Plastic Design of a Three-Story Steel Frame

OSCAR DE BUEN

THIS PAPER DESCRIBES the plastic analysis and design of a three-story steel frame for an office building located in Mexico City.

Two types of design loads were considered: vertical live and dead loads, classified as "permanent loads" and horizontal earthquake loads, classified as "accidental loads". (The Mexico City area is subject to intense earthquakes; consequently wind loads play a secondary role for design purposes in this type of building.)

A conventional plastic design method was used for the analysis of vertical loading and the method of plastic distribution of moments was used for seismic analysis, with certain modifications for adaptation to the particular problem under study.

The design method basically follows the latest AISC Specification, Part 2. However, because the steel frame is more than two stories high and is out of the range of these specifications, the column design formulas were modified to comply with our particular requirements.

Vertical and horizontal loads and load factors were established following the Mexico City Building Code recommendations. Load factors were $\lambda_1 = 1.85$ for "permanent" loads and $\lambda_2 = \lambda_1/1.50 = 1.23$ for the combination of "permanent" and "accidental" loads.

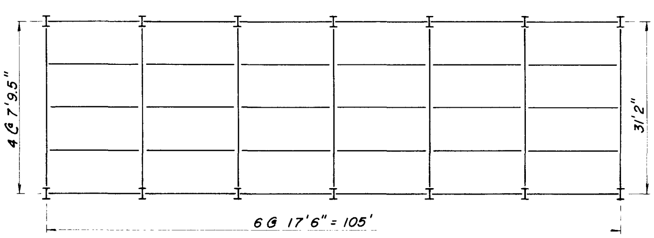

Fig. 1. Floor plan

Oscar de Buen is a Consulting Engineer and Professor of Civil Engineering, University of Mexico, Mexico City, and is a Professional Member of AISC.

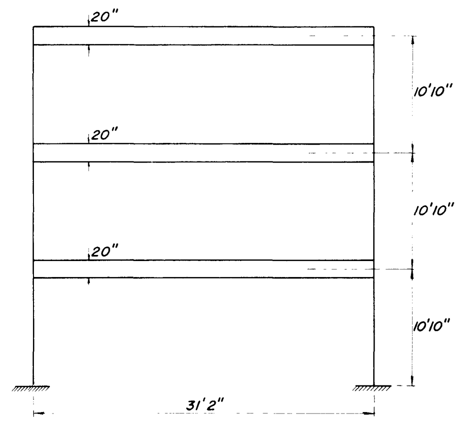

Fig. 2. Cross section

BUILDING DESCRIPTION

Figure 1 shows a typical floor plan arrangement. Figure 2 illustrates a typical cross-section of the steel frame.

Secondary floor beams were designed with both ends simply supported and working in composite action with the concrete floor slab.

LOADS

Total vertical design load (dead plus live) is 154 lbs/sq ft for the first and second floors and 135 lbs/sq ft for the third floor. Live load is only one-third of the total load for the first and second floors and only one-seventh of the total load for the third floor. Consequently, it was not considered necessary to analyze alternate live load distributions and design was based on full dead plus live loading applied throughout the steel frame.

Seismic design coefficient was assumed as 0.07, and seismic accelerations were considered with straight line variations in intensity having a maximum value at the top.

VERTICAL LOAD ANALYSIS

Girders (λ_1 = **1.85**)—Secondary floor beams originate concentrated loads on the main girders; however, for design purposes these loads were substituted with a uniformly distributed load, with a total value represented by the total vertical load applied on the area located between two parallel transverse bents.

Plastic moments on the girders were computed assuming the formation of a mechanism with plastic hinges at both ends and at the center of the span. Theoretical lengths center to center of column axis were considered for all computations.

Example—Design considerations for the 31 ft-2 in. main girders on first and second floors were as follows:

$$\text{Total vertical load: } 17.5 \times 31.167 \times 0.154 = 83.6 \text{ kips}$$

$$83.6 \times 1.85 = 154.5 \text{ kips}$$

$$M_p = \frac{154.5 \times 31.167}{16} = 302 \text{ kip-ft}$$

Columns—Columns should be capable of supporting moments and axial loads transmitted by the girders until mechanisms are formed in the girders. Moments and axial loads are shown in Fig. 3. Moments on the columns were computed following the method of elastic distribution of the moments transmitted by the girders at the formation of mechanisms. (Weight of the walls located along both longitudinal axes of the building is included as part of the axial loads considered in the design.)

Moments due to vertical loads, transmitted by the longitudinal beams framed to the columns are insignificant for interior columns, because the adjacent spans are the same and the live load is a small percentage of the total load. These moments were therefore disregarded.

Figure 4 shows the plastic moments due to the vertical loads considered in the main frame.

SEISMIC ANALYSIS (λ_2 = 1.23)

The steel frame was designed to withstand the horizontal seismic loads established by the Mexico City Code, using a 0.07 seismic coefficient, acting simultaneously with the vertical (dead and live) loads, both multiplied by the load factor λ_2 = 1.23.

Transverse Bent—Design loads are shown in Fig. 5 and a possible collapse mechanism is shown in Fig. 6, assuming plastic hinges at the column bases, and at the middle and right end sections of the girders. Moments shown at the plastic hinges are the M_p values required for vertical loading and moments shown at the left end of each girder are the moments corresponding to a distribution of moments in equilibrium.

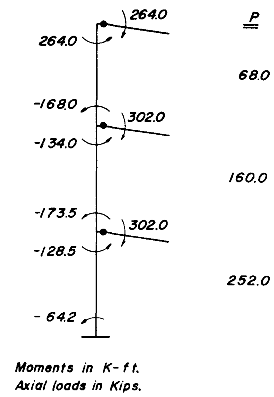

Fig. 3. Vertical loading: moments and axial loads in columns

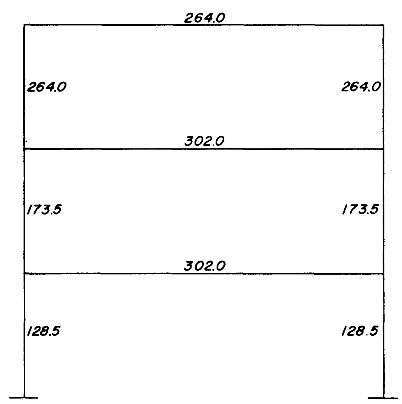

Fig. 4. Vertical loading: plastic moments, in kip-ft

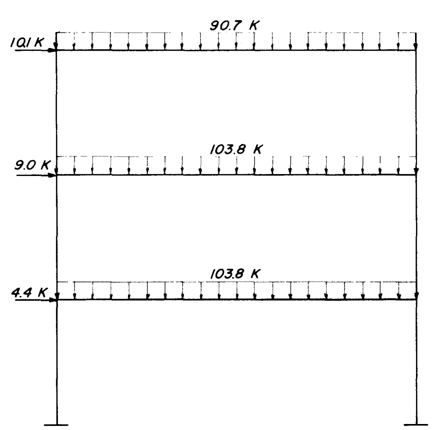

Fig. 5. Vertical + seismic loading: design loads

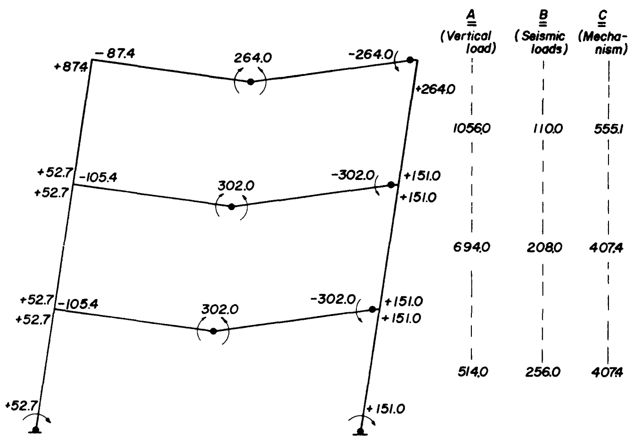

Fig. 6. Vertical + seismic loading: possible collapse mechanism (moments in kip-ft)

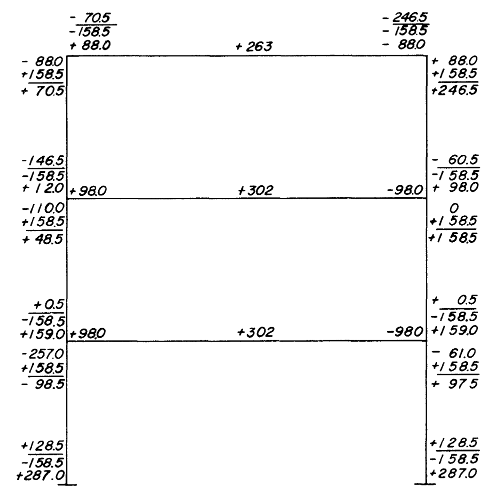

Fig. 7. Vertical + seismic loading: plastic moment distribution

A check was made to find if the transverse bent is capable of supporting the loads shown in Fig. 5 with the plastic moments necessary to support the vertical loads, or if this is not the case, to find which members need to be reinforced. This condition was checked as described below.

Columns A, B and C of Fig. 6 show the following data:

Column A: Sum of plastic moments available at both ends of both columns at each floor line, in accordance with the conditions assumed for vertical loading (M_p values necessary to support vertical loading.)

Column B: Sum of the existing earthquake moments at both ends of both columns at each floor level (product of seismic shear force existing on each floor level times the height of the columns on that floor).

Column C: Sum of moments at both ends of the columns at each floor level, corresponding to the mechanism as assumed in Fig. 6.

Comparing moments in columns C and B, it is noted that the mechanism of Fig. 6 is probably not going to be developed, because the values of column C are much larger than those of column B, which are the moments due to seismic loading. If a comparison is made of the moments in columns B and A it is seen that the loading combination under study is probably not going to be critical, since the plastic moments required on the columns due to vertical loading are much larger than the moments due to seismic loading. (It must be remembered that two different load factors are used for the two different load combinations.)

The next step was to try to find a distribution of moments in equilibrium with the existing exterior loads, with values smaller or equal in every section to the moments necessary for vertical loading.* The following method was applied (Fig. 7):

1. Moments were applied at the ends of all the girders, with the necessary values to reduce the moment at the center to the vertical load M_p value, with horizontal corrective diagrams.
2. Equilibrium of joints was obtained, starting from top to bottom, satisfying simultaneously the equilibrium of horizontal loads existing in every floor (the sum of moments at both ends of all columns should be equal to the value of the moments shown under column B in Fig. 6). It was found that the moments at the first floor columns were larger than those necessary for vertical loading.
3. Moments at the lower end of the first story columns were reduced to the value required for vertical loading, by applying at that end a moment of -158.5 kip-ft and a moment of $+158.5$ kip-ft at the upper ends, in order to maintain the equilibrium of horizontal loads.
4. Equilibrium of joints was obtained, starting from the bottom, satisfying simultaneously the equilibrium of horizontal loads, with the application of a -158.5 kip-ft moment at the lower ends of columns and the application of a $+158.5$ kip-ft moment at the upper ends.

Following this method, a moment distribution corresponding to the frame subject to vertical and earthquake loads with a load factor $\lambda_2 = 1.23$ was obtained, satisfying equilibrium conditions, with all moments smaller or equal to the corresponding moments required for vertical loading. Consequently, the critical loading condition for design of every member of the frame under study is the vertical load alone, with a load factor $\lambda_1 = 1.85$.

* *Comparison of moments shown in columns A, B and C gives a good indication of the behavior of the frame, permitting quick results using plastic distribution of moments. The application of this method to seismic analysis in other frames has given satisfactory results in every case.*

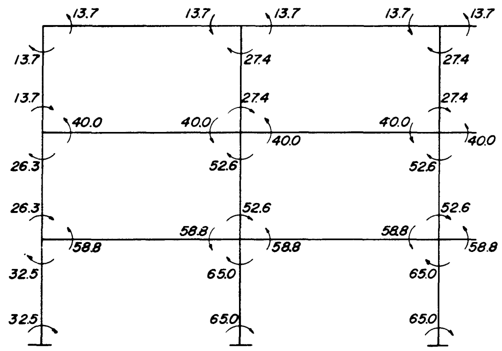

Fig. 8. Moments due to longitudinal seismic forces (kip-ft)

Longitudinal Frames—Design of longitudinal frames was made in such a way as to satisfy the moment distribution shown in Fig. 8. Moments indicated at the mid-span of the girders are the simply supported moments with a load factor $\lambda_2 = 1.23$, because a distribution of moments was chosen in such a way that corrections at mid-span were not necessary.

DESIGN

A36 steel was assumed for the hot rolled beams and A7 steel was assumed as the basis of the design for the members fabricated from steel plates. This last selection was forced upon the designer because of conditions concerning availability of steel in Mexico at the time this analysis was made.

Girders—Total girder height was limited to a maximum of 20 in. because of architectural considerations. Hot rolled I beam sections were chosen for the spandrel girders and secondary beams spanning 17 ft-6 in., and welded I beam sections were selected for the 31 ft-2 in. main girder spans. These selections were necessary because Mexican mills do not roll I beam sections deeper than 15 in.

Design of a Main Girder:

$M_p = 302$ kip-ft
$L = 31$ ft-2 in.

A fabricated welded I beam section was chosen using 8 in. $\times$ 5/8 in. plates for the flanges and a web consisting of a 13 in. $\times$ 1/4 in. plate. Plastic moment of this section is 315 kip-ft.

Shear:

$V_u = 78.5$ kips
$0.00055 \times 33 \times 0.25 \times 18$
$= 81.5 \text{ kips} > 78.5 \text{ kips}$ **ok**

Width-thickness ratios:

Flanges: $\dfrac{4}{5/8} = 6.4 < 8.5$ **ok**

Web: $\dfrac{18}{1/4} = 72 \cong 70$ **ok**

Lateral bracing: The girder is supported laterally all along its upper flange by the concrete slab and at secondary beam connecting points; consequently, there is no need to install any additional bracing at the central hinge.

The lower flange is in compression at the hinges located at the ends of the girder.

Connections of secondary floor beams were designed to provide adequate lateral support to the lower flange of the main girder; then:

$$L_{cr} = \left[60 - 40\left(-\frac{137}{302}\right)\right]1.93 =$$
$$(60 + 18.2)1.93 \cong 152 \text{ in.} > 7 \text{ ft-}9\tfrac{1}{2} \text{ in.}$$

Columns—Columns were designed using formulas (7a) and (7b) of the AISC Specification, for members under axial compression and bending, modified to be employed with ultimate bending moments and axial loads.

The building does not have nonstructural elements providing lateral restraint against sidesway; consequently, this restraint must be provided by the frame.

Formula (20) of the AISC Specification, Part 2, was not considered suitable because it indicates only the cases in which the strength of the columns is not reduced by sidesway of the frame, and it does not provide any way of computing the reduced strength in cases in which sidesway does exist.

The effect of sidesway on the strength of columns was considered by the introduction of a coefficient, larger than 1.0, multiplying the first term of formula (7a), obtained by the application of Merchant's formula[1,2] for frame buckling, as it is shown in the following numerical example. (It is not the intention of this article to propose a definite solution of the design problem originated by the columns in frames in which sidesway is not prevented; however, it has been found that this approach to the problem is a logical one and it could be employed until more complete theoretical and experimental information is obtained.)

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1. *Merchant, W., Rashid, C. A., Bolton, A. and Salem, A.* The Behavior of Unclad Frames *Proceedings, Fiftieth Anniv. Conf., Inst. of Structural Engineers, London, 1958*

2. *Horne, M. R.* Instability and the Plastic Theory of Structures *Engineering Inst. of Canada, Transactions 1960, Vol. 4, No. 2, p. 31*

Design formulas are then:

$$\alpha \frac{P}{P_0} + \frac{M_x}{M_{cr_x}} \left[\frac{1}{1 - P/P_{ex}} \right] C_m + \frac{M_y}{M_{cr_y}} \left[\frac{1}{1 - P/P_{e_y}} \right] C_m \leq 1.0 \quad (1)$$

$$\frac{M}{M_p} = 1.18 \left(1 - \frac{P}{P_y} \right) \leq 1.0 \quad (2)$$

where

$P_0 = P_{cr}$ = critical axial load (without moment)

M_{cr_x}, M_{cr_y} = critical moments which could be supported by the column (without axial load)

P_{ex}, P_{ey} = elastic critical axial loads

C_m = a coefficient whose value is, in this case, 0.85 (frames subject to joint translation)

P_y = $A\sigma_y$

α = a coefficient which takes into account the sidesway of the frame

Example—Design of the first floor columns:

Vertical load: ($\lambda_1 = 1.85$)	P	= 252 kips
	M_x	= 128.5 kip-ft
Vertical load + seismic load:	P	= 168.5 kips
($\lambda_2 = 1.23$)	M_x	= 85.6 kip-ft
	M_y	= 65.0 kip-ft

Try a section composed of three welded plates, 12 in. × ¾ in. in flanges and 10 in. × ½ in. in the web.

$A = 23.1$ in.^{2}	$I_x = 563$ in.^{4}	$S_x = 98$ in.^{3}
$Z_x = 108$ in.^{3}	$r_x = 5.03$ in.	$L = 118$ in.
$I_y = 217$ in.^{4}	$S_y = 36.1$ in.^{3}	$Z_y = 53.7$ in.^{3}
$r_y = 3.06$ in.	$K_x \cong K_y \cong 1.6$	

(K values were obtained using the nomogram recommended by the AISC.)

$P_o = 666$ kips $M_{cr_x} = M_{p_x} = 298$ kip-ft;

$P_{ex} = 11{,}600$ kips $M_{cr_y} = M_{yp} = 148$ kip-ft;

$P_{ey} = 4{,}400$ kips

Coefficient α is computed employing Merchant's formula

$$\frac{1}{\lambda_{cr}} \leq \frac{1}{\lambda_p} + \frac{1}{\lambda_{EC}}$$

where λ_{cr} is the column true load factor, λ_p the simple plastic theory load factor and λ_{EC} the elastic load factor. (In fact, λ_p has not been computed, but is, necessarily, greater than 1.85 (or 1.23), because it corresponds to the simple plastic theory, without taking into account the effect of axial loads in columns. Conservative values of α are therefore obtained; most probably, in this particular problem, frame buckling does not reduce the structure's real load factor.)

$$P_{crE} = \frac{\pi^2 E I_y}{(KL)^2} = 2{,}850 \text{ kips}, \qquad \lambda_{EC} = \frac{2{,}850}{252/1.85} = 20.9$$

$$\text{Vertical load: } \frac{1}{\lambda_{cr}} = \frac{1}{1.85} + \frac{1}{20.9} = 0.591 = \frac{1}{1.68}$$

$$\lambda_{cr} = 1.68; \text{ then } \alpha = \frac{1.85}{1.68} = 1.10$$

$$\text{Vertical + seismic loads: } \frac{1}{\lambda_{cr}} = \frac{1}{1.23} + \frac{1}{20.9} = \frac{1}{1.16}$$

$$\alpha = \frac{1.23}{1.16} = 1.06$$

Then, from Equations (1) and (2):

$$\text{Vertical load: } \frac{252}{666}(1.10) + \frac{128.5}{298}\left[\frac{0.85}{1 - 252/11{,}600}\right] =$$

$$0.416 + 0.375 = 0.791 < 1.00$$

Vertical load + longitudinal seismic load:

$$\frac{168.5}{666}(1.06) + \frac{85.6}{298}\left[\frac{0.85}{1 - \frac{168.5}{11{,}600}}\right] + \frac{65}{148}\left[\frac{0.85}{1 - \frac{168.5}{4{,}400}}\right]$$

$$= 0.269 + 0.248 + 0.386$$

$$= 0.903 < 1.00$$

$$M = 1.18\left(1 - \frac{252}{765}\right) M_p = 1.18 \times 0.670 \times 298$$

$$= 236 \text{ kip-ft} > 128.5$$

The section is correct.