# A General Solution for Eccentric Loads on Weld Groups

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Eccentric loads on weld groups traditionally have been analyzed by elastic methods in which the individual effects of an axial load (applied through the centroid of the weld group) and a pure moment (in the plane of the weld group) were combined.[1] In the 7th Edition *Manual of Steel Construction*[2] of the American Institute of Steel Construction, as in earlier editions, such a procedure was used to create tables which could be used to determine allowable eccentric loads on selected weld groups.

These tables were criticized as having non-uniform factors of safety when compared to the actual ultimate loads which the welds could support. An "ultimate strength method" was proposed[3] in which the resulting force per unit of length of each weld element is calculated from

$$R = R_{\text{ult}}(1 - e^{-\mu\Delta})^{\lambda}$$

and the total resisting forces and moments are predicted by summing the elemental forces. In the 8th Edition Manual,[4] such a procedure was used to create tables similar to those in earlier editions, but with different numerical coefficients.

Butler, Pal, and Kulak[3] describe the ultimate strength method in some detail. For the sake of completeness in this report, this procedure is summarized as follows:

1. Choose an instantaneous center of rotation. (Fig. 1)
2. Assume that the resisting force on any weld element acts perpendicularly to a radius connecting that element to the instantaneous center.
3. Calculate the angle, $\theta$, between the elemental force and the axis of the weld element. (Angle $\theta$ is expressed in degrees.)
4. Determine the ultimate deformation which can occur on each weld element from

$$\Delta_{\max} = 0.225(\theta + 5)^{-0.47}$$

5. The weld element which will reach its ultimate defo mation first is the one for which the ratio of $\Delta_{\max}$ d vided by the radius to the instantaneous center is th smallest. It is assumed that deformations vary linearl with distance from the instantaneous center.
6. Consistent deformations ($\Delta$) at all other weld elemen are then found from

$$\Delta_i = r_i(\Delta_{\max}/r)_{\min}$$

7. The following parameters are then calculated for ea weld element:

$$R_{\text{ult},i} = \frac{10 + \theta_i}{0.92 + 0.0603\theta_i}$$

$$\mu_i = 75e^{0.0114\theta_i}$$

$$\lambda_i = 0.4e^{0.0146\theta_i}$$

$$R_i = R_{\text{ult},i}(1 - e^{-\mu_i\Delta_i})^{\lambda_i}$$

8. By statics, calculate the corresponding applied force a moment which hold the weld forces in equilibrium.

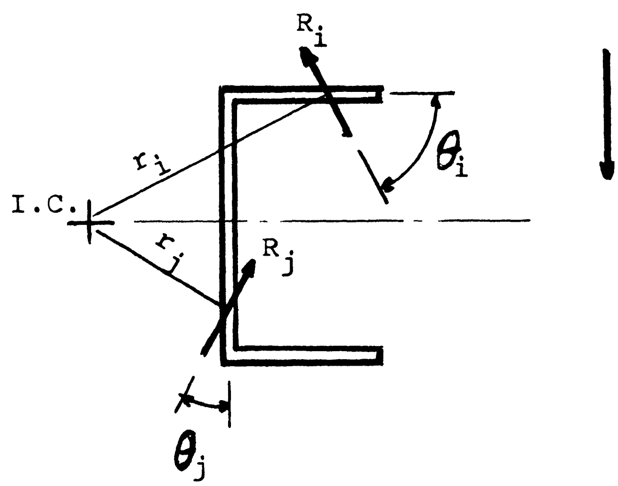


*Figure 1*

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It should be noted that the formulas given above correspond to a 1/4-in.weld made using E60 electrodes. The $R_i$ values are pounds per linear inch of weld. The final forces and moments are ultimate values.

Tide[5] described the way in which these values were converted to allowable load coefficients for the AISC tables. In summary, this requires:

1. Converting from 1/4-in. weld to 1/16-in. weld by dividing by 4.
2. Converting from E60 electrode to E70 electrode by multiplying by 70/60.
3. Introducing a safety factor conforming to AISC Specification Sect. 1.5.3 by multiplying by 0.3.
4. Checking the shear stress on the most highly stressed weld element and, if it exceeds 21.0 ksi, reducing the load by the ratio of 21.0 divided by that shear stress.

The papers by Tide[5] and by Butler et al[3] indicate that solutions have been obtained using computer programs. Butler states that the method is general, but shows details for only a C-shaped weld subjected to loading parallel to a principal axis. Tide shows results for a pair of parallel line welds and a C-shaped weld with the loads again parallel to principal axes. The AISC tables include a pair of parallel line welds, rectangular box welds, C-shaped welds, and L-shaped welds; loads are parallel to a principal axis in the first three types and parallel to a leg of the L in the last.

In a previous paper,[6] the author showed how rapid solutions could be obtained for any eccentrically loaded bolt groups. The same method can be extended to weld groups. In essence, the method involves:

1. Directly finding the instantaneous center corresponding to elastic behavior of any weld arrangement for any eccentric load.
2. Directly determining the elastic solution for the maximum permissible load.
3. Directly determining an *approximate* value for the ultimate load.
4. Iterating to improve the approximate value.
5. Using the same procedure described by Tide to convert the ultimate load to an allowable load consistent with the AISC tables.

Inasmuch as welds are continuous, it is necessary to discretize into a finite number of weld elements. A moderately large number of discrete elements is required if reasonable accuracy is to be achieved, so the rather long procedure for each weld element described above probably makes the procedure too laborious for manual calculations. Accordingly, the procedure is given here as a FORTRAN computer program. (See Appendix.)

Engineers familiar with FORTRAN should have no particular difficulty in understanding the program after reading the following discussion.

## DISCUSSION OF THE COMPUTER PROGRAM

**Computations for the Elastic Solution**—The length of each weld element, $W$, is calculated, and using these discrete elements, the centroid is located, the polar moment of inertia, $J$, is calculated, and the moment, $M_o$, of the applied unit force about the centroid is calculated.

A mapping function called FACTOR is used to transform forces to distances and it is equal to $J$ divided by the product of the total length of weld and moment $M_o$.

The instantaneous center is located by adding to the $X$ and $Y$ coordinates of the centroid the quantities $-P_y \times$ FACTOR and $+P_x \times$ FACTOR, respectively.

The radius vector, $D$ to the center of each weld element is calculated, and the largest one noted. The allowable moment about the instantaneous center of all elemental forces is $\Sigma WD^2/D_{\max}$ times the allowable force per inch of weld. The allowable (elastic) load is that moment divided by the moment of the applied unit force about the instantaneous center. This load and the identification number of the critical weld element are printed.

**Computations for the First Approximate Ultimate Load**—The instantaneous center located above is used as the first trial center. Calculations follow the procedure attributed to Butler[3] described earlier in this paper. (Angle $\theta$ is determined as follows: Form the dot product of the radius vector and the weld element vector. Divide this by the product of the magnitude of the two vectors; this quotient is the cosine of the angle between the vectors. Its complement is angle $\theta$.)

The (first approximate) ultimate load is the moment about the instantaneous center of all the elemental forces ($R_i$ times $W_i$ times $D_i$) divided by the moment about the instantaneous center of the applied unit load. This ultimate load, and the identification number of the critical weld (having the largest force per inch) are printed, along with the coordinates of the current instantaneous center.

**Iterating to Improve the Approximate Ultimate Load**—When the instantaneous center is correctly located, not only will the sum of the moments about that center be zero, but the vector sum of the forces will be zero as well.

For the ultimate strength case, the $X$ component of the force on each weld element due to the unit load can be found from

$$R_{xi} = -\frac{D_{yi}}{D_i} R_i W_i \frac{1}{P_{\text{ult}}}$$

and similarly for the $Y$ component.

The sum of all the $R_x$ values plus $P_x$ will not be zero unless the instantaneous center is correct. The unbalanced force is $F_{xx}$. Similarly for the $Y$ direction. The vector sum of $F_{xx}$ and $F_{yy}$ is $F$, the unbalanced force. $F_{xx}$ , $F_{yy}$ and $F$ are printed when the approximate ultimate load is printed.

It has been found that components of a desirable shift from the previous trial center can usually be predicted from the same formulas used to locate the elastic instantaneous center. Thus, if $x_1$ and $y_1$ represent the coordinates of the instantaneous center for which the unbalanced force components are $F_{xx}$ and $F_{yy}$ , the coordinates of the next center should be:

$$x_2 = x_1 - (F_{yy} \times \text{FACTOR})$$

$$y_2 = y_1 + (F_{xx} \times \text{FACTOR})$$

New values for the radius vectors to all weld elements are now calculated and the ultimate strength solution is repeated. In most cases, the unbalanced force decreases rapidly, and the ultimate load stabilizes after a few iterations. In the program, when the unbalanced force is less than 1% of the applied unit force, the solution is declared found.

The ultimate load is converted to permissible load on a $\frac{1}{16}$-in. weld made using E70 electrodes in the manner attributed to Tide[5] described earlier. This value is printed.

It should be observed in using the program that, as in the case of AISC tables, the permissible load on the weld group is obtained by multiplying the permissible load given by the program by the number of sixteenths in the weld size. Furthermore, if the electrodes used are other than E70 (having $F_v$ = 21 ksi), the permissible load should be multiplied by the allowable shear divided by 21 ksi. Unlike the AISC tables, no multiplication by any length should made.

**Input**—In creating the program, provision was initially made for describing the weld configuration in two different ways. (Both of these have been retained.)

1. If the first columns on a data card contain the word LINE, the description of a weld consists of the coordinates of its starting point and ending point, and the number of equal segments into which the weld is to be subdivided.
2. If the first columns on the data card contain the word ELEMENT, the description of a weld element consists of the coordinates of its center and the length of its $X$ and $Y$ projections on the coordinate axes.

If the first columns on the data card contain the word LOAD, the description of a normalized load (magnitude = 1) consists of its X and Y projections on the coordinate axes, and the coordinates of a point on its line of action.

Observe that there may be as many data cards of LINE and/or ELEMENT type as required to describe the entire weld. Such cards may be in any order or any mixture. The *last* data card must be a LOAD card, since this is also a signal that all data has been given. (Format for all three types of data cards is A2, 8X, 5F10.3. A fourth form of data is discussed later.)

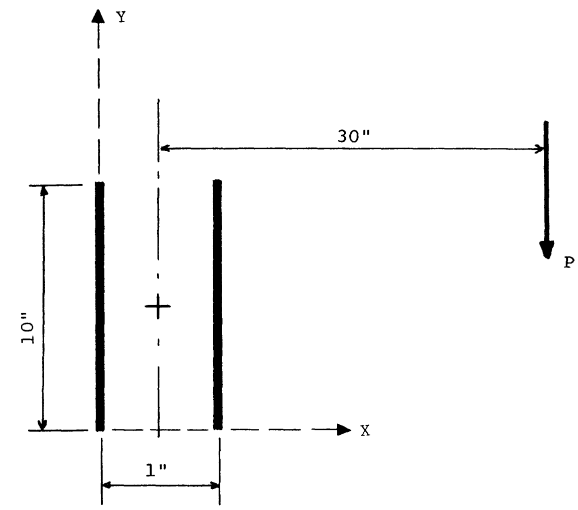


*Figure 2*

The program converts weld descriptions of either type into weld elements. These are numbered consecutively and printed, giving their identification numbers, center coordinates, and X and Y projected lengths. The LOAD data is also printed.

**Using the Program**—In all the examples, the maximum permitted elastic load, $P_e$, the ultimate load, $P_u$, and the maximum permitted load, $P_p$, derived from the ultimate load, are listed.

*Example 1 (see Fig. 2):*

Data cards were prepared using the LINE and LOAD descriptions. Actual computer output is shown in Fig. 3 for a case where each line is subdivided into 10 equal elements.

A reasonable question arises in every problem regarding the number of subdivisions required to get good accuracy. For the weld geometry and load of Example 1, computer runs were made dividing each line into 2, 5, 10, 20, and 40 segments. The results are shown in Table 1.

**Table 1**

| Subdivisions | 2 | 5 | 10 | 20 | 40 |
|---|---|---|---|---|---|
| $P_e$ | 1.539 | 1.253 | 1.151 | 1.100 | 1.075 |
| $P_u$ | 26.304 | 24.783 | 24.741 | 24.593 | 24.499 |
| $P_p$ | 1.565 | 1.472 | 1.469 | 1.460 | 1.455 |

Values obtained from the 7th Edition AISC Manual (for the elastic solution) and from the 8th Edition AISC Manual (for the ultimate strength solution) are, respectively, $P_e$ = 1.05 and $P_p$ = 1.45.

```
DATA FOR DESCRIPTION OF WELDS AND LOAD

LINE            C.C000    C.0000    0.0000   10.0000    10.0
LINE            1.0000    C.0000    1.C000   10.0000    10.0
LOAD            0.0000   -1.0000   30.5000    0.0000

   NUMBER OF WELD ELEMENTS =      20
COORDINATES AND PROJECTICNS
   1      0.0000     0.5000     0.0000     1.0000
   2      0.0000     1.5000     0.0000     1.C000
   3      0.0000     2.5000     0.0000     1.0000
   4      0.0000     3.5C00     0.0000     1.0000
   5      0.0000     4.5000     0.0000     1.0000
   6      0.0000     5.5000     C.0000     1.0000
   7      0.0000     6.5000     0.0000     1.0000
   8      0.C000     7.5000     0.0000     1.C000
   9      0.C000     8.5000     0.0000     1.0000
  10      0.0000     9.5000     0.0000     1.0000
  11      1.C000     0.5000     0.0000     1.0000
  12      1.C000     1.5000     0.0000     1.0000
  13      1.0000     2.5000     0.0000     1.0000
  14      1.C000     3.5000     0.0000     1.0000
  15      1.C000     4.5000     0.0000     1.0000
  16      1.0000     5.5C00     0.0000     1.0000
  17      1.0000     6.5000     0.0000     1.0000
  18      1.0000     7.5000     0.0000     1.0000
  19      1.C000     8.5000     0.0000     1.0000
  20      1.C000     9.5000     0.0000     1.C000

PX=       0.C000,PY=     -1.0000, POLX =     30.5000,POLY =       0.0000

ELASTIC VALUE FOR MAXIMUM LOAD IS      1.151
        MULTIPLY THIS BY NUMBER OF SIXTEENTHS
        AND BY ALLOWABLE KSI / 21.0
CRITICAL ELEMENT IS NUMBER   11


AT TRIAL CENTER NO.    1
        X=      0.2139     Y=      5.0000
                   FX =     -0.0000    FY =      0.3313
                             F =    0.3313
                   APPRCXIMATE ULTIMATE LOAD =      24.748
        CRITICAL ELEMENT IS NUMBER     1
        ON WHICH ULTIMATE FORCE PER INCH IS     15.6361
```

*Figure 3. Computer printout for Example 1*

```
AT TRIAL CENTER NO.    2
        X=      0.3087     Y=      5.0000
                   FX =     -0.0000    FY =     -0.1052
                                F =      0.1052
                   APPROXIMATE ULTIMATE LOAD =      24.743
        CRITICAL ELEMENT IS NUMBER    1
        ON WHICH ULTIMATE FORCE PER INCH IS    15.6238


AT TRIAL CENTER NO.    3
        X=      0.2786     Y=      5.0000
                   FX =     -0.0000    FY =      0.0344
                                F =      0.0344
                   APPROXIMATE ULTIMATE LOAD =      24.740
        CRITICAL ELEMENT IS NUMBER    1
        ON WHICH ULTIMATE FORCE PER INCH IS    15.6277


AT TRIAL CENTER NO.    4
        X=      0.2884     Y=      5.0000
                 . FX =     -0.0000    FY =     -0.0112
                                F =      0.0112
                   APPROXIMATE ULTIMATE LOAD =      24.741
        CRITICAL ELEMENT IS NUMBER   10
        ON WHICH ULTIMATE FORCE PER INCH IS    15.6264


AT TRIAL CENTER NO.    5
        X=      0.2852     Y=      5.0000
                   FX =     -0.0000    FY =      0.0036
                                F =      0.0036
                   APPROXIMATE ULTIMATE LOAD =      24.741
        CRITICAL ELEMENT IS NUMBER    1
        ON WHICH ULTIMATE FORCE PER INCH IS    15.6269


INSTANTANEOUS CENTER HAS BEEN LOCATED
        MAXIMUM PERMISSIBLE LOAD IS       1.469
        MULTIPLY THIS BY NUMBER OF SIXTEENTHS
        AND BY ALLOWABLE KSI / 21.0
```

*Fig. 3 (cont'd)*

*Example 2:*

Use the same weld configuration and load as in Example 1. Intuition indicates that more subdivisions should give better results, but there are two effects. One is simply the closer approximation to continuity achieved by using smaller elements. The other is the fact that the maximum stress occurs on the outside edge of a weld element rather than at its center. Obviously, more subdivisions of uniform size means that the edge of an element and its center will be closer together. It is, however, possible to divorce the two effects by using non-uniform weld elements. For example, the top and bottom 0.001 in. of each of the welds in Example 2 are entered as individual elements, and the remainder subdivided as in Example 1. The new results are shown in Table 2.

**Table 2**

| Subdivisions | 2 | 5 | 10 | 20 | 40 |
|---|---|---|---|---|---|
| $P_e$ | 0.797 | 1.009 | 1.039 | 1.047 | 1.049 |
| $P_u$ | 23.867 | 23.977 | 24.307 | 24.374 | 24.388 |
| $P_p$ | 1.417 | 1.423 | 1.443 | 1.447 | 1.448 |

It can be observed that the solutions stabilize with fewer subdivisions, and that the approach to the true solution is from the safe side rather than the unconservative side. The same behavior has been observed in a dozen other test cases for welds of different configurations.

*Example 3:*

Use the same weld configuration and load as in Example 1. It should obviously make very little difference if the small element welds are introduced in addition to (or even overlapping) the original welds. Therefore, if the critical weld element was unknown before the first analysis, only two additional cards need be prepared and added to the previous data for Example 1 to achieve the precision of Example 2. In Example 3, the data is:

| | | | | | |
|---|---|---|---|---|---|
| ELEMENT | 0. | 10. | 0. | 0.001 | |
| ELEMENT | 1. | 10. | 0. | 0.001 | |
| LINE | 0. | 0. | 0. | 10. | 10. |
| LINE | 1. | 0. | 1. | 10. | 10. |
| LOAD | 0. | −1. | 30.5 | 0. | |

The results are $P_e = 1.040$, $P_u = 24.312$, $P_p = 1.444$.

*Example 4:*

Use the same weld configuration and load as in Example 1. In order to avoid the extra effort involved in specifying data for additional elements, an automatic insertion of an extra "fake" element 1/1000th as long as the subdivision size can be made at each end of any line by using the word

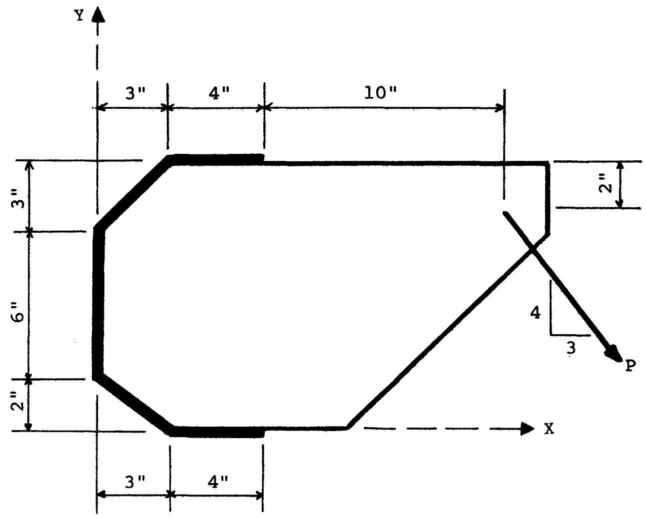


*Figure 4*

AUTOMATIC in place of LINE. When this was done for both line welds, the results were as shown in Table 3.

**Table 3**

| Subdivisions | 2 | 5 | 10 | 20 | 40 |
|---|---|---|---|---|---|
| $P_e$ | 0.800 | 1.010 | 1.040 | 1.047 | 1.049 |
| $P_u$ | 23.919 | 23.997 | 24.317 | 24.378 | 24.391 |
| $P_p$ | 1.420 | 1.424 | 1.444 | 1.447 | 1.448 |

*Example 5 (see Fig. 4):*

Welds need not be vertical or horizontal, or even symmetrical. The applied load can be applied in any direction. Consider the bracket shown.

Making use of the AUTOMATIC feature, and selecting the number of subdivisions for each line so that "real" elements were about 1-in. long, the data looked like this:

| | | | | | |
|---|---|---|---|---|---|
| AUTOMATIC | 7.0 | 11.0 | 3.0 | 11.0 | 4.0 |
| AUTOMATIC | 3.0 | 11.0 | 0.0 | 8.0 | 4.0 |
| AUTOMATIC | 0.0 | 8.0 | 0.0 | 2.0 | 6.0 |
| AUTOMATIC | 0.0 | 2.0 | 3.0 | 0.0 | 4.0 |
| AUTOMATIC | 3.0 | 0.0 | 7.0 | 0.0 | 4.0 |
| LOAD | 0.6 | −0.8 | 17.0 | 9.0 | |

From the program, the value of $P_e$ is 3.869; since the weld size is $5/16$, the allowable elastic load is $5 \times 3.869 = 19.345$ kips. The value of $P_p$ is 5.452; the permissible load is $5 \times 5.452 = 27.260$ kips.

**REFERENCES**


1. *Salmon, C. G. and J. E. Johnson* Steel Structures—Design and Behavior *Second Edition, Harper and Row, 1980, Chap. 5.*
2. Manual of Steel Construction *Seventh Edition, American Institute of Steel Construction, 1970.*

3. *Butler, L. J., S. Pal, and G. L. Kulak.* Eccentrically Loaded Welded Connections *Journal of the Structural Division, ASCE, Vol. 98, ST5, May 1972, pp. 989–1005.*
4. Manual of Steel Construction *Eighth Edition, American Institute of Steel Construction, 1980.*
5. *Tide, R. H. R.* Eccentrically Loaded Weld Groups—AISC Design Tables *Engineering Journal, American Institute of Steel Construction, Vol. 17, No. 4, 4th Quarter 1980.*
6. *Brandt, G. Donald* Rapid Determination of Ultimate Strength of Eccentrically Loaded Bolt Groups *Engineering Journal, American Institute of Steel Construction, Vol. 19, No. 2, 2nd Quarter 1982.*

## APPENDIX

```
$JOB            LIST,KP=29,TIME=600,PAGES=90
$IBFTC
C       PROGRAM TO LOCATE INSTANTANEOUS CENTERS OF WELD GROUPS
C        AND OBTAIN ELASTIC AND ULTIMATE RESISTANCES
        REAL J, M, MP, MPCG, IX, IY, MU, LAMBDA
        INTEGER ELEM, TYPE
        INTEGER AUTO
        DIMENSION TEMP(5)
        DIMENSICN  X(100), Y(100), DX(100), DY(100), D(100),
       1    THETA(100), R(100), W(100), WX(100), WY(100)
        FALLOW = 0.9281
  200   CCNTINUE
C       GENERALIZED INPUT
        WRITE ( 6,6000 )
 6000   FORMAT ( 1H1,  'DATA FOR DESCRIPTION OF WELDS AND LOAD ', /   )
        I = 0
  300   CONTINUE
        READ ( 5, 5CC )    TYPE,   ( TEMP(JJ),JJ= 1,5 )
  500   FORMAT ( A2,  8X,   5F10.3 )
        DATA     LINE / 'LI'/ ,    ELEM / 'EL' /,   LOAD /'LC' /
        CATA   AUTO / 'AU' /
        IF ( TYPE .EQ. LINE )  GO TO 301
        IF ( TYPE .EQ. AUTO )   GO TO 301
        IF ( TYPE .EQ. ELEM )  GO TO 302
        IF ( TYPE .EQ. LOAD )  GO TO 303
        WRITE  ( 6, 6004 )    TYPE, ( TEMP(JJ),JJ=1,5 )
 6004   FCRMAT (1HO,'UNABLE TO DECODE THIS CARD  ', /,
       1     10X, A2, 8X, 5F10.3, /, '    TYPE NOT STANDARD ', /, /, /  )
        GC TO 300
  301   CCNTINUE
        WRITE(6, 6001 )          ( TEMP(JJ),JJ=1,5   )
 6001   FORMAT ( 1H , 'LINE', 6X, 4F10.4, F8.1 )
        IF ( TYPE .EQ. LINE )   GO TO 306
        WRITE ( 6, 6005 )
 6005   FORMAT (1H ,'SHORT WELDS ADDED AUTOMATICALLY TO LINE ABOVE',/,/)
  306   CONTINUE
C       CCNVERT TEMP INTO LINE DATA
        SX = TEMP(1)
        SY = TEMP(2)
        EX = TEMP(3)
        EY = TEMP(4)
        NSEG = TEMP(5)
C       XP1, YP1 = PROJECTED LENGTH OF 1 ELEMENT
        XP1 = ( EX - SX ) / NSEG
        YP1 = ( EY - SY ) / NSEG
C       XBACK, YBACK = COORDINATE OF POINT HALF STEP BEYOND START
        XBACK = SX - XP1/2.
        YBACK = SY - YP1/2.
        IF (TYPE .EQ. LINE)   GO TO 304
        FAKEX = XP1 / 1000.
        FAKEY = YP1 / 1000.
        I = I + 1
        WX(I) = FAKEX
        WY(I) = FAKEY
        X(I) = SX
        Y(I) = SY
```

```
  304  CONTINUE
       DO 1001 .  K = 1, NSEG
       I = I + 1
       WX(I) = XP1
       WY(I) = YP1
       X(I) = XBACK + K * XP1
       Y(I) = YBACK + K * YP1
  1001 CONTINUE
       IF ( TYPE .EQ. LINE )    GO TU 305
       I = I + 1
       WX(I) = FAKEX
       WY(I) = FAKEY
       X(I) = EX
       Y(I) = EY
  305  CONTINUE
       GO TO 300
  302  CONTINUE
       WRITE ( 6, 6002 )          ( TEMP(JJ),JJ = 1,4 )
  6002 FORMAT ( 1H  ,  'ELEMENT' , 3X, 4F10.4 )
C      CONVERT TEMP TO ELEMENT DATA
       I = I + 1
       X(I) = TEMP(1)
       Y(I) = TEMP(2)
       WX(I) = TEMP(3)
       WY(I) = TEMP(4)
       GO TO 300
  303  CONTINUE
       WRITE ( 6, 6003 )          ( TEMP(JJ),JJ= 1, 4   )
  6003 FORMAT ( 1H  ,  'LOAD' ,  6X,  4F10.4 )
C      CONVERT TEMP TO LOAD DATA
       PX = TEMP(1)
       PY = TEMP(2)
       POLX = TEMP(3)
       POLY = TEMP(4)
       N = I
       WRITE (6,602) N,(I,X(I),Y(I), WX(I), WY(I),   I=1,N )
  602  FORMAT ( 1H1, '    NUMBER OF WELD ELEMENTS = ', I6, /,
      1 ' COORDINATES AND PROJECTIONS   ', /,
      2    (1X, I4, 4F10.4)    )
       WRITE (6,603)  PX, PY, POLX, POLY
  603  FORMAT ( 1H0,'PX= ',F10.4,   ',PY= ' , F10.4,
      1     ', POLX = ', F10.4,   ',POLY = ', F10.4   )
C      CALCULATE LENGTH OF EACH WELD ELEMENT
       DO 101    I = 1, N
       W(I)   =   SQRT ( WX(I) ** 2 + WY(I) ** 2 )
  101  CONTINUE
C      LOCATE CENTER OF GRAVITY OF WELD GROUP
       SUMX = 0.
       SUMY = 0.
       SUMW = 0.
       DO 102   I=1,N
       SUMX = SUMX + X(I) * W(I)
       SUMY = SUMY + Y(I) * W(I)
       SUMW = SUMW + W(I)
  102  CONTINUE
       XCG = SUMX / SUMW
       YCG = SUMY / SUMW
C      CALCULATE MOMENT OF P ABOUT CG
       MPCG = PY * (POLX - XCG) - PX * (POLY - YCG)
C      CALCULATE J
       J = 0.
       DO 103  I = 1,N
       DX(I) = X(I) - XCG
       DY(I) = Y(I) - YCG
```

```
         IX = W(I) / 12. * WY(I) ** 2 + W(I) * DY(I) ** 2
         IY = W(I) / 12. * WX(I) ** 2 + W(I) * DX(I) ** 2
         J = J + IX + IY
  103    CONTINUE
         FACTOR = J / ( SUMW * MPCG )
         XIC =-PY * FACTOR + XCG
         YIC =+PX * FACTOR + YCG
         ITER = 0
         FPREV = 1.E+12
  201    CONTINUE
         RECALCULATE DX, DY, D  AND DELTA FROM TRIAL CENTER
         DO 104    I=1,N
         DX(I) = X(I) - XIC
         DY(I) = Y(I) - YIC
C        CALCULATE LENGTH OF RADIUS VECTOR
         D(I) = SQRT ( DX(I)**2 + DY(I)**2 )
  104    CONTINUE
C        CALCULATE MOMENT OF P
         MP = PY * (POLX-XIC) - PX*(POLY-YIC)
         IF ( ITER .GE. 1 )   GO TO 204
C        GET THE ELASTIC SOLUTION
         BIGD = 0.
         SUMWD2 = 0.
         DO 107 I = 1, N
         IF(D(I) .LE. BIGD )   GO TO 203
         BIGD = D(I)
         IBIGD = I
  203    CONTINUE
         SUMWD2 = SUMWD2 + W(I) * D(I) ** 2
  107    CONTINUE
         ELPMAX=ABS ( SUMWD2 * FALLOW / ( MP * BIGD ) )
         WRITE (6,604)    ELPMAX, IBIGD
  604    FORMAT ( 1H0, 'ELASTIC VALUE FOR MAXIMUM LOAD IS ', F10.3, /,
     1      10X, 'MULTIPLY THIS BY NUMBER OF SIXTEENTHS ', /,
     2      10X, 'AND BY ALLOWABLE KSI / 21.0 ', / ,
     3      1H , 'CRITICAL ELEMENT IS NUMBER ',  I4, / )
  204    CONTINUE
         DLDMIN = 1.E+12
         DO 105   I = 1,N
C        CALCULATE  ANGLE BETWEEN RADIUS VECTOR AND WELD AXIS
         PHI=(ARCOS(ABS(( WX(I) * DX(I) + WY(I) * DY(I)) /
     1         (W(I) * D(I)))))* 180. / 3.1415927
C        CALCULATE ANGLE BETWEEN FORCE AND WELD AXIS
         THETA(I) = 90. - PHI
C        CALCULATE PRELIMINARY DELTA VALUES
         DELTA =  0.225 * ( THETA(I) + 5.0) ** ( -0.47 )
         DELD  =   DELTA / D(I)
C        FIND SMALLEST DELTA/D
         IF ( DELD     .GE.  DLDMIN )   GO TO 105
         DLDMIN = DELD
         DMAX = D(I)
         BIGDEL = DELTA
  105    CONTINUE
         BIGR = 0.
         SUMM = 0.
C        CALCULATE REVISED DELTA, MU, LAMBDA, RULT, R, M
         DO 106   I = 1, N
         DELTA  =   BIGDEL * D(I) / DMAX
         MU  =   75. * EXP ( 0.0114 * THETA(I) )
         LAMBDA =  0.4 * EXP ( 0.0146 * THETA(I) )
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      RULT  =   ( 10. + THETA(I) ) / ( 0.92 + 0.0603 * THETA(I) )
      R(I) = RULT    * ( 1. - EXP ( -MU    * DELTA    )) ** LAMBDA
C     CALCULATE LARGEST R
      IF ( R(I) .LE. BIGR )   GO TO 202
      BIGR = R(I)
      JBIG = I
  202 CONTINUE
      M =  R(I) * D(I) * W(I)
      SUMM=SUMM + M
  106 CONTINUE
C     CALCULATE RUNIT DUE TO UNIT LOAD, CALCULATE PULT
      RUNIT = -MP / SUMM
      PULT = ABS ( SUMM / MP )
C     CALCULATE RX/P AND RY/P
      SUMRX = 0.
      SUMRY = 0.
      DO 108  I=1,N
      RX  =   -DY(I) / D(I) * R(I) * RUNIT * W(I)
      SUMRX = SUMRX + RX
      RY  =   DX(I) / D(I) * R(I) * RUNIT * W(I)
      SUMRY = SUMRY + RY
  108 CONTINUE
      FX = PX + SUMRX
      FY = PY + SUMRY
      F = SQRT( FX**2 + FY**2 )
      ITER = ITER + 1
      WRITE (6,601)    ITER, XIC, YIC, FX, FY, F, PULT, JBIG, BIGR
  601 FORMAT (1H0, 'AT TRIAL CENTER NO. ',  I4, / ,
     1    10X, 'X= ', F10.4, '     Y= ', F10.4, /,
     2    20X, 'FX = ', F10.4, '    FY = ', F10.4, /,
     3    30X, 'F = ',  F10.4, /
     4    20X, 'APPROXIMATE ULTIMATE LOAD = ', F10.3, /,
     5    10X, 'CRITICAL ELEMENT IS NUMBER ',  I4, /,
     6    10X, 'ON WHICH ULTIMATE FORCE PER INCH IS ', F10.4, / )
      IF ( ABS(F) .LE. 0.01 )    GO TO 206
      IF ( ITER .GE. 30 )   GO TO 208
C     TRY NEW CENTER
      IF ( F .LT. FPREV )   GO TO 205
      FACTOR = FACTOR / 2.
  205 CONTINUE
      FPREV = F
      XIC = XIC - FY * FACTOR
      YIC = YIC + FX * FACTOR
      GO TO 201
  206 CONTINUE
C     CONVERT PULT TO 1/16 WELD OF E70 ELECTRODE
      PMAX = PULT / 4. * 70. / 60.
C     INTRODUCE SAFETY FACTOR
      PMAX = 0.3 * PMAX
      RLIM = FALLOW / 0.3 * 4.0 * 60. / 70
      IF ( BIGR .LE. RLIM )   GO TO 207
      PMAX = PMAX * RLIM / BIGR
  207 CONTINUE
      WRITE (6,605)   PMAX
  605 FORMAT ( 1H0,  'INSTANTANEOUS CENTER HAS BEEN LOCATED ', /,
     1    10X,'MAXIMUM PERMISSIBLE LOAD IS ' , F 10.3, /,
     2    10X, 'MULTIPLY THIS BY NUMBER OF SIXTEENTHS ', /,
     3    10X, 'AND BY ALLOWABLE KSI / 21.0 ', /  )
      GO TO 200
  208 WRITE (6,606)  ITER
  606 FORMAT ( 1H0,  'ITERATIONS =  ', I5,'...TRIALS TERMINATED.')
      GO TO 200
      END

$DATA
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