Shell Shapes Framed in Steel

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IN CONSIDERING the use of steel framing for shell shapes, one's attention turns immediately to the problems of detailing members and designing joints. The author had occasion to consider these problems while doing the structural design for the Johnson Wax Pavilion at the New York World's Fair (Fig. 1). An architectural feature of this pavilion consists of six "petals" or shell shapes which flare out from the top of columns that also support the theatre. It was decided to frame these petals in steel (Figs. 2 and 3). The framing for one petal consists of a "spine" which is in effect a continuation of the column, curved "ribs" transverse to this spine, and two warped channels and a warped angle which serve as edge beams. In addition to doing the structural design, the author also supplied the fabricator with the geometry required for detailing these members. From this data individual pieces were detailed and shop drawings made.

Fig. 1. The Johnson Wax Pavilion

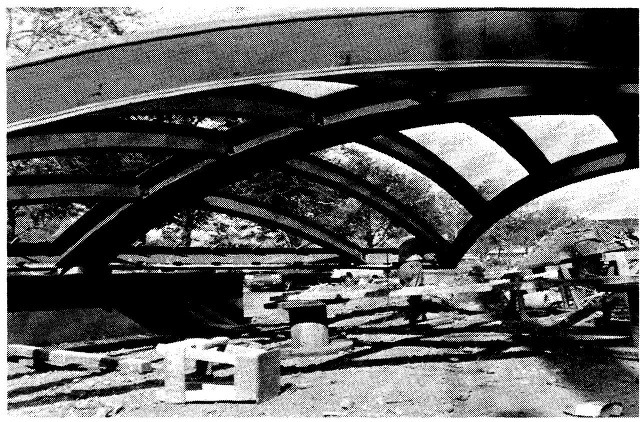

Fig. 2. Steel framing for the petals

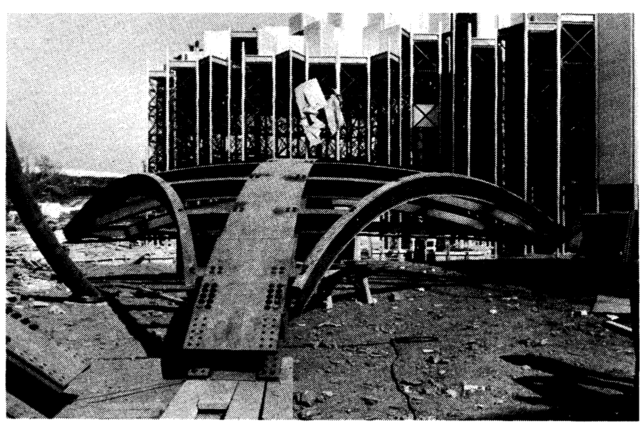

Fig. 3. Alternate view of the framing

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GENERAL ENGINEERING CONSIDERATIONS

When using steel framing for shell shapes, the design of joints capable of transmitting torsion, shear, moment and axial load is greatly simplified by keeping continuous structural members within the surface of the shell. Consequently the flanges of these members are kept at a tangent to this surface. At the point of intersection, these members meet in a plane tangent to the shell, and the joint is no more complicated than if the members themselves were to lie in a flat plane. The angles that these members make with one another in this plane can easily be determined by using methods of vector analysis. The use of descriptive geometry is not advisable since the change in scale from the drawing to the prototype in large structures would result in errors that could not be tolerated in the fabrication process. For this reason an analytical approach is recommended.

Techniques from vector analysis were used instead of formulas from analytic geometry, since these principles are more basic and hence more incisive. Rather than force the problem to fit pre-existing formulas, the necessary relations were derived directly.

This procedure is discussed below in relation to the petals for the Johnson Wax Pavilion. General principles will be discussed at the end of this paper.

DETAILING OF THE PETALS

The petals are segments of a sphere cut by three planes—two of which are vertical and one of which is perpendicular to a vertical plane (the x-y plane in Fig. 4). This surface is represented in spherical coordinates by the variables R, θ, φ with R equal to a constant. The relationship between these variables and the x, y, z coordinates of the cartesian system is given by:

$$x = R \sin\theta \cos\varphi \tag{1}$$

$$y = R \sin\theta \sin\varphi \tag{2}$$

$$z = R \cos\theta \tag{3}$$

$$R = (x^2 + y^2 + z^2)^{1/2} \tag{4}$$

$$\theta = \cos^{-1} \frac{z}{R} = \tan^{-1}\left[\frac{(x^2 + y^2)^{1/2}}{z}\right] \tag{5}$$

$$\varphi = \cos^{-1}\left[\frac{x}{R \sin\theta}\right] = \tan^{-1} \frac{y}{x} \tag{6}$$

The central spine lies on a great circle of the sphere: R = constant, φ variable, $\theta = 90$ degrees. The ribs also lie on great circles with R = constant, φ = constant and θ variable. The ribs are symmetrical in relation to the spine, as are the two vertical cutting planes. One of these planes is represented by points **a-b-a$_H$**. This cutting plane cuts the sphere in a circle of radius r which is less than R, the radius of the sphere. The other cutting plane is represented by points **a-c-c′-a′** and is perpendicular to the vertical plane $\theta = 90$ degrees.

Points **a** and **a′** are control points. The x, z, and R coordinates of these points are given; the unknown coordinates θ, φ and y are determined from Equations (5), (6) and (2) in that order. Vectors **u** and **u′** of adjacent petals are tangent at these points. The tangent vector is represented by **u**. The horizontal projection of this vector (indicated by $\mathbf{u_H}$), makes an angle of 30 degrees with the axis of the petal. Thus six petals form a closure of 360 degrees. The components of vector $\mathbf{u_H}$ are known. The vector **u** has the same x and z components but the y component is unknown. Since **u** is tangent to the spherical surface, in the spherical coordinates its R component is zero. Making a coordinate transformation from cartesian to spherical coordinates gives the R component of **u** in terms of the x, y, z components of **u**. By setting this equal to zero the unknown y component is determined. In a similar manner the unknown y component of **v** is also found. By dividing these vectors by their respective lengths the corresponding unit vectors are determined and the cosine of the angle between these vectors obtained by taking the dot product. (The dot product is a standard mathematical technique in vector analysis.) This is the angle between the built-up angle and the built-up channel in section L-1, Fig. 5. (Point **1** corresponds to point **a** in the discussion above). This section lies in the plane tangent to the shell at this point, i.e. the plane formed by vectors **u** and **v**. The unit vector along the axis of the 10 W is simply the base vector $\mathbf{e}_\theta$ (R and φ are constant); hence dot products

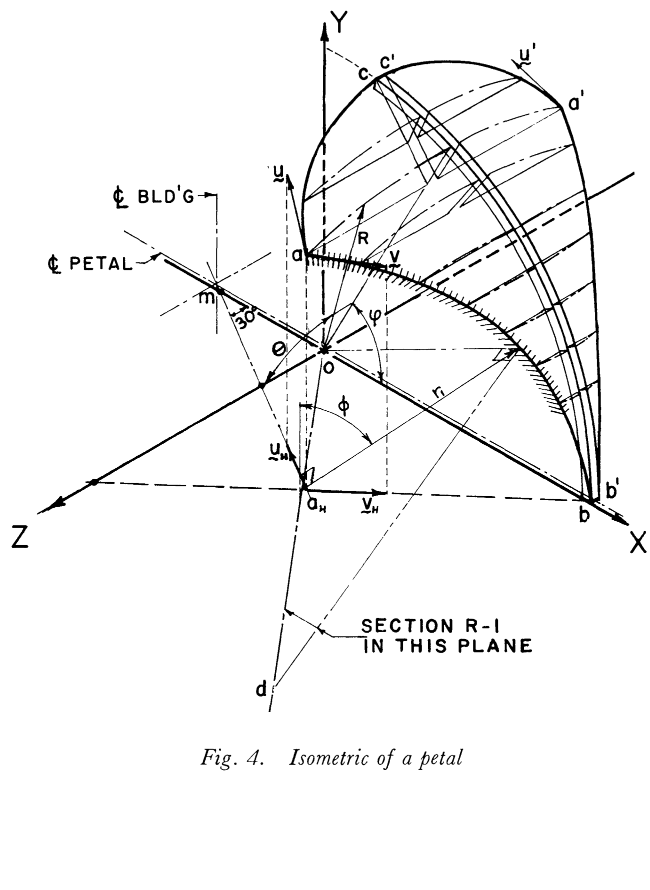
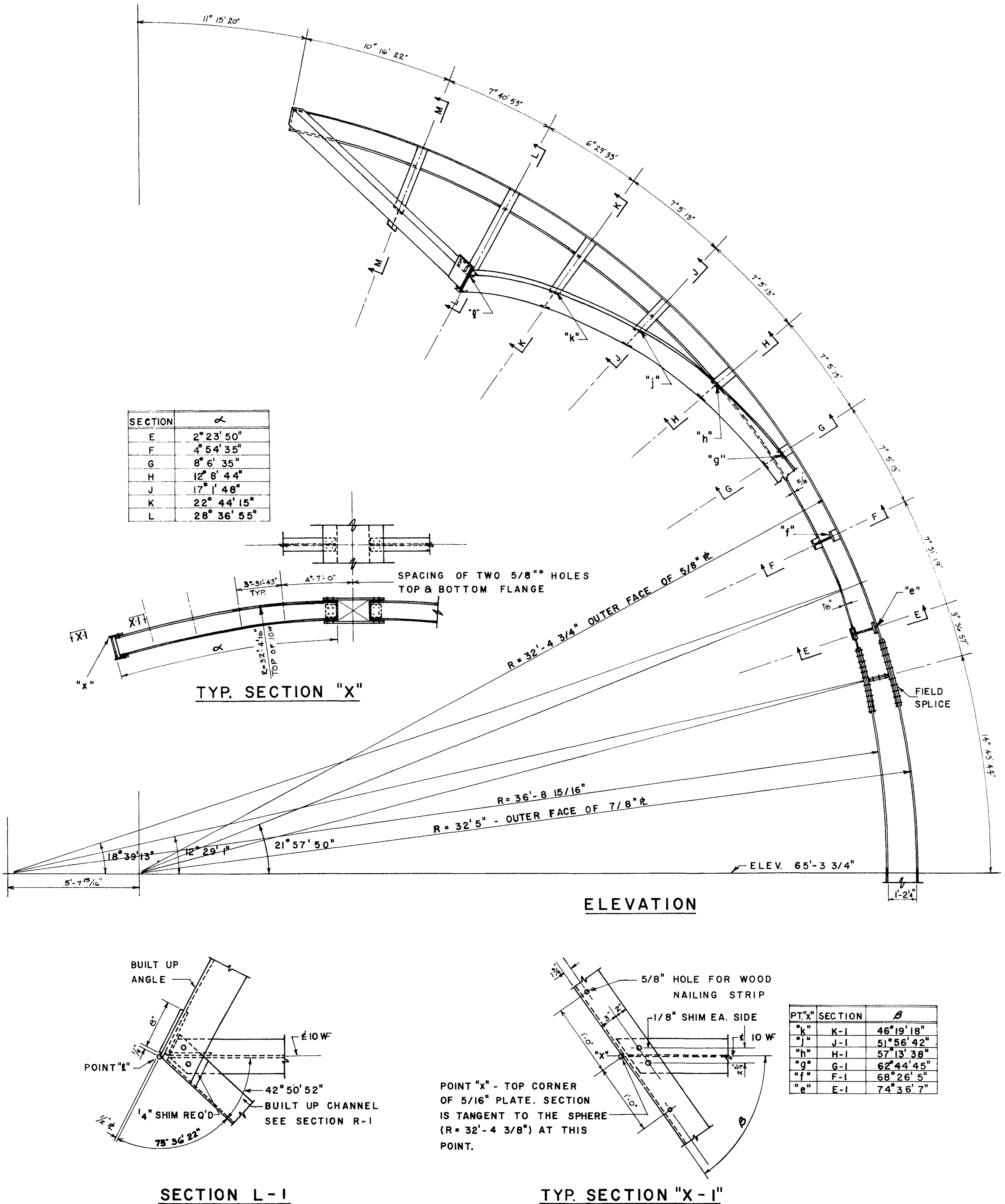


Fig. 4. Isometric of a petal



SECTION	α
E	2° 23' 50"
F	4° 54' 35"
G	8° 6' 35"
H	12° 8' 44"
J	17° 1' 48"
K	22° 44' 15"
L	28° 36' 55"

PT. "X"	SECTION	β
"k"	K-1	46° 19' 18"
"j"	J-1	51° 56' 42"
"h"	H-1	57° 13' 38"
"g"	G-1	62° 44' 45"
"f"	F-1	68° 26' 5"
"e"	E-1	74° 36' 7"

Fig. 5. Geometry of petals

easily give the angles between these members. The length of arc to point **l** — Section L (see typical Section X) — is simply $90 - \theta = \alpha$.

In a similar manner the members shown in Sections E through K (typical Section X) were detailed. For these members the angle φ was given (Elevation—Fig. 5), then the angle $\alpha = 90 - \theta$ was determined (typical section X). The unit vector tangent to the sphere and lying in the cutting plane was next found and from the dot product with $\mathbf{e}_\theta$ the angle β of typical Section X-1 was determined.

The built-up channel into which these 10 WF ribs fit is detailed in Fig. 6. Section R lies in the cutting plane previously described. The location of Sections E-1 through L-1 is also shown in this section. This built-up channel was formed from plates cut to one radius in the plane of the plate and rolled to another radius in the plane perpendicular to the plate (Section R-1). In this manner the flanges and the web were fabricated and then welded together. This was possible since the flanges and web effectively lie on the surface of concentric cones (Fig. 4 and Section R-1 of Fig. 6). The warped angle lying along the arc of **a-c-c′-a′** of Fig. 4 was fabricated in a similar manner.

In addition to giving arc lengths of all members and angles of intersection of these members, bolt holes were located so that these members could be joined together in the field. Bolt holes for wood nailing strips were also located.

The shop drawings were produced from plans similar to Figs. 5 and 6. The pieces were then fabricated to the required lengths and offsets with angles properly cut off and all holes in place. The pieces were then brought together and although curving through space in different directions the bolts dropped into place and neither field cutting nor welding was required.

Wood nailing strips were then bolted in place and to these were attached two layers of ¼-in. plywood (Fig. 7). The plywood was given its finished coats of paint on the ground and then the petals were lifted for the field

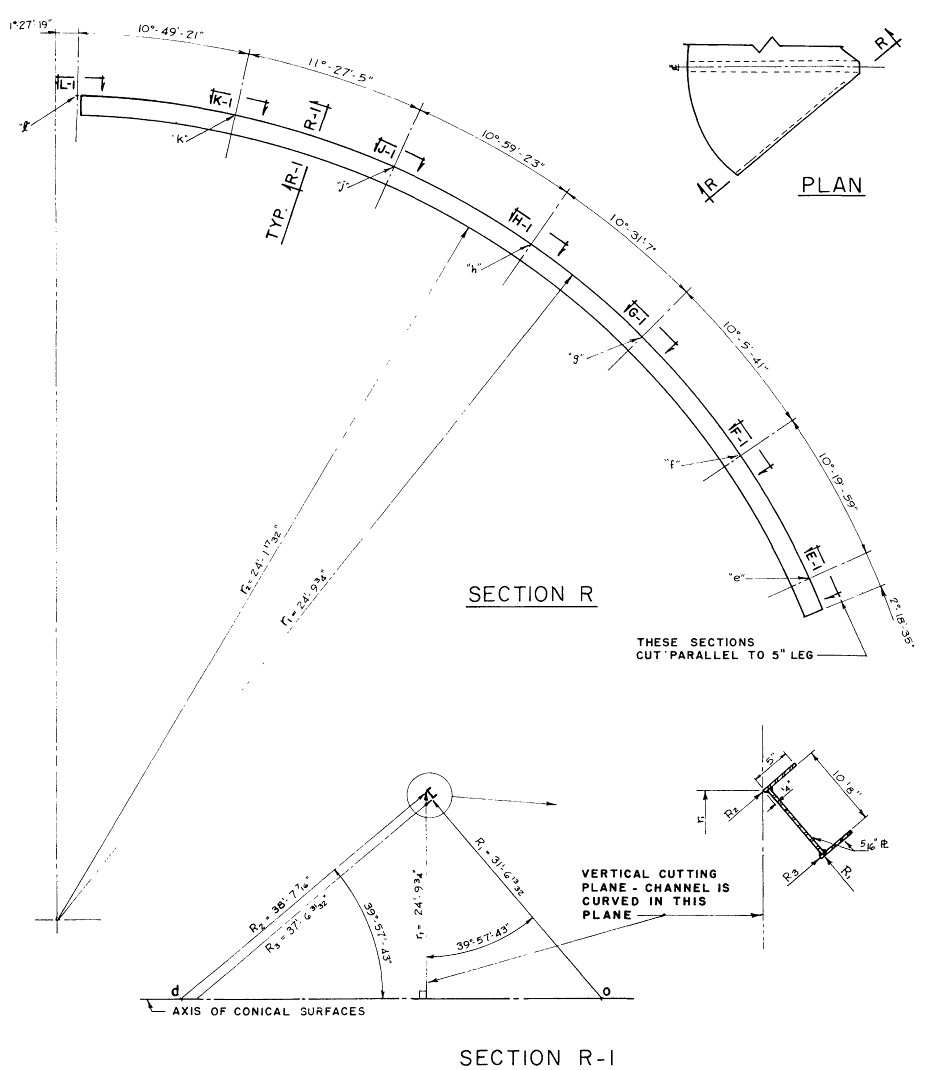


Fig. 6. *Details of warped channel*

connection to be made approximately 63 ft above grade (Fig. 8). Only one crane was used. One petal was lifted at a time and then the field splice made at the top of the column. As the crane released its load, the column deflected inward due to the cantilever of the petal. After the field splice of adjacent petals had been made, the connection between them was secured. This occurred at point **a** of Fig. 4 (point **l** of Section L-1). Since the petals were tangent at this point, the connection consisted simply of two flush ½-in. plates 8 in. wide protruding 4 in. above the surface of the shell (Fig. 5). The plate from one petal had two 1-in. wide by 2-in. long slots perpendicular to the shell surface, and for the other they were tangent to this surface. These slots received ⅞-in. diameter high strength bolts. After five petals had been erected and interconnected the ring was opened by cables attached to trucks, the sixth petal was placed, the ring closed and the interconnection between the petals completed.

The final sheathing was placed around the field splice and in the openings provided for the lifting of the petals. The crane boom was used for this purpose.

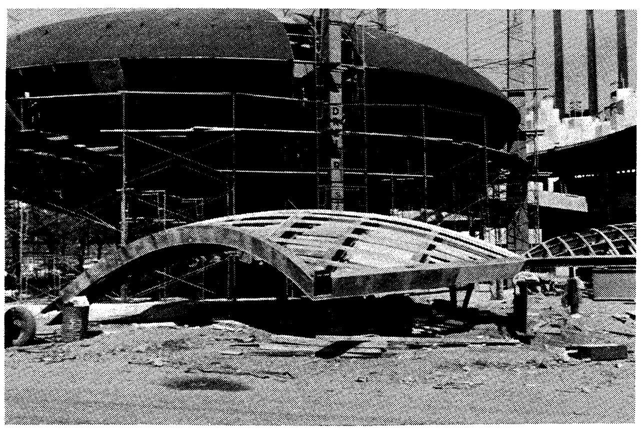


Fig. 7. Sheathing of the petal

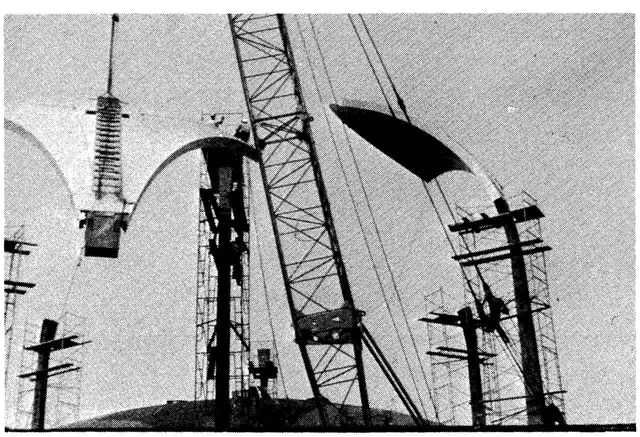


Fig. 8. Erection of petals

GENERAL CONCLUSIONS

The details of the fabrication and erection of the petals of the Johnson Wax Pavilion are interesting in themselves, but more important are the principles that have general application to the steel framing of shell shapes.

From the previous discussion it can be concluded that by keeping those members that are continuous within the surface of the shell (i.e., the ribs, spine and edge beams of the petal), the design and fabrication of joints is greatly simplified. In addition the skin or enclosing surface may be easily attached to these members since blocking up to the shell surface is not required.

Whenever possible, continuous members should lie on the coordinate lines of the curvilinear coordinate system used to describe the shell surface. Usually this will mean that these members are curved in a plane. When the coordinate system is orthogonal these members will also intersect at a right angle as do the ribs and the spine of the petals. Under these conditions the length of these members is a function of only one of the curvilinear coordinates. Because of this the intersection with members that do not lie on coordinate lines, such as edge beams, is easily determined. The intersection of the ribs with the warped channel is a case in point.

What is gained in the simplification of joint design and the attachment of the enclosing surface is lost in the fabrication of certain members. This is particularly the case when these members have to be built up from curved plates. In general this occurs when the normal to the shell surface does not lie in the plane in which the member is curved and arises because of the requirement that the flanges of these members must lie within the surface of the shell. Such is the case with the warped channels which serve as edge beams for the petals. It would also occur for those members of a shell of revolution that lie in planes perpendicular to the axis when the normal to the shell is not perpendicular to this axis. Because of the added expense in the fabrication of these members, the continuity in this direction should be developed in this manner only when its necessity is dictated by structural or fabrication considerations.

STEEL FRAMING FOR AN AUDITORIUM

Figure 9 shows an extension of these principles to the steel framing of an auditorium. The arches radiating from the center and those that serve as edge beams are continuous and are detailed in the manner previously discussed. The outward thrust of the arches that intersect the edge beam is transmitted by ties or by a reinforced concrete membrane to three very stiff arches that extend to the point supports. The continuous edge beams transmit the reaction component in their plane, and the bending moment, due to the continuity of the connection, to these same point supports.

If this were a reinforced concrete shell, a rigorous analysis would not be possible due to the edge conditions. These same edge conditions also create large bending moments which in part destroy the efficient behavior of the continuous shell surface. In addition the pouring of this surface would also require expensive formwork and although an enclosing surface would be obtained, extremes of climate in the United States would often make an additional waterproofing membrane necessary.

By building such a shell shape in steel:

1. A rigorous analytical solution can easily be obtained through the use of an electronic digital computer.
2. Members in bending can be efficiently designed for this purpose.
3. Formwork is not required.
4. An enclosing surface can easily be attached to the steel.
5. A much lighter structure would be obtained.

If the concrete membrane were used, the ties could be eliminated, composite action would result, and a very efficient structure would be obtained. By leaving the steel lattice exposed against the concrete membrane a striking architectural feature would result.

The suggestions made in this paper should enable steel to be used in many previously untried and imaginative ways, thus giving the architect greater freedom in his choice of forms while at the same time producing more economical structures.

PROJECT CREDITS—JOHNSON WAX PAVILION

Architects: Lippincott & Margulies, Inc. Industrial Designers
Structural Engineer: Severud, Elstad, Krueger Assoc.
Contractor: Turner Construction Co.
Structural Steel Fabricator: Dreier Steel Co., Inc.

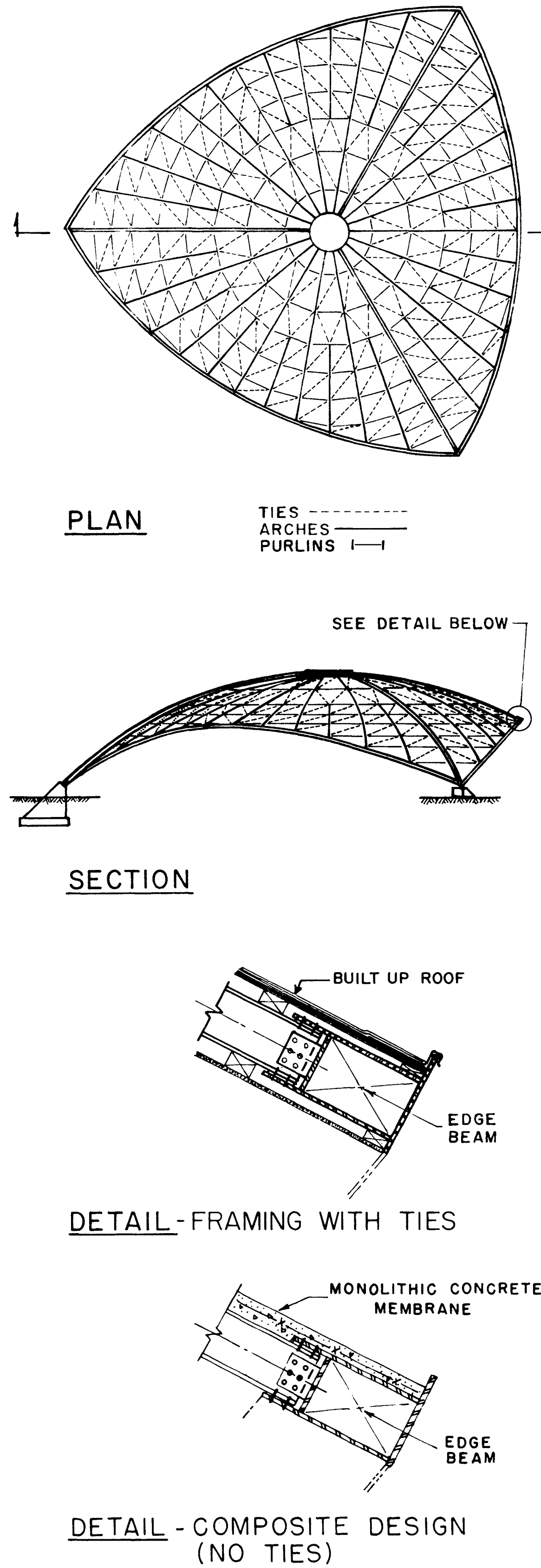


Fig. 9. Steel framing for an auditorium