Stability of Metal Structures—A World View

STRUCTURAL STABILITY RESEARCH COUNCIL
EUROPEAN CONVENTION FOR CONSTRUCTIONAL STEELWORK
COLUMN RESEARCH COMMITTEE OF JAPAN
COUNCIL OF MUTUAL ECONOMIC ASSISTANCE

PART B. APPROACHES AND DESIGN PROCEDURES

CHAPTER		CONTRIBUTORS			
No.	Title	Japan	North America	West Europe	East Europe
1	Compression Members	M. Wakabayashi	R. Zandonini, L. Tall	G. W. Schulz	O. Halasz, M. Ivanyi
2	Built-up Members	T. Suzuki	B. G. Johnston	L. Finzi	O. Halasz, M. Ivanyi
3	Beams	Y. Fukumoto	T. V. Galambos	J. Lindner	O. Halasz, M. Ivanyi
4	Plate and Box Girders	F. Nishino	A. Ostapenko	C. Massonnet	O. Halasz, M. Ivanyi
5	Beams-Columns	H. Akiyama	W. F. Chen	J. Strating	O. Halasz, M. Ivanyi
6	Frames	S. Morino	L. W. Lu	U. Vogel, C. Massonnet	O. Halasz, M. Ivanyi
7	Triangulated Structures	T. Nakamura	T. V. Galambos	P. Dubas, C. Urbano	O. Halasz, M. Ivanyi
8	Shells	S. Kobayashi	A. Chajes	D. Vandepitte	O. Halasz, M. Ivanyi
9	Composite Members	M. Wakabayashi	G. Winter	P. J. Dowling	—
10	Cold-formed Steel	—	S. J. Errera	J. W. Stark	O. Halasz, M. Ivanyi

This comprehensive comparison of the current state of the art in research and design for structural stability, as viewed in four major regions of the world, has been published in serial form in the AISC Engineering Journal. This final installment contains Chapters 8, 9 and 10 of Part B, and Part C. A List of Abbreviations and a Glossary of Terms covering the complete report is contained in the first installment (3rd Quarter 1981).

Coordinating Editors:
Duiliu Sfintesco, CTICM, France
Lynn S. Beedle, Lehigh University, USA
Gerald W. Schulz, University of Innsbruck, Austria
Riccardo Zandonini, Technical University of Milan, Italy

Regional Editors:
Japan: Ben Kato, University of Tokyo
North America: T. V. Galambos, Washington University in St. Louis
West Europe: Duiliu Sfintesco, CTICM
East Europe: Otto Halasz, Technical University of Budapest

Chapter 8. Shells

a. REGIONAL RECOMMENDATIONS

JAPAN

(There are no regional recommendations in Japan.)

NORTH AMERICA

(Key Document: SSRC Guide[1])

CYLINDRICAL SHELLS—HYDROSTATIC PRESSURE

The critical pressure in the elastic range is

$$p_{cr} = \frac{C_1 8E}{\sqrt{3(1-\mu^2)}}\left(\frac{t}{d}\right)^2$$

where

E = Young's modulus
t = thickness
d = diameter
μ = Poisson's ratio
C_1 = a constant which can be determined from a curve given in the Guide

Commentary—The above formula applies to elastic buckling of unstiffened shells or elastic buckling between stiffeners of stiffened shells. Additional procedures are given in the Guide to calculate the inelastic buckling pressure and to deal with overall buckling of stiffened shells.

SPHERICAL SHELLS—EXTERNAL PRESSURE

The critical pressure is

$$p_{cr} = \frac{8}{3}\eta E\left(\frac{t_m}{r}\right)^2 \times \left\{\left[0.21\left(\frac{\Delta}{t_m}\right)^2 + 0.0715\left(\frac{t_B}{t_m}\right)^{3/2}\right]^{1/2} - 0.459\frac{\Delta}{t_m}\right\}$$

where

η = plasticity reduction factor
E = Young's modulus
Δ = deviation from perfect surface
t_m = effective membrane thickness
t_B = effective bending thickness

A method is given in the Guide for calculating η, t_m and t_B.

Commentary—This formula applies to stiffened and orthotropic shells as well as to unstiffened isotropic shells. The formula can account for imperfections of any magnitude.

WEST EUROPE

(Key Document: ECCS Recommendations[24])

The March 1978 edition of the ECCS Recommendations does not contain any rules regarding the design of shells. A first draft of such rules is to be found in Chapter 10 of the ECCS Manual[2], used as the Introductory Report for the Second International Colloquium on Stability. The following summarizes the February 1979 draft of those rules.

UNSTIFFENED CYLINDERS SUBJECT TO MERIDIONAL COMPRESSION

For the rules to apply, l/r may not exceed $0.95\sqrt{r/t}$ and the maximum inward imperfection measured from a template of length $l_r = 4\sqrt{rt}$ may not exceed $0.02l_r$. If it exceeds $0.01l_r$, the given reduction factors α must be halved.

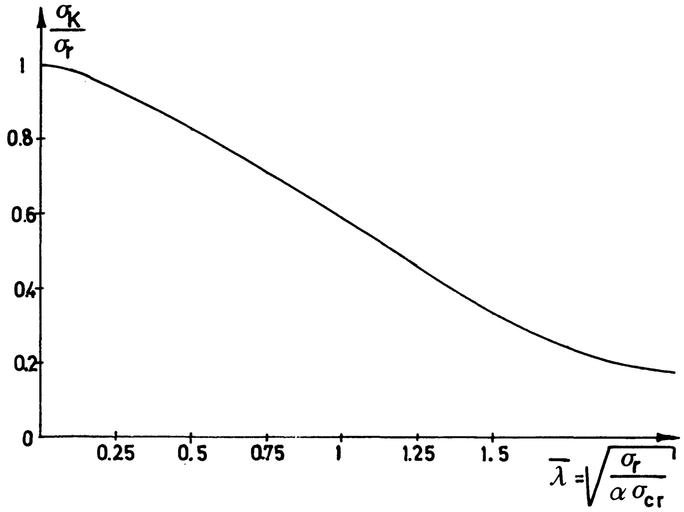


Figure WE8.1

The strength condition $\sigma_d \leq \sigma_K$ must be checked, σ_d and σ_K being the factored acting meridional compressive stress and the design value of the buckling stress, respectively. The ratio σ_K/σ_r is given (Fig. WE8.1) as a function of the slenderness parameter $\bar{\lambda} = \sqrt{\sigma_r/\alpha\sigma_{cr}}$, where

$$\sigma_{cr} = 0.605E(t/r) \qquad \text{(WE8.1)}$$

$$\sigma_K/\sigma_r = 0.75/\bar{\lambda}^2 \quad \text{when } \bar{\lambda} \geq \sqrt{2} \qquad \text{(WE8.2)}$$

$$= 1 - 0.4123\bar{\lambda}^{1.2} \quad \text{when } \bar{\lambda} \leq \sqrt{2} \qquad \text{(WE8.3)}$$

The reduction factor α is given numerically as a function of r/t for:

1. Axially loaded cylinders.
2. Cylinders subject to pure bending.
3. Cylinders subject to compression and bending.
4. Cylinders subject to axial compression and an internal pressure p.

In case 4, α depends also on $pr\sqrt{r}/Et\sqrt{t}$. The values of α are derived from the lower bound of a cloud of experimental points. Equation (WE8.2) contains an additional partial safety factor $\gamma = 4/3$ which accounts for the high imperfection sensitivity of meridionally compressed cylinders.

CYLINDERS UNDER UNIFORM EXTERNAL PRESSURE

The rules apply if the out-of-roundness does not exceed 0.5% of the radius. The strength requirements are:

For the cylinder: $p_d \leq p_K$ where p_d and p_K are the design values of the acting external pressure and of the buckling pressure.

For the stiffening rings: Considering both direct stresses and bending stresses, the latter due to an assumed out-of-roundness of 0.5%, the pressure $4p_d/3$ shall not cause the stiffener flange to yield.

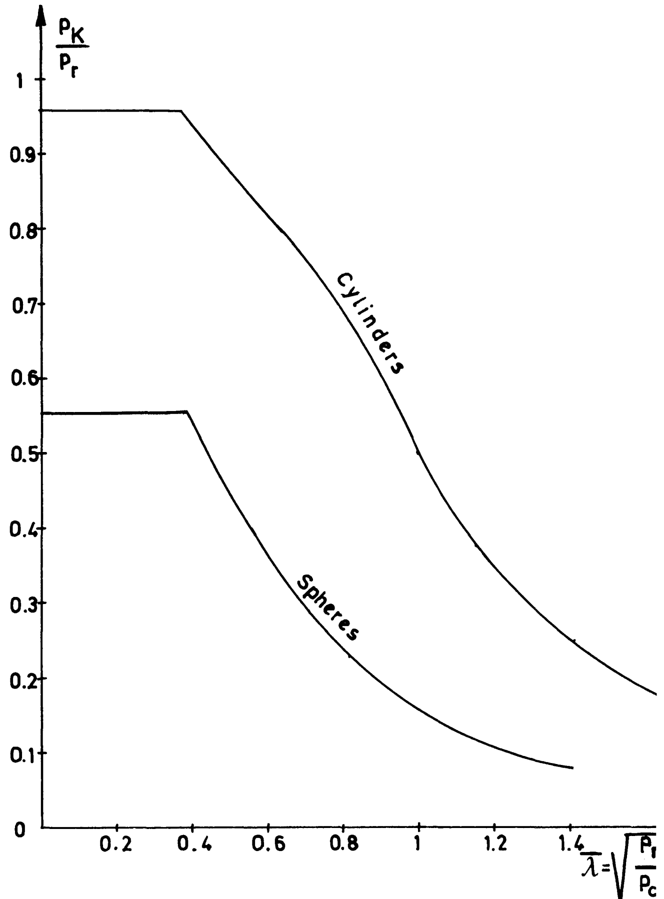


Figure WE8.2

The requirement relating to the ring is detailed for "light" and for "heavy" stiffeners, the latter being capable of retaining circularity. As to the cylinder wall proper, p_K/p_r is obtained from Fig. WE8.2, where p_r is the pressure causing the highest circumferential direct stress in the wall to reach σ_r , and p_{cr} is the lowest bifurcation pressure on a perfect elastic cylinder, given by a formula which is a slight modification of the von Mises formula. The abscissa in Fig. WE8.2 is

$$\bar{\lambda} = \sqrt{p_r/p_{cr}} \qquad \text{(WE8.4)}$$

The curve in Fig. WE8.2 represents the lower bound of many experimental results.

CYLINDERS UNDER COMBINED AXIAL LOAD AND EXTERNAL PRESSURE

The strength requirement

$$\frac{\sigma_d}{\sigma_K} + \frac{p_d}{p_K} \leq 1 \qquad \text{(WE8.5)}$$

reflects Dunkerly's straight interaction line.

UNSTIFFENED SPHERICAL SHELLS UNDER UNIFORM RADIAL PRESSURE

The strength condition $p_d \leq p_K$ must be checked. The ratio of the design value p_K of the buckling pressure to $p_r = 2t\sigma_r/r$ is obtained from Fig. WE2, where the abscissa is

$$\bar{\lambda} = \sqrt{\frac{p_r}{p_{cr}}} = \sqrt{\frac{r\sigma_r}{0.605tE}} \qquad \text{(WE8.6)}$$

The ordinates of the curve in Fig. WE2 are those of a lower bound of experimental points, divided by the additional partial safety factor $\gamma = 4/3$ in order to take into account the high imperfection sensitivity of radially compressed spheres. The design curve only applies to spheres which are spherical to within 1% on radius and whose radius of curvature, based on an arc length of $2.4\sqrt{rt}$, does not exceed the nominal value by more than 30%.

EAST EUROPE

(No regional recommendations on shells from East Europe)

b. SPECIFICATIONS AND CODES

JAPAN

AIJ STANDARD FOR STEEL STRUCTURES

Diameter-to-thickness limitations of steel circular tubes:

a) Allowable stress design:

$$\frac{D}{t} \leq \frac{23{,}540}{F} \qquad \text{(J8.1)}$$

b) Plastic design:

$$\frac{D}{t} \leq \frac{11{,}770}{F} \qquad \text{(J8.2)}$$

where F is the specified minimum yield stress in MPa.

Commentary—Equation (J8.1) is specified under the condition that a tube subject to axial compression should not buckle until the axial stress reaches the yield stress.

Equation (J8.2) is specified under the condition that a tube should not buckle and unload until the axial strain attains eight times the yield strain.

NORTH AMERICA

North American codes contain little if any information regarding shells.

WEST EUROPE

Very few national codes in Western Europe contain rules relating to shells.

Great Britain (BS 5500, 1978)

BS 5500 provides rules for stiffened cylinders and cones subject to external pressure and for radially loaded spheres or dished ends of pressure vessels. The specifications relating to cylinders are equivalent with the ECCS draft. In fact the ECCS draft is a rewrite of BS 5500. The British specification relating to spheres differs only from the ECCS draft by the use of a somewhat lower overall safety factor. Meridionally compressed cylinders are not dealt with in BS 5500.

Austria (Önorm B 4650—Teil 4, 1977)

The norm considers only cylinders. Axial load, bending, combined axial load and bending, external pressure, combined axial or eccentric load, and external or internal pressure are treated. For the basic case of axial loading, Önorm B 4650 does not differ substantially from the ECCS draft, but it does provide a rule for long cylinders. The Austrian rules for stiffened cylinders under external pressure are simpler than the ECCS rules. The code states that geometrical imperfections unavoidable in construction practice are covered by the reduction factors, but no specific limitation of the shape imperfections is laid down. Specifications concerning stepped cylinders are being prepared by the Austrian code-writing body.

Germany (Draft)
The German Arbeitsgruppe Stabilität-Schalen is preparing a code pertaining to shells. To the best of the writer's knowledge the specifications will be in keeping with the general ECCS philosophy and the ratio of the buckling stress σ_K or pressure p_K to σ_r or p_r will be given as a function of the ratio of σ_r or p_r to σ_{cr} or p_{cr}.

Norway (Det Norske Veritas rules regarding offshore structures)
The rules apply only when the structural imperfections are within stated limits. They deal very comprehensively with cylinders: unstiffened cylinders, ring stiffened cylinders, stringer stiffened cylinders, and ring and stringer stiffened cylinders, and with axial compression, bending, external pressure, torsion, shear, or a combination thereof. Plasticity is taken into account by means of a correction factor given as a function of a slenderness parameter $\bar{\lambda}$ which is quite similar to the one used in the ECCS recommendations.

EAST EUROPE

Soviet Union

1. Limit stress (σ_{us}) of axially compressed cylindrical shells:

   In the elastic range:

$$\sigma_{us} = \sigma_{cr} = c\,(Et/R) \qquad \text{(EE8.1)}$$

   where c is given in Fig. EE8.1a, based on experiments.

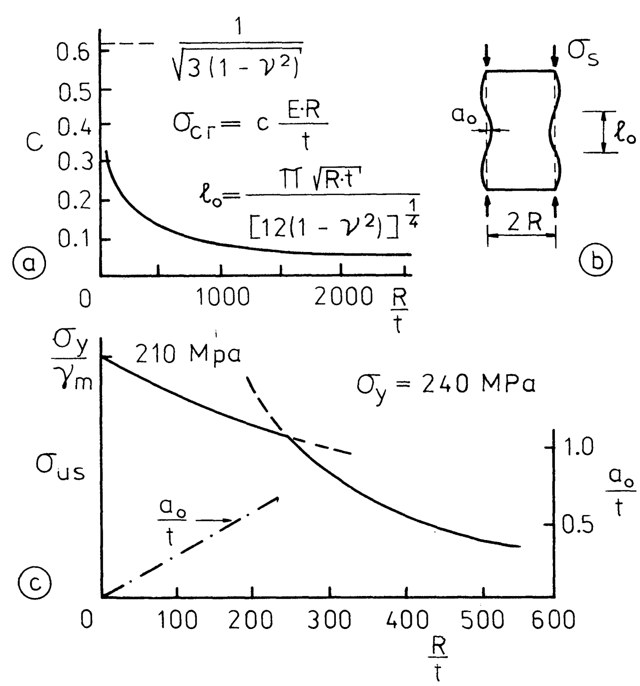


Figure EE8.1

In the inelastic range a lower limit is given according to Broude's theory,[299] reducing the actual problem to the analysis of an eccentrically compressed beam-column on elastic foundation of effective length $l^x = \sqrt{2}\,l_o$ (Fig. EE8.1b) and initial eccentricity a_o as represented by Fig. EE8.1c. By further approximations, results are given as limit stress $\sigma_{u,ecc}$ [Chapter 5, Eq. (EE5.8)] of a beam-column with fictitious slenderness

$$\lambda = 3.23\sqrt{R/t} \qquad \text{(EE8.2)}$$

and initial eccentricity

$$\left.\begin{aligned} e &= e_o(W/A) \\ e_o &= 1.2(a_o/t) \end{aligned}\right] \qquad \text{(EE8.3)}$$

Formulae can be used only if tolerances in fabrication and erection given in the corresponding codes are not surpassed.

2. Limit stress of cylindrical panels under uniform meridional compression:

   If $b^2/Rt < 20$, as represented in Fig. EE8.2

   If $b^2/Rt > 20$, given by Eqs. (EE8.1) and (EE8.2)

3. Limit stress $\sigma_{ur} = p_u/Rt$ of a cylindrical shell of length l under uniform external radial pressure p:

   If $0.5 \leq l/R \leq 10$:

$$\sigma_{ur} = 0.6\,\frac{\pi\sqrt{2}}{3\sqrt{3}}\,\frac{E}{(1-\nu^2)^{3/4}}\,\frac{R}{l}\left(\frac{t}{R}\right)^{3/2}$$

$$= 0.55\,\frac{R}{l}\left(\frac{t}{R}\right)^{3/2} \quad \text{(Ref. 251)} \qquad \text{(EE8.4)}$$

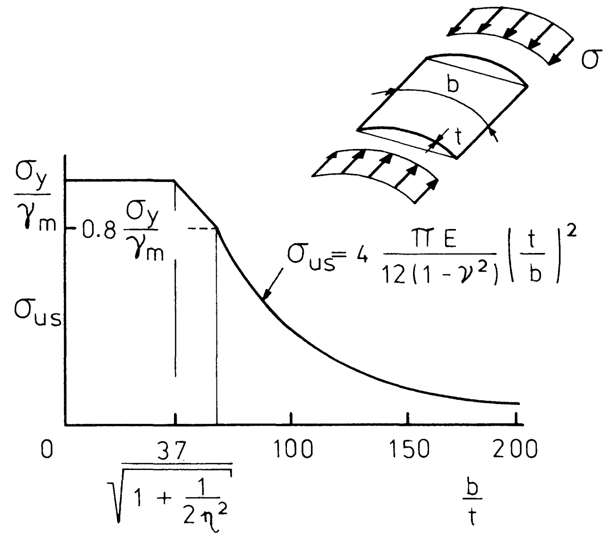


Figure EE8.2

If $l/R \geq 20$:

$$\sigma_{ur} = 0.6 \frac{E}{4(1-\nu^2)} \left(\frac{t}{R}\right)^2$$

$$= 0.17\, E\left(\frac{t}{R}\right)^2 \qquad \text{(EE8.5)}$$

Effect of initial imperfections is taken into account in both equations by factor 0.6.

4. Interaction of meridional and radial compression is expressed by

$$\frac{\sigma_s}{\sigma_{us}} + \frac{\sigma_r}{\sigma_{ur}} \leq 1$$

5. Limit load of a conical shell is given by Eq. (EE8.6), σ_{us} being the limit stress of a cylindrical shell of radius R_e. In case of external pressure Eq. (EE8.4) can be used replacing R by R_e (Fig. EE8.3).
6. Limit stress $\sigma_{usp} = p_u R_s/2t$ of a spherical shell under uniform external radial pressure p:

$$\sigma_{usp} = 0.1E(t/R_s) \qquad \text{(EE8.7)}$$

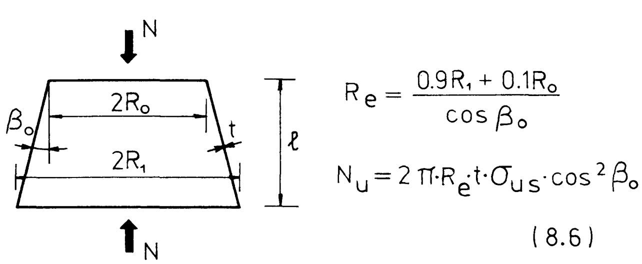


Figure EE8.3

replacing factor $1/\sqrt{3(1-\nu^2)} = 0.606$ in theoretical formula by 0.1 because of effect of imperfections.

In all cases further diminishing factors can be applied taking into account special circumstances.

Czechoslovakia
Similar methods are adopted with concise formulae for all steel grades, using notation

$$\eta = \sqrt{\frac{240}{\sigma_y}} \qquad (\sigma_y \text{ in MPa})$$

1. If $R/t \leq 50\eta^{4/3}$:

$$\sigma_{us} = \sigma_y/\gamma_m \qquad \text{(EE8.8)}$$

If $R/t \geq 180\eta^{4/3}$:

$$\sigma_{us} = \left(\frac{155t}{R}\right)^{3/2} \cdot \eta^2 \qquad \text{(EE8.9)}$$

with linear interpolation between the two limits (Fig. EE8.4).

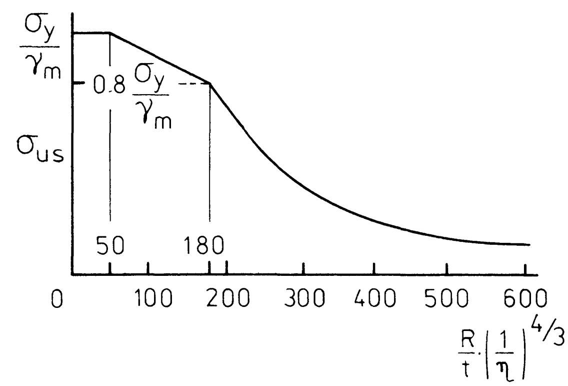


Figure EE8.4

2. If $b^2/Rt < 4$, the cylindrical panel is treated as a plate.

If $4 \leq b^2/Rt \leq 20$:

For $b/t \leq \dfrac{37}{\sqrt{1 + \dfrac{1}{2\eta^2}}}$:

$$\sigma_{us} = \sigma_y/\gamma_m \qquad \text{(EE8.10)}$$

For $b/t \geq 58\eta$:

$$\sigma_{us} = \left(\frac{52t}{b}\right)^2 \eta^2(\sigma_y/\gamma_m) \qquad \text{(EE8.11)}$$

with linear interpolation.

If $b^2/Rt > 20$, Eqs. (EE8.8) and (EE8.9) are to be applied.

3. If $0.5 \le l/R \le 10$:

For $\frac{l}{R}\left(\frac{R}{t}\right)^{3/2} \le 330\eta^2$:

$$\sigma_{ur} = \sigma_y/\gamma_m \qquad \text{(EE8.12)}$$

For $\frac{l}{R}\left(\frac{R}{t}\right)^{3/2} \ge 510\eta^2$:

$$\sigma_{ur} = 408\,\frac{R}{l}\left(\frac{t}{R}\right)^{3/2}\frac{1}{\eta^2}(\sigma_y/\gamma_m) \qquad \text{(EE8.13)}$$

If $l/R \ge 20$:

For $\frac{R}{t} \le 9\eta$:

$$\sigma_{ur} = \sigma_y/\gamma_m \qquad \text{(EE8.14)}$$

For $\frac{R}{t} \ge 13\eta$:

$$\sigma_{ur} = 126\left(\frac{t}{R}\right)^2\eta^2\sigma_y \qquad \text{(EE8.15)}$$

with linear interpolation between the limits and for $10 < l/R < 20$ as well.

4. Interaction formula

$$\frac{\sigma_s}{\sigma_{us}} + \frac{\sigma_r}{\sigma_{ur}} \le \sqrt{1 + \frac{\sigma_s\sigma_r}{(\sigma_y/\gamma_m)^2}} \qquad \text{(EE8.16)}$$

is adopted.

In case of interaction of axial compression σ_s and internal pressure p_i, limit stresses σ'_{us} and σ''_{us} are:

If $\sigma'_r = p_i\frac{R}{t} \ge 1.33\,\sigma'_{us}$:

For $\frac{R}{t} \le 360\eta^2$:

$$\sigma'_{us} = \sigma_y/\gamma_m \qquad \text{(EE8.17)}$$

For $\frac{R}{t} \ge 560\eta^2$:

$$\sigma'_{us} = 450\,\frac{t}{R}\,\eta^2\sigma_y/\gamma_m \qquad \text{(EE8.18)}$$

If $\sigma'_r > 1.33\sigma'_{us}$:

$$\sigma''_{us} = \sigma_{us} + (\sigma'_{us} - \sigma_{us})\sqrt{\frac{\sigma'_r}{2\sigma'_{us}}} \qquad \text{(EE8.19)}$$

c. COLLOQUIUM CONTRIBUTIONS (COMMENTARIES)

JAPAN

S. KOBAYASHI (TOKYO)[252]

Buckling of cantilever conical shells, which are clamped at the small radius end and free at the large radius end, under external lateral pressure:

Toda and Komatsu[253] proposed the following approximate formula

$$p_{cr} = (0.4\psi + 0.55)p_e$$

where p_e is determined by Eq. (J8.6) and $\psi = 1 - r_1/r_2$ is the taper ratio. See Fig. J8.1.

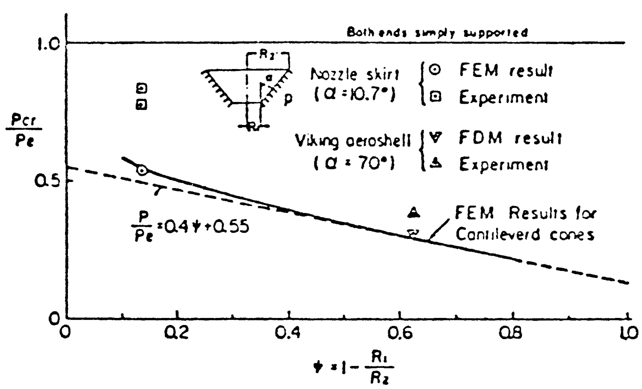


Figure J8.1

Proposed design formula for the buckling of isotropic truncated conical shells (symbols shown in Fig. J8.2):

1. Axial compression:

$$p_{cr} = \frac{1}{3}p_{cyl}(\cos\alpha)^2 \qquad \text{(J8.3)}$$

where

$$p_{cyl} = \frac{2\pi E t^2}{\sqrt{3(1-\nu^2)}}$$

2. Axial compression and internal pressure:

For $\bar{p}_i < 1$:

$$p_{cr} - \pi p_i r_1{}^2 = \frac{1}{3}(1 + 2\bar{p}_i)p_{cyl}(\cos\alpha)^2 \qquad \text{(J8.4)}$$

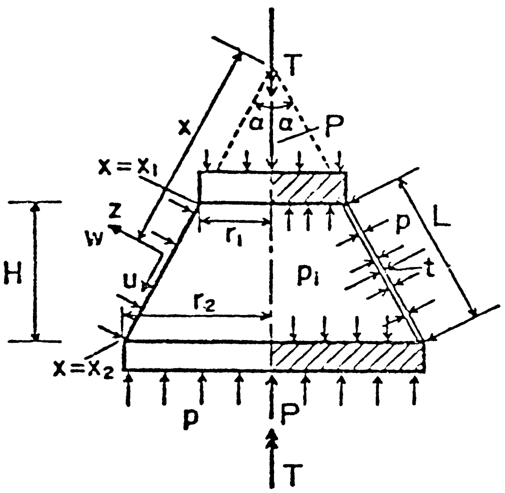


Figure J8.2

For $\overline{p}_i > 1$:

$$p_{cr} - \pi p_i r_1{}^2 = p_{cyl}(\cos\alpha)^2$$

where

$$\overline{p}_i = \frac{p_i}{E}\left(\frac{r_1}{t\cos\alpha}\right)^2$$

3. External pressure:

$$p_{cr} = 0.75 p_e \qquad \text{(J8.5)}$$

where

$$p_e = \frac{0.92E}{(\mathrm{L}/\rho_{av})(\rho_{av}/t)^{5/2}} \qquad \text{(J8.6)}$$

and

$$\rho_{av} = \frac{r_1 + r_2}{2\cos\alpha}$$

4. Torsion:

$$T_{cr} = 0.67\overline{T}_{cr} \qquad \text{(J8.7)}$$

where

$$\overline{T}_{cr} = 1.70\pi^3 D(1-\nu^2)^{3/8}\sqrt{t/H}\times\left[\left\{1+\sqrt{\frac{1}{2}\left(1+\frac{r_2}{r_1}\right)} - \frac{1}{\sqrt{\frac{1}{2}\left(1+\frac{r_2}{r_1}\right)}}\right\}\frac{r_1\cos\alpha}{t}\right]^{5/4}$$

and

$$D = \frac{Et^3}{12(1-\nu^2)}$$

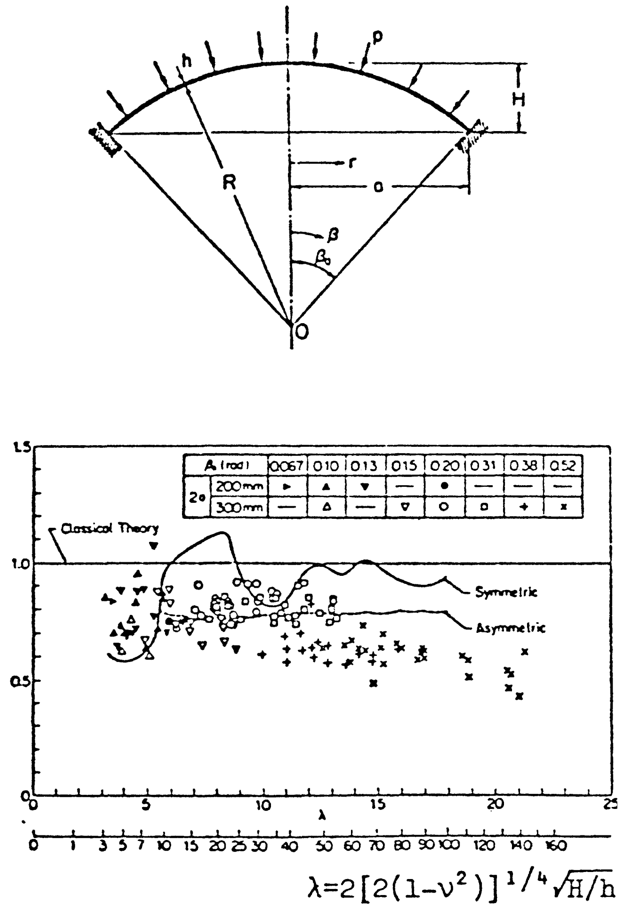


Figure J8.3

5. Interaction curves:

$$\frac{P}{P_{cr}} + \frac{p}{p_{cr}} + \left(\frac{T}{T_{cr}}\right)^2 = 1 \qquad \text{(J8.8)}$$

where

P_{cr} = critical axial compressive force calculated from Eq. (J8.3)
p_{cr} = critical hydrostatic pressure calculated from Eq. (J8.5)
T_{cr} = critical torque calculated from Eq. (J8.7)

Experimental buckling pressure of spherical caps by Sunakawa and Ichida:

In Fig. J8.3,

$$p_{cl} = \frac{2}{\sqrt{3(1-\nu^2)}}E\left(\frac{h}{R}\right)^2$$

is the classical buckling pressure.

The test specimens were formed from Poly Methyl Methacrylate plate by the use of air vacuum forming process at elevated temperature.

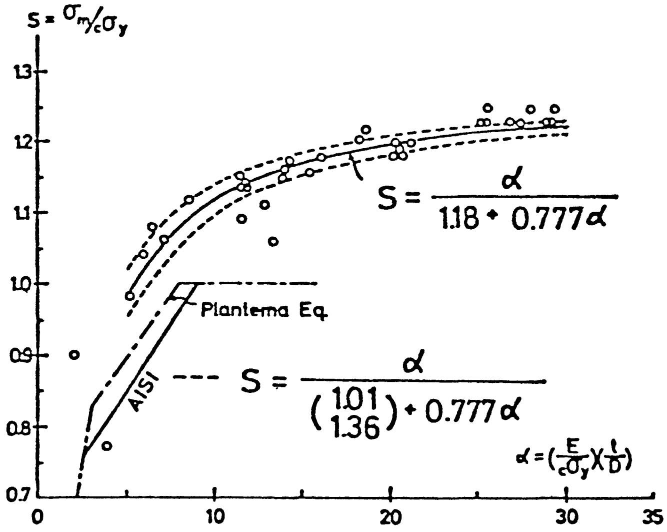


Figure J8.4

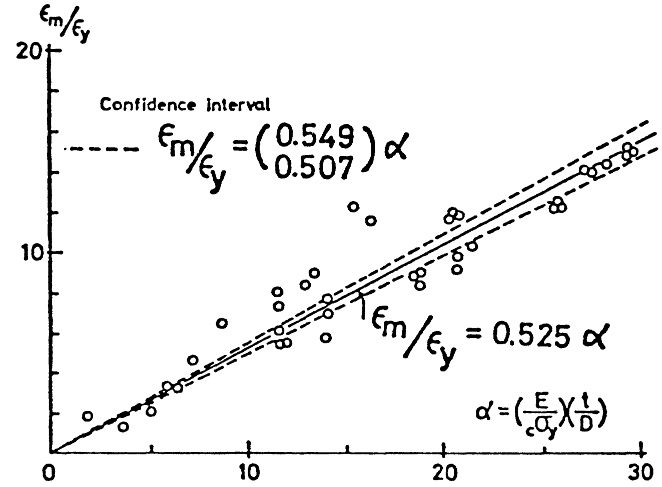


Figure J8.5

B. KATO (WASHINGTON)[254]

Empirical formulae for ultimate stress and strain of circular steel tubes (cold-formed, seamed by electric resistance welding) subject to wall buckling. Dashed lines are 95% confidence limits on the basis of the Student's t-distribution. The information on the maximum stress can be used to predict the rotation capacity of tubular members subject to bending. ${}_c\sigma_y$ is related to the tensile yield stress of the base material ${}_v\sigma_y$ as

$$ {}_c\sigma_y = [1.38 - 0.009(D/t)]\, {}_v\sigma_y $$

1. Maximum stress σ_m: See Fig. J8.4.
2. Maximum strain ϵ_m: See Fig. J8.5.

In Figs. J8.4 and J8.5:

D = diameter of tube
t = wall thickness of tube
σ_m = maximum compressive stress
${}_c\sigma_y$ = compressive yield stress of tube (from stub-column test)
ϵ_m = maximum strain corresponding to σ_m
ϵ_y = ${}_c\sigma_y/E$

NORTH AMERICA

K. BUCHERT (WASHINGTON)[255]

The paper compares the provisions in the ECCS Manual with those in the SSRC Guide for buckling of spherical shells under external pressure. The main differences are that the ECCS Manual restricts itself to unstiffened shells, whereas the SSRC Guide covers a variety of different types. The ECCS formula is based on a lower bound to experimental results, while the SSRC formula is theoretically obtained. The ECCS relation is valid only for small imperfections below a given size, whereas the SSRC formula makes it possible to consider imperfections of any size.

A. CHAJES (WASHINGTON)[256]

The paper compares the provisions in the ECCS Manual with those in the SSRC Guide for buckling of cylindrical shells under external pressure. Both publications base their formulas on the von Mises theory. They differ in that the ECCS Manual gives an allowable pressure based on a lower bound to test results, while SSRC simply gives a theoretical value of the critical pressure. One formula handles both elastic and inelastic buckling in the ECCS Manual, whereas two different procedures are given for these cases in the SSRC Guide.

WEST EUROPE

Among the papers presented at Liege, the one by Saal[257] and the one by Esslinger, Geier and Wood[258] corroborate specific points in Chapter 10 of the ECCS Manual.

The paper by Galletly[259] and the one by Massonnet and Baltus[260] bring to light a discrepancy between certain experimental buckling pressures and buckling pressures predicted by the BOSOR 5 program, which can handle elastic-plastic shells of revolution. So far the discrepancy is unexplained.

At the Liege colloquium, Gachon[261] gave an account of an interesting test on a beam with a corrugated web. The comparison with the behavior of a flat web does not seem to reveal any superiority of the former.

Bornscheuer[262] complained about the lack of coherence in the presentation of the ECCS Recommendations, as initially drafted, relating to cylinders under axial load and to those under external pressure, and also wanted to switch from the allowable load to the limit load concept, while Wood[263] pointed out a lack of consistency in the safety factors: 2 for a very thick, axially loaded cylinder when it is considered as a shell, 1.5 when it is considered as a column. The writer believes that the three points brought up by Bornscheuer and Wood have been corrected in the February 1979 version of the ECCS Recommendations.

At the Budapest colloquium, White[264] presented the results of measurements of residual stresses in cylinders due to flame-cutting and to welding and found that their order of magnitude can be predicted theoretically.

EAST EUROPE

L. KOLLAR (BUDAPEST)[265]

In his General Report on "Shells," the author comments on Colloquium Contributions.

M. CERNY (BUDAPEST)[266]

Finite element method is applied to axisymmetric shells by replacing the shell with a series of conical rings for the analysis of the linear critical load. Linear viscoelastic material is also dealt with.

P. BROZ (BUDAPEST)[267]

Linear critical load and post-buckling behavior of cylindrical panels with simply supported curved edges and arbitrary conditions along straight edges is analyzed by using mixed (trigonometrical and power) series. Non-uniform curvature can also be dealt with. Numerical results and comparison to other methods are given.

E. DULACSKA (BUDAPEST)[268]

Approximate method is given for estimating the upper critical load of spherical and cylindrical shells in the inelastic range. Decrease in flexural rigidity is taken into account according to the behavior of the most affected cross section. Results are presented by graphs for engineering practice.

F. HUNYADI AND M. IVANYI (BUDAPEST)[269]

Paper presents a useful practical approach for the computation of the linear buckling stress of cylindrical shells with longitudinal stiffeners under combined axial load and internal pressure (of importance in metal silos). Results of field measurements regarding the structural behavior of silos are reported as well.

S. FERNEZELY AND M. IVANYI (BUDAPEST)[270]

Results of full-scale tests with built-up aluminum arches of two corrugated plates are reported and compared to analytical data. Special care was given to the effect of local buckling and decreasing effective width on structural behavior and overall stability.

S. FERNEZELY, L. KRISTOF, AND A. SZITTNER (BUDAPEST)[271]

Above mentioned investigations were carried through with a different type of aluminum arch, giving special care to the behavior of shear connectors.

J. NAGY (BUDAPEST)[272]

The influence of shear creep on the critical load of shallow sandwich arches with soft core is investigated. General equations and their solutions for different loading conditions are given, presented by graphs. General conclusions could be drawn concerning the importance of shear creep.

Z. F. BACZYNSKI (BUDAPEST)[273]

An analysis of stability problems of open-top cylindrical steel tanks with floating roofs is presented, including wind pressure.

J. ZIOLKO (BUDAPEST)[274]

Distribution and magnitude of wind pressure and suction upon cylindrical steel shells (tanks) were investigated by wind tunnel model tests.

Z. BYCHAWSKI (BUDAPEST)[275]

Proper directions in investigating rheological stability of spatial structures as shells and enclosures are dealt with.

Lacking stability criteria for rheological systems, some new concepts are given regarding energetical premises. Special features of rheological stability as compared to classical elastic approach are pointed out.

A. SZITTNER (BUDAPEST)[276]

Paper points out the importance of small-scale tests related to stability problems, defining the field of their economic application and gives a survey of many interesting small-scale experiments carried out in the Department for Steel Structures, Technical University Budapest.

d. CURRENT STUDIES

JAPAN

(*No current studies on shells were reported for Japan.*)

NORTH AMERICA

Subjects currently under investigation include:

- Analysis of shells with openings and intersecting shells
- Buckling under dynamic loadings
- Inelastic instability of shells
- Development of more efficient methods for investigating nonlinear systems
- Buckling of shells under combined loads
- Effect of residual stresses on shell-like structures

WEST EUROPE

Buckling of cylinders under wind pressure, of cylinders locally anchored to a concrete jacket and subject to external pressure, and of externally pressurized cones is under investigation at the Technical University at Graz, Austria.

An experimental program on hydrostatically loaded cones is being carried out at Ghent University, Belgium.

The research which is under way in the Federal Republic of Germany bears upon axially loaded and externally pressurized cones, stringer stiffened cylinders in pure bending, unstiffened long cylinders at the transition between shell and column buckling, cylindrical sandwich panels, and axially loaded cylinders containing granular material, at the Aeronautic and Space Research Centre at Braunschweig, and upon long unstiffened, axially or eccentrically compressed cylinders and horizontal simply supported cylinders partially filled with liquid, at the Technical University at Darmstadt.

Computer routines for analysis of elastic and elastic-plastic buckling by the finite element method are developed and their results compared with tests, at the Nuclear Centre at Saclay, France.

In Great Britain the following topics are being studied: elastic and elastic-plastic buckling of internally pressurized ellipsoidal and torispherical shells at the Universities of Liverpool and of Manchester, external pressure on composite cylinders at the latter university, unstiffened or stiffened cylinders and cylindrical panels related to offshore structures at Imperial College and at University College, London, stiffened cylinders at Glasgow, and spheres at Oxford University.

Task Group VIII-9 experiences considerable difficulty in formulating practically usable and safe rules, for axially loaded, stiffened cylinders, in spite of the many published mainly theoretical studies on the subject. Another legitimate question is whether the rules drafted for steel shells are applicable to aluminum structures.

Chapter 9. Composite Members

a. REGIONAL RECOMMENDATIONS

JAPAN

(There are no regional recommendations in Japan.)

NORTH AMERICA

(Key Document: SSRC Guide[1])

In regard to composite columns, there are no "Regional Recommendations" in the formal sense, since Chapter 19 of the SSRC Guide does not contain any specific recommendations. It merely reviews the field and recent research, mostly other than North American.

On the other hand, Task Group 20 of SSRC, in May 1978, was given the task of working with Prof. R. Furlong in developing the approach of his Washington Colloquium contribution. The aim is to incorporate this method in the AISC Specification. Considerable progress has been made in this work which, in an informal sense, can be regarded as a regional recommendation.

WEST EUROPE

(Key Document: ECCS Recommendations[24])

No clauses relating to the design of composite columns are contained in the ECCS Recommendations. Information relating to this topic appears, however, in the ECCS Manual on Stability[2] and in the draft Joint ECCS-CEB-IABSE Committee Recommendations on Composite Construction. The ultimate load design method outlined therein covers concrete-filled steel tubular columns as well as concrete-encased steel section columns. They relate to equivalent pin-ended columns, the effective lengths being determined by appropriate frame design methods.

Basically a definition of nondimensional slenderness is proposed which allows the European Column Curves to be used for the design of axially loaded imperfect composite columns. This slenderness factor was suggested by Virdi and Dowling[277] and is

$$\bar{\lambda} = L/L_E \qquad \text{(WE9.1)}$$

where

$$L_E = \pi \sqrt{\frac{EI_{concrete} + EI_{steel}}{F_u}}$$

F_u = squash load of the composite reinforced steel-concrete section

Beam-columns are designed using parabolic interaction formulae which can be used for columns subjected to any ratios of factored end moments and axial loads of given slenderness and concrete contribution factor. Basu and Sommerville[278] showed that the same interaction curves could be used for composite columns possessing the same slenderness and concrete contribution factor α, where

$$\alpha = \frac{A_c f_c}{A_s f_s + A_{sr} f_{sr} + A_c f_c} \qquad \text{(WE9.2)}$$

Typically, an interaction formula is of the form

$$\frac{M}{M_u} = f_n \left(\frac{p}{p_u}\right) \qquad \text{(WE9.3)}$$

It is, of course, necessary to define the plastic moment of resistance M_u of the composite section in the above nondimensionalized formula. Simple expressions and graphs are given to help designers evaluate these moments.

Biaxial bending is accounted for by use of a modified Bresler type interaction formula, proposed by Basu and Sommerville[278] and modified by Virdi and Dowling.[277]

Practical details of casing reinforcement and concrete quality are also discussed.

Commentary—The work in the Manual on Stability is based mainly on theoretical work carried out at Imperial College, but which has been validated against experimental work done in Belgium (Liege), Germany, USA, and Japan, as well as Great Britain. Since then, an alternative approach for beam-column design has been proposed by researchers at the University of Bochum. Both approaches to beam-column design are contained in the draft Recommendations for Composite Construction.

EAST EUROPE

Regional Recommendations give no specific rules for composite columns. Design is usually based on principles given in recommendations for reinforced concrete structures.

b. SPECIFICATIONS AND CODES

JAPAN

AIJ STANDARD FOR STRUCTURAL CALCULATION OF STEEL-REINFORCED CONCRETE STRUCTURES (See Fig. J9.1)

When the maximum slenderness ratio of columns exceeds 50, the section shall be proportioned for axial force and bending moment increased by the following coefficient:

$$\alpha = \frac{100}{150 - \lambda} \qquad \text{(J9.1)}$$

where $\lambda = h/r$ is effective slenderness ratio of columns, h is the distance between lateral supports of a column and r is minimum radius of gyration of concrete section. Maximum permitted slenderness ratio of columns is 100.

Commentary—This method will be revised in the future to the same method used in the new AIJ Standard for Structural Calculation of Mixed Tubular Steel-Concrete Composite Structures.

AIJ STANDARD FOR STRUCTURAL CALCULATION OF MIXED TUBULAR STEEL-CONCRETE COMPOSITE STRUCTURES (draft for revision; see Fig. J9.2)

When the effective length exceeds 12 times the depth of the cross section, the section of columns shall be determined by the following formulas:

For $N \leq {}_rN$ or $M \leq {}_sM_o/\delta$:

$$\left.\begin{aligned} N &= {}_rN \\ M &= {}_sM_o/\delta' + {}_rM \end{aligned}\right] \qquad \text{(J9.2)}$$

For $N > {}_rN_o$ or $M < {}_sM_o/\delta$:

$$\left.\begin{aligned} N &\leqq {}_rN_o + {}_sN \\ M &= {}_sM/\delta \end{aligned}\right] \qquad \text{(J9.3)}$$

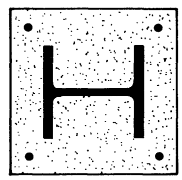
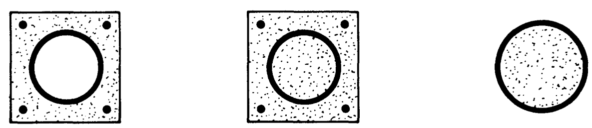


Figure J9.1



(a) encased (b) encased and infilled (c) infilled

Figure J9.2

where

$$\left.\begin{aligned} \delta &= 1\Bigg/\left(1 - \frac{{}_r\nu \cdot {}_rN_o}{N_E}\right) \text{ and } \delta \geq 1 \\ \delta' &= 1\Bigg/\left(1 - \frac{{}_r\nu \cdot {}_rN}{N_E}\right) \text{ and } \delta' \geq 1 \end{aligned}\right] \qquad \text{(J9.4)}$$

for encased columns.

In the case of concrete encased and infilled steel tubed columns, infilled concrete shall be neglected.

In the case of concrete infilled steel tubed columns, the formulas are:

For $N \leq {}_cN_o$ or $M \geq {}_sM_o/\delta$:

$$\left.\begin{aligned} N &= {}_cN \\ M &\leq {}_sM_o/\delta' + {}_cM \end{aligned}\right] \qquad \text{(J9.5)}$$

For $N > {}_cN_o$ or $M < {}_sM_o/\delta$:

$$\left.\begin{array}{l} N \leq {}_cN_o + {}_sN \\ M - {}_sM/\delta \end{array}\right] \tag{J9.6}$$

where

$$\left.\begin{array}{l} \delta = 1\Big/\left(1 - \dfrac{{}_c\nu \cdot {}_cN_o}{N_E}\right) \text{ and } \delta \geq 1 \\ \delta' = 1\Big/\left(1 - \dfrac{{}_c\nu \cdot {}_cN}{N_E}\right) \text{ and } \delta' \geq 1 \end{array}\right] \tag{J9.7}$$

In Eqs. (J9.2) through (J9.7),

$$N_E = \frac{\pi^2}{l^2}\left(\frac{{}_cE \cdot {}_cI}{5} + {}_sE \cdot {}_sI\right) \tag{J9.8}$$

l = effective length of composite column

${}_sN$ = allowable axial force for steel tubed long column subjected to bending moment ${}_sM$ and shall be computed by AIJ Standard for Steel Structures

${}_rN, {}_cN$ = allowable axial forces for covering reinforced concrete and infilled concrete long beam-columns, respectively, and shall be computed by AIJ Standard for Structural Calculation of Reinforced Concrete Structures assuming that their cross-sections are subjected to ${}_r\delta \cdot {}_rM$ and ${}_c\delta \cdot {}_cM$, respectively

$$\left.\begin{array}{l} {}_r\delta = 1\Big/\left(1 - \dfrac{{}_r\nu \cdot {}_rN}{{}_rN_E}\right) \\ {}_c\delta = 1\Big/\left(1 - \dfrac{{}_c\nu \cdot {}_cN}{{}_cN_E}\right) \end{array}\right] \tag{J9.9}$$

$$\left.\begin{array}{l} {}_rN_E = \dfrac{2\pi^2 {}_cE \cdot {}_cI}{5l^2} \\ {}_cN_E = \dfrac{\pi^2 {}_cE \cdot {}_cI}{5l^2} \end{array}\right] \tag{J9.10}$$

${}_rN_o, {}_cN_o$ = allowable axial force for covering reinforced concrete and infilled concrete long columns, respectively, under minimum permitted eccentricity

${}_r\nu, {}_c\nu$ = safety factors for reinforced concrete and infilled concrete portions, respectively; 1.5 under permanent load

${}_cE, {}_sE$ = Young's moduli of concrete and steel cross section, respectively

${}_cI, {}_sI$ = moment of inertia of concrete and steel cross sections, respectively

Minimum permitted eccentricity is 5% of the depth of the cross section.

Commentary—For short composite beam-columns, a method of superposition has been used in AIJ standards. In the present Standard for composite tubed columns, bending moment and axial force in a long column are multiplied by a factor $\alpha = 100/(150 - \lambda)$, as in the present AIJ standard for Structural Calculation of Steel-Reinforced Concrete Structures [see Eq. (J9.1)]. This method will be revised by March 1980, as shown above, along the line of the recommendation presented in the two papers to the Colloquium.

If $\delta = \delta' = 1$, Eqs. (J9.2), (J9.3), (J9.4), and (J9.6) become identical with the equations of superposition for short columns.

NORTH AMERICA

ACI BUILDING CODE

In the USA, the design of composite columns is covered only in the American Concrete Institute Building Code, ACI 318-77.

The Code is written in the Load and Resistance Factor (LRFD) or Limit State format; that is, loads are multiplied by appropriate load factors L.F. (larger than 1.0) and nominal member strengths by strength reduction factors ϕ (smaller than 1.0). The design method for composite columns is substantially identical with that for reinforced concrete columns, with minor modifications.

For composite columns loaded axially or with small eccentricity, the design strength of the cross section is

$$N_d = K\phi N_n = K\phi(0.85 f'_c A_c + A_s f_{ys} + A_r f_{yr})$$

where

N_n = nominal axial strength

K = 0.85

ϕ = 0.70 (except ϕ = 0.75 for spiral-reinforced members)

f'_c = concrete cylinder strength

A_c = concrete area

A_s = area of steel shape

f_{ys} = yield strength of steel shape, not to be taken larger than 50,000 psi

f_{yr} = yield strength of reinforcing bars

A_r = area of reinforcing bars, if any

For composite columns subject to simultaneous compression plus bending, the nominal compression and mo-

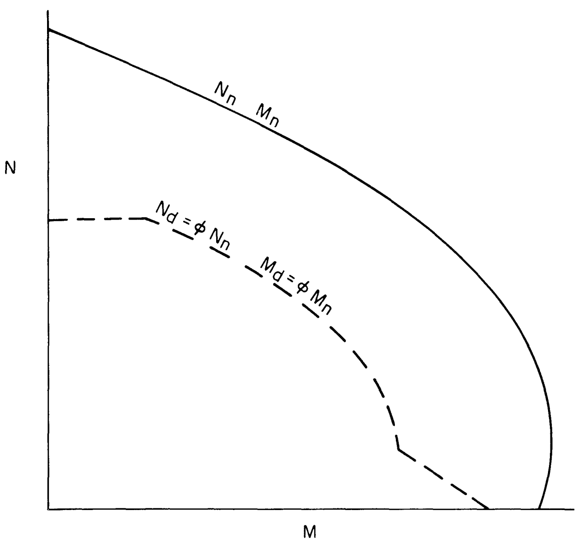


Figure NA9.1

ıent strengths N_n and M_n are calculated from strain ompatibility.

The design strengths N_d and M_d are then

$$N_d = \phi N_n \text{ and } M_d = \phi M_n$$

ˏhere ϕ is either 0.70 or 0.75, as defined above.

The relationships so obtained between N_n, N_d, M_n and M_d are schematically shown in Fig. NA9.1 for a typical oncrete-encased W-shape.

Slenderness effects, i.e., instability effects, are accounted or by a moment magnification factor δ applied to M_d. For olumns with small or no calculated moments, a minimum ccentricity $(0.6 + 0.03h)$ is prescribed, where h is the ertinent width or depth of the cross section. This moment nagnification factor is

$$\delta = \frac{C_m}{1 - P_u/(\phi P_{cr})} \geq 1$$

vhere

P_u = appropriately factored applied loads
ϕ = as defined above
C_m = 1.0 for columns not braced against sidesway
= $0.6 + 0.4M_1/M_2 \geq 0.4$ for braced columns, where M_2 is the numerically larger of the two end moments
$P_{cr} = \pi^2 EI/(kL_u)^2$
KL_u = effective length
$EI = (E_c I_g/5 + E_s I_s)/(1 + \beta)$
β = a factor accounting for creep, depending on dead-to-live load ratio

Commentary—This design method for composite columns, essentially identical with that for reinforced concrete columns, in most cases is excessively conservative for the following reasons: (a) The same low ϕ-factors are applied to composite as to reinforced columns, in spite of the fact that the fabrication and erection methods of steel construction produce a much better geometrical accuracy than *in situ* concrete construction. (b) The same holds for the additional reduction factor K which is presumed to account for accidental end eccentricities. (c) The creep factor β is applied not only to the concrete portion of the section which does creep, but also to the steel section which does not. (d) The same minimum eccentricity for slenderness effects is used for composite as for reinforced columns.

This conservatism is probably the main reason for the fact that very little use is made of composite columns in the USA.

CANADIAN STANDARD CSA S16.1

The Canadian Standard CSA S16.1-1974, written in the LRFD or Limit State Design format, contains the following provision, applicable only to hollow, concrete-filled structural sections:

$$N_{d(comp)} = N_{ds} + N_{dc}$$

where

N_{ds} = factored (i.e., multiplied by ϕ) axial compression strength of steel member
N_{dc} = factored compressive strength provided by concrete area
$= f_c A_c$
A_c = concrete area

$$f_c = \phi\, 0.85 f'_c \left(\frac{\sqrt{(\pi^2 E/f_{ys})} - (Kl/r)}{\sqrt{(\pi^2 E/f_{ys})}} \right)$$

for $Kl/r \leq \sqrt{\pi^2 E/f_{ys}}$
$= 0$ for $Kl/r > \sqrt{\pi^2 E/f_{ys}}$

$\phi = 0.67$
Kl/r = slenderness ratio of steel section

For compression plus bending, bending is to be resisted entirely by the steel section, which is to be proportioned as a beam-column to carry the total moment plus $N_{ds} = N_{d(comp)} - N_{dc}$.

Commentary

(a) Any concrete contribution to the stiffness EI is neglected.
(b) The Standard contains no provisions for concrete-encased structural steel shapes.

WEST EUROPE

The approach proposed in the draft recommendations has been adopted in a modified or draft form by the following countries:

United Kingdom: BS5400, Part 5 (1979)

German Federal Republic: DASt-UA Verbundkonstruktionen, Tragfaihigkeit von Berbundstuetzen (Draft Nov. 1978)

Switzerland: Normenentwurf SIA 161 (1978)

EAST EUROPE

Design rules for composite columns (concrete-encased steel sections) are given mainly in specifications for reinforced concrete structures, using similar principles as for ordinary reinforced concrete columns, for both centrally compressed members and beam-columns, adopting the same reduction factor depending on slenderness ratio.

- **Hungarian** specifications do not allow taking into account complete plastification of steel section in bending: an elastic core around central axis is to be supposed.
- **German** specifications deal with concrete-filled built-up steel columns. Slenderness ratio and ultimate stress are calculated taking into account cross-sectional data of steel components only; calculating effective stresses, a reduced contribution of concrete core can be considered.

c. COLLOQUIUM CONTRIBUTIONS (COMMENTARIES)

JAPAN

M. WAKABAYASHI (TOKYO)[279]

New design formulas for long composite columns and beam-columns applying a method of superposition are proposed and it is shown that the errors involved are sufficiently small. In the proposed method the ultimate load-carrying capacity of a beam-column is obtained by adding the capacities of steel and reinforced concrete long columns. This method has an advantage that steel and reinforced concrete portions can be designed using independent design specifications. See Fig. J9.3.

M. WAKABAYASHI (WASHINGTON)[280]

Concerning the application of the author's method of superposition for the ultimate strength design of slender composite beam-columns, considerations on the effect of creep and end moment ratio are presented. Modified design formulas applicable to the allowable strength design are presented. The errors involved in the proposed formulas are examined for concrete-encased and concrete-filled beam-columns. See Fig. J9.4.

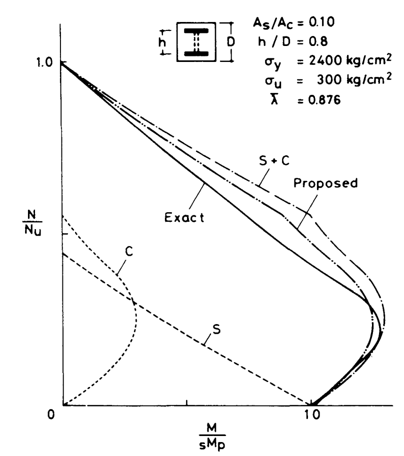


Figure J9.3

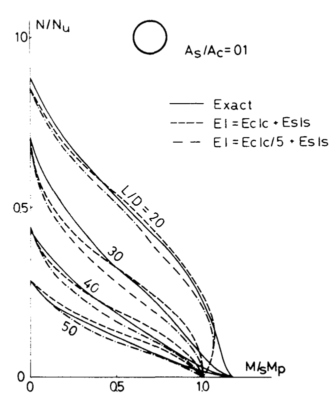


Figure J9.4

M. TOMII, K. YOSHIMURA AND Y. MORISHITA (WASHINGTON)[281]

This paper presents the results of an experimental investigation on concrete-filled steel tubular stub columns under concentric loading, to investigate the effects of shape and size of steel tube and mechanical properties of concrete on their structural behavior. Ultimate strength of circular and octagonal columns are considerably higher than their nominal squash load N_o due to triaxial effects, but this is not the case in square columns, as shown in Fig. J9.5.

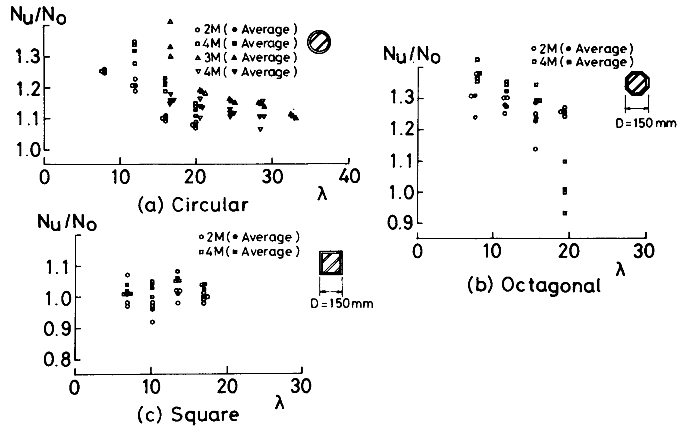


Figure J9.5

NORTH AMERICA

See Sect. d, Current Studies, which also reports on the Colloquium Contribution made by R. W. Furlong.

WEST EUROPE

J. P. GRIMAULT AND J. JANSS (LIEGE)[282]

In this paper the problem of local buckling of the walls of a concrete-filled steel tubular column is treated. It is suggested that columns with tubes where walls are more slender than those to which the method in the Manual on Stability[2] is limited can be treated using an effective area concept to allow for premature local buckling.

P. J. DOWLING, H. F. CHU AND K. S. VIRDI (LIEGE)[283]

A more accurate treatment of columns subjected to biaxial bending than that referred to in the Regional Recommendations is given. The treatment can be used in conjunction with a bilinear interaction curve approach to uniaxial bending, which is simpler than the parabolic interaction curve suggested earlier. A comparison of results using this improved design method with exactly calculated results shows that there is a significant advantage to be gained by its use, especially for columns failing biaxially.

A. K. BASU (LIEGE)[284]

This paper gives a more rational treatment of the effects of frame action on composite column behavior. It is shown that the simple concept of equivalent pin-ended columns may not always prove to be satisfactory in design.

H. S. BIUTENKAMP AND J. H. WENDRICH (LIEGE)[285]

In the prepared discussion the authors presented a method for simulating buckling curves for composite columns, using a Monte Carlo method. It is suggested that the use of the ECCS curves as outlined in the Manual does not give the same consistency as was obtained for bare steel columns. This compromise is one that the originators of the ECCS proposals were aware of, and is acknowledged in the paper by Dowling, Chu, and Virdi.[283]

EAST EUROPE

(No colloquium contributions on composite members from East Europe.)

d. CURRENT STUDIES

JAPAN

An experimental study on the ultimate load-carrying capacity of concrete-encased steel columns under concentric and eccentric compression is being carried out by Wakabayashi *et al.*[286] So far, the correlation between test results and theoretical prediction is satisfactory.

An analytical study on the load-carrying capacity of concrete-encased steel columns under combined axial load and bending was performed by Morino, Huang, and Lee.[287] A series of interaction diagrams will be available.

NORTH AMERICA

This discussion briefly sketches an approach which has since been greatly elaborated by Prof. R. W. Furlong with the initiative and guidance of Task Group 20 of SSRC, with the specific aim of eventual inclusion in the AISC Specification. Consequently, the following condensation of Furlong's Report to Test Group 20 of Oct. 1978 represents, simultaneously, a report on his Colloquium Contribution[288] and a report on Current Studies and Informal Recommendations.

The basic approach is to take the AISC Specification provisions for steel columns and steel beam-columns, and to modify the appropriate quantities in these provisions to reflect the strengthening and stiffening effects of concrete in the composite member.

In contrast to the ACI and CSA Codes, the AISC Specification is formulated in the allowable stress format. In the following, quantities with the subscript m are those which appear without such subscript in the AISC Specification for steel members and which are modified to reflect the effect of concrete in composite members.

Axially Loaded Columns—The allowable stress on the structural steel section of the composite member is given by the following two equations, the first of which applies in the range of low and moderate slendernesses, and the second for slender columns in the Euler range:

$$F_{am} = (1/S.F.)[1 - (Kl/r_m)^2/2C_{cm}{}^2]F_{ym}$$

$$F_{am} = (1/S.F.)[\pi^2 E_m/(Kl/r_m)^2]$$

where

$S.F.$ = AISC safety factor, varying from 1.67 at $Kl/r_m = 0$ to 1.92 when Kl/r_m exceeds C_{cm}

$C_{cm} = \sqrt{2\pi^2 E_m/F_{ym}}$

For concrete-encased structural shapes:

$$r_m = r_s$$

$$F_{ym} = F_{ys} + 0.7F_{yr}(A_r/A_s) + 0.6f'_c(A_c/A_s)$$

$$E_m = 29 \times 10^6 + 0.2E_c(A_c/A_s)$$

For concrete-filled pipe or tube:

$r_m = r_s$, but $\geq 0.3h_2$, where h_2 = overall concrete thickness of encased columns perpendicular to plane of bending

$$F_{ym} = F_{ys} + F_{yr}(A_r/A_s) + 0.85f'_c(A_c/A_s)$$

$$E_m = 29 \times 10^6 + 0.4E_c(A_c/A_s)$$

In the above,

r_s = radius of gyration of steel shape

F_{ys} = specified yield strength of steel shape $\leq$ 55,000 psi

F_{yr} = specified yield strength of reinforcing bars $\leq F_{ys}$

E_c = modulus of elasticity of concrete

Eccentrically Loaded Columns (Beam-Columns)—The AISC Specification deals with beam-columns by means of a linear interaction equation between the allowable axial stress F_a when $M = 0$ and the allowable bending stress F_b when $N = 0$. This linear interaction equation is not applicable to composite columns, since it is known that for them (similarly as for reinforced concrete columns) the interaction plot is curvilinear of the general shape of the solid curve of Fig. NA9.1. (See Ref. 1, pp. 534 and 537.) Consequently, it is proposed to adopt the general form of the AISC interaction equation, except that the first term is squared, resulting in a parabolic curve of the desired general shape. Hence, the interaction equation reads:

$$\left(\frac{f_a}{F_{am}}\right)^2 + \frac{C_m}{1 - \dfrac{f_a}{F'_{em}}}\left(\frac{f_{bm}}{F_{bm}}\right) \leq 1.0$$

where

$f_a = N/A_s$

$f_b = M/S_m$

$$S_m = S_s + 1/3\, A_r(h_2 - 2c_r) + \left(h_2/2 - \frac{A_w F_{ys}}{1.7f'_c h_1}\right)A_w$$

C_r = average of distance from compression or tension face to reinforcing bars next to that face

A_w = web area of structural shape (A_w = 0 for steel tubes)

h_1 = thickness of concrete in plane of bending of encased column

S_s = section modulus of steel shape

$$F'_{em} = (1/1.92)\frac{\pi^2 E_m}{(Kl/r_m)^2}$$

F_{am} = as defined for axially loaded columns, above

F_{bm} = $0.75F_{ys}$ for concrete-filled pipe or tube

= $0.6F_{ys}$ for encased steel shapes

C_m = as given in the AISC Specification

and all other terms are as defined for axially loaded columns.

Comparison with Test Data—Comparison of allowable loads with test loads yielded:

For 73 axially loaded concrete filled tubes:
Avg. N_{test}/N_a = 2.04
Standard dev. = 17%

For 29 axially loaded encased shapes:
Avg. N_{test}/N_a = 2.17
Standard dev. = 15%

For 32 eccentrically loaded filled tubes:
Avg. N_{test}/N_a = 2.52
Standard dev. = 16%

For 60 eccentrically loaded encased shapes:
Avg. N_{test}/N_a = 2.16
Standard dev. = 15%

Commentary—Comparison with test results, as given above, shows satisfactory and slightly conservative agreement, except for eccentrically loaded tubes (see below). A test load ratio of about 2.1, compared with the AISC nominal safety factor which varies from 1.67 to 1.92, seems acceptable in view of the greater uncertainty introduced by the composite action of the two materials.

The following points need to be made: (a) Some lower limit needs be set for the steel ratio of the structural shape or pipe, in order for a column to qualify as composite rather than reinforced. Evidently, four small angles, one in each corner and lightly laced together, will not endow the column with the qualities of a composite member. A minimum structural steel percentage of 4 to 6% may be in order. (b) The third term in the equation for S_m , containing both A_w and f'_c , is supposed to account for the relatively minor contribution of the steel web and the concrete to the simple bending strength. Since, for steel tubes, A_w = 0, this third term drops out. If, in addition, A_r = 0 as usual for filled steel tubes, one obtains $S_m = S_s$, i.e., no increase in simple bending capacity of the composite tube as compared to the bare tube. If this interpretation is correct, it would account at least in part for the high test load ratio of 2.52 for eccentrically loaded tubes. Clarification of this third term is needed.

WEST EUROPE

Work on encased composite columns loaded biaxially with stub beams has been reported recently from Warwick University, England.

Research on circular tubular columns with centrifugally cast concrete annulus is in progress at Imperial College.

Both CIDECT and the Tubes Division of the British Steel Corporation have commissioned design manuals on concrete-filled steel tubular columns at Liege and imperial College, London, respectively.

Work on the fire-resistance of concrete-filled tubular columns using steel fibers as reinforcement for the concrete filling has also been commissioned by CIDECT.

EAST EUROPE

Further development of design rules based on actual tests is needed.

Chapter 10. Cold-formed Steel

a. REGIONAL RECOMMENDATIONS

JAPAN

(There are no regional recommendations in Japan.)

NORTH AMERICA

(Key Document: SSRC Guide[1]*)*

The design philosophies applied to cold-formed steel in North America are discussed in Chapter 9 of the Third Edition of the SSRC Guide. In general, post-buckling strength of thin plate elements with large width-thickness ratios is considered either directly or indirectly in the design expressions; allowable loads are based on the ultimate strength of the member. Von Karman's effective width concept, modified by Winter and others on the basis of experimental results, is used to determine the strength of *stiffened* compression elements; thus the post-buckling strength is considered directly. For *unstiffened* compression elements and for flat elements subjected to bending and shear (such as beam webs), design stresses are a function of critical buckling stresses, and the post-buckling strength is considered indirectly by using a reduced apparent safety factor. To handle interaction between local buckling of individual elements of a compression member and overall buckling of that member, "Q-factors" are used to account for the reduced effective areas and reduced allowable stresses of the stiffened and unstiffened elements.

Efforts are currently underway to formulate design procedures in terms of effective widths for unstiffened compression elements and for webs of beams.

In North America cold-formed steel construction is governed mainly by three design specifications:

1. *Specification for the Design of Cold-Formed Steel Structural Members*, American Iron and Steel Institute
2. *CSA Standard S136-1974*, *Cold-Formed Steel Structural Members*, Canadian Standards Association
3. *Specification for the Design of Light-Gage Cold-Formed Stainless Steel Structural Members*, American Iron and Steel Institute

In most respects the AISI and CSA documents use the same approach to the basic buckling problems in cold-formed steel design. However, the newer CSA Standard uses, for example, revised formulas for effective width of stiffened elements, and different determinations of stiffener adequacy. The Canadian Standard also offers the option of either working stress or limit states design. AISI is currently working on a load and resistance factor design (LRFD) formulation.

Current working stress procedures are discussed individually below for specific problems. For convenience, the AISI Cold-Formed Steel Specification is considered as the "regional recommendation."

AISI RECOMMENDATIONS (1968)

Design Yield Stress—The increase in yield stress due to cold forming can be utilized based on tests or on a formula presented.

Stiffened Compression Elements—The effective width of stiffened compression elements is

$$b = 1.9t\sqrt{\frac{E}{f}}\left(1 - \frac{0.415}{w}\sqrt{\frac{E}{f}}\right)$$

where w = width, t = thickness, E = Young's Modulus, and f is the compression stress computed on the basis of effective width.

Effective Thickness of Multiple-Stiffened Plate Elements

$$t_s = (12I_s/w_s)^{1/3}$$

where w_s is the whole width and I_s is the moment of inertia of the full area of the multiple-stiffened element, including the intermediate stiffeners, about its own centroidal axis.

Edge Stiffeners

$$I_{min} = 1.83t^4(W^2 - 4000/F_y)^{1/2} \geq 9.2t^4$$

where $W = w/t$

Intermediate Stiffeners—". . . not less than twice the minimum allowable moment of inertia specified for edge stiffeners."

Beams—Lateral Buckling—The AISI and CSA specifications include provisions to prevent lateral buckling of I-shapes, channels and zees, and box- and hat-shaped beams based on various approximations of theoretical behavior, as discussed in Chapter 3.

Two specific problems related to lateral torsional-flexural buckling of beams are discussed in Section c, Colloquium Contributions, Chapter 3.

Beams—Bending Stresses in Webs

$$\text{AISI and CSA:} \quad F_{bw} = \frac{520{,}000}{(h/t)^2} \le F_b$$

The theoretical critical buckling stress for a web plate in bending is

$$\sigma_{cr} = K \frac{\pi^2 E}{12(1-\mu^2)(w/t)^2}$$

For a steel web plate in bending, $K = 23.9$ and $\sigma_{cr} = 640{,}000/(h/t)^2$.

Only a small factor of 1.23 has been applied to the critical stress to obtain the allowable bending stress. This small safety factor is sufficient to prevent large web deflections at design loads; the necessary strength reserve is provided by the post-buckling strength. Current research is attempting to define the ultimate strength of beam webs more precisely.

Web Crippling—Empirical formulas are used to determine allowable concentrated loads or reactions to avoid crippling of beam webs.

Axially Loaded Compression Members

$$P = AF_{a1}$$

$$F_{a1} = 0.522QF_y - \left(\frac{QF_yKL/r}{1494}\right)^2 \text{ for } \frac{KL}{r} < \frac{C_c}{\sqrt{Q}}$$

$$F_{a1} = \frac{12\pi^2 E}{23(KL/r)^2} \qquad \text{for } \frac{KL}{r} \ge \frac{C_c}{\sqrt{Q}}$$

The AISI and CSA specifications govering flexural buckling are both based on the SSRC method, with inclusion of a local buckling factor Q. A recent investigation aimed at refining this procedure is described in the Colloquium Contributions, later in this chapter.

A unique feature of the AISI cold-formed steel specifications is the treatment of torsional buckling and torsional-flexural buckling. For singly-symmetric shapes of open cross section which may be subject to torsional-flexural buckling and which are not braced against twisting and not subject to local buckling, the average axial stress, P/A, shall not exceed F_{a1} above, or F_{a2} given below:

$$\sigma_{TFO} > 0.5F_y: \quad F_{a2} = 0.522F_y - \frac{F_y^2}{7.67\sigma_{TFO}}$$

$$\sigma_{TFO} \le 0.5F_y: \quad F_{a2} = 0.522\sigma_{TFO}$$

where

F_{a2} = allowable average compression stress under concentric loading, ksi

σ_{TFO} = elastic torsional-flexural buckling stress under concentric loading which shall be determined as follows:

For members whose cross-sections have one axis of symmetry (x-axis), σ_{TFO} is less than both σ_{ex} and σ_t and is equal to:

$$\sigma_{TFO} = \frac{1}{2\beta}[(\sigma_{ex} + \sigma_t) - \sqrt{(\sigma_{ex} + \sigma_t)^2 - 4\beta\sigma_{ex}\sigma_t}]$$

where

$$\sigma_{ex} = \frac{\pi^2 E}{(KL/r_x)^2}\text{, ksi}$$

$$\sigma_t = \frac{1}{Ar_o^2}\left[GJ + \frac{\pi^2 EC_w}{(KL)^2}\right]\text{, ksi}$$

$\beta = 1 - (x_o/r_o)^2$

A = cross-sectional area

$r_o = \sqrt{r_x^2 + r_y^2 + x_o^2}$ = polar radius of gyration of cross section about the shear center, in.

r_x, r_y = radii of gyration of cross section about centroidal principal axes, in.

E = modulus of elasticity = 29,500 ksi

G = shear modulus = 11,300 ksi

K = effective length factor

L = unbraced length of compression member, in.

x_o = distance from shear center to centroid along the principal x-axis, in.

J = St. Venant torsion constant of the cross section, in.[4] For thin-walled sections composed of n segments of uniform thickness: $J = (1/3)(l_1t_1^3 + l_2t_2^3 + \ldots + l_it_i^3 \ldots + l_nt_n^3)$

t_i = steel thickness of the member for segments i, in.

l_i = length of middle line of segment i, in.

C_w = warping constant of torsion of the cross section, in.[6]

A method of dealing with simultaneous local and torsional-flexural buckling also is presented.

Compression Plus Bending—Interaction equations are used to determine allowable loads under combined axial and bending stresses, including a treatment when torsional-flexural buckling may occur.

WEST EUROPE

(Key Document: ECCS/CEEPF Recommendations (Draft, May 1979)

Load factor design. The maximum load carrying capacity is checked by an elastic method. The maximum stress under factored load must not exceed the design yield strength σ_r of the element considerated.

1.1 Design Yield Stress—It is permitted to take into account the increase of the yield strength due to hardening as a result of cold forming. The design yield strength may either be based on tests or calculated with the formula of Lind and Shroff:

$$\Delta\sigma_{rg} = C.\alpha(\sigma_{ub} - \sigma_{rb})\,(Nt^2/A)$$

$$\sigma_{rg} = \sigma_{rb} + \Delta\sigma_{rg}$$

where

σ_{rg} = average design yield strength of a section
$\Delta\sigma_{rg}$ = average increase of yield strength
σ_{rb} = specified yield strength of basic material
σ_{ub} = specified ultimate strength of basic material
N = number of bends
t = thickness of the sheet
A = cross-sectional area
C = coefficient depending on forming process:
$C = 5$ for press-braking
$C = 7$ for cold-rolling
α = coefficient depending on angle of bending (for 90°: $\alpha = 1$)

1.2 Local Buckling of Flat Compression Elements—The design strength of unstiffened elements, being flat parts having one laterally unsupported edge, is based on theoretical initiation of buckling. The design compressive stress σ_c must be multiplied by a local buckling coefficient α_B before being checked against the yield stress σ_r. Formulae for α_B are given.

For stiffened elements, post-buckling strength is taken into account by using the effective width concept. The basic effective width formula for elements without intermediate stiffeners supported on two webs is:

$$b_{eff} = 56.1\, b_o \Big/ \left(15.3 + \frac{b_o}{t}\sqrt{\frac{\sigma_c}{235}} \leq b_o\right)$$

If the flat element includes a stiffener, any deformation of the plate will reduce shear-stress transfer, causing a kind of "shear lag". This is taken into account by reducing the effective cross sections of stiffeners and plate elements. Formulae for the reduction factors are given. (See Fig. WE10.1.)

Requirements for adequate stiffness of stiffeners:

Edge stiffeners:

$$b_l \geq 2.8\, t \sqrt[6]{(b_o/2)^2 - \frac{0.13}{\sigma_r/E}} \geq 4.8t$$

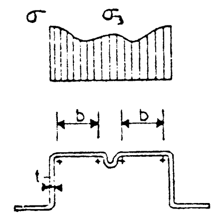


Figure WE10.1

$$I_s \geq 1.83t^4 \sqrt{(b_o/t)^2 - \frac{0.13}{\sigma_r/E}} \geq 9.2t^4$$

Intermediate stiffeners:

$$I_s \geq 3.66t^4 \sqrt{(b'_o/t)^2 - \frac{0.13}{\sigma_r/E}} \geq 18.4t^4$$

1.3 Lateral Buckling—The critical elastic stress σ_{cr} of the ideal member is first calculated for the relevant loading and end conditions. To take into account the influence of imperfections, the following procedure is used:

Calculate:	$\lambda_{eff} = \sqrt{\pi^2 E/\sigma_{cr}}$
Find:	Limit stress σ_k as the value corresponding to λ_{eff} in ECCS buckling curve B
Check:	$\sigma_{cM} \leq \sigma_k$

As a simple design method, lateral buckling in these shapes can be treated as buckling perpendicular to the y-y axis of the compression flange: $\lambda_{cf} = l/i_{cf}$. See Fig. WE10.2.

This method is also suitable for calculation of the buckling load of laterally unsupported flanges in sections, as shown in Fig. WE10.3. In that case:

$$\lambda_{cf} = \frac{\pi}{i_{cf}}\sqrt[4]{\frac{EI_{cf}}{4k_o}}$$

where k_o = spring constant = F/δ.

In both methods, for slender compressed flat elements, the width has to be reduced to the effective width (see Sect. 1.2).

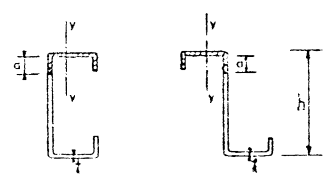


Figure WE10.2

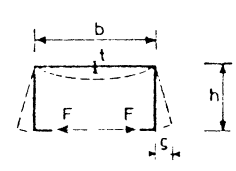


Figure WE10.3

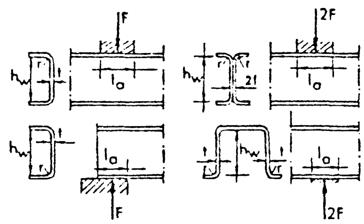


Figure WE10.4

1.4 Web Crippling (see Figs. WE10.4 and WE10.5)

Beams with single unreinforced webs:

Category loads ①:

$$F_{max} = t^2\sigma_r\left[4 + 0.175\left(1 - \frac{h_w}{191t}\right)\left(\frac{l_a}{t} + 0.5\right)\right] \times \left(1.33 - 0.33\frac{\sigma_r}{235}\right)\left(1.15 - 0.15\frac{r}{t}\right)$$

Category loads ②:

$$F_{max} = t^2\sigma_r\left[7.4 + 0.096\left(1 - \frac{h_w}{256t}\right)\left(\frac{l_a}{t} + 55.5\right)\right] \times \left(1.22 - 0.22\frac{\sigma_r}{235}\right)\left(1.06 - 0.06\frac{r}{t}\right)$$

Beams made of two channels connected back to back:

Category loads ①:

$$F_{max} = t^2\sigma_r\left(7.4 + 0.93\sqrt{\frac{l_a}{t}}\right)$$

Category loads ②:

$$F_{max} = t^2\sigma_r\left(11.1 + 2.41\sqrt{\frac{l_a}{t}}\right)$$

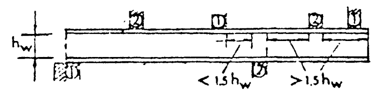


Figure WE10.5

1.5 Centrally Compressed Members—If the section has unstiffened flat elements, the one with the largest b_o/t ratio is taken and its local buckling coefficient α_B is calculated (see Sect. 1.2). For each stiffened flat element and for a stress value of σ_r/α_B , the effective width is calculated. For the effective cross section and the relevant direction of buckling:

$$i_{eff} = \sqrt{I_{eff}/A_{eff}} \rightarrow \lambda_{eff} = \frac{l}{i_{eff}\sqrt{\alpha_B}}$$

σ_k (from ECCS buckling curve b) $\rightarrow \alpha = \sigma_r/\sigma_k$

Design buckling load: $N = \dfrac{A_{eff}\sigma_r}{\alpha \cdot \alpha_B}$

If the section has laterally unsupported flanges α_{cf} is calculate (see Sect. 1.3). The higher value of α_{cf} and α_B is then used. For angles with equal legs, a special design method is given.

1.6 Members Subjected to Compression and Bending—The method is based on a linear combination of the effect of single compression without bending and simple bending.

General formula:

$$\frac{\sigma_{cN}}{\sigma_k} + \frac{\sigma_{cM_x}}{\sigma_{kD}} + \frac{\sigma_{cM_Y}}{\sigma_r} \leq \frac{1}{\alpha_B} \text{ or } \frac{1}{\alpha_{cf}}$$

where

σ_k = in accordance with Sect. 1.5
σ_{kD} = in accordance with Sect. 1.3

EAST EUROPE

(Key Documents: INCERC Design Guidelines for Steel; CMEA Recommendations for Aluminum Structures)

STEEL

Flexural-torsional buckling of thin-walled compressed members is to be checked using interaction formula (for monosymmetric sections):

$$N\frac{y_\omega^2}{r_\omega^2} + (N - N_{Ey})(N - N_\omega) = 1$$

$$\frac{1}{\rho^2} = \frac{N}{N_{Ey}} \qquad \text{(EE10.1)}$$

where

y_ω = distance between gravity center and shear center
r_ω = polar radius of gyration regarding the shear center
N_{Ey} = Euler load for buckling around the axis of symmetry
$N_\omega = \dfrac{1}{r_\omega^2}\left(GJ_t + \dfrac{\pi^2 EJ_\omega}{l_e^2}\right)$
GJ_t = torsional rigidity
J_ω = warping modulus

σ_u is to be taken from column curves, substituting

$$\lambda = \rho\lambda_y \qquad \text{(EE10.2)}$$

ALUMINUM

Special rules for effective width, h, of compressed cold-formed elements are given (see Fig. EE10.1).

1. For flat plates with thickness t, a slightly modified version of Eq. (EE10.3) is given [see Eq. (EE10.4)], including the effect of plastic deformations:

$$h_e/h = \sqrt{\sigma_{cr}/\sigma_{max}} \quad \text{(EE10.3)}$$

$$h_e = 0.6\lambda_o t \quad \text{(EE10.4)}$$

where

σ_{max} = stress in the effective portion of the cross section

λ_o = a fictitious slenderness, taken from diagram in Fig. EE4.2, supposing $\sigma_{up} = \sigma_{max}$

2. For ondulated and corrugated plates:

If $a/h \geq 3$, using plastic reduction $E_t/E \approx 0.225$:

$$h_e = 2.08\sqrt{\frac{s}{vt\sigma_{max}}(\sqrt{D_xD_y} + D_{xy})} \quad \text{(EE10.5)}$$

where

D_x = flexural rigidity per unit length, in longitudinal direction

$$D_y = \frac{s}{v}\frac{Et^3}{12(1-\nu^2)}$$

$$D_{xy} = \nu D_y + \frac{v}{s}\frac{Gt^3}{6}$$

If $a/h \leq 3$:

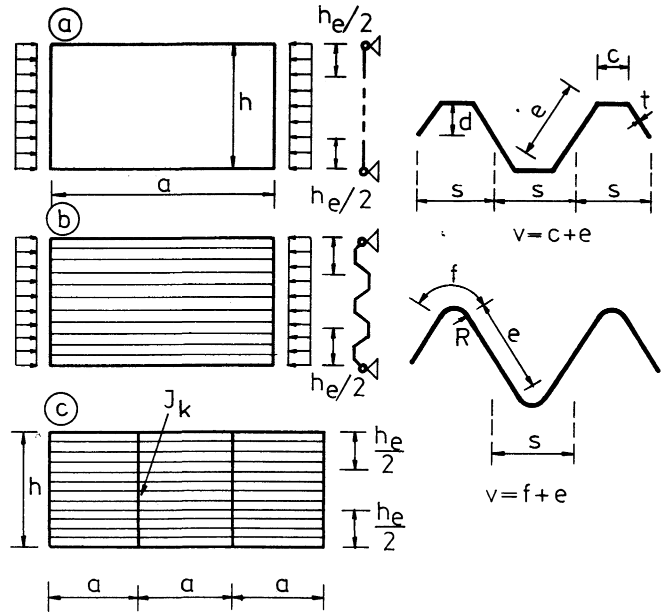
$$h_e = 1.48\sqrt{\frac{s}{vt\sigma_{max}}\left(D\frac{h^2}{x_a3} + 2D_{xy} + D_y\frac{a^2}{h^2}\right)} \quad \text{(EE10.6)}$$

Minimum rigidity of transverse ribs is

$$EJ_k \geq \frac{D_x}{4}\frac{h^4}{a^3} \quad \text{(EE10.7)}$$

Having less rigid ribs, Eq. (EE10.6) is to be applied, with

$$D_y = \frac{s}{v}\frac{Et^3}{12(1-\nu^2)} + \frac{EJ_k}{a} \quad \text{(EE10.8)}$$



e/f	0.25	0.5	1.0	2.0	4.0
k_1	4.79	5.39	6.14	6.92	7.57

Figure EE10.1

3. Local instability requirements, using plastic reduction $\sqrt{E_t/E} \approx 0.5$, are (Fig. EE10.1):

For corrugated plates in compression:

$$\sigma \leq 1.8E/(e/t)^2 \quad \text{(EE10.9)}$$

For corrugated plates in bending:

$$\sigma \leq 2.4E/(e/t)^3 \quad \text{(EE10.10)}$$

For ondulated plates:

$$\left.\begin{array}{l} \sigma \leq k_1\,[0.45E/(e/t)^2 \\ \text{and} \\ \sigma \leq 0.06Et/R \end{array}\right] \quad \text{(EE10.11)}$$

where k_1 is to be taken from Fig. EE10.1.

b. SPECIFICATIONS AND CODES

JAPAN

(No specifications and codes information from Japan.)

NORTH AMERICA

CSA STANDARD S136–1974: COLD-FORMED STEEL STRUCTURAL MEMBERS

Some differing requirements are as follows:

Stiffened Compression Elements (Fig. NA10.1)

$$\text{CSA:} \quad b = 1.64t\sqrt{E/f}$$

Effective Thickness of Multiple-Stiffened Plate Elements

$$\text{CSA:} \quad t_s = \left[\frac{1}{2}\frac{w_s}{p} + \frac{3I_s{}^{1/2}}{pt^3}\right]^{1/3}$$

in which w_s and p are the whole width and developed width, respectively, of the multiple-stiffened element, and I_s is the moment of inertia of the full area of the element, including the intermediate stiffeners, about its own centroidal axis.

Edge Stiffeners

$$\text{CSA:} \quad I_{min} = (2W - 13)t^4 \geq 9t^4$$

where $W = w/t$

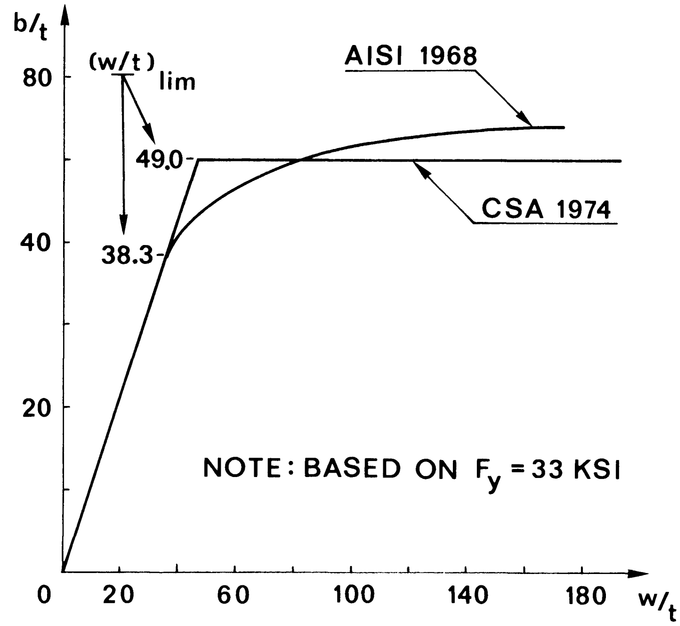


Figure NA10.1

WEST EUROPE

FEDERAL REPUBLIC OF GERMANY

A translated version of the AISI specification is used.

FRANCE

Recommendations drafted by CTICM (Centre Technique Industrieel de la Construction Métallique) are very similar to ECCS-CEEPF Recommendations.

THE NETHERLANDS

Recommendations drafted by TNO. Effective width concept is used for both stiffened and unstiffened flat elements.

$$b_e/b = \sqrt{\frac{\sigma_{kr}}{\sigma_c}}\left(1 - 0.22\sqrt{\frac{\sigma_{kr}}{\sigma_e}}\right)$$

where

$$\sigma_{kr} = \frac{k_d \pi^2 E}{12(1 - \nu^2)(b/t)^2}$$

k_d = initial buckling factor

SWEDEN

A code for light-gage sections in steel and aluminum is under preparation.

Effective width formula for stiffened elements based on AISI specification. For unstiffened elements, an effective *thickness* approach is developed. Calculation of an element with intermediate stiffeners is based on a model in which the stiffener is regarded as a beam-column elastically supported by transverse strips.

UNITED KINGDOM

Addendum No. 1 (March 1975) to BS 449 (Part 2). Post-buckling strength is taken into account by using a local plate stress factor (C_L), comparable with b_e/b. Values of C_L are tabulated for a wide range of plate width-to-thickness ratios b/t. For a detailed commentary, reference is made to Walker.[289]

Most steel specifications give design rules similar to Eqs. (EE10.1) and (EE10.2).

CZECHOSLOVAKIA

Specifications provide approximate formulas for flexural-torsional buckling of nonsymmetric cross sections as well.

SOVIET UNION

Specification allows partially closed cross sections (section "c" in Fig. EE10.2 with batten plates along the open side) to be analyzed as built-up sections, based on Eq. (EE2.1) replacing "single chord" by the half-section.

Largest allowable width-to-thickness ratios of unstiffened flanges (s/t), stiffened flanges (s_o/t), and lips (s_1) are summarized in Fig. EE10.3; for webs (h_o/t), see Fig. EE1.3.

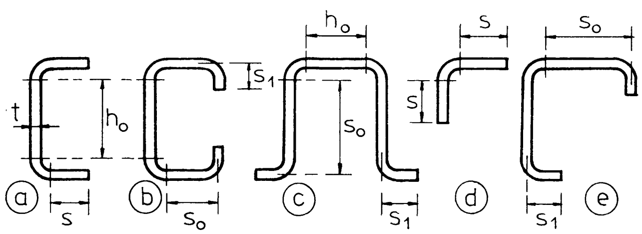


Figure EE10.2

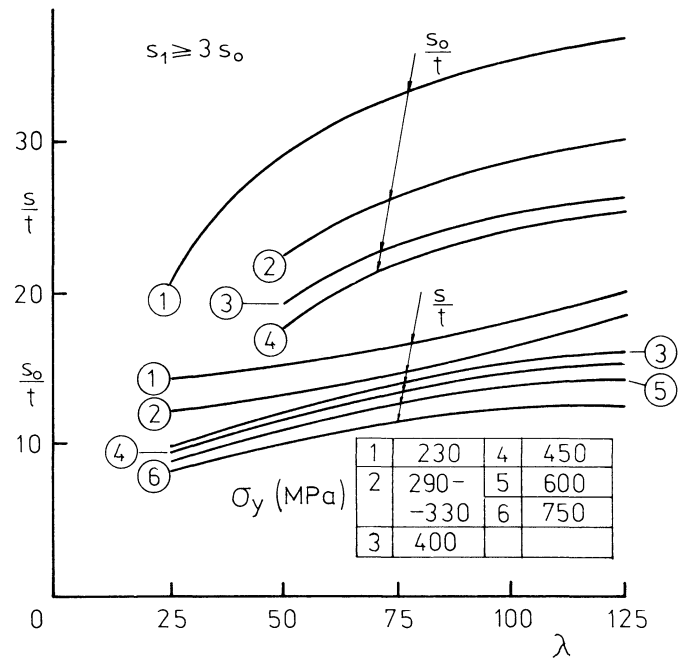


Figure EE10.3

c. COLLOQUIUM CONTRIBUTIONS (COMMENTARIES)

JAPAN

(No colloquium contributions on cold-formed members from Japan.)

NORTH AMERICA

V. KALYANARAMAN, T. PEKOZ AND G. WINTER (WASHINGTON)[290]

A review of an experimental investigation to study the interaction of load and overall buckling of thin-walled columns is presented. Both the tangent modulus and SSRC methods, when modified to take into account the reduction in stiffness as a result of local buckling, can be used to predict the flexural buckling strength.

$$P_c = A_{eff}\sigma_{eff}$$

$$\sigma_{eff} = \frac{\sigma_y - \sigma_y{}^2(L/r_e)^2}{4\pi^2 E} \quad \text{when } L/r_e \leq \sqrt{2}\,L/r$$

$$\sigma_{eff} = \frac{\pi^2 E}{L/r_e} \quad \text{when } L/r_e \geq \sqrt{2}\,L/r$$

where P_c = calculated column capacity, and A_{eff} and r_e are the effective area and effective radius of gyration, respectively.

S. T. WANG AND R. S. WRIGHT (WASHINGTON)[291]

A solution scheme is presented for the analysis of torsional-flexural buckling of locally buckled beams and columns, based on the finite element method and the concept of effective width. A considerable amount of post-local-buckling strength may be available in a locally buckled beam or column, but the use of full section properties to predict the strength of such members may be unsafe.

R. A. LA BOUBE AND W. W. YU (WASHINGTON)[292]

This report discusses development of an "effective depth" equation for web elements of cold-formed beams subjected to bending stress. Reference to the work by Bergfelt and Thomasson is included. An expression for effective depth is obtained from a regression analysis of test results of 56 beam specimens with stiffened compression flanges. A similar study is underway for beams with unstiffened flanges.

WEST EUROPE

A. REIS AND J. ROORDA (LIEGE)[293]

The elastic stability of imperfect thin-walled beams subject to the interaction between lateral torsional and local plate buckling was studied. Imperfections in the shape of the overall mode were assumed, and the determination of the uncoupled and coupled paths was done by the principle of virtual work. Experimental and numerical results of this study are presented in Fig. WE10.6.

J. RHODES AND J. HARVEY (LIEGE)[294]

The authors investigated the interaction of local and column (Euler) buckling in plain channel columns. The results of a theoretical approach were compared with test results. The results showed very good agreement.

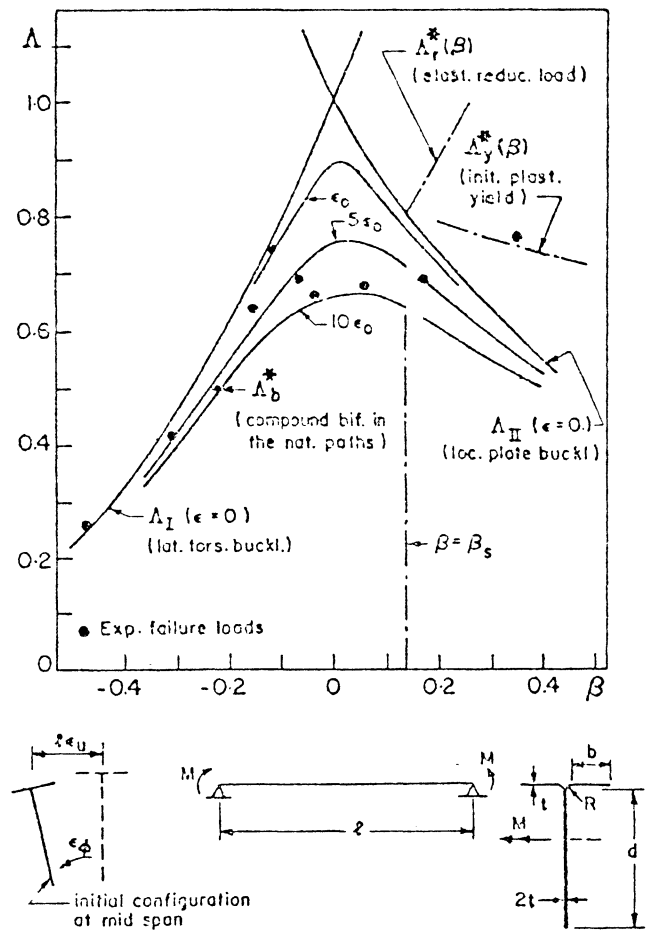


Figure WE10.6

EAST EUROPE

M. SKALOUD AND J. NAPRSTEK (LIEGE)[295]

The authors investigated the interaction between column and plate buckling. They simulated on a computer the performance of thin-walled steel columns with a rectangular hollow cross section. The columns were given an initial curvature and the effect of plate buckling was taken into account by using an effective width according to Winter's formula. It was confirmed that initial imperfections and load eccentricity and the interaction between column and plate buckling very significantly affected the performance of the members.

K. SZILASSY (BUDAPEST)[296]

Paper analyzes the buckling of columns of cold-formed rectangular tubular cross section, taking into account the effect of the strain-hardening caused by cold-forming process, resulting in variable mechanical properties along the cross section. Based on stub-column tests and yield stress measurements within the cross section, an effective stress-strain equation for the whole cross section

$$\epsilon = 0.00167\ \sigma/\sigma_y + 0.002(\sigma/\sigma_y)^{12}$$

is used for the theoretical investigations, and buckling curves for different initial eccentricities are calculated, differing from the multiple column curves given in the Hungarian specifications but in good accordance with the results of 21 buckling tests presented in the paper.

d. CURRENT STUDIES

JAPAN

(No current studies reported for Japan.)

NORTH AMERICA

As indicated above, work is underway on: effective width formulations for design of unstiffened compression elements and webs of beams; an AISI load and resistance factor design procedure for cold-formed members; examination of stiffener requirements; and the interaction of local buckling with (1) lateral-torsional buckling of beams and (2) flexural buckling or torsional-flexural buckling of columns.

Studies are also in progress on fabricated tubular columns, cylindrical shells, storage rack structures, and the combined effects of web bending and crippling.

WEST EUROPE

The most up to date review of current research in this field is presented in the papers of the International Conference on Thin-Walled Structures, held at the University of Strathclyde, Glasgow, Scotland from April 3 to April 6, 1979.

EAST EUROPE

(No current studies reported for East Europe.)

PART C. COMPARISONS AND CONCLUSIONS

The design concepts throughout the world are currently undergoing a change from allowable stress design to limit states design. The material collected in Parts A and B shows that this transition progresses differently in the geographical regions investigated.

In the Western and Eastern European codes, the change to limit states design is quite advanced and, from an overall point of view, unified specifications can be expected in the near future. Japanese and North American design practice is still more or less based on allowable stress design principles, with the exception of Canada, where alternative limit states design rules have been introduced.

These differences in design practice and the use of the new and more advanced design principles in no way reflects the status of research in the various geographical regions. In all four regions, the ultimate strength approach, together with probabilistic safety considerations, is the commonly accepted basis of all recent research in the field of structural stability.

The results of these worldwide research efforts are not fully implemented in the form of limit states design rules, and in many cases remain camouflaged in an allowable stress design format. One reason for this has certainly been seen in the different ways in which specifications are established in the various countries through public authorities. It seems that when government, industry, and research institutions share the responsibility, and when the resulting code is commonly obligatory, these conditions accelerate the implementation process. Another reason is the heterogeneous progress in the various partial fields of stability theory, nationally and internationally, and the different importance that eventually less developed theoretical areas have for the entire design concept, as well as for the specific design needs in the various geographical regions.

In this concluding section, the discrepancy between research achievements and code implementation will be investigated, based on the material in Parts A and B; reference is also made to a recent investigation by Massonnet and Maquoi.[297]

No attempt is made in this discussion to make a quantitative comparison of economy of the different design approaches. The national differences in loading conditions and the multitude of applied "load", "resistance", and "safety" factors make such a comparison on an international level prohibitive.

Only major research gaps will be mentioned here. A detailed listing of needed research is given in Section d, Current Studies, of each chapter in Part B.

1. CENTRALLY COMPRESSED MEMBERS

The basic case of the centrally compressed column reflects quite well the general changes in the theoretical treatment of stability problems, as well as the status of subsequent implementation of the achieved theoretical progress in design practice throughout the world.

For a more realistic assessment of column strength, the tangent modulus method based on bifurcation theory has been replaced by a theoretical model, where stability is considered as a problem of divergence of equilibrium and is treated as inelastic second order analysis of columns with initial geometrical and material imperfections. This ultimate strength analysis of imperfect structural members has become the commonly accepted basis of all recent research and of the corresponding design clauses in limit states design concepts.

These principles are also the basis of the design recommendations of the Structural Stability Research Council (SSRC), USA; of the European Convention of Constructional Steelwork (ECCS), West Europe; and of the Council of Mutual Economic Assistance (CMEA), East Europe. These have been presented as "Regional Recommendations" in Part B. Although the design clauses in the three recommendations differ in details, they all provide multiple column curves for the design of centrally compressed columns.

In West and East Europe this idea of multiple ultimate strength curves has been widely accepted and has been adopted in the course of introduction of limit states design in most of the national codes. The North American and Japanese design practice, largely still allowable stress design oriented, relates the column strength throughout the various building and bridge codes to one single column curve, based either on tangent modulus theory or ultimate strength theory.

This continued use of one single column design curve, together with the tangent modulus concept, has to be understood from the role of the column curve in an allowable stress design concept. Since stability is treated as a bifurcation problem, the design of structural members and systems can be related by means of the effective length method to the column curve of the hinged column. The rather critical attitude in such a design concept towards the use of multiple column curves has to be viewed with regard to their application for beam-column and frame design. For such applications, the significance of multiple column curves for the design results can eventually diminish, in particular when the structural members under consideration are stocky or of low slenderness ratio.

Quite different is the role of multiple column curves in a limit states design concept based on ultimate strength principles. The application of the design curve is mainly restricted to the design of structures, where the different assessment of column strength through multiple column curves significantly influences the design. For the design of frames in such a limit states design concept, as shown in Part B, the column curves act only as reference curves for the determination of initial geometrical imperfections for a subsequent analysis according to a second order plastic hinge method.

The introduction of a multiple column curve design scheme based on ultimate strength principles depends, therefore, not only on the state of the art of column research, but on the readiness to introduce similar advanced methods for the design of other structural members and of systems.

With regard to the present status of column research, the ultimate strength of conventional shapes in their as-rolled or as-welded condition seems to be sufficiently investigated. However, the refined assessment of column strength through multiple column curves makes it necessary to observe the technological progress and to correct and update the selection charts. For economic reasons, the beneficial effect of cold-straightening should be recognized in the design clauses.

More information is needed for a theoretical prediction of the residual stress distribution in unconventional shapes. More investigations are also needed for those cases where longitudinal and circumferential residual stresses have to be recognized for the ultimate strength analysis, as in the cases of large fabricated tubular members and cold-formed tubes.

2. BUILT-UP MEMBERS

The design approach for battened or laced built-up members has undergone a development similar to that of axially loaded simple members. Since the critical load analysis of perfectly straight built-up columns does not supply realistic data for the design of chord and web members, the new design concepts are based on the ultimate strength analysis of built-up members with an initial out-of-straightness.

Such a concept has been adopted by ECCS and is described in Part B. However, most of the other regional recommendations and national codes still require an elastic critical load analysis by using an effective slenderness ratio.

It is hoped that future limit states design codes will adopt the ultimate strength principles for the design of built-up members, not only to achieve a consistent theoretical treatment, but in order to replace empirical formulae through a realistic assessment of the axial forces, moments, and shear forces acting in the member.

Considering the importance of this type of column for industrial buildings, future investigations should center around an optimum design for built-up columns.

3. BEAMS

In the regional recommendations, the design clauses to prevent lateral torsional failure of laterally unsupported beams are either based on elastic critical loads, relating beam behavior in the inelastic range to column buckling by means of an equivalent slenderness ratio procedure (SSRC, INCERC), or the design clauses provide a separate design curve as the result of ultimate strength calculations (ECCS). Similarly wide is the range of applied theoretical models in the various national codes. Eventual unification might be encouraged through further investigations of the ultimate strength behavior of beams.

Recent research, including several Colloquium contributions, have investigated the role that support conditions, the type of loading and level of load application, shape of cross section, residual stresses and geometrical imperfections have on the ultimate strength of beams. In the ECCS Recommendations, a single design curve and a modified slenderness account for these parameters. Additional investigations should clarify whether further differentiations analogous to the column design procedures are desirable. It is already evident that some of these parameters influence the ultimate strength of beams quite differently than that of columns. In particular, residual stresses seem to be less significant for beam behavior, and further investigations should settle the question of whether or not a distinction has to be made between rolled and welded beams.

4. PLATE AND BOX GIRDERS

Like those used for many other components, the design approaches adopted for plate and box girders are going through a transition phase from the elastic allowable stress approach to the limit state. The ECCS Recommendations contain provisional clauses which are based on a modified critical elastic buckling approach to stiffened plated structures. It is true to say that many apparently elastic methods are in reality elasto-plastic methods couched in elastic terms. A good example of the present state of affairs is the comparison which has been drawn recently between codified methods for box girder compression flange design and presented at the Cardiff Conference on the new British bridge code. It was concluded that no method was truly plastic, but that all methods reviewed had some sections based on plastic considerations and others on elastic considerations. The latter approach is necessary, as insufficient information exists on the complete collapse behavior of a wide compression flange and all its components. The same can be said of other elements, such as orthogonally stiffened webs for plate or box girders.

Despite this, tremendous advances have been made over the past 10 years in our understanding of the behavior beyond service loading of plate and box girders. This is one of the positive spin-offs from the tragic collapses of box girders bridges in the late 60's and early 70's. The tremendous volume of research triggered by these collapses advanced the understanding of the ultimate behavior of plated structures at a rate which would not have been possible were it not for the mistakes which were made.

Most attention has been focussed on box girders, rather than plate girders, but the lessons learned have in many cases been applicable to both. The inelastic strut approach used for compression flanges is now incorporated in the new

USA draft AASHTO recommendations for box girder bridge design, as well as in the new UK code BS5400, Part 3, and Norwegian DnV Rules for offshore structures. Alternative approaches were first explored in depth within the Preliminary Report to the 2nd International Colloquium on Stability and the information was widely disseminated at all of the Colloquia. The tension field-method of design, also reviewed in depth in the same Report and discussed widely at the Symposia, has also been taken on board by the U.S., U.K., Swiss, and other steel codes, although limitations have been placed on its application to various geometries.

The work on the inelastic stability of plate and box girders is being used as a basis for several draft codes in countries including Germany, The Netherlands, Belgium and, indeed, the EEC countries as a whole, in the context of the new Eurocode 3 for structural steelwork. It is also likely to be used as source material in countries as far ranging as Japan and Brazil. In the latter country a limit state code for steel buildings has recently been drafted and one for bridges is currently being prepared.

There are, of course, many problems which remain to be solved before ultimate limit models can be adopted for all plate and box girder configurations. Among the stability problems which are still being tackled on pseudo-plastic bases in the majority of codes are the following:

- local elasto-plastic tripping behavior of stiffeners for flanges and webs
- overall stiffener sizing for deep webs of plate and box girders
- redistribution of stresses from webs to flanges in girders with deep thin stiffened webs
- shear lag effects in very wide flanges
- wide flanges subjected to combined inplane and lateral loading

The encouraging aspect is that there is a definite convergence of approach to design of box and plate girders, using a proper understanding of elasto-plastic large-deflection behavior to formulate rational and simple yet accurate methods to determine their true ultimate limit state.

5. BEAM-COLUMNS

All three Regional Recommendations and the Japanese codes use linear interaction equations both for uniaxial and biaxial bending. Flexural-torsional buckling is treated in an approximate way by relating it to the failure mode of lateral buckling of beams.

For biaxial bending, the SSRC Recommendations also suggest nonlinear interaction equations, providing a better fit to the actual interaction curves and removing the conservatism of the linear interaction equations.

The modification of the conventional linear interaction equations in the ECCS Recommendations also aims for a more economic design. As shown in Part B, the column buckling load is replaced in these formulae through a fictitious eccentricity applied to the axial load N. This has particular advantages for larger slenderness ratios, since it accounts for the reduction of the axial force N at the presence of bending moments.

More economy can also be expected from recent developments to adjust the design formulae more closely to the specific load-carrying capacity of the different cross-sectional shapes, quite analogous to the multiple column curve philosophy for simple columns. In this line are the Canadian Specifications, where for the biaxial bending case the given linear interaction equations for H-sections are modified for the use for square tubular sections. Nonlinear interaction equations developed recently (see Part B) distinguish between wide-flange shapes and box sections. Extensions to other structural shapes can be expected.

Similarly stronger recognition of shape characteristics seems to have affected the design practice in some East European countries, where the limit fibre stress is related by means of an effective slenderness to the limit stress of a fictitious centrally compressed member.

The problem of applying the solutions for the "isolated" beam-column to the actually restrained beam-column in a framework by other than the effective length method has been attacked in different codes. The ECCS Recommendations limit the application of the beam-column interaction formulae to braced frames. For unbraced frames, a second order analysis of the entire system is required. The code of the German Federal Republic completely eliminates the use of interaction equations and replaces it by second order elastic analysis or by second order plastic hinge analysis.

This is in compliance with the present tendency in research towards a full system analysis according to ultimate strength principles, which in return should lead to practicable design tools for the proportioning of beam-columns as parts of an entire framework or of subassemblages.

6. FRAMES

The regional recommendations and the national codes reflect in different degrees the present tendency to reduce the application of traditional frame design procedures based on effective length concept and beam-column interaction formulae to nonsway problems. The conservatism of these design methods, as well as unsafe design results for certain applications, have led to an introduction of more refined design methods for overall system analysis, based on second order theory.

For elastic frame design, the SSRC and the INCERC Recommendations allow the effective length procedure for both braced and unbraced frames, while the ECCS Recommendations restrict the use of this concept to the case of braced frames. SSRC in addition provides P-Δ procedures and methods based on story drift control. For unbraced frames, ECCS requires a full second order analysis, assuming initial imperfections.

For plastic frame design, the SSRC procedures require the direct inclusion of P-Δ moments in the calculation of the ultimate strength of the frame. According to the ECCS Recommendations, braced frames may be designed using beam-column interaction formulae. Unbraced frames in the ECCS concept may be designed according to simple plastic theory, modified Merchant-Rankine formula, or second order plastic hinge method, depending on the given criteria (see Part B).

The Japanese codes generally provide the effective length procedure for frame design, but, due to the special wind and earthquake loadings, stability aspects are less dominant for the design considerations.

The tendency towards full system design will certainly continue in the future. The further development of practicable design methods based on second order theory is needed, together with a calibration of such design procedures vis-a-vis design examples executed in design practice. More emphasis should also be given to recognition of the behavior of three-dimensional structures in the relevant design clauses.

Of importance is the further clarification of the influence of geometrical and material imperfections on the ultimate strength of frames, considering different system configurations and loading conditions. This is particularly relevant for a realistic estimation of the representative out-of-plumb, which is introduced in most second order design procedures.

7. TRIANGULATED STRUCTURES

Despite the general trend towards more refined analytical methods, the design of the majority of trusses, which are trusses where the panel points are braced normal to the plane of the truss, will continue to follow the conventional procedures adopted by the present codes. These simplified methods assure the stability of the system by considering the member stability only.

In these methods the members are idealized as hinged struts and the eventual restraint supplied at the joints is accounted for by introducing an effective length. The corresponding effective length factors are derived in nearly all recommendations and codes on the basis of bifurcation theory.

In the case of limit states design concepts, the following question should be considered: to what extent are the effective length factors based on inelastic bifurcation theory, and meant for application in an allowable stress design concept, still valid at ultimate load level.

By simplifying the actual conditions, the behavior of a member in a truss at ultimate load level will depend on the slenderness range, the stiffness ratio of the restraints, and the interaction of the axial forces present in the member with the bending moments caused by local deformations of the member as well as by global deformations of the truss. The presence of residual stresses, in particular pronounced zones of compressive residual stresses (for example, at the joints), will lead to earlier plastification and subsequent increase of deformations. It seems obvious that the conventional bifurcation theory does not account for this behavior. A cautious use of effective length factors derived on this basis seems appropriate.

This is supported by Colloquium contributions (see Part B). As a consequence, the new Swiss code SIA 161, which is based on limit states design, allows for buckling in the plane of the truss a K-factor of 0.8 for web members and only $K = 0.9$ for chord members, as compared with $K = 0.8$ for all truss members in a previous edition of the code.

Further investigations of the ultimate strength of trusses should therefore be encouraged in order to check the numerical validity of effective length factors.

For an economical truss design, and in order to maintain a coherent level of structural safety at the same time, not only should the application of effective length factors be emphasized, but full use should be made of the advantages of multiple column design curves for truss design.

8. SHELLS

Generally speaking, regional recommendations and national specifications tend to rely on fairly straightforward formulas, rather than involved procedures. An exception is the ECCS set of rules concerning ring-stiffened cylinders under external pressure, which is a rewrite of the corresponding section of British Standard BS 5500.

Since certain types of shell are very imperfection sensitive under certain types of loading, it stands to reason that the magnitude of the imperfections is a decisive factor in most specifications. This is achieved in two different ways. In the ECCS Recommendations and in the British and Soviet codes, for instance, the rules apply only when the imperfections do not exceed certain tolerances, while the SSRC Guide gives critical loads which are a direct function of the imperfections and account for imperfections of any magnitude.

The scope of various sets of specifications is quite different. The ECCS text restricts itself to unstiffened shells, except for ring-stiffened cylinders under external pressure, and the Austrian code mentions only cylinders. The SSRC Guide applies to a wide variety of stiffened and orthotropic shells. The Soviet and the Czechoslovakian codes consider cylindrical panels and conical shells, while the ECCS rules do not.

European standards, in West Europe as well as in East Europe, are mainly based on lower bounds of experimental results. The formulas in the SSRC Guide, on the other hand, were theoretically obtained.

On the face of it, the Soviet code is predominantly oriented elastically. Elastic-plastic buckling appears to be considered more explicitly in the ECCS Recommendations.

The ECCS rules and the Soviet standards both use Dunkerly's straight interaction line for the case of combined axial load and radial pressure on cylinders. A somewhat more optimistic interaction curve, located above the straight line, appears in the Czechoslovakian specifications. Since there do not seem to be much experimental data available for the combined loading cases, at least in the elastic-plastic range, and since it is a case of much practical importance in offshore work, it would be interesting to know the origin and the basis of the Czechoslovakian rule.

9. COMPOSITE MEMBERS

No design clauses exist in current regional recommendations. However, documents elaborated by relevant working commissions indicate the design trends in the various geographical regions.

In the ECCS Manual on Stability and in the draft Joint ECCS-CEB-IABSE Recommendations on Composite Construction, an ultimate strength approach is proposed for encased columns and concrete-filled tubes. For the design of axially loaded columns, a modified slenderness is defined that relates the design to the ECCS multiple column curves for steel sections. For the design of beam-columns, parabolic interaction formulae are proposed.

Japanese codes are presently under revision, but future composite design will be based on a method of superposition, according to which the strength of a composite column or beam-column is calculated as the sum of the strength of the component material columns. A stated advantage of this procedure is that the strength of the steel and concrete portions can be determined by using independent design specifications.

In the USA, the design of composite members is so far only covered in the American Concrete Institute Building Code. The design approach specified is essentially identical with that for reinforced concrete members and leads to a quite conservative design of composite members. On initiative of SSRC, developments are underway to relate composite design to the procedures given in the AISC specifications for steel column and steel beam-column design.

No official design clauses are given in Eastern European codes.

Opposite these quite diversified design approaches for composite members, it is hoped that joint international ventures of steel and concrete associations, such as the Joint ECCS-CEB-IABSE Committee, will succeed in promoting a fairly unified design philosophy.

10. COLD-FORMED STEEL

Codes for the design of light-gage cold-formed steel structures are available in North America, West and East Europe. There are no Japanese rules in this field.

One of the most important factors in the design of thin-walled members is to account for local buckling in compressed plate-elements. The effect of the postcritical reserve of strength is invariably utilized. The effective-width concept has been adopted in each of the above-mentioned codes, although the formulas that are used are slightly different.

An effective-width approach has also been adopted for plates with multiple stiffeners (orthotropic plates). For compressed plates with one or two intermediate stiffeners, both the ECCS draft, CSA and the AISI specifications use a similar approach by giving minimum requirements for the second moment of area. Both CSA and AISI use a similar approach also for edge stiffeners, while the ECCS draft treats the edge stiffener as a compressed elastically supported column where the limit stress is determined by ECCS buckling curve b.

Unstiffened elements (plate-elements with one edge free) are treated both in the AISI code and the ECCS draft with the aid of a design compressive stress which is a function of the critical buckling stress. Efforts are under way to formulate effective width design procedures for this case also.

Concerning centrally compressed members, the AISI and CSA specifications governing flexural buckling are both based on the SSRC method, with inclusion of a local buckling factor Q. The slenderness ratio is to be calculated for the gross cross section. In the ECCS draft, a slenderness ratio based on effective cross section is to be used for determining the buckling stress from ECCS column curve b.

Current research in the field of light-gage steel structures includes the following topics:

- Interaction of local and overall buckling (including torsional-flexural buckling)
- Effect of intermediate stiffeners
- Web buckling and web crippling