Allowable Stress for Bending Members

NIRMAL MAITRA

The allowable bending stress is a very important design parameter. It controls not only the design of beams, but also of columns when subjected to bending in addition to axial load. The following design examples in the AISC *Manual*[1] show how the allowable bending stress controls the design:

A. Beam design
   M_R method: p. 2–4
   S_x method: p. 2–4

B. Column design
   Combined stress: p. 3–11

(Nomenclatures used are found at the end of this paper.)

The magnitude of the allowable bending stress is a major parameter in the design process. But since its value is not constant (see later detailed discussion), it often becomes a major impediment in quick design of a member. The AISC *Manual*[1] has a table for allowable stress of compression members. But it does not have a similar table for bending members. As a result, the design engineer has to labor through formulas. This process is not only time consuming but also it is susceptible to human errors. This paper presents a method which eliminates that procedure. It includes a table and a simple formula, and their use is explained in this paper.

The basic theory on which the table and the formula have been constructed will first be discussed.

The allowable bending stress changes from one stress behavior to another in three distinct stages, as shown in Fig. 1. These stages and the corresponding AISC Specification formulas for the stress behavior are:

Case 1: $F_b = 0.66F_y$

When laterally unsupported length of the beam compression flange, l, does not exceed L_c,
where $L_c = 76\ b_f \div \sqrt{F_y}$, or
$L_c = 20{,}000 \div (d/A_f)F_y$,
whichever is smaller. (Sect. 1.5.1.4.1.5)

Nirmal Maitra is a Design Leader with Foster Wheeler Energy Corporation, Livingston, New Jersey.

Case 2: $F_b = 0.6F_y$

When laterally unsupported length of the compression flange exceeds L_c but is less than L_u,
where $L_u = r_T\sqrt{102 \times 10^3 C_b/F_y}$, or
$L_u = 20{,}000\ C_b \div (d/A_f)F_y$,
whichever is larger. (Sect. 1.5.1.4.5.2)

Case 3: When the lateral support length of the compression flange exceeds L_u, the largest value of F_b computed from the following formulas is used for the design, but not more than $0.6F_y$:

$$F_b = \left[\frac{2}{3} - \frac{F_y(l/r_T)^2}{1530 \times 10^3 C_b}\right] F_y \qquad (1.5\text{-}6a)$$

when

$$\sqrt{\frac{102 \times 10^3 C_b}{F_y}} \leq \frac{l}{r_T} \leq \sqrt{\frac{510 \times 10^3 C_b}{F_y}}$$

$$F_b = \frac{170 \times 10^3 C_b}{(l/r_T)^2} \qquad (1.5\text{-}6b)$$

$$\text{when } \frac{l}{r_T} \leq \sqrt{\frac{510 \times 10^3 C_b}{F_y}}$$

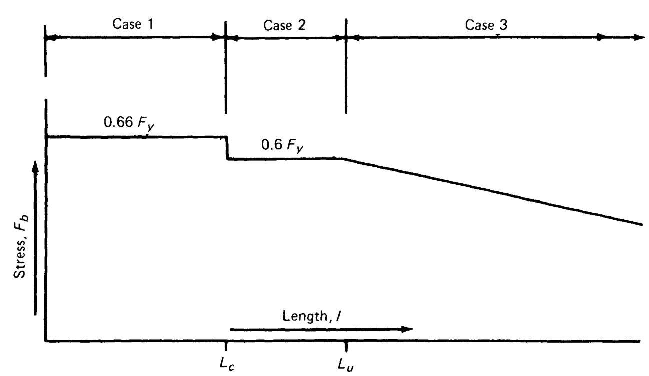

Fig. 1. Allowable bending stress

$$F_b = \frac{12 \times 10^3 C_b}{l \cdot d/A_f} \qquad (1.5\text{-}7)$$

As shown in the above relationships, the value of L_c and L_u determines the stress behavior stage. The first job for the design engineer is to work out the value of L_c and L_u from formulas given above. However, the AISC *Manual*[1] provides values of L_c and L_u for all W, S, and M shapes. This obviates the necessity of calculating them from the formulas.

With values of L_c and L_u known, the value of F_b can now be worked out. When l, laterally unsupported length of beam compression flange, is less than L_u, the solution for F_b is straightforward. F_b is $0.66F_y$ when $l < L_c$, i.e., Case 1. $F_b = 0.6F_y$ when $L_c < l < L_u$, i.e., Case 2.

When l is larger than L_u, the situation becomes complicated. The traditional way is to solve the equations (1.5-6a) or (1.5-6b), and (1.5-7) and the highest value obtained less than or equal to $0.6F_y$ is accepted for the design.

This procedure is very time consuming. When there are a large number of beams to be designed or when trial cases are to be repeated, loss of time is frustratingly heavy.

Table 1 presented with this paper is aimed to do away with this time-consuming calculation process. This table gives the allowable stresses for A36 bending members, assuming $C_b = 1$, for different values of l/r_T. It is similar in format to the AISC Table 3-36 (p. 5-74, Ref. 1) which gives allowable stresses for compression members. There is, however, a difference between AISC Table 3-36 and Table 1 presented with this paper. While the former is applicable for all cases of compression members, the latter is good only in certain conditions, i.e., when Eq. (1.5-7) does not govern the design of the bending member. The author presents a method to check through a simple calculation. This check immediately indicates whether Table 1 should be used or not. In case the check indicates that Table 1 is not applicable and Eq. (1.5-7) governs, the author presents a substitute for Eq. (1.5-7) so that the provisions of this equation are satisfied and calculation is completed at faster speed.

The proposed design procedure is fully explained here. Illustrative examples will demonstrate its use.

Table 1. Allowable Stress (ksi) for Bending Members (F_y = 36 and C_b = 1)

l/r_T	F_b (ksi)	l/r_T	F_b (ksi)	l/r_T	F_b (ksi)	l/r_T	F_b (ksi)	l/r_T	F_b (ksi)
l	24.0	81	18.2	111	13.3	141	8.6	171	5.8
53	22.0	82	18.1	112	13.1	142	8.4	172	5.7
54	21.3	83	17.9	113	12.9	143	8.3	173	5.7
55	21.2	84	17.8	114	12.8	144	8.2	174	5.6
56	21.1	85	17.6	115	12.6	145	8.1	175	5.6
57	21.0	86	17.5	116	12.4	146	8.0	176	5.5
58	20.9	87	17.3	117	12.2	147	7.9	177	5.4
59	20.8	88	17.2	118	12.0	148	7.8	178	5.4
60	20.7	89	17.1	119	11.8	149	7.7	179	5.3
		90	16.9	120	11.8	150	7.6	180	5.2
61	20.6								
62	20.5	91	16.7	121	11.6	151	7.5	181	5.2
63	20.4	92	16.6	122	11.4	152	7.4	182	5.1
64	20.3	93	16.4	123	11.2	153	7.3	183	5.1
65	20.2	94	16.3	124	11.1	154	7.2	184	5.0
66	20.1	95	16.1	125	10.9	155	7.1	185	5.0
67	20.0	96	16.0	126	10.7	156	7.0	186	4.9
68	19.8	97	15.8	127	10.5	157	6.9	187	4.9
69	19.7	98	15.6	128	10.4	158	6.8	188	4.8
70	19.6	99	15.5	129	10.2	159	6.7	189	4.8
		100	15.3	130	10.1	160	6.6	190	4.7
71	19.5								
72	19.4	101	15.1	131	9.9	161	6.6	191	4.7
73	19.2	102	14.9	132	9.8	162	6.5	192	4.6
74	19.1	103	14.8	133	9.6	163	6.4	193	4.6
75	19.0	104	14.6	134	9.5	164	6.3	194	4.5
76	18.9	105	14.4	135	9.3	165	6.2	195	4.5
77	18.7	106	14.2	136	9.2	166	6.2	196	4.4
78	18.6	107	14.1	137	9.1	167	6.1	197	4.4
79	18.5	108	13.9	138	8.9	168	6.0	198	4.3
80	18.3	109	13.7	139	8.8	169	6.0	199	4.3
		110	13.5	140	8.7	170	5.9	200	4.2

DESIGN PROCEDURE

1: Obtain from the AISC *Manual*[1] the following properties of the beam: L_c, L_u, d/A_f, r_T

2: Find the value of L_r, where $L_r = 1.18(d/A_f)(r_T)^2$
Note: L_r is in ft

3a: If $L_r \geq l$, use Table 1 to find F_b

3b: If $L_r \leq l$, use following relationship to find F_b:
$F_b = 0.6F_y(L_u/l)$

ILLUSTRATIVE EXAMPLES

Example 1

Problem: Find F_b for W14x22 when $l = 8'$ ($F_y = 36$ ksi)

Solution:

1. From AISC *Manual*, $L_c = 5.3'$, $L_u = 5.6'$
$d/A_f = 8.2'$, $r_T = 1.25$

As $l > L_u$, this problem belongs to Case 3 (explained earlier).

2. $L_r = 1.18(d/A_f)(r_T)^2 = 1.18(8.2)(1.25)^2$
$= 15.12 > 8' \rightarrow$ Use 3a

3a. $l/r_T = 8 \times 12/1.25 = 77$
From Table 1, $F_b = 18.7$ ksi

Example 2

Problem: Solve Example 1 when $l = 16'$

Solution:

1: Same as Example 1

2: $L_r = 15.12 < 16' \rightarrow$ Use 3b

3b: $F_b = 0.6F_y(L_u/l) = 21.6\ (5.6/16) = 7.56$ ksi

CONCLUSION

This paper presents a table and a formula which would help to find the value of the allowable bending stress, F_b, expeditiously. The table and formula have been derived from AISC equations. Examples illustrate their use.

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NOMENCLATURE

d = Depth of column, beam or girder (in.)
l = For beams, distance between cross-sections braced against twist or lateral displacement of the compression flange (in.)
r_T = Radius of gyration of a section comprising the compression flange plus one-third of the compression web area taken about an axis in the plane of the web (in.)
A_f = Area of compression flange (sq. in.)
C_b = Bending coefficient
F_b = Bending stress permitted in a prismatic member in the absence of axial force (ksi)
F_y = Specified member yield stress of the type of steel being used (ksi)
L_c = Maximum unbraced length of compression flange at which the allowable bending stress may be taken as $0.66F_y$ (ft)
L_u = Maximum unbraced length of compression flange at which the allowable bending stress may be taken as $0.6F_y$ (ft)
L_r = Maximum unbraced length of compression flange at which the allowable bending stress may be calculated from AISC equations (1.5-6a) and (1.5-6b) (ft)
M_r = Beam resisting moment (kip-ft)
S_x = Elastic section modulus about the x-x axis (in.^{3})

REFERENCES

1. Manual of Steel Construction *Eighth Edition, America Institute of Steel Construction, Inc., Chicago, Ill., 1980.*
2. *Roy, Ranjit* Stress Reduction Factors for Unsupporte Lengths *AISC Engineering Journal, 2nd Quarter, 1981.*