# Computing The Effective Plastic Moment

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Imagine there are two geometrically identical rolled beams sitting side by side, one having a yield strength, $F_y$, equal to 36 ksi and the other having $F_y$ equal to 50 ksi. Imagine further that the flange and web proportions are such that the $F_y = 36$ ksi beam exactly satisfies the American Institute of Steel Construction (AISC) Specification[1] Part 2 slenderness (width-to-thickness) requirements for plastic design. Under these rules, the $F_y = 50$ ksi beam would be excluded from plastic design.

The interesting possibility of pretending that the $F_y = 50$ ksi beam has an effective yield strength of only 36 ksi was suggested during the development of the Autostress Design method.[2] The thesis put forth was that the web and compression flange could each be considered to have an effective yield strength, $F_{ywe}$ and $F_{yfe}$, that could be employed in the computation of an effective negative plastic moment, $M_{pne}$. For a compact shape under the AISC Part 2 rules, $M_{pne} = M_{pn}$, where $M_{pn}$ is the full plastic moment in negative bending; for a shape noncompact by these rules, it could be conceived that $M_{pne} < M_{pn}$. In no case could $M_{pne}$ exceed $M_{pn}$. A distinction is made here between the plastic moments in negative and positive bending because they are only equal for special cases, such as a rolled beam; for a section with dissimilar flanges or a composite flange, these moments are generally unequal.

To demonstrate that this hypothesis was suitable for design use, the $F_y = 50$ ksi beam would have to possess at least as much inelastic-rotation capacity when measured at $M_{pne}$ as the compact $F_y = 36$ ksi beam possesses at $M_{pn}$. (In this example, $M_{pne}$ of the $F_y = 50$ ksi beam would be about equal to $M_{pn}$ of the $F_y = 36$ ksi beam.) The results of 49 tests in the literature showed[2] that this was generally true—only one of these beams had a slightly lower (7% less) capacity. As shown in Appendix A of Ref. 3, the required inelastic rotation of a beam increases with $F_y$. Therefore, because the equations to be given for the effective plastic moment were derived for $F_y = 50$ ksi, they should not be used for steels with $F_y > 50$ ksi without additional study; the equations are valid for $F_y \leq 50$ ksi.

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Thus, the effective plastic moment is suggested as an extension of the present AISC Part 2 plastic design rules. $M_{pne}$ is merely substituted for $M_{pn}$. The use of $M_{pne}$ does not preclude the use of any beam geometry and, furthermore, permits existing structures to be rated with plastic-design methods, even though their beam geometry does not meet the present AISC Part 2 slenderness requirements. This paper presents the equations for computing $M_{pne}$, two numerical examples, and a discussion of optimum proportions.

## EQUATIONS FOR $M_{pne}$

Figure 1 defines the geometry of a section that may be homogeneous, hybrid, or composite. The section is assumed to be in negative bending corresponding to the condition near the interior support of a continuous beam. The plastic neutral axis is computed at $M_{pn}$. According to the AISC Part 2 rules, the compression flange is considered compact for a yield strength of 50 ksi if the flange slenderness, $b_f/2t_f$, is equal to or smaller than 7.0. For other yield strengths, the requirement is approximately

$$b_f/2t_f \leq 7.0\sqrt{50/F_{yf}} = 49.5/\sqrt{F_{yf}} \tag{1}$$

where $F_{yf}$ equals the specified minimum yield strength of the compression-flange material in ksi. Rather than limit the maximum value of the flange slenderness according to the specified minimum yield strength, we define an effective (reduced) yield strength of the flange material, $F_{yfe}$, ac-

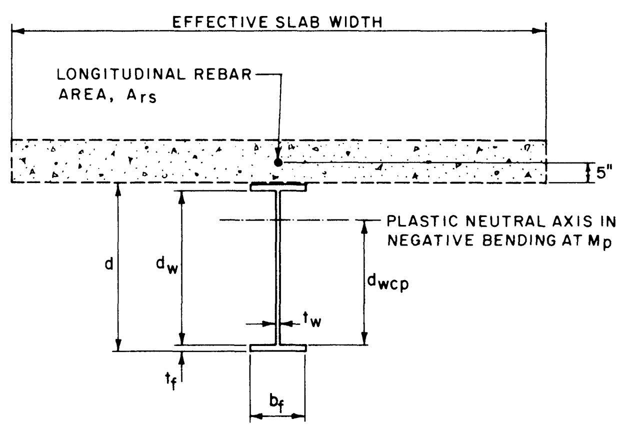


*Fig. 1. Cross section*

cording to the actual compression-flange slenderness, so that

$$F_{yfe} = 9800(t_f/b_f)^2 \le F_{yf} \quad (2)$$

Because AISC gives web slenderness ratios only for symmetrical sections, a few adjustments are needed to account for the shift of the neutral axis in composite and unsymmetrical sections. We define the effective web slenderness at the full plastic moment of the composite section as $2d_{wcp}/t_w$, where $d_{wcp}$ is defined in Fig. 1; for a symmetrical shape, $2d_{wcp} = d_w$. The AISC web requirement is based on the overall depth of the section rather than the web depth. The actual depth of the web between the flanges, $d_w$, is about 95% of the overall depth, $d$, for most rolled beams. Thus, with $d = d_w/0.95$, the AISC requirement can be restated as

$$2d_{wcp}/t_w \le 0.95(412/\sqrt{F_{yw}}) \quad (3)$$

In a manner similar to the flanges, we replace the specified yield strength of the web material, $F_{yw}$, by the effective yield strength of the web, $F_{ywe}$, to obtain

$$F_{ywe} = 38{,}300(t_w/d_{wcp})^2 \le F_{yf} \quad (4)$$

The upper limit of $F_{yfe}$ and of $F_{ywe}$ is $F_{yf}$, because flange and web buckling are controlled primarily by flange strain rather than the web yield strain. Finally, we normalize both $F_{yfe}$ and $F_{ywe}$ with respect to the actual flange yield stress and obtain the reduction factors $R_f = F_{yfe}/F_{yf}$ and $R_w = F_{ywe}/F_{yf}$ for the flanges and web.

The negative plastic moment, $M_{pn}$, of the section is the sum of the contribution of the flanges including the rebars, $M_{pnf}$, and the web, $M_{pnw}$, so that

$$M_{pn} = M_{pnf} + M_{pnw} \quad (5)$$

The effective plastic moment, $M_{pne}$, is then obtained by multiplying the components of $M_{pn}$ by their respective reduction factors

$$M_{pne} = R_f M_{pnf} + R_w M_{pnw} \quad (6)$$

It is suggested that $M_{pne}$ be used instead of $M_{pn}$ in conventional plastic design.

Equations (2) and (4) were derived from the AISC Part 2 slenderness limits. However, other limits could have been selected. For example, suggested Canadian limits[4] would only change the constants in Eqs. (2) and (4) to 11,700 and 46,200; the procedure for computing $M_{pne}$ would not change.

## TWO NUMERICAL EXAMPLES

$M_{pne}$ will be computed for two composite rolled beam sections ($F_{yf} = F_{yw} = F_y$) with varying amounts of longitudinal rebar area, $A_{rs}$, ranging from 0 to 8 in.$^2$, and for $F_y$ = 36 and 50 ksi. The rebar area is assumed to have a yield strength of 60 ksi and to be located 5 in. above the beam. The rolled beam fillets are neglected for simplicity. The two rolled beam sections considered—a W30x132 and a W36x135—are within 2% of having the same area. However, the W30x132 is a relatively stocky section, and the W36x135 is a relatively slender section.

**Table 1. Slenderness Values**

| | Web $d/t_w$ | Compression Flange $b_f/2t_f$ |
|---|---|---|
| AISC Part 2 limit: | | |
| $F_y$ = 36 ksi | 68.7 | 8.5 |
| $F_y$ = 50 ksi | 58.3 | 7.0 |
| W30x132 | 49.3 | 5.27 |
| W36x135 | 59.2 | 7.56 |

In Table 1 the shapes are compared with the AISC Part 2 (Sect. 2.7) slenderness limits for zero axial load. These limits do not directly address a nonsymmetrical section, such as a rolled beam composite with rebars. However, by applying those limits, the W30x132 would be acceptable for both yield strengths, whereas the W36x135 would be acceptable only for the 36 ksi yield strength.

$M_{np}$ and $M_{pne}$ are plotted in Figs. 2 and 3 for the two shapes at both yield strengths; the yield strength is indicated parenthetically. For the W30x132, $F_{yfe}$ = 88.1 ksi, and because $F_y$ = 36 and 50 ksi, the flanges are fully effective for both yield strengths. The web is also fully effective for both yield strengths at low values of .he rebar area, $A_{rs}$.

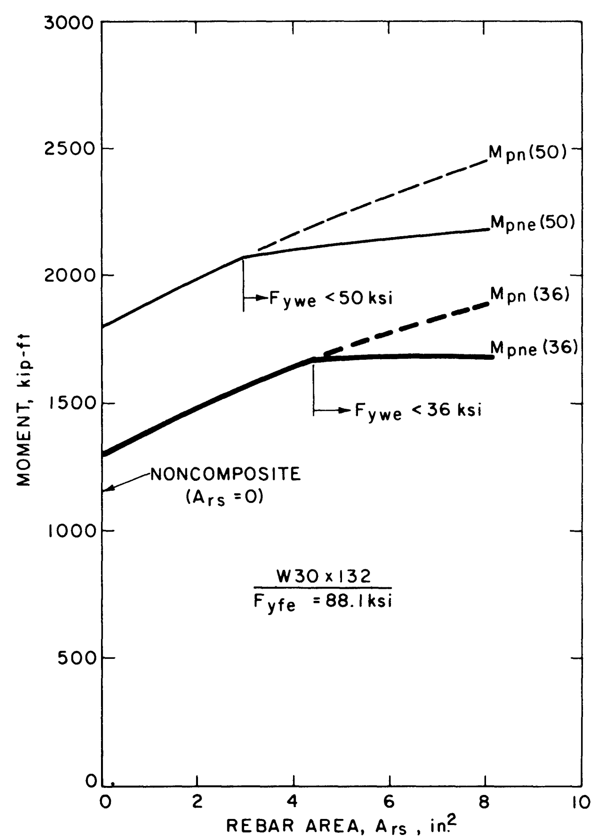


*Fig. 2. Moment vs Rebar Area for W30x132*

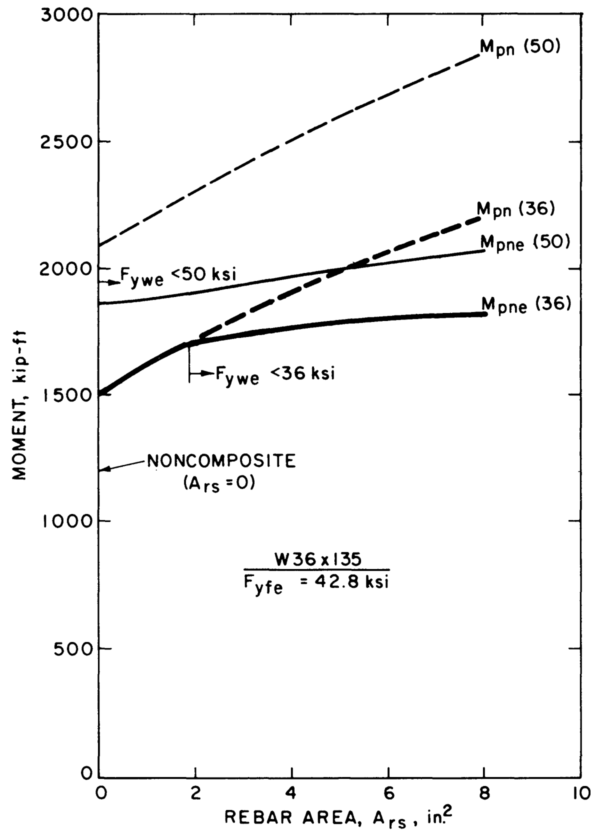


*Fig. 3. Moment vs Rebar Area for W36x135*

Thus, the $M_p$ and $M_{pe}$ curves are coincident for low values of $A_{rs}$. When $F_{ywe}$ becomes less than $F_y$ (as $A_{rs}$ and, concomitantly, $D_{wcp}$ increase), the $M_{pe}$ curve shows little or no increase with increasing $A_{rs}$ because the increased rebar force is offset by both the reduced distance to the neutral axis and the reduced $F_{ywe}$ caused by an increased $D_{wcp}$.

For the more slender W36x135, the curves for $F_y = 36$ ksi are similar to those for the W30x132. The compression flange is fully effective and the web is fully effective for low values of $A_{rs}$. However, for $F_y = 50$ ksi, the noncomposite shape ($A_{rs} = 0$) has an $M_{pne}$ below $M_{pn}$ because $F_{ywe} =$ 47.8 ksi and $F_{yfe} = 42.8$ ksi. As $A_{rs}$ increases, $M_{pne}$ increases slightly as before. In fact, for other than small values of $A_{rs}$, $M_{pne}$ of the W30x132 is larger than that of the W36x135, both at $F_y = 50$ ksi. In other words, for $F_y =$ 50 ksi, the shallower W30x132 is generally (except for small values of $A_{rs}$) a superior plastic-design composite section compared with the deeper W36x135 when the effective plastic moment concept is used.

## OPTIMUM CROSS SECTION FOR $M_{pne}$

For a given amount of material and depth of section, the optimum material distribution to maximize $M_{pne}$ is obtained by first proportioning the compression flange such that $F_{yfe} = F_{yf}$. This is best achieved by reproportioning the flange width and thickness. When this condition is satisfied, it will usually be beneficial to move some material from the web to the flanges. The optimum $M_{pne}$ is reached when the incremental increase in flange moment is balanced by the decrease in web moment.

This can be illustrated by considering a noncomposite rolled beam of yield strength $F_y$ that has $F_{yfe} = F_y$. For simplicity the moment arm between the flanges is assumed to be $d_w$. The effective plastic moment is then

$$M_{pne} = b_f t_f d_w F_y + t_w d_w{}^2 F_{ywe}/4 \qquad (7)$$

Letting $\alpha = d_w/t_w$, from Eq. (4), with $d_{wcp} = d_w/2$,

$$\alpha = 391.4/\sqrt{F_{ywe}} \qquad (8)$$

Letting the cross-sectional area $A = 2b_f t_f + d_w t_w$, we can substitute into Eq. (7) to obtain

$$M_{pne} = A d_w F_y/2 - d_w{}^3 F_y/2\alpha + 391.4^2 d_w{}^3/4\alpha^3 \qquad (9)$$

Considering all values on the right side of Eq. (9) to be constant except $\alpha$, we can determine the optimum value of $\alpha$, $\alpha_{opt}$, by setting the first derivative equal to zero.

$$\frac{dM_{pne}}{d\alpha} = d_w{}^3 F_y/(2\alpha^2) - 3(391.4)^2 d_w{}^3/4\alpha^4 \qquad (10)$$

Letting $\alpha_p$ be the value of $\alpha$ when the web is fully effective, $\alpha_p = 391.4/\sqrt{F_y}$ from Eq. (8). We can then solve for $\alpha$ as $\alpha_{opt}$ from Eq. (10).

$$\alpha_{opt} = 1.225\alpha_p \qquad (11)$$

Thus, the optimum $M_{pne}$ for this section occurs when the web depth/thickness ratio is 22.5% above the ratio that makes the web fully effective, or stated differently, when 22.5% of the web material is moved to the flanges.

## CONCLUSIONS

The effective-plastic-moment approach does not exclude any shapes from plastic design; additionally, it permits plastic-design methods to be employed in the rating of existing members. For plastic design, the most efficient distribution of material occurs when the effective flange yield strength is not below the actual compression flange yield strength. Furthermore, for a symmetrical section of a given depth, the effective plastic moment is optimum when 22.5% of the web area is moved from the web to the flanges, starting from a web area that makes the web fully effective. The effective-plastic-moment approach is applicable to sections that are symmetric, unsymmetric, hybrid, and composite in negative bending.

## NOMENCLATURE

$A$ = Cross-sectional area
$A_{rs}$ = Longitudinal rebar area
$b_f$ = Compression flange width
$d$ = Overall depth of steel section

$d_w$ = Depth of web
$d_{wep}$ = Depth of web in compression at $M_p$
$F_y$ = Actual yield strength
$F_{yf}$ = Actual yield strength of compression flange
$F_{yfe}$ = Effective yield strength of compression flange
$F_{ywe}$ = Effective yield strength of web
$M_{pn}$ = Full negative plastic moment
$M_{pne}$ = Effective negative plastic moment
$M_{pnf}$ = Contribution to $M_{pn}$ of the flanges
$M_{pnw}$ = Contribution to $M_{pn}$ of the web
$R_f$ = Reduction factor for flanges
$R_w$ = Reduction factor for web
$t_f$ = Thickness of compression flange
$t_w$ = Web thickness

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## DISCLAIMER



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