framework, beams tend to bend as the columns buckle, thus reducing the rotational stiffness of the connections. Therefore, the flexibility effect of beams on connection stiffness has to be taken into account in determining the effective length of the column.

Consider a rectangular frame as shown in Fig. 13. Beams having moment of inertia I_g and length L_g are connected

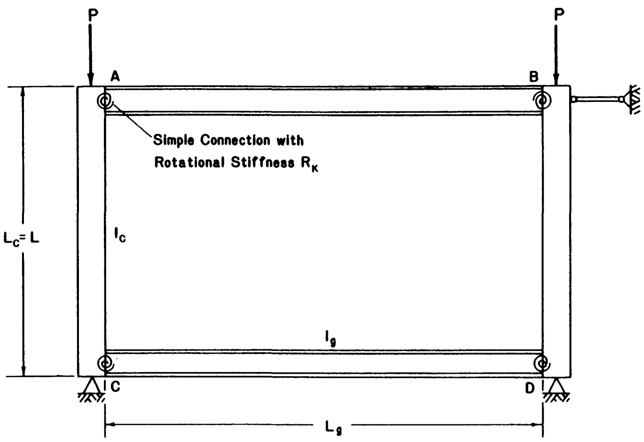


Fig. 13. Rectangular frame with semi-rigid connections

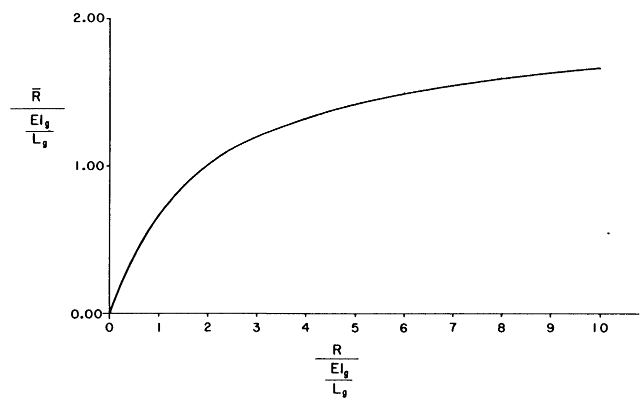


Fig. 14. $\overline{R}$ versus R

to columns by simple beam-to-column connections having rotational stiffness R_k. The relative rotational stiffness $\overline{R}_k$ due to the flexibility of beams is equal to:

$$\overline{R}_k = \frac{2EI_g}{L_g}\left[\frac{1}{1+\dfrac{2\,EI_g}{L_gR_k}}\right] \tag{5}$$

Equation (5) is true only if the adjoining beam bends in single curvature with rotation at the near end equal and opposite to the rotation at the far end. It will be conservative if the far end of the beam is fixed or if the beam bends in double curvature. A graphical representation of Eq. (5) is shown in Fig. 14. Equations (3a) and (3b) and Fig. 12 can still be used to estimate the K-factor if beam flexibility is taken into account. However, α has to be replaced by $\overline{\alpha}$, in which

$$\overline{\alpha} = \overline{R}_k/M_{pc} \tag{6}$$

where M_{pc} is defined as before in Eq. (2).

K-VALUES FOR DESIGN

From the above discussion, it is clear that the determination of effective length for columns with simple end restraints involves the consideration of a number of factors. Among these are the rotational stiffness of the connections, the unbraced lengths of the columns and the girders and the section properties of these members. This is very undesirable as far as design is concerned, because the designer would need to know or guess the type of connection and the size of the members before proceeding with the design. To overcome this, Tables 1 and 2[10] list some typical values of rotational stiffness of two commonly used simple beam-to-column connections (double web angle and top and seat angles).

By using these tables, with appropriate comparable sizes and lengths of columns and girders, together with Figures 12 and 14, it appears that a K-value of 0.95 for strong axis bending and 0.90 for weak axis bending will be conservative for most cases, except for very low λ. This is not surprising, because at very low λ the strengthening effect of end restraints is negligible (see Figs. 6, 7, and 8). Therefore, it is advisable to use a K-value of unity for both axes of bending if λ_o is less than or equal to 0.5.

Table 1. R_k for Double Web Angles

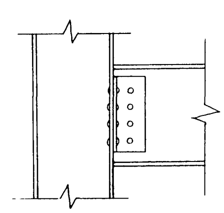
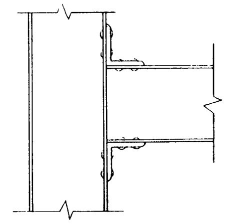
	No. of Rows of Fasteners	R_k (kip-in./rad)
	3	3.23×10^4
	4	7.69×10^4
	5	2.86×10^5
	6	5.00×10^5
	7	9.09×10^5
	8	1.33×10^6
	9	1.92×10^6
	10	2.86×10^6

Table 2. R_k for Top and Seat Angles

Angles 6 x 4 x 3/4 x 0′-8″	Depth of Beam (in.)	R_k (kip-in./rad)
	8	2.17×10^6
	10	2.78×10^6
	12	3.57×10^6
	14	4.35×10^6
	16	5.56×10^6
	18	7.14×10^6
	21	8.33×10^6
	24	1.00×10^7
	27	1.28×10^7
	30	1.52×10^7
	33	1.82×10^7
	36	2.17×10^7

To sum up, the recommended K-values are:
For strong axis bending:

$$K = 1.00 \quad (\text{for } \lambda_o \leq 0.5)$$
$$K = 0.95 \quad (\text{for } \lambda_o > 0.5) \tag{6}$$

For weak axis bending:

$$K = 1.00 \quad (\text{for } \lambda_o \leq 0.5)$$
$$K = 0.90 \quad (\text{for } \lambda_o > 0.5) \tag{7}$$

where

$$\lambda_o = \frac{\lambda}{K} = \frac{1}{\pi}\sqrt{\frac{F_y}{E}}\left(\frac{L}{r}\right)$$

and L is the unbraced length of the column.

BASIC COLUMN CURVE FOR DESIGN

To obtain a good limit state design, a good basic column curve is needed. The SSRC Curve 2 is preferable to the CRC curve, because the former takes into account both initial crookedness and residual stresses, whereas the latter only considers residual stresses in their development. The SSRC Curve 2 is one of a set of three multiple column curves[3,14] developed for pin-ended columns. In its original form, five equations and 12 coefficients are necessary to describe its complete shape. Rondal and Maquoi[19] proposed a single equation with only one parameter to approximate this curve. The equation is given as follows:

$$\frac{P_{max}}{P_y} = 1.000 \quad (\text{for } 0 \leq \lambda \leq 0.15)$$

$$\frac{P_{max}}{P_y} = \frac{1}{2\lambda^2}\{[1 + \eta + \lambda^2] - \sqrt{[1 + \eta + \lambda^2]^2 - 4\lambda^2}\} \quad (\text{for } \lambda \geq 0.15) \tag{8}$$

where

λ = non-dimensional slenderness ratio
$\eta = 0.293\ (\lambda - 0.15)$

The parameter η represents a global imperfection integrating geometrical imperfections as well as residual stresses. SSRC Curve 2 described by Eq. (8) was shown[19] to agree well with the original curve, with a maximum error of only 2.12%. Figure 15 shows a comparison of the original SSRC Curve 2 and that described by Eq. (8).

By using SSRC Curve 2 [Eq. (8)] and appropriate K-values [Eqs. (6) and (7)], all three important factors (residual stresses, geometrical imperfections and end restraints) that influence the ultimate strength of centrally loaded columns can be taken into account in a limit state design. The limit state design to be used is called Load and Resistance Factor Design (LRFD).

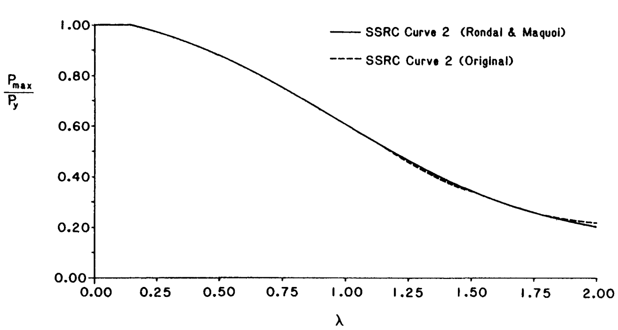


Fig. 15. Comparison of SSRC Curve 2 (original and Ref. 19 versions)

COLUMN DESIGN USING LRFD

The general format of LRFD is:

$$\phi R_n \geq \Sigma\ \gamma_i\ Q_{ni} \tag{9}$$

where

ϕ = resistance factor
R_n = nominal strength
γ_i = load factors
Q_{ni} = nominal load effect

The resistance factor ϕ and the load factor γ are statistically based safety factors to account for the uncertainties and variabilities inherent in the determination of the nominal strengths and load effects due to natural variation in material properties and load effects. The use of these safety factors is further justified when the accuracy of the theory and the precision of the analysis are reduced due to assumptions and simplifications made. Note that the resistance factor ϕ is always less than unity, while the load factor γ is always greater than unity.

In the case of end-restrained columns, the authors, based on the study of the safety of columns by Bjorhovde,[4] recommend a ϕ value of 0.85. The nominal strength R_n can be determined from Eqs. (6), (7) and (8). The load factors and load combinations [right side of Eq. (9)] are listed in Table 3.[12]

The SSRC Curve 2 scaled down by a ϕ-factor of 0.85 is shown in Fig. 16. This curve, together with Eqs. (6) and (7), enable a designer to determine the maximum load that an end-restrained column can carry. The procedures are illustrated in the design examples that follow.

Table 3. Load Factors

$1.4D_n$
$1.2D_n + 1.6L_n$
$1.2D_n + 1.6S_n + (0.5L_n \text{ or } 0.8W_n)$
$1.2D_n + 1.3W_n + (0.5L_n)$
$1.2D_n + 1.5E_n + (0.5L_n \text{ or } 0.2S_n)$
$0.9D_n - (1.3W_n \text{ or } 1.5E_n)$

D_n = Dead load
L_n = Live load
W_n = Wind load (50 yr mean recurrence interval map)
S_n = Snow load (50 yr mean recurrence interval map)
E_n = Earthquake load

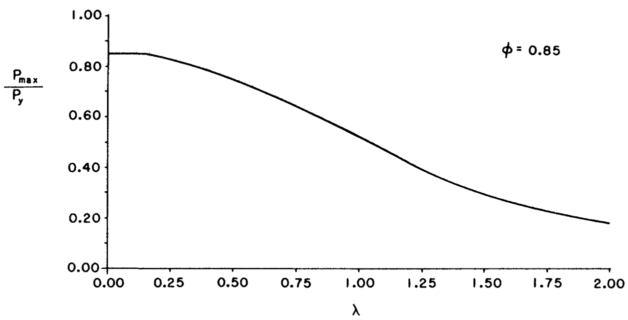


Fig. 16. SSRC Curve 2 with ϕ = 0.85

DESIGN EXAMPLES

Two examples using Eqs. (6) to (9) and $\phi = 0.85$ will be shown here.

Example 1

Given:

Rectangular frame as shown in Fig. 13. The frame is braced against out of plane buckling and sidesway is prevented. Beams AB and CD are connected to the column flanges by simple beam-to-column connections (Type 2 construction). Lengths AC and BD are both 15 ft long.

Find:

Select the lightest W section to carry a live load of 100 kips and a dead load of 60 kips.
Use A36 steel.

Solution:

From Table 3:

$$\Sigma \gamma_i Q_{ni} = (1.2 \times 60) + (1.6 \times 100) = 232 \text{ kips}$$

Required resistance [Eq. (9)] is:

$$\phi R_n = \phi P_{max} \geq 232$$

Strong axis bending controls; use Eq. (6).

Section	r_x	λ_o	$\lambda = K\lambda_o$	$\frac{\phi P_{max}}{P_y}$	ϕP_{max}
W8x28	3.45	0.585	0.556	0.731	217
W10x30	4.38	0.461	0.461	0.764	243
W8x31	3.47	0.582	0.553	0.733	241

Use W10x30 ($\phi P_{max} = 243 > 232$ **o.k.**)

Note that $K = 0.95$ was used in column 4 for sections W8x28 and W8x31 because of strong axis bending and $\lambda_o > 0.5$. For section W10x30, $K = 1$ was used, because $\lambda_o < 0.5$. Values in column 5 can be obtained from Fig. 16.

Example 2

Given:

Same as Example 1 except that the beams are now framed to the column webs.

Solution:

The only difference is that the columns will now buckle in the weak axes. Therefore, use Eq. (7).

Section	r_y	λ_o	$\lambda = K\lambda_o$	$\frac{\phi P_{max}}{P_y}$	ϕP_{max}
W8x31	2.02	0.999	0.899	0.573	188
W8x35	2.03	0.994	0.895	0.576	214
W10x39	1.98	1.020	0.918	0.563	233

Use W10x39 ($\phi P_{max} = 233 > 232$ **o.k.**

K was taken as 0.90 in column 4 because of weak axis bending and $\lambda_o > 0.5$.

LRFD VERSUS ASD

The development of the ASD (Allowable Stress Design) and LRFD formats are based on different philosophies. Direct correlation between the two is therefore impossible. However, if a special case is considered, one can compare the maximum stress each approach can offer to resist the load to see how well the latter calibrates against the former.

The ASD has the general format:

$$\frac{R_n}{F.S.} \geq \Sigma Q_{ni} \tag{10}$$

where

R_n, Q_n = nominal resistance and load, respectively
$F.S.$ = factor of safety
$\frac{R_n}{F.S.}$ = allowable stress from AISC column design curve

The LRFD format is defined previously and is repeated here as:

$$\phi R_n \geq \Sigma \gamma_i Q_{ni} \tag{9}$$

For the special case of having dead load D_n and live load L_n only, with $L_n = 2D_n$, Eq. (9) can be written as:

$$\frac{R_n}{1.726} \geq \Sigma Q_{ni} \tag{11}$$

By using Eqs. (10) and (11) and assuming $F_y = 36$ ksi, Table 4 can be constructed.

A comparison of column 3 with columns 5 and 7 in Table 4 indicates that for strong axis bending with $L/r < 140$,

Table 4. Comparison of LRFD and ASD

$\frac{L}{r}$	λ_o	ASD F_{allow} (ksi)	LRFD Strong Axis $\lambda = k\lambda_o$	LRFD Strong Axis F_{max} (ksi)	LRFD Weak Axis $\lambda = K\lambda_o$	LRFD Weak Axis F_{max} (ksi)
20	0.224	20.60	0.224	20.39	0.224	20.39
40	0.449	19.19	0.449	18.84	0.449	18.84
60	0.673	17.43	0.639	17.16	0.606	17.48
80	0.897	15.36	0.852	14.68	0.807	15.25
100	1.122	12.98	1.066	11.86	1.010	12.59
120	1.346	10.28	1.279	9.33	1.211	10.08
140	1.570	7.62	1.492	7.36	1.413	8.03
160	1.794	5.83	1.704	5.89	1.615	6.46
180	2.019	4.61	1.918	4.79	1.817	5.27
200	2.243	3.73	2.131	3.97	2.019	4.37

and for weak axis bending with $L/r < 120$, the LRFD format gives a lower permissible value for stress than the ASD format. However, this reduction is not significant. If the live load-to-dead load ratio decreases, this reduction will be negligible. This is the case for a high-rise building when the live load reduction factor allowed by ANSI[1] can be used. It should be pointed out that for high L/r (e.g., $L/r > 140$ for strong axis bending and $L/r > 120$ for weak axis bending in this special case), the LRFD format gives a higher permissible stress, because the strengthening effect of end-restraint becomes more noticeable.

Finally, it is interesting to point out that if the two design examples shown in the previous section, with $L_n/D_n = 1.67$, were worked out using the ASD format, the same wide-flange sections would be chosen.

FURTHER RESEARCH

It has been shown that end restraint does play an important role in determining the load-carrying capacity of columns. The magnitude of end fixity, which has a significant influence on column strength, depends highly on the connection stiffness and beam flexibility. Therefore, a systematic study of the behavior of connections and all other factors affecting their behavior are imperative in order to fully assess the strength of these end-restrained columns. Although experiments on beam-to-column connections were conducted at various times in the past five decades, much of the data reported in these investigations needs to be updated. With better understanding of various shear and moment connections, different K-factors can be used in the design procedure which will lead to a more rational and economical design.

SUMMARY AND CONCLUSIONS

A brief summary of recent research on the effect of end restraints on column stability was given. It is concluded that end restraint from simple beam-to-column connections has a significant strengthening effect on column strength. The incorporation of this small end restraint in column design was proposed [Eqs. (6) and (7)] by using the effective length concept.

The SSRC Curve 2 (in Rondal and Maquoi mathematical form) was chosen as the base line curve, not only because it takes into account residual stresses and initial crookedness in its development, but it also gives better end points for the beam-column interaction formula.[26]

By using the SSRC Curve 2 together with the appropriate K-factors, column design examples using LRFD format were given. Upon comparison (of a special case) with the ASD format, it is believed that both approaches give a comparable design at low or median slenderness ratios, whereas, for high slenderness ratios, the LRFD format will result in a more economical design.

Since the behavior of end-restrained columns is greatly influenced by the types of connections and their stiffnesses, a systematic study of the behavior of these connections is essential in order to fully assess the strength of these columns.

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REFERENCES

1. Minimum Design Loads for Buildings and Other Structures (ANSI A58.1-1972) *American National Standards Institute, New York, N.Y.*
2. *Bergquist, D. J.* Test on Columns Restrained by Beams with Simple Connections *Department of Civil Engineering, University of Texas at Austin, Jan. 1977.*

3. *Bjorhovde, R.* Deterministic and Probabilistic Approaches to the Strength of Steel Columns *thesis presented to Lehigh University, at Bethlehem, Pa., in 1972 in partial fulfillment of the requirements for the degree of Doctor of Philosophy.*
4. *Bjorhovde, R.* The Safety of Steel Columns *Journal of the Structural Division, ASCE, Vol. 104, No. ST3, March 1978, pp. 463–477.*
5. *Chapuis, J. and T. V. Galambos* Restrained Crooked Aluminum Columns *Journal of the Structural Division, ASCE, Vol. 108, No. ST3, March 1982, pp. 511–524.*
6. *Chen, W. F. and T. Atsuta* Theory of Beam-Column: Vol. 1—In-Plane Behavior and Design *McGraw-Hill Publishing Co., Inc., New York, N.Y., 1976.*
7. *Chen, W. F., and T. Atsuta* Theory of Beam-Column: Vol. 2—Space Behavior and Design *McGraw-Hill Publishing Co., Inc., New York, N.Y., 1977.*
8. *Chen, W. F. and F. Cheong-Siat-Moy* Limit States Design of Steel Beam-Columns, A State-of-the-Art Review *Journal Solid Mechanics Archives, Vol. 5, Issue 1, Noordhoff, Leyden, The Netherlands, Feb. 1980, pp. 29–73.*
9. *Chen, W. F.* End Restraint and Column Stability *Journal of the Structural Division, ASCE, Vol. 106, No. ST11, Proc. Paper 15796, Nov. 1980, pp. 2279–2295.*
10. *DeFalco F. and F. J. Marino* Column Stability in Type 2 Construction *Engineering Journal, American Institute of Steel Construction, Vol. 3, No. 2, April 1966, pp. 67–71.*
11. *Frye, M. J. and G. A. Morris* Analysis of Flexibly Connected Steel Frames *Canadian Journal of Civil Engineers, Vol. 2, No. 3, Canada, Sept. 1975, pp. 280–291.*
12. *Galambos, T. V.* Load and Resistance Factor Design *Engineering Journal, American Institute of Steel Construction, 3rd Quarter, 1981, pp. 73–82.*
13. *Jones, S. W., P. A. Kirby and D. A. Nethercot* Columns with Semirigid Joints *Journal of the Structural Division, ASCE, Vol. 108, No. ST2, Feb. 1982, pp. 361–372.*
14. *Johnston, B. G. (ed.)* Guide to Stability Design Criteria for Metal Structures *3rd edition, John Wiley and Sons, Inc., New York, N.Y., 1976.*
15. *Lewitt, C. W., E. Chesson and W. H. Munse* Restraint Characteristics of Flexible Riveted and Bolted Beam-to-Column Connections *Engineering Experiment Station Bulletin No. 500, University of Illinois, Jan. 1969.*
16. *Lui, E. M. and W. F. Chen* Strength of H-Columns with Small End Restraints *The Journal of the Institution of Structural Engineers, Vol. 60, London, 1982 (to appear).*
17. *Rathbun, J. C.* Elastic Properties of Riveted Connections *Transactions of the American Society of Civil Engineers, 1936, Vol. 101, pp. 524–563.*
18. *Romstad, K. M. and C. V. Subramanian* Analysis of Frames with Partial Connection Rigidity *Journal of the Structural Division, ASCE, Vol. 96, No. ST11, Nov. 1970, pp. 2283–2300.*
19. *Rondal, J. and R. Maquoi* Single Equation for SSRC Column-Strength Curves *Technical Notes, Journal of the Structural Division, ASCE, Vol. 105, No. ST1, Jan. 1979, pp. 247–250.*
20. *Shen, Z. Y. and L. W. Lu* Analysis of Initially Crooked, End Restrained Steel Columns *Fritz Engineering Laboratory Report No. 471.2, Lehigh University, Bethlehem, Pa., Dec. 1981.*
21. *Sommer, W. H.* Behavior of Welded Header Plate Connections *M. A. Sc. thesis, University of Toronto, 1969.*
22. Specification for the Design, Fabrication and Erection of Structural Steel for Buildings *American Institute of Steel Construction, Nov. 1978.*
23. *Steel Structures Research Committee* First, Second and Final Reports *Department of Scientific and Industrial Research, HMSO, London, 1931, 1934, 1936.*
24. *Sugimoto, H. and W. F. Chen* Small End Restraint Effects on Strength of H-Columns *Journal of the Structural Division, ASCE, Vol. 108, N. ST3, March 1982, pp. 661–681.*
25. *Vinnakota, S.* Effect of Imperfections on Planar Strength of Restrained Beam-Columns *SSRC-TG23 Report, May 1981.*
26. *Vinnakota, S.* Verification of the SSRC Interaction Formula for Lateral Buckling of Beam-Columns *SSRC-TG23 Report, March 1982.*
27. *Young, C. R. and K. B. Jackson* The Relative Rigidity of Welded and Riveted Connections *Canadian Journal of Research, 1934, 11, No. 1, pp. 62–100, and 1934, 11, No. 2, pp. 524–563.*