Design of W-Shapes for Combined Bending and Torsion

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Numerous sources of information are available for the calculation of stresses in W-shapes in combined bending and torsion.[1-5] Of these, only Ref. 3 gives much attention to effects of structural details, torsional restraint of attached members, the difficulty of achieving end-warping restraint, or the importance of twist angle when a masonry wall is supported.

Nine graphical charts are provided herein to permit rapid design checks for a variety of load types and end conditions involving W-shapes in combined bending and torsion. The graphs are based on solutions that are tabulated on pgs. 76 and 77 of Ref. 2. For a very adequate review of torsion theory as applied to structural members, the reader may refer to Chapter 8 of the text by C. G. Salmon and J. E. Johnson.[1]

The graphical charts permit evaluation of:

1. The angle of twist, ϕ, resulting from a given torque applied to a member. Permissible magnitudes of twist are not covered by specifications.

2. The twist stiffness (M/ϕ). This is important if the member that applies torque is rigidly attached and has bending stiffness of its own. Compatibility of twist due to torque and end rotation of the beam applying the torque would have to exist for reduction of the effective torque. *Stress due to torsion can develop only to the degree that twist is permitted by contiguous framing members.*

3. Maximum direct stress due to warping restraint. For each chart covering the evaluation of ϕ, there is a companion chart for the same load and end conditions, permitting the calculation of the maximum direct stress due to warping restraint that is additive to the direct stress due to bending caused by direct vertical loading.

If twist due to torsion is a real problem that cannot be minimized by rearrangement of framing layout, or by participation of framing members, the designer should consider replacing an open cross section, such as a W-shape, with a box-shaped cross section. Such a member will probably be more than 50 times stronger and stiffer than the W-shape having the same unit weight. If a channel section is loaded by hanger rods, the details should be arranged so that the load is applied through the shear center.[5]

To define terminology, a simple case of torsion is shown in Fig. 1 with exaggerated distortion. If a simply supported beam is subjected to a torque due to an eccentric load at midspan, the flanges will tilt at the ends with opposite rotational sense, as shown. The tilt is termed *warping.* At midspan the rotation angle ϕ is a maximum, but the warping displacement is zero as as it reverses in sense, i.e., there is *complete warping restraint* at midspan. If z is taken as the coordinate distance along the beam axis, $d\phi/dz$, or ϕ', is the rate of change of the twist angle. This also becomes zero at midspan where it changes sign.

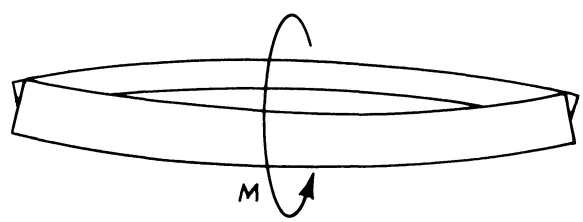

Figure 1

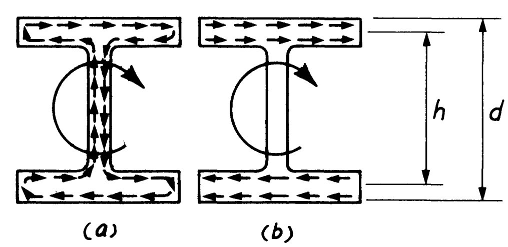

Figure 2

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At all locations except midspan the torsional resistance is provided by a superposed combination of two patterns of shear stress, shown in Fig. 2. The pattern in Fig. 2a is that of *St. Venant*, which is the only type present if a member is in *pure* (uniform) torsion, with no warping restraint at either end. This never occurs in combined bending and torsion. The shear pattern in Fig. 2b is developed in proportion to the degree that there is restraint against warping. Torsional twist is then "non-uniform", the flanges bend in an opposed sense and, as shown in Fig. 3, flange lateral bending stresses f_w develop that are additive to the stress f_b due to in-plane bending moment.

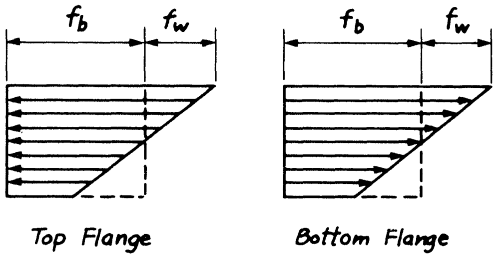

Figure 3

In structural design of W-shapes under static load, there is no need to evaluate the shear stress due to St. Venant torsion. Unacceptable twist deformation would develop long before the allowable stress in shear is reached. For example, if a 10-ft long segment of a W36x300 shape were twisted in pure torsion, the total twist angle would be more than 5° before the maximum shear stress in the main portion of the flange reached 15 ksi.

To calculate either ϕ or M/ϕ for a given value of total torque M applied to a beam, the graphs in Appendix Charts A1, A3, A5, A7, and A9 provide an evaluation of the quantity $\phi GJ/Ma$ for various values of L/a, various load distributions, and various beam end conditions, i.e., simple, fixed, or free, where

ϕ = total angle of twist at a designated location
G = shearing modulus of elasticity (11,200 kips/in.^{2} for steel)
J = St. Venant torsion constant (in.^{4})
a = torsion bending constant = $(EC_w/GJ)^{1/2}$ (in.)

Values of J and a are tabulated in the "Torsion Properties" tables, AISC Manual, 8th Ed., for all W shapes. The values for a are in the columns headed $(EC_w/GJ)^{1/2}$, where C_w is the *torsion warping constant.* The constant a, a function of cross section properties, provides a rough measure of the length of beam from any fully restrained location which would provide appreciable resistance to warping. The coefficient a was used in the early work of Timoshenko and is used in the solutions tabulated in Ref. 2. In Ref. 3, $\lambda = 1/a$ is used in similar equations.

To determine the flange bending stress due to warping restraint, it is necessary to relate beam rotation ϕ to lateral flange deflection u. In Fig. 4, for small rotations,

$u = \phi h/2$ = flange deflection, in.
M_f = flange bending moment, kip-in.
$I_f = I_y/2$ = moment of inertia of one flange, in.^{4}
z = distance along beam axis

$$\frac{d^2u}{dz^2} = u'' = \frac{M_f}{EI_f} = \frac{2M_f}{EI_y}$$

$$u'' = \phi'' h/2$$

whence

$$M_f = \frac{EI_y u''}{2} = \frac{EI_y h\phi''}{4}$$

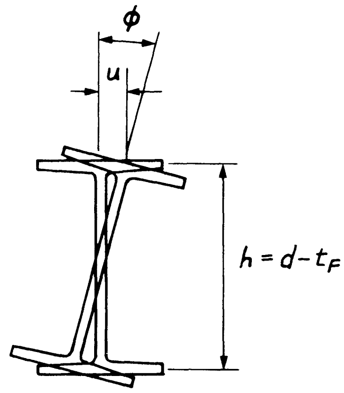

Figure 4

For various load and end conditions, Charts A2, A4, A6, A8 and A9 provide the magnitude of the maximum value of $\phi'' GJa/M$, a dimensionless function.

$$M_f = \frac{EI_y h}{4} \times \frac{M}{GJa} \times \left[\frac{\phi'' GJa}{M} \text{ (from chart)}\right]$$

Substituting $EI_y/GJ = 4a^2/h^2$,

$$M_f = \frac{Ma}{h} \times \left(\frac{\phi'' GJa}{M}\right)$$

The maximum flange bending stress, f_w, due to warping restraint, is:

$$f_w = \frac{2M_f}{S_y} = \frac{2aM}{hS_y} \times \left(\frac{\phi'' GJa}{M}\right) \quad (1)$$

Coefficient a pertains only to W-shapes or built-up doubly-symmetric I-shaped members. For general application to other than W or I shapes, a may be replaced in the chart functions by $(EC_w/GJ)^{1/2}$.

SUMMARY OF CASES CONSIDERED

If available, Refs. 2, 4, and 5 can provide useful supplementary information, as will be briefly discussed in conjunction with a summary of cases considered herein. Numerically, the results given in Refs. 2 and 4 agree exactly with those in this presentation.

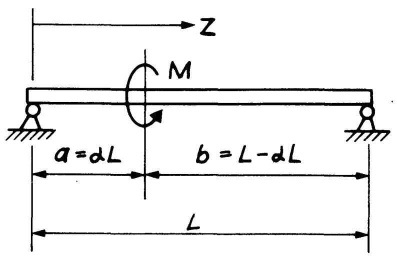

Figure 5

Simple Beam with Torque at Any Location (Fig. 5)—Charts A1 and A2 provide graphs for computation of maximum ϕ or f_w, respectively, at the torque location, for a range of $0.5 < L/a < 6.0$, with curves plotted for α values of 0.10, 0.15, 0.20, 0.25, 0.30, 0.40, and 0.50. If $L/a < 1$, a conservative approximate estimate of f_w may be made. Divide the torque M by h and consider each flange as loaded by a concentrated force M/h. Then calculate the value of $M_f = (M/h)\alpha(1 - \alpha)L$ and calculate $f_w = 2M_f/S_y$.

Reference 2 contains separate charts for α values of 0.10, 0.30, and 0.50 on which curves for various values of L/a are plotted for the *full length of the member.* These charts provide for the calculation of both ϕ and f_w and are especially useful if the combined effect of more than one torque at different locations needs to be superposed. However, interpolation for intermediate values of α is difficult, since it must be made between charts on different pages.

Reference 4 provides very complete tables for the calculation of f_w at or away from any torque location. Interpolation is convenient. But no tabular information is provided for the calculation of ϕ; hence, torsional stiffness and framing participation cannot be evaluated. The procedure in Ref. 4 was originally developed by C. G. Salmon[1] and differs from that used in Ref. 2 or in this presentation. In place of L/a, λL is used, as is the case in Refs. 1 and 3. Then moments in each flange are calculated for a force of M/h as if each flange were a separate beam. For a short member, this procedure results in a good approximation, shown in Fig. 6 by the dashed line. The correct moment, shown in Fig. 6 by the solid line, equals the ordinate to the dashed line multiplied by β for that location, as tabulated in Table 1 of Ref. 4. Thus, with this table at hand, one can calculate by superposition the magnitude of f_w due to a number of concentrated torques applied anywhere along a simple beam. Lacking Ref. 4, one can make a crude but conservative approximation on the basis of the dotted line, as will be shown in Design Example 4.

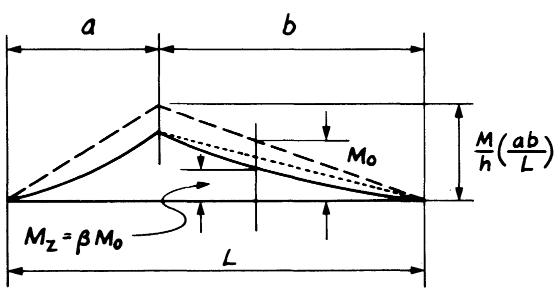

Figure 6

Fixed-ended Beam with Torque at Any Location—Charts A3 and A4 of this report provide for the calculation of ϕ at $z = \alpha L$ and for the maximum f_w, respectively, the latter occurring at the support nearest the applied torque location. Chart A4 permits superposition to determine maximum f_w for any number of concentrated torques applied at various locations. To estimate ϕ for more than one torque application, Ref. 2 could be used. For $L/a < 2$, the previously described approximate procedure provides an adequate solution. In this case, at the left end of the beam in Fig. 7, $M_f = Ma^2b/hL^2$ and at the right end $M_f = Mab^2/hL^2$.

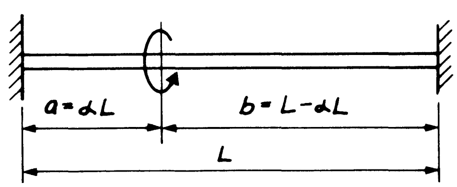

Figure 7

Simple and Fixed-ended Beams with Third Point, Quarter Point, or Uniform Load—Charts A5, A6, A7, and A8 provide for calculation of maximum ϕ and maximum f_w. Design Examples 3, 5, 6, and 7 illustrate applications including superposition when loads are not equal.

Cantilever Beam with Uniform or Concentrated End Load—Chart A9 provides for calculation of maximum ϕ and maximum f_w. The approximate procedure can be used if $L/a < 0.5$.

DESIGN EXAMPLES

Several of the following design examples are presented not only to demonstrate the use of the graphical charts in the appendix, but also to show how torsion may be reduced by very simple consideration of the effect of contiguous framing participation. They are similar to situations the writer has encountered in reviewing actual designs. In using examples as a basis for discussion, they are obviously not presented in the compact style that would be typical of design office computation sheets. The first three examples include load and end conditions that are identical to those of the first three examples of Ref. 4.*

Example 1a

Given:

A simple beam with span $L = 15$ ft carries a load of 9 kips concentrated at midspan, with an eccentricity of 6 in. (In Ref. 4 a W10x54 shape was found satisfactory and the combined bending and warping stress at midspan was found to be 20.59 ksi.)

Solution:

W10x54 properties from AISC Manual, 8th Ed.:

$h = d - t_f = 10.09 - 0.61 = 9.48$ in.
$S_x = 60.0$ in.^{3}
$S_y = 20.6$ in.^{3}
$J = 1.82$ in.^{4}
$a = 57.4$ in.

Torsional moment $M_t = 9 \times 6 = 54$ kip-in.

Bending moment $M_b = \dfrac{9 \times 15 \times 12}{4} = 405$ kip-in.

Stress due to bending $f_b = 405/60 = 6.75$ ksi

$$L/a = \frac{15 \times 12}{57.4} = 3.14$$

$\alpha = 0.5$ (α is the proportionate distance from end of span to torque location)

From Chart A2: $\dfrac{\phi'' GJa}{M} = 0.459$

Then, by Eq. (1):

$$f_w = \frac{2 \times 57.4 \times 54 \times 0.459}{9.48 \times 20.6} = 14.57 \text{ ksi}$$

The maximum combined stress due to bending and warping restraint is:

$f = 6.75 + 14.57 = 21.32$ ksi **o.k.**

The combined stress of 21.32 ksi is 3.5% greater than the stress determined in Example 1 of Ref. 4. The difference is in large part due to the fact that, in Ref. 4, h is taken as the full depth (d) of the shape instead of $(d - t_f)$, which is the distance between flange warping shear resultants.

Example 1b

Given:

Same load, span, and apparent eccentricity as Example 1a, but with consideration of a hypothetical framing situation in which the 9 kip load is applied by a W6x9 column 12 ft in height. To minimize torsional stress, the column will be framed to the beam with a rigid connection as shown in Fig. 8. Assume that the column is hinged at the top.

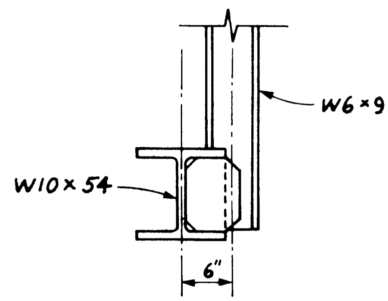

Fig. 8. Example 1b

Solution:

Properties of W6x9:

$A = 2.68$ in.^{2}
$I_x = 16.4$ in.^{4}
$r_x = 2.47$ in.

The rotational bending stiffness of the column at its point of connection to the beam is:**

$$\frac{M}{\phi} = \frac{3EI}{L} = \frac{3 \times 29000 \times 16.4}{144} = 9908 \text{ kip-in./radian}$$

This stiffness should be multiplied by $[1 - (f_a/F'_e)]$ to reduce it for axial load effect.

$f_a = 9/2.68 = 3.36$ in.^{2}

From AISC Specification Table 9, Appendix A, for $L/r = 144/2.47 = 58.3$, $F_e' = 43.94$ ksi. The reduced column stiffness is:

$$9908 \times \left(1 - \frac{3.36}{43.94}\right) = 9150 \text{ kip-in./radian}$$

* *Reference 4 was based on properties listed in the 7th Edition of the AISC Manual.*

** *Beam bending stiffnesses for simple cases are used in the moment-distribution procedure.*[5]

From Chart A1, for the beam:

$$\frac{\phi GJ}{Ma} = 0.33$$

from which the torsional stiffness is

$$M/\phi = \frac{11{,}200 \times 1.82}{57.4 \times 0.33} = 1076 \text{ kip-in./radian}$$

Calculate torque distribution factors to beam and column:

	Stiffness	Dist. Factor
Beam	1076	0.105
Column	9150	0.895
	10226	1.000

Hence the torsional moment actually resisted by the beam has been reduced from 54 kip-in. to 0.105 × 54 = 5.67 kip-in. The direct stress due to warping has been correspondingly reduced from 14.57 ksi to 0.105 × 14.57 = 1.53 ksi.

The column must be designed for a combined axial load of 9 kips and an end moment of 54 − 5.7 = 48.3 kip-in. The moment at the column base will induce a lateral force on the beam:

$$P = 48.3/144 = 0.34 \text{ kips}$$

which will cause a bending moment about the weak axis of the beam of

$$M_y = \frac{0.34 \times 15 \times 12}{4} = 15.3 \text{ kip-in.}$$

and a stress due to bending of

$$f_b = 15.3/20.6 = 0.74 \text{ ksi.}$$

In summary, the maximum direct stresses are:

Due to vertical load bending		6.75 ksi
Due to induced lateral load		0.74 ksi
Due to torsion warping restraint		1.53 ksi
	Total	9.02 ksi

Obviously, a smaller section can be used. However, the foregoing analysis has omitted the magnification of the secondary effects of lateral bending and warping restraint stresses that become significant as the lateral buckling load is approached. The situation is analogous to that of an arch or beam-column for which the effects of bending moment are augmented by a magnification factor as in AISC interaction formula (1.6–1a). Chu and Johnson[6] have shown that for a bent beam in torsion the warping restraint stress is magnified for a W-shape as the lateral buckling load is approached. Such magnification would occur only if no lateral restraint were present at the load point. In the present example, the column applies load but does not provide lateral restraint. (In all other examples in this report, lateral restraint is assumed to be present at load points.)

When no lateral restraint is present, the following two requirements are suggested:

1. For the range of l/r_T for which Formula (1.5–6a) is applicable:

$$f_{bx} \leq F_b \text{ by Formula (1.5–6a)}$$

2. In all cases:

$$f_{bx} + (f_{by} + f_w)\left[\frac{1}{1 - (f_{bx}/F_b)}\right] \leq 0.6\, F_y$$

where F_b is the maximum value given by Formula (1.5–6b) or Formula (1.5–7). (As in column design, this bending stress magnification is based on elastic behavior.)

The foregoing will now be applied to a revised beam selection.

After some preliminary trials, a W12x30 beam was found to be satisfactory. From the AISC Manual:

$h = 12.34 - 0.44 = 11.90$ in.
$r_T = 1.73$ in.
$S_x = 38.6$ in.^{3} $\qquad d/A_f = 4.30$ in.$^{-1}$
$S_y = 6.24$ in.^{3} $\qquad L/r_T = 180/1.73 = 104.0$
$J = 0.46$ in.^{4} $\qquad Ld/A_f = 4.30 \times 180 = 774$
$a = 63.9$ in. $\qquad L/a = 180/63.9 = 2.82$

From Chart A1: $\phi GJ/Ma = 0.265$

From Chart A2: $\phi'' GJa/M = 0.442$

Torsional stiffness at load point:

$$M/\phi = \frac{11{,}200 \times 0.46}{63{,}9 \times 0.265} = 304 \text{ kip-in./radian}$$

Calculate distribution factors:

	Stiffness	Dist. Factor
Beam	304	0.032
Column	9150	0.968

Calculate F_b, assuming $C_b = 1$ and $L = 180$ in.:

By AISC Formula (1.5–6b):

$$F_b = 170{,}000/(104)^2 = 15.72 \text{ ksi}$$

By AISC Formula (1.5–7):

$$F_b = 12{,}000/774 = 15.50 \text{ ksi}$$

Beam moment: 0.032 × 54 = 1.7 kip-in.

Column moment: 0.968 × 54 = 52.3 kip-in.

Induced lateral load: 52.3/144 = 0.363 kips

Lateral bending moment:

(0.363 × 180)/4 = 16.33 kip-in.

Stress due to vertical load: 405/38.6 = 10.49 ksi

Amplified stress due to lateral load:

$$\frac{16.33}{6.24}\left[\frac{1}{\left(1-\frac{10.49}{15.72}\right)}\right]=7.89 \text{ ksi}$$

Amplified stress due to warping restraint, by Eq. (1):

$$f_w=\frac{2\times 1.7\times 63.9\times 0.442}{11.90\times 6.24}\left[\frac{1}{\left(1-\frac{10.49}{15.72}\right)}\right]$$

$$=3.88 \text{ ksi}$$

In summary, the maximum direct stress in the flange is:

$10.49 + 7.89 + 3.88 = 22.26$ ksi **o.k.**

The revised beam selection based on consideration of column participation has permitted a 44% weight reduction as compared with the original design that neglected rotational restraint.

Example 2

Given:

A beam with span of 10 ft is assumed fixed at ends both as to bending and flange warping restraint. At midspan it carries a concentrated load of 20 kips with 6-in. eccentricity. (In Ref. 4, the maximum combined stress at end or center using a W12x53 was found to be 18.97 ksi.)

Solution:

From the AISC Manual:

$h = 12.06 - 0.58 = 11.48$ in.
$S_x = 70.6$ in.^{3}
$S_y = 19.2$ in.^{3}
$J = 1.58$ in.^{4}
$a = 72.0$ in.

$L/a = 120/72 = 1.67$

Applied torque, $M = 20 \times 6 = 120$ kip-in.

Moment due to vertical load:

$M_x = 20 \times 120/8 = 300$ kip-in.

Stress due to vertical bending:

$f_b = 300/70.6 = 4.25$ ksi

From Chart A4:

$$\frac{\phi'' GJa}{M}=0.198$$

Stress due to warping restraint by Eq. (1):

$$f_w=\frac{2\times 72.0\times 120\times 0.198}{11.48\times 19.2}=15.52 \text{ ksi}$$

The combined stress $f = 4.25 + 15.52 = 19.77$ ksi, which is 4.3% greater than by Ref. 4 because d is 4.8% greater than h.

With L/a less than 2, the simple procedure will give an approximate answer:

Concentrated flange load $= 120/11.48 = 10.45$ kips

$$\text{Flange moment } M_f=\frac{PL}{8}=\frac{10.45\times 120}{8}$$

$$=156.8 \text{ kip-in.}$$

Approximate warping restraint stress,

$$f_w=\frac{2\times 156.8}{19.2}=16.3 \text{ ksi} \quad (5.0\% \text{ too high})$$

The foregoing would also be the initial part of the "β correction method" of Ref. 4. From Table 2 of Ref. 4, the interpolated value of β for $\lambda L = L/a = 1.67$ is 0.9425, with the result that $f_w = 0.9425 \times 16.3 = 15.36$ ksi as compared with 15.52 ksi. In this modification of the β method, d was replaced by h.

Two questions in connection with the foregoing example should be considered:

1. As in the case of Example 1, is rotational stiffness provided by the member that introduces the torque?
2. How are the ends restrained against flange warping? Warping restraint is much more difficult to achieve than end-fixity for bending moment. If the span is one of several of a continuous beam, with each span similarly loaded, there is inherent end-fixity both for beam action and flange warping. But if the beam is an isolated span, Ojalvo[7] and others have shown that a closed box made up of several plates, or a channel, is required at the ends to approximate full warping restraint. Simply welding to an end plate or to a column flange is insufficient. (See Fig. 9).

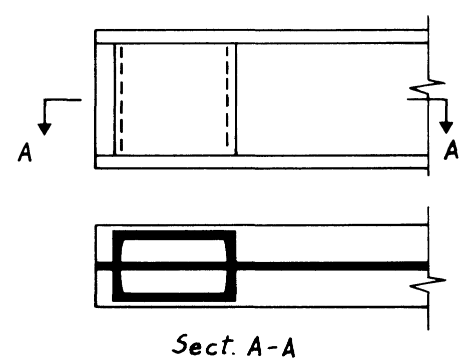

Figure 9

Turning to the first question, from Chart A3,

$$\frac{\phi GJ}{Ma} = 0.0228$$

and the torsional stiffness,

$$\frac{M}{\phi} = \frac{11{,}200 \times 1.58}{0.0228 \times 72.0} = 10{,}780 \text{ kip-in./radian}$$

In this case the twist stiffness of the beam is about 10 times that of the beam initially checked in Example 1 and 46 times stiffer than the final selection. Even so, for the same column arrangement as for Example 1b, the warping restraint stress would be reduced appreciably.

Example 3a

Given:

A beam with a length of 22 ft carries a uniform load of 2 kips/ft, 6 in. eccentric to the plane of the web. The W14x103 selected in Example 3 of Ref. 4 is a discontinued section. Try a W14x99.

Solution:

$h = 14.16 - 0.78 = 13.38$ in.
$S_x = 157$ in.^{3}
$S_y = 55.2$ in.^{3}
$J = 5.37$ in.^{4}
$a = 93.1$ in.
$L/a = 22 \times 12/93.1 = 2.84$

Total torsional moment:

$$M = 2 \times 6 \times 22 = 264 \text{ kip-in.}$$

Check maximum stresses:

Due to vertical bending:

$$M = \frac{2.1 \times 22^2 \times 12}{8} = 1524.6 \text{ kip-in.}$$

$$f_b = \frac{1524.6}{157} = 9.71 \text{ ksi}$$

From Chart A6: $\dfrac{\phi'' GJa}{M} = 0.192$

By Eq. (1):

$$f_w = \frac{2 \times 93.1 \times 264 \times 0.192}{13.38 \times 55.2} = 12.78 \text{ ksi}$$

Combined stress:

$$f = 9.71 + 12.78 = 22.49 \text{ ksi}$$

As anticipated, this is somewhat greater than the stress of 20.45 ksi of Ref. 4, due both to the use of a lighter weight section and the use of d instead of h as the couple arm of the flange resultant shears.

Example 3b

Given:

To illustrate problems that may arise when a uniform eccentric load involves a masonry wall, the conditions in Example 3a will be modified to those in Fig. 10. A span of 22 ft is again used. The reinforced cinderblock wall, filled with concrete, is 6-ft high and supports a concrete slab that introduces a load on the wall of 2.5 kips/ft. The wall center line is 3 in. eccentric to the beam web. Try the same section as used in Example 3a.

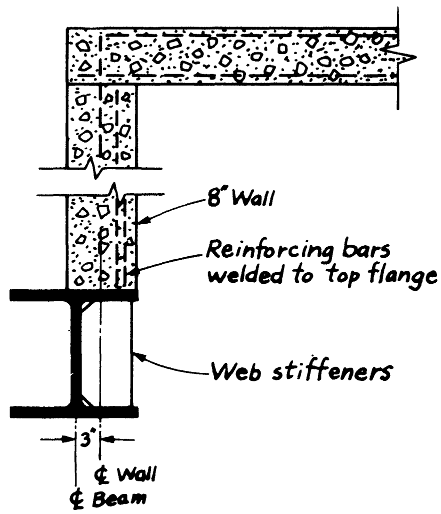

Fig. 10. Example 3b

Solution:

Weight of beam:	0.1 kip/ft
Weight of wall:	0.5 kip/ft
Slab load:	2.5 kip/ft
Total:	3.1 kip/ft

Stress due to vertical bending:

$$M_b = \frac{3.1 \times 22^2 \times 12}{8} = 2250.6 \text{ kip-in.}$$

$$f_b = \frac{2250.6}{157} = 14.34 \text{ ksi}$$

Torsional direct stress due to warping restraint:

$$M = 3.0 \times 3 \times 22 = 198 \text{ kip-in.}$$

$$f_w = \frac{2 \times 93.1 \times 198 \times 0.192}{13.38 \times 55.2} = 9.58 \text{ ksi}$$

Check maximum twist rotation with only the wall load in place:

Total torsional moment:

$$M = 0.5 \times 3 \times 22 = 33 \text{ kip-in.}$$

Referring to Chart A5, for $L/a = 2.84$:

$$\frac{\phi GJ}{Ma} = 0.165$$

$$\phi = \frac{0.165 \times 33 \times 93.1}{11{,}200 \times 5.37} = 0.0084 \text{ rad.}$$

Without temporary shoring, the top of the wall would tend to deflect laterally by $0.0084 \times 79 = 0.66$ in. at midspan, as hollow blocks were filled with wet concrete.

When the additional slab load is superposed, the total maximum lateral deflection at the top of the wall (if it were permitted) would be $6 \times 0.66 = 3.96$ in.—which would be intolerable in addition to doubling the torsional load at midspan. Obviously, attachment of the slab at the top of the wall would prevent such movement. However, unsightly horizontal cracks might develop and the structure would appear to be in distress.

The solution to the problem is to make the steel beam and the wall into an integral unit. This could be done by adding vertical reinforcing bars as shown in the sketch. Two effects now join to *eliminate* the torsion problem. Compared to the steel beam alone, the torsional rigidity of the wall-beam unit will be increased 100-fold. Secondly, the bending stiffness of the reinforced wall would, by itself, absorb nearly all of the torsional load. Calculations are omitted, but it was found that a W14x82 member would be more than adequate. The steel beam should be shored up during construction to prevent premature twist prior to concrete setup.

Example 4

Given:

Similar to Example 4 of Ref. 4, but with modified loads to adjust for a numerical error. Span is 25 ft and the loads of 210 and 280 kips, as shown, are 3 in. eccentric to the plane of the web. (See Fig. 11.) It will be assumed that the torsion cannot be mitigated by the stiffness of the contiguous structural elements that introduced the load. The problem illustrates the superposition of effects due to more than one concentrated torque, and added computations when a welded built-up section is used. The section will be the same as that used in Example 4 of Ref. 4 (see Fig. 12).

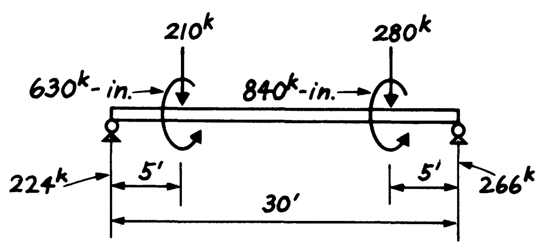

Fig. 11. Example 4

Solution:

Cross section properties:

$$h = 36 - 2 = 34 \text{ in.}$$
$$S_x = 1309 \text{ in.}^3$$
$$I_y = 1947 \text{ in.}^4$$
$$S_y = 216 \text{ in.}^3$$
$$J \simeq \tfrac{1}{3}(2 \times 18 \times 2^3 + 32 \times 1^3) = 106.7 \text{ in.}^4$$

$$a = \frac{h}{2}\sqrt{\frac{EI_y}{GJ}} = \frac{34}{2}\sqrt{\frac{29{,}000 \times 1947}{11{,}200 \times 106.7}} = 116.9 \text{ in.}$$

$$L/a = 25 \times 12/116.7 = 2.57$$

Check flange stress at 280 kip load point:

$$M_b = 266 \times 5 \times 12 = 15{,}960 \text{ kip-in.}$$

$$f_b = \frac{15{,}960}{1309} = 12.19 \text{ ksi}$$

Applied torques:

At 280 kip load point:

$$M = 280 \times 3 = 840 \text{ kip-in.}$$

At 210 kip load point:

$$M = 210 \times 3 = 630 \text{ kip-in.}$$

From Chart A2, for $\alpha = 0.2$:

$$\frac{\phi'' GJa}{M} = 0.317$$

By Eq. (1), at the 280 kip load point:

$$f_w = \frac{2 \times 116.7 \times 840 \times 0.317}{34 \times 216} = 8.46 \text{ ksi}$$

Calculate f_w at 210 kip load and assume straight line reduction to location of 280 kip load:

$$f_w = 0.25 \times \frac{2 \times 116.7 \times 630 \times 0.317}{34 \times 216} = 1.59 \text{ ksi}$$

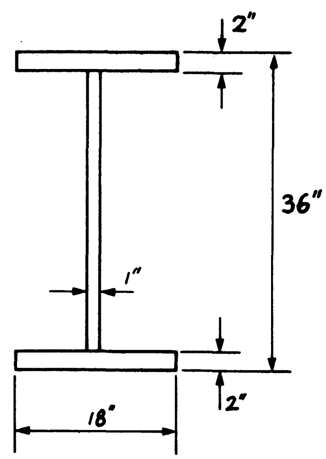

Fig. 12. Example 4

Combined stress at 280 kip load point:

$f = 12.19 + 8.46 + 1.59 = 22.24$ ksi

More accurately, if Ref. 4 is available, calculate the unreduced flange moment due to the 210 kip load at 280 kip location:

$$M_f = 0.25 \times \frac{630}{34} \times 0.2 \times 0.8 \times 25 \times 12 = 222.4 \text{ kip-in.}$$

From Table 1 of Ref. 4, interpolating to obtain β for $\lambda L = L/a = 2.57$:

$\beta = 0.4422$, and the corrected flange moment

$M_f = 0.4422 \times 222.4 = 98.4$ kip-in.

and the contribution of the 210 kip load to flange stress due to warping restraint at the 280 kip location is

$f_w = (2 \times 98.4)/216 = 0.91$ ksi

The corrected combined stress is 21.56 ksi. The procedure used here has overestimated the stress by 3.2%. The error will decrease as L/a becomes smaller, but will be greater for large values of L/a.

If torques are applied at the third-points, Example 5 shows the procedure to get an accurate evaluation of ϕ or f_w at the most stressed load point of a simply supported beam. For quarter-point loading, Example 6 shows the procedure to get accurate results for ϕ and f_w at the center. The procedure followed in the preceding example can be used to make a conservative estimate of ϕ and f_w at the quarter-point if a heavy load at that location indicates that maximum values might be at that location. For a fixed-ended beam the maximum stress is always at a fixed end and the total f_w due to any number of torques can be readily superposed by use of Chart A4.

Example 5 (procedure only)

A simple beam has concentrated torques of 400 kip-in. and 200 kip-in., respectively, at third-points A and B (Fig. 13). Maximum f_w will occur at A. From Chart A6, determine $\phi'' GJa/M$ for 200 kip-in. torques at both A and B, and from Chart A2 for an additional 200 kip-in. torque at A. Calculate the stress f_w for each of the two conditions. Maximum stress is the sum the two calculated stresses at A.

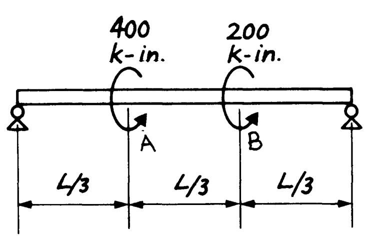

Fig. 13. Example 5

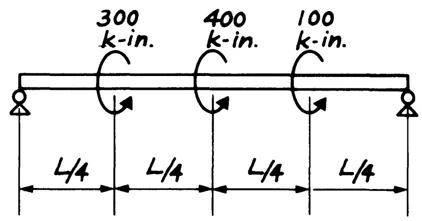

Fig. 14. Example 6

Example 6 (procedure only)

A simple beam with torques of 300, 400, and 100 kip-in., applied as shown in Fig. 14. The torques at the quarter-points will contribute to the midspan torsional warping restraint to the same degree as if they were 200 kip-in. each. Using Chart A6, determine $\phi'' GJa/M$ for quarter-point loading and from Chart A2, determine $\phi'' GJa/M$ for midspan loading. Use these values to calculate f_w for a total of 600 kip-in. for a set of three quarter-point loads, and add to f_w for a single torque of 200 kip-in. at midspan. Add the total f_w to f_b for vertical load.

Example 7a

Given:

In Example 1b the effectiveness of column rotational stiffness in inhibiting twist was demonstrated. This example will show how beams may serve the same function. The beam shown in cross section in this example has a span of 30 ft with simple twist-resistant end supports. It is loaded at the third-points by hanger rods that carry 40 kips each. Lateral support and rotational stiffness are provided by auxiliary W21x50 beams of 18-ft span that frame into the load points and introduce additional vertical loads of 35 kip each. See Fig. 15. To provide participation in resisting the torque loads, moment-resisting connections are provided

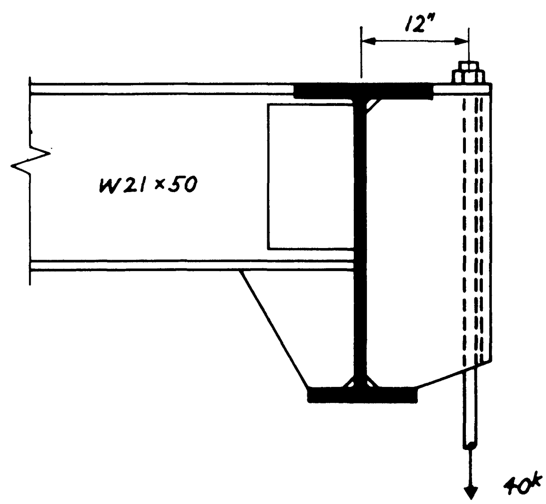

Fig. 15. Example 7a

as shown, but the far ends of the 18-ft beams will terminate in non-moment-resisting web angle connections. In anticipation that the torsional effects will be minimized, the beam selection will be based on a trial stress of 20 ksi for vertical loads alone.

Solution:

Assume beam weight of 0.15 kip/ft

D. L. moment: $0.15 \times 30^2/8 = 16.9$ kip-ft

Moment due to third point loading:

$75 \times 10 = 750$ kip-ft

Trial section modulus required:

$S = 766.9 \times 12/20 = 460$

Minimum weight selection from AISC Manual for a W36x150:

$$h = 35.85 - 0.94 = 34.91 \text{ in.}$$
$$S_x = 504 \text{ in.}^3$$
$$S_y = 45.1 \text{ in.}^3$$
$$J = 10.1 \text{ in.}^4$$
$$a = 145 \text{ in.}$$

For the W21x50 beams:

$$I_x = 984 \text{ in.}^4$$

Determine distribution factors for the applied torques:

$$L/a = 30 \times 12/145 = 2.48$$

From Chart A5, for third-point loading:

$$\frac{\phi GJ}{Ma} = 0.167$$

Rotational moment: $2 \times 40 \times 12 = 960$ kip-in.

Rotational stiffness: $$\frac{M}{\phi} = \frac{11{,}200 \times 10.1}{0.167 \times 145} = 4671 \text{ kip-in./rad.}$$

Rotational stiffness of the two W21x50 beams, assuming the far ends to be hinged:

$$\frac{M}{\phi} = \frac{2 \times 3EI}{L} = \frac{2 \times 3 \times 29{,}000 \times 984}{18 \times 12} = 792{,}667 \text{ kip-in./rad.}$$

	Stiffness	Dist. Factor
W36x150, in torsion:	4671	0.006
W21x50, in bending:	792,667	0.994
Total:	797,338	

Obviously, torsion may be omitted as a problem. However, the solution will be carried through to include calculation of the warping restraint stress.

From Chart A6, for the third-point loading:

$$\frac{\phi'' GJa}{M} = 0.268$$

Torque distributed to the W36x150:

$$M = 0.006 \times 960 = 5.8 \text{ kip-in.}$$

By Eq. (1), warping restraint stress:

$$f_w = \frac{2 \times 145 \times 5.8 \times 0.268}{34.91 \times 45.1} = 0.29 \text{ ksi}$$

In resisting most of the torque, an additional end reactic is added to the vertical load on the W36x150 beams:

$$\frac{954}{2 \times 12 \times 18} = 2.2 \text{ kips}$$

causing added moment in the W36x150 of 2.2x10 = 2 kip-ft, making the total M_b = 788.9 kip-ft, and the tot: direct stress:

$$f = \frac{788.9 \times 12}{504} + 0.29 = 19.07 \text{ ksi.}$$

Example 7b

Given:

Same load as in Example 7a. Determine beam selectio with lateral support at third-points, but without mc ment-resisting connections between the W21x50 beams an the W150x36 beam.

Solution:

Calculations are omitted, as they are completely similar t those of Example 7a. A W36x300 was found to be adequat with stress f_w = 12.44 ksi and f_b = 8.43 ksi, totalling 20.8 ksi. Compared to the previous example, the weight of th 36-in. beam has been doubled.

In the previous examples no shear stress calculation have been made. The shear stress in flanges that accom panies flange moment due to warping restraint should b checked only when torques are applied very near a fixe end or at the end of a very short cantilever beam. Even then flange bending stress due to torque will usually govern. A rule of thumb would be to check flange shear only if th distance from a fixed end (or the length of a cantilevei beam) is appreciably less than $a/2$. Example 8 will illustrate.

Example 8

Given:

In a mill building, a W14x145 column 22 ft in heigh supports a crane runway girder and may be stressed as a column to an average stress of 12 ksi. If used as an anchor to move equipment, a chain hoist hook may be temporarily attached to one flange 18 in. above the base to pull in the weak direction of the column. Determine the maximum allowable pull if the stress due to flange bending stress is allowed to go to 10 ksi.

Solution:

$a = 73.6$ in.

$L/a = 18/73.6 = 0.24$

Use the approximate procedure and check for moment of $18P$ in one flange:

$$10 = \frac{2 \times 18P}{87.3} \quad \text{or} \quad P = 24.2 \text{ kips}$$

Check maximum flange shear, assuming distribution as in a rectangle, i.e., $f_{v(max)} = 1.5 f_{v(avg)}$:

$$A_f = 1.09 \times 15.5 = 16.90 \text{ in.}^2$$

$$f_v = \frac{1.5 \times 24.2}{16.90} = 2.15 \text{ ksi} \quad \textbf{o.k.}$$

Compare foregoing with the more accurate solution:

W14x145 properties:

$h = 14.78 - 1.09 = 13.69$
$S_y = 87.3$ in.^{3}
$a = 73.6$ in.
$J = 15.2$ in.^{4}
$\alpha = 1.5/22 = 0.068$

$L/a = 22 \times 12/73.6 = 3.59$

$$M = \frac{24.2 \times 13.69}{2} = 165.6 \text{ kip-in.}$$

From Chart A4, assuming both ends fixed:

$$\frac{\phi'' GJa}{M} = 0.185$$

By Eq. (1):

$$f_w = \frac{2 \times 73.6 \times 165.6 \times 0.185}{13.69 \times 87.3} = 3.77 \text{ ksi}$$

Due to bending:

$$f_b = \frac{24.2 \times 18}{87.3} = 4.99 \text{ ksi}$$

Maximum combined stress:

$$f = 4.99 + 3.77 = 8.76 \text{ ksi}$$

as compared with 10 ksi by the approximate procedure.

CONCLUSION

In some of the foregoing design examples, the reduction or virtual elimination of the torsion problem through participation of contiguous structural members may seem so obvious as to be trivial. Nevertheless, the following three cases are cited:

1. In a midwestern city, a modern school with brick walls developed large cracks because of the torsional deflection of supporting spandrel beams.
2. In a high-rise steel building in a major city, exterior beams were designed for full torsional loading in a situation very similar to that shown in Example 7.
3. In a western industrial plant, beams were designed for full torsional load in a situation similar to that shown in Example 1b.

In the interest of both safety and economy, it is important to know when and when not to design for torsion, and if torsion must be considered, to be able to evaluate its effective magnitude and the additional warping restraint stresses that result.

DEDICATION

This paper is planned for reprinting in a commemorative volume of papers at the occasion of the 60th birthday of Prof. Dr. Bruno Thürlimann. The writer had the privilege of association with Dr. Thürlimann in the 1940's, at Lehigh University. At that time, as a graduate student, Dr. Thürlimann showed ample evidence of the superior quality that foreshadowed his successful career.

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3. *Hotchkiss, J. G.* Torsion of Rolled Steel Sections in Building Structures *AISC Engineering Journal, 1966, Vol. 3, No. 1, p. 19.*
4. *Lin, P. H.* Simplified Design for Torsional Loading of Rolled Steel Members *AISC Engineering Journal, 3rd Quarter 1977, Vol. 14, No. 3, p. 98.*
5. *Johnston, B. G., F. J. Lin and T. V. Galambos* Basic Steel Design *2nd Ed., Prentice-Hall, Englewood Cliffs, N.J., 1980.*
6. *Chu, K. H. and R. B. Johnson* Torsion in Beams with Open Sections *Journal of the Structural Div., ASCE, Vol. 100, No. ST7, July 1974, p. 1397.*
7. *Ojalvo, M.* Discussion of ASCE Proc. Paper 10390, "Warping and Distortion of I-Section Joints" by P. Vacharajittiphan and N. S. Trahair *Journal of the Structural Div., ASCE, Vol. 101, No. ST1, Jan. 1975, p. 343.*

APPENDIX

In all of the graphical charts that follow, the term M with no subscript refers to the *total torsional moment* applied to the entire span. The graphs for various load and end conditions are based on solutions of the following differential equation, which states that the total torsional moment resisted by a structural shape is the sum of St. Venant torque and warping shear torque:

$$M_z = GJ\frac{d\phi}{dz} - EC_w\frac{d^3\phi}{dz^3}$$

where M_z = torsional resisting moment at any location z.

The development of the equation will be found in many texts, including Ref. 1. The solutions may be written in terms of either exponential or hyperbolic functions, usually the latter, and are tabulated in Ref. 2. The most complex of these, for cases presented in Charts A1, A2, A3, and A4, were obtained by C. P. Heins and P. A. Seaburg[2].

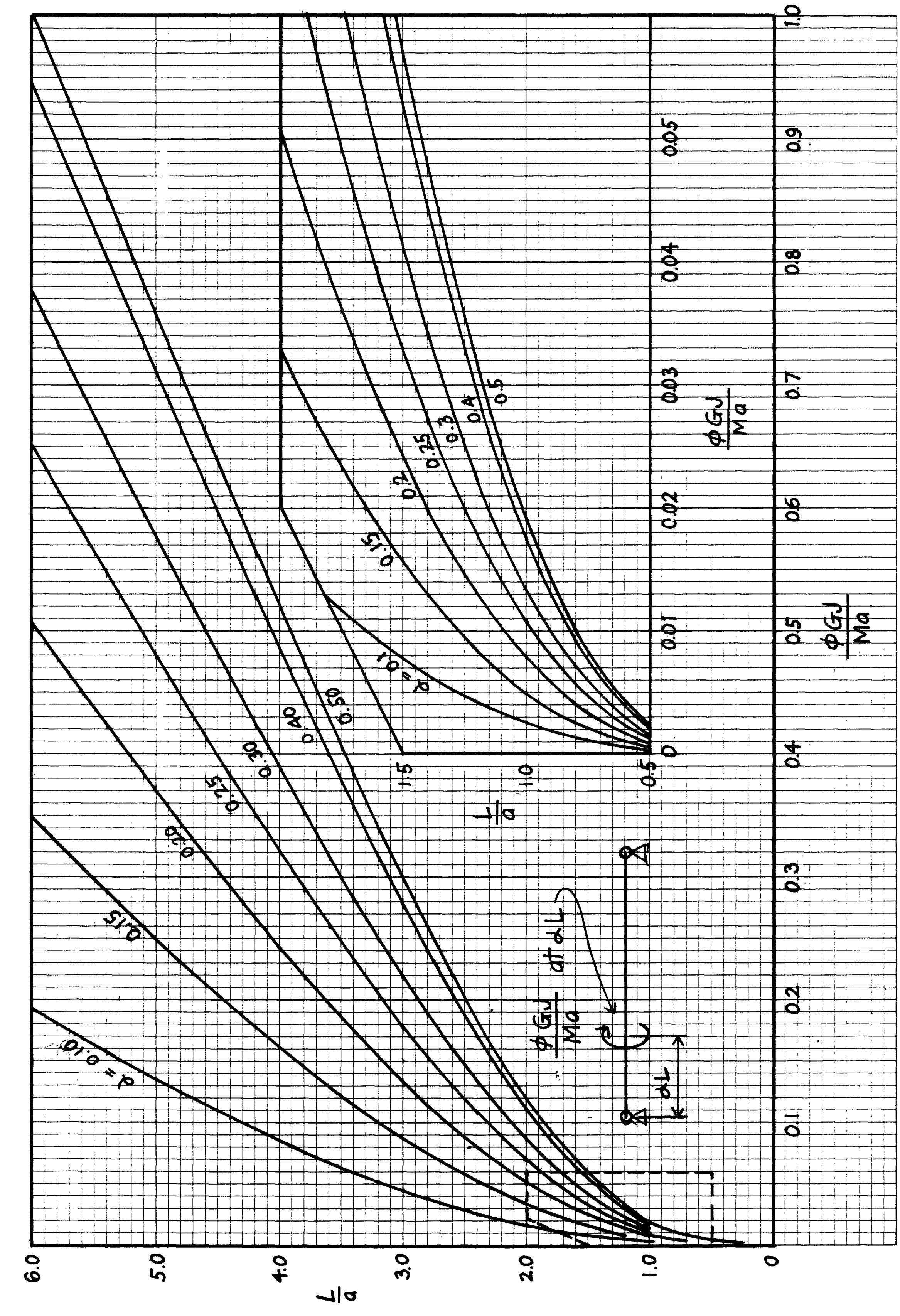
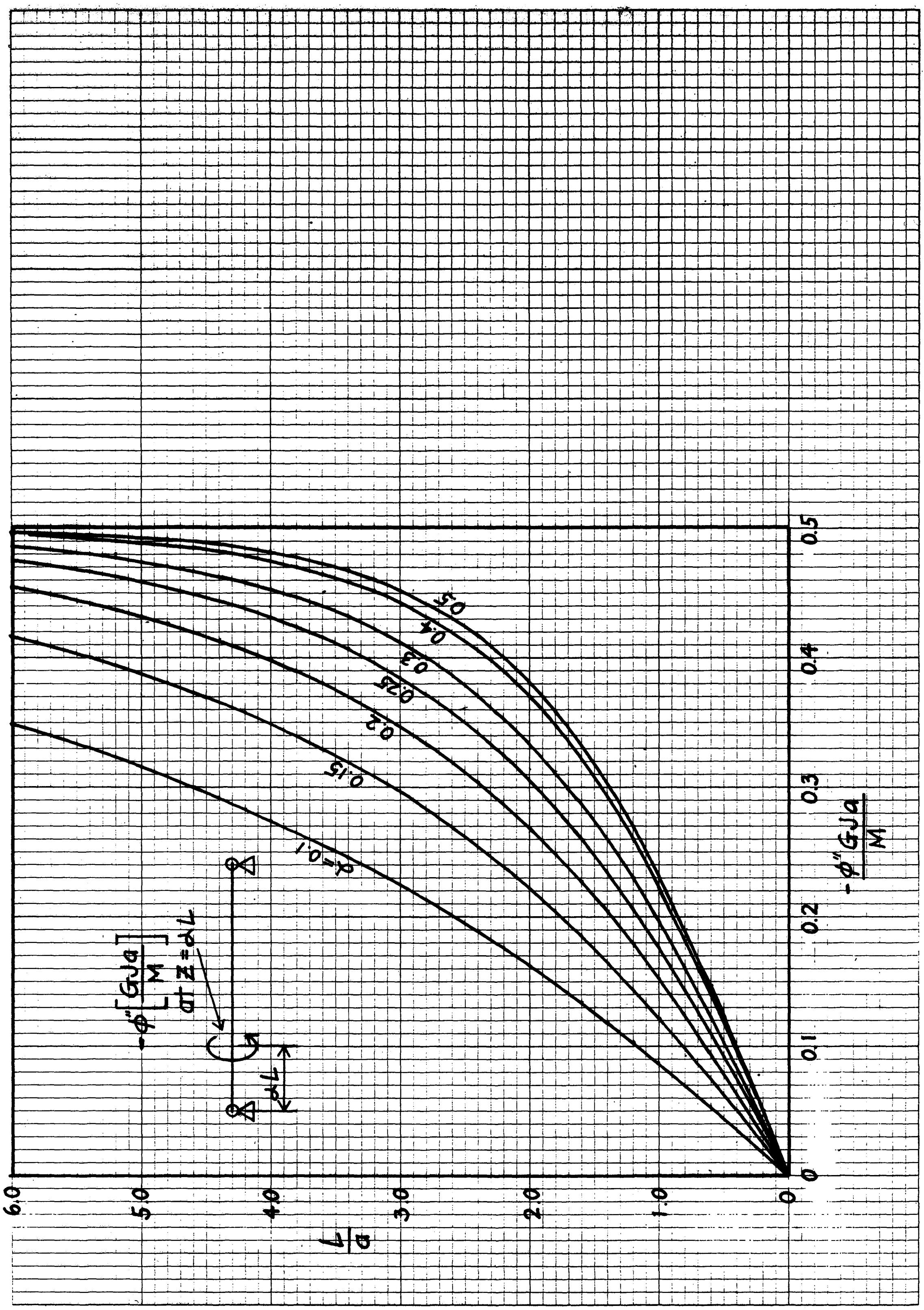

L/a
φGJ/Ma
α = 0.10
0.15
0.20
0.25
0.30
0.40
0.50
φGJ/Ma at αL
αL
α = 0.1
0.15
0.2
0.25
0.3
0.4
0.5

Chart A1



$-\phi'' \left[\frac{GJa}{M}\right]$
at $z = \alpha L$
αL
$\alpha = 0.1$
0.15
0.2
0.25
0.3
0.4
0.5
$\frac{L}{a}$
$-\phi'' \frac{GJa}{M}$
0
0.1
0.2
0.3
0.4
0.5
1.0
2.0
3.0
4.0
5.0
6.0

Chart A2

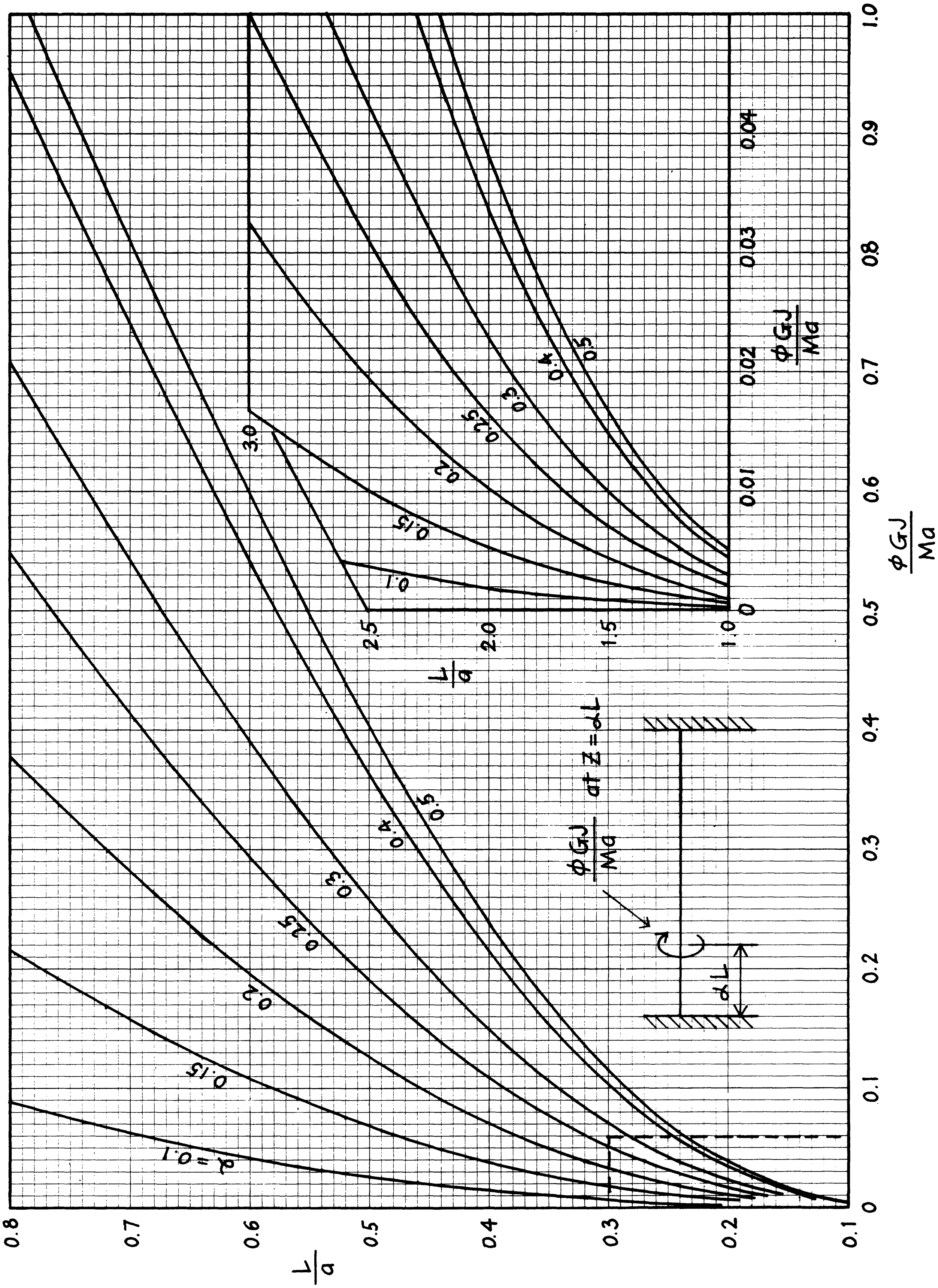

$\frac{L}{a}$
$\frac{\phi GJ}{Ma}$
$\frac{\phi GJ}{Ma}$ at $Z = \alpha L$
αL
$\alpha = 0.1$
0.15
0.2
0.25
0.3
0.4
0.5
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
3.0
2.5
2.0
1.5
1.0
0
0.01
0.02
0.03
0.04

Chart A3

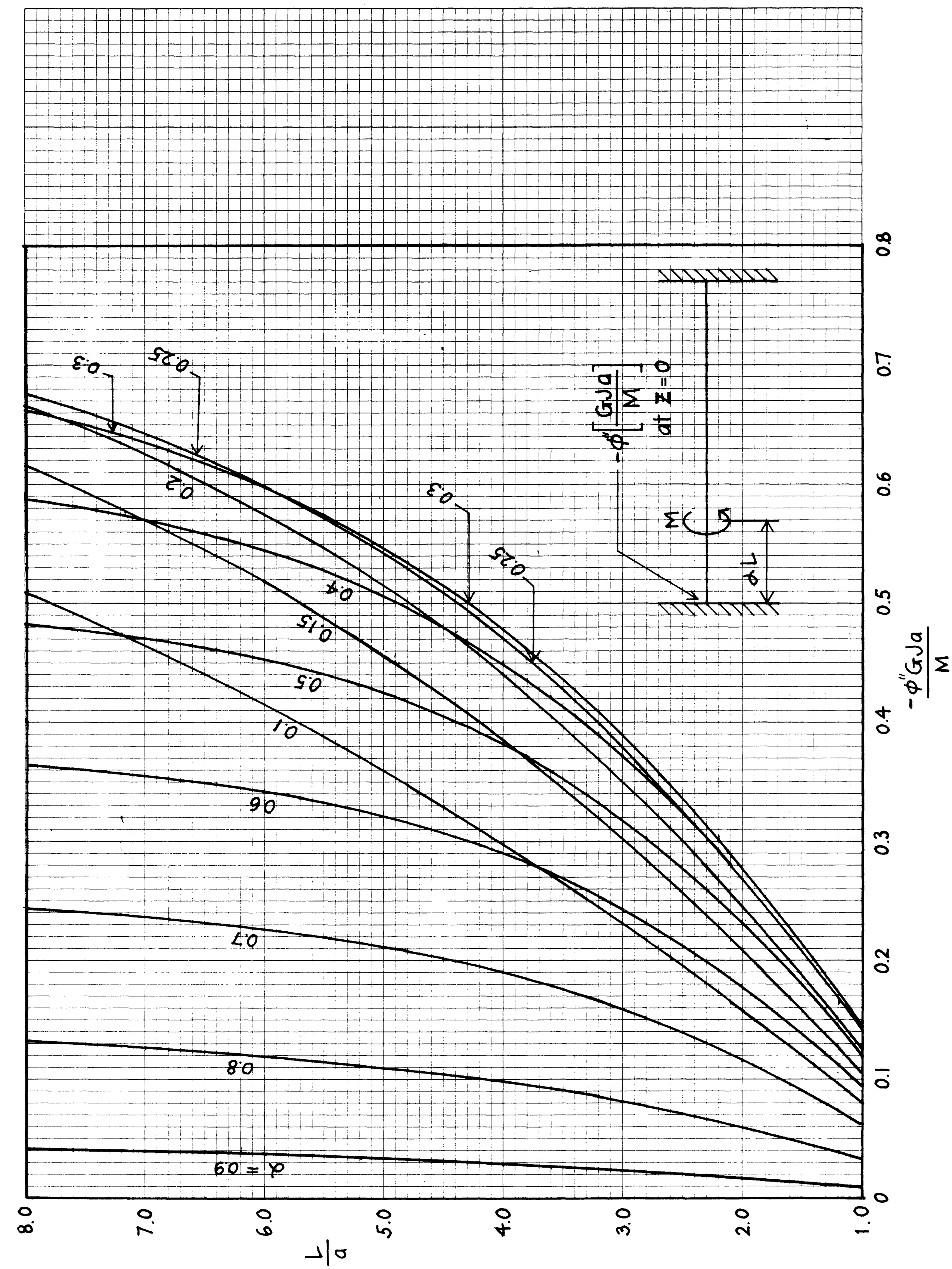
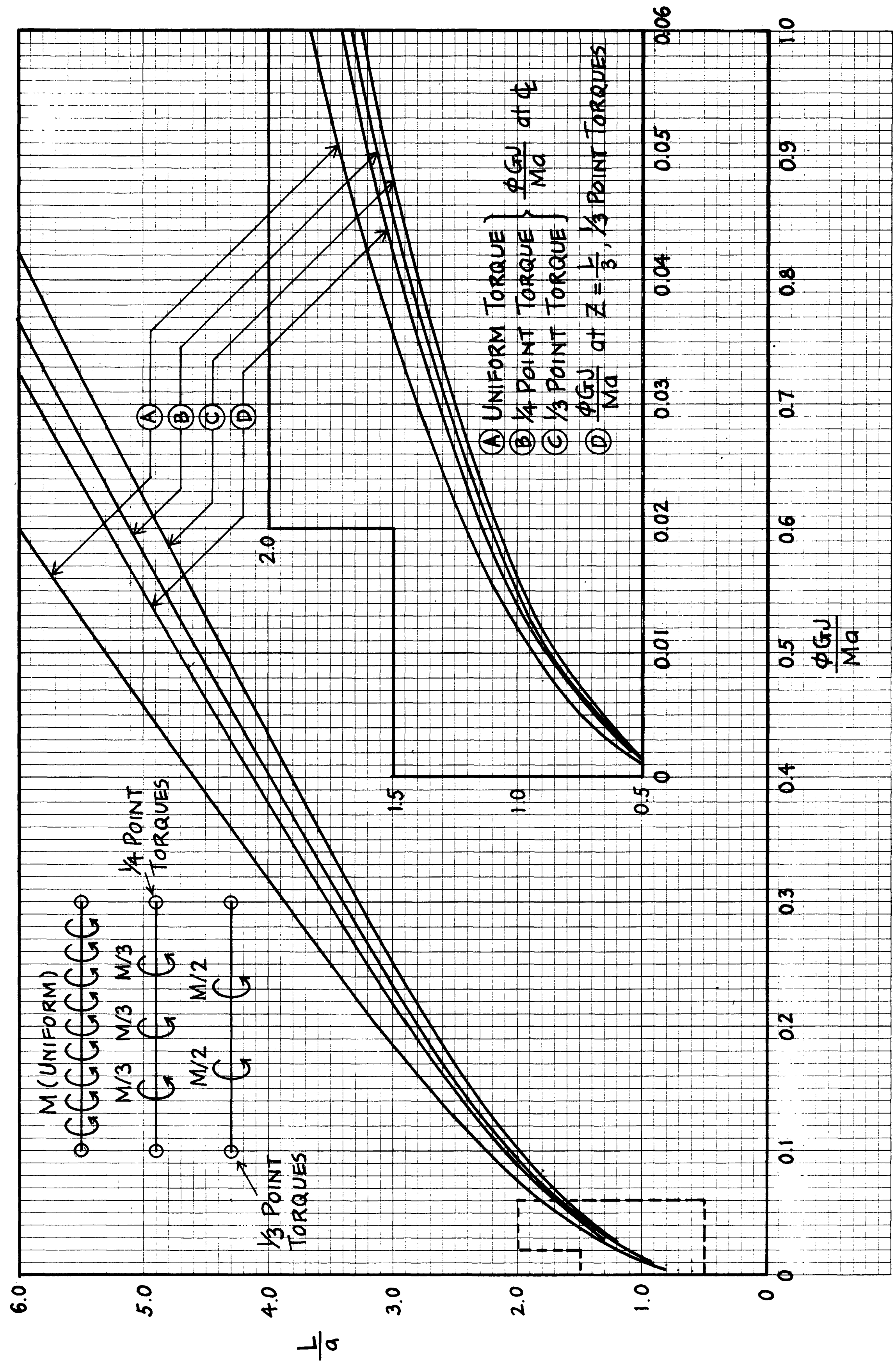
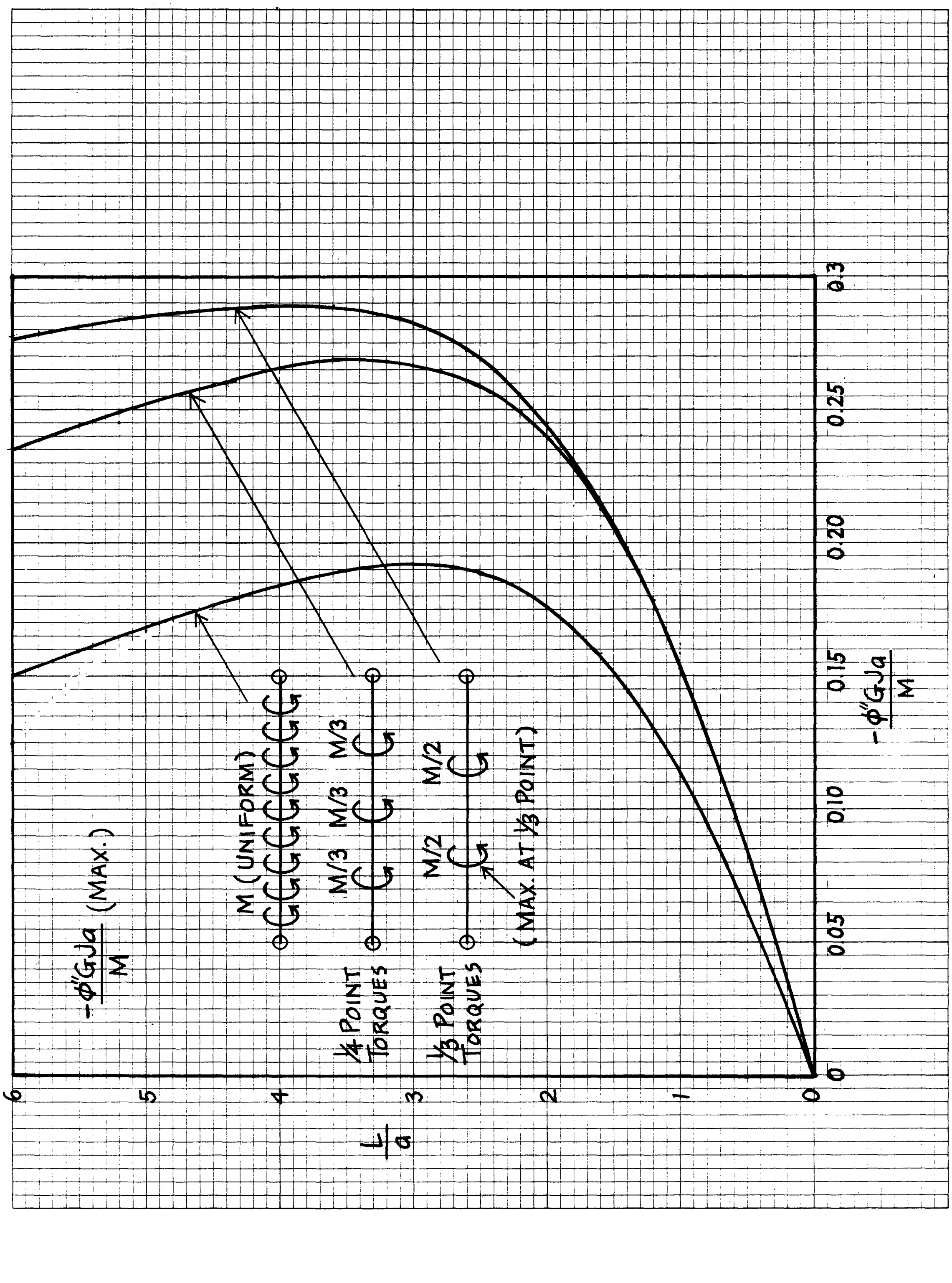

$-\phi''\frac{GJa}{M}$
$\frac{L}{a}$
$-\phi''\left[\frac{GJa}{M}\right]$ at $z=0$
M
αL
$\phi = 0.9$
0.8
0.7
0.6
0.5
0.4
0.3
0.25
0.2
0.15
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0

Chart A4



M (UNIFORM)
M/3 M/3 M/3
M/2 M/2
1/4 POINT TORQUES
1/3 POINT TORQUES
Ⓐ UNIFORM TORQUE
Ⓑ 1/4 POINT TORQUE
Ⓒ 1/3 POINT TORQUE
$\frac{\phi GJ}{Ma}$ at ℄
Ⓓ $\frac{\phi GJ}{Ma}$ at $Z = \frac{L}{3}$, 1/3 POINT TORQUES
$\frac{\phi GJ}{Ma}$
$\frac{L}{a}$
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
0 0.01 0.02 0.03 0.04 0.05 0.06
0 1.0 2.0 3.0 4.0 5.0 6.0
0.5 1.0 1.5 2.0

Chart A5



$-\frac{\phi'' G J a}{M}$ (MAX.)
$\frac{L}{a}$
M (UNIFORM)
1/4 POINT TORQUES
M/3 M/3 M/3
1/3 POINT TORQUES
M/2 M/2
(MAX. AT 1/3 POINT)
0
0.05
0.10
0.15
0.20
0.25
0.3
$-\frac{\phi'' G J a}{M}$
0
1
2
3
4
5
6

Chart A6

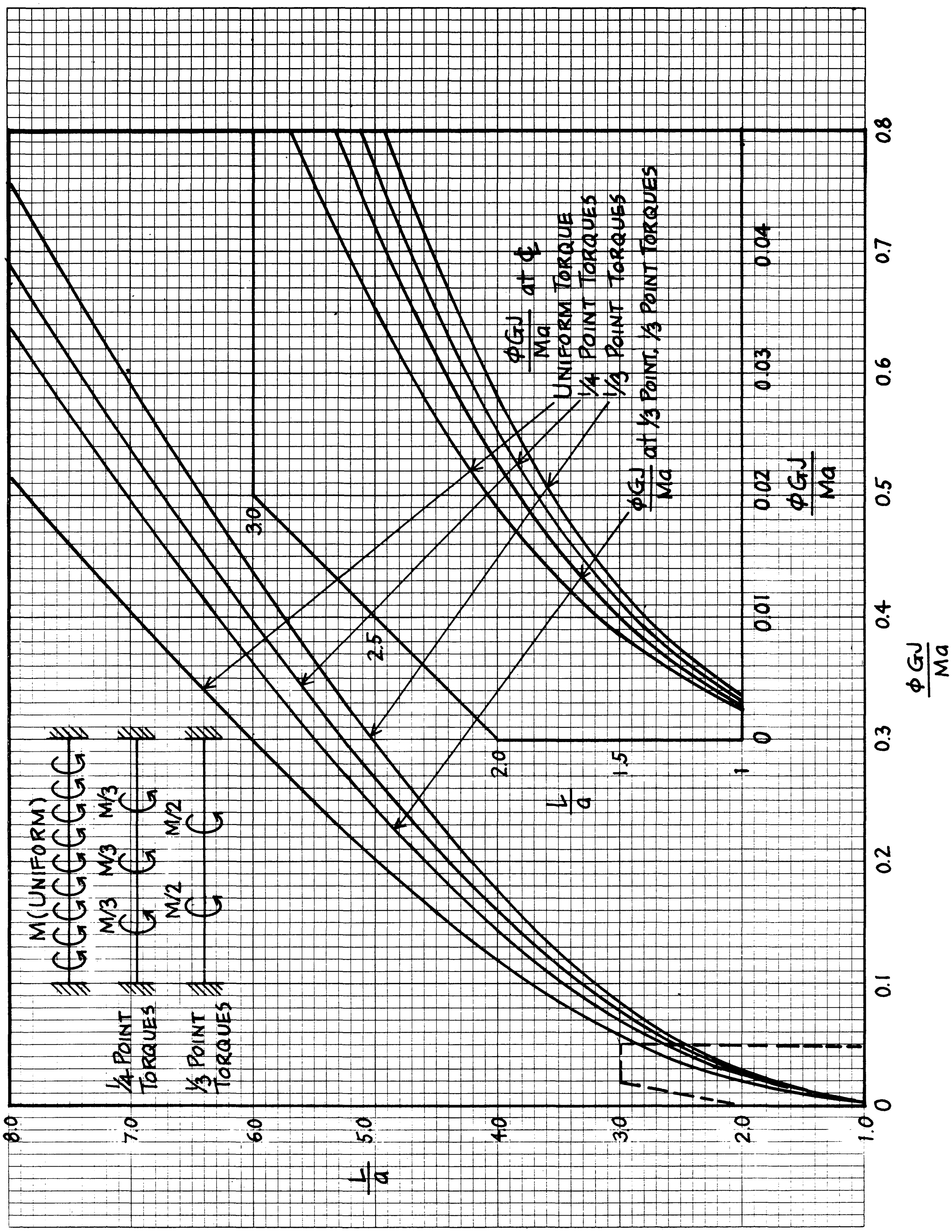

M(UNIFORM)
¼ POINT TORQUES
M/3 M/3 M/3
⅓ POINT TORQUES
M/2 M/2
L/a
φGJ/Ma at ℄
UNIFORM TORQUE
¼ POINT TORQUES
⅓ POINT TORQUES
φGJ/Ma at ⅓ POINT, ⅓ POINT TORQUES
φGJ/Ma
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
0 0.01 0.02 0.03 0.04
1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0
1 1.5 2.0 2.5 3.0

Chart A7

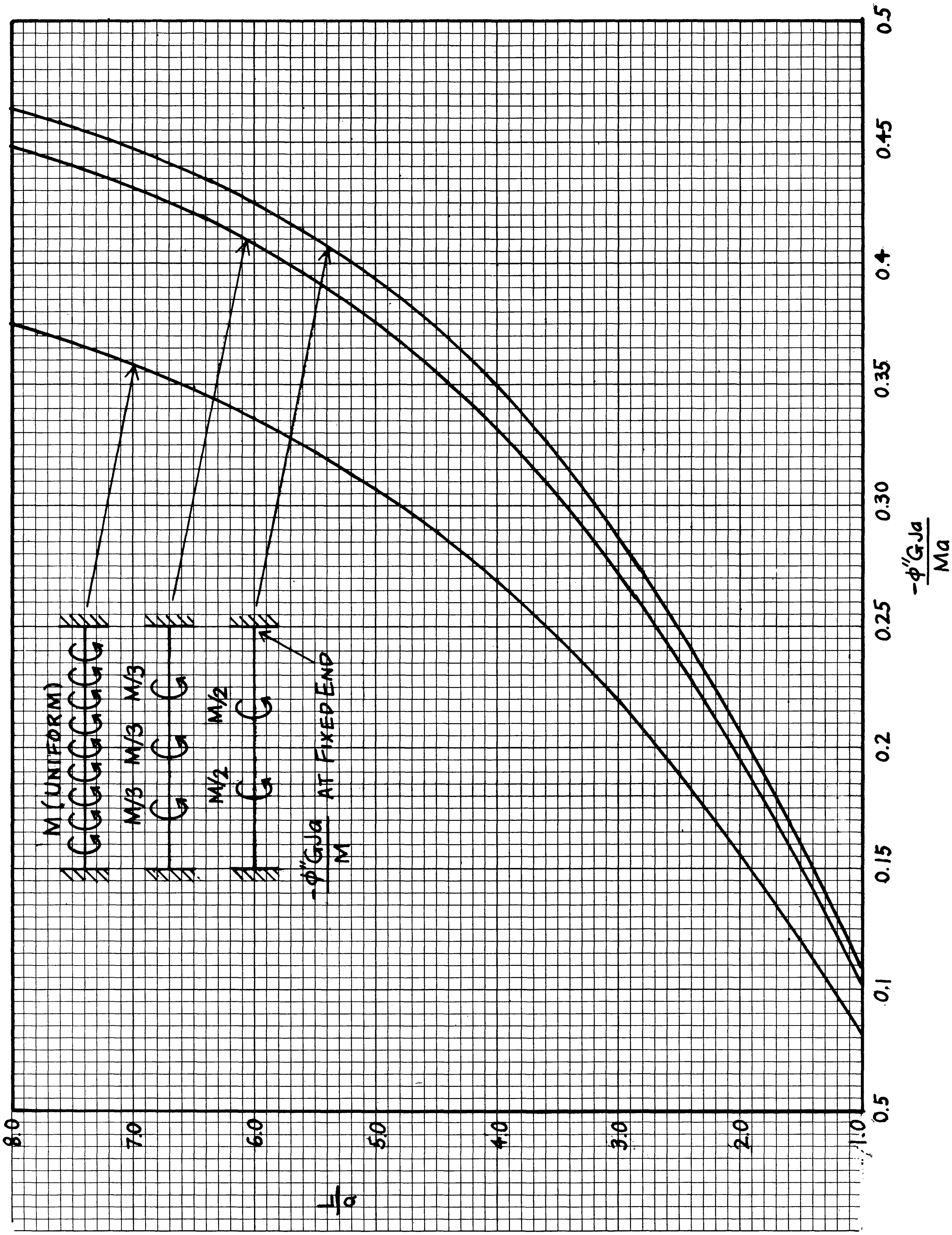

M (UNIFORM)
M/3 M/3 M/3
M/2 M/2
$-\frac{\phi''GJa}{M}$ AT FIXED END
$\frac{L}{a}$
$-\frac{\phi''GJa}{Ma}$
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.5
0.1
0.15
0.2
0.25
0.30
0.35
0.4
0.45
0.5

Chart A8

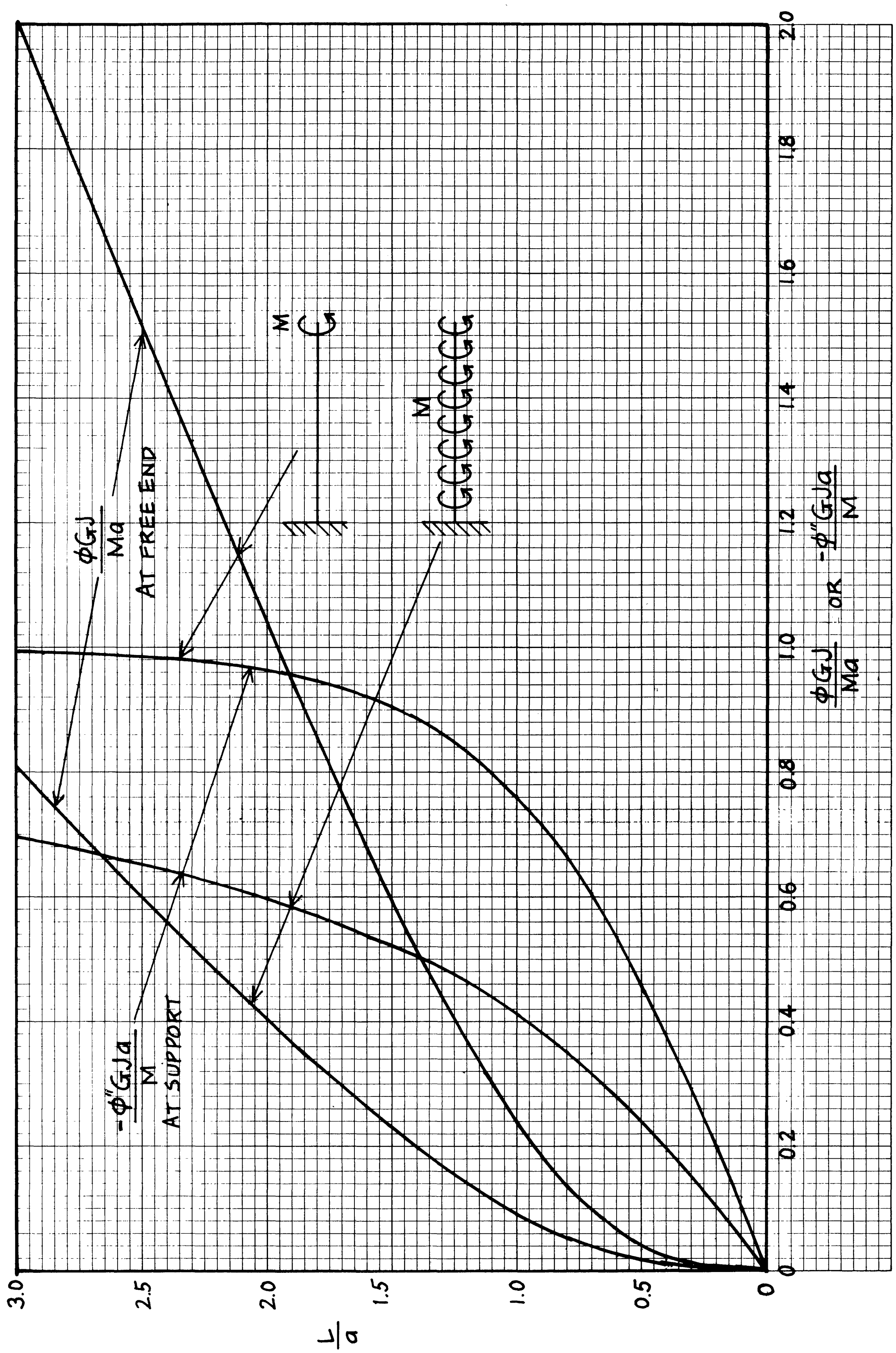

$\frac{\phi GJ}{Ma}$
AT FREE END
$\frac{-\phi'' GJa}{M}$
AT SUPPORT
M
M
$\frac{\phi GJ}{Ma}$ OR $\frac{-\phi'' GJa}{M}$
L/a
0
0.5
1.0
1.5
2.0
2.5
3.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0

Chart A9