Plastic Behavior of Beams with Mid-Depth Web Openings

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It has become common practice to cut holes in the webs of steel beams in building structures in order to provide for the passage of service facilities such as ducts, water and sewage pipes, etc. This results in a considerable reduction in the depth between the ceiling and the floor above. Consequently, considerable savings in building costs can be achieved.

The plastic analysis of steel I-beams with a single rectangular mid-depth hole has been extensively studied.[3,4,5,6,10,11] Analysis was carried out by assuming the predicted normal and shear stresses at failure in the top and bottom parts of the beam, and obtaining the failure load (bending moment and shearing force at the center of the hole) by considering the equilibrium of the external and the assumed internal forces. These analyses are somewhat lengthy compared to the authors' analysis.

This paper presents a new approach to the problem, in which plastic hinges are assumed at the corner of the hole. Formulas for the predicted failure loads have been derived and can be solved by hand or by small calculators.

Analyses of beams with double or multiple holes have also been investigated.[2,9] The authors' analysis can be extended to provide a simpler solution for such beams.

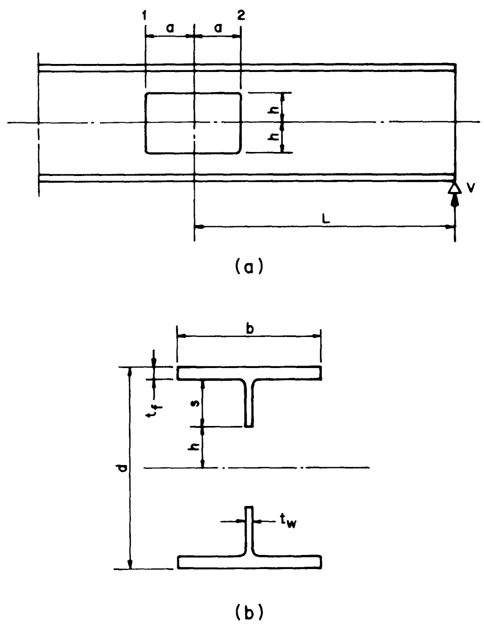

Fig. 1. Beam with single rectangular hole

INTERACTION DIAGRAMS

When a hole is cut in the web of a steel I-beam as shown in Fig. 1, the moment M and shear V at the center of the hole are reduced. This reduction depends on the size of the hole. Interaction between M and V does develop. The bending moment and the shearing force are usually non-dimensionalized in interaction diagrams by dividing M and V by the fully plastic moment M_p and shear V_p of the uncut section, respectively.

$$M_p = \sigma_y Z_p = \sigma_y[bt_f(d - t_f) + 0.25\, t_w\, (d - 2t_f)^2] \quad (1)$$

$$V_p = (\sigma_y/\sqrt{3})(d - 2t_f)t_w \quad (2)$$

in which

d = the total depth of the beam
b = flange width
t_f = flange thickness
t_w = web thickness
σ_y = yield stress
Z_p = plastic modulus of section

It is the purpose of this paper to find a simple and practical method to obtain an interaction diagram relating M/M_p and V/V_p at the center of the hole.

ANALYSIS

Consider a steel beam with single unreinforced hole (Fig. 1). The center line of the hole is at a distance L from the simple support of the beam.

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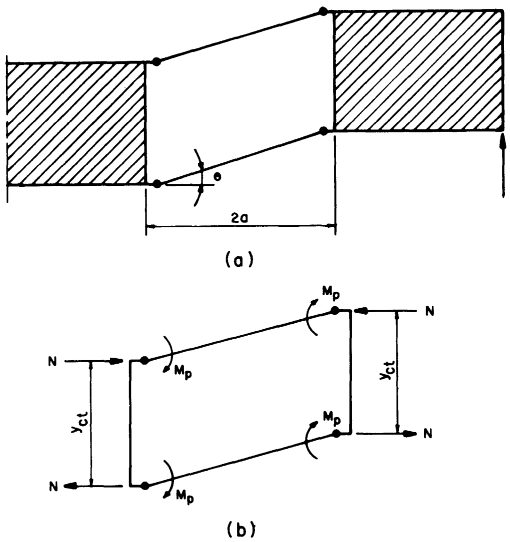


Fig. 2. Diagram showing position of plastic hinges

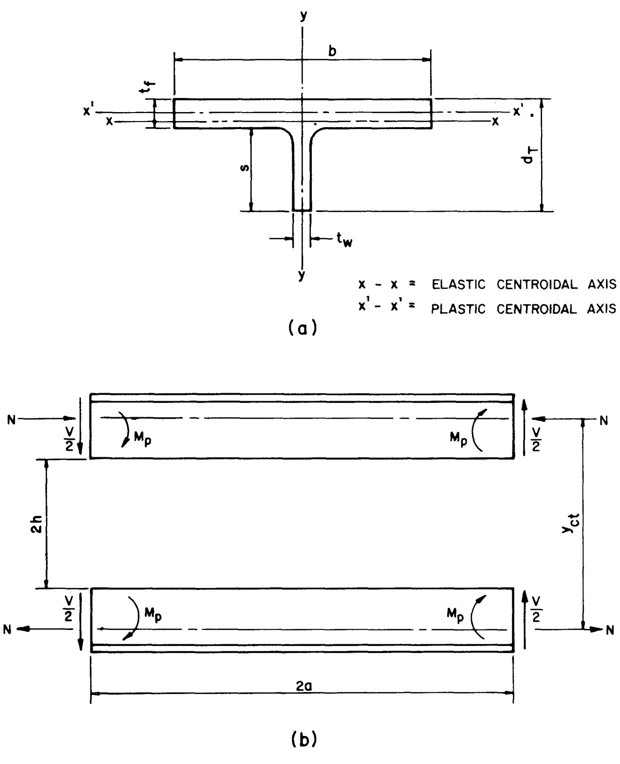


Fig. 3. Forces and moments on top and bottom tees

At collapse, four plastic hinges must be developed on both of the cross-sections, 1 and 2. Fig. 2 shows the positions of the plastic hinges and the mechanism failure.

At ultimate load, the following forces and moments are acting in the top and bottom tees (Fig. 3):

1. A normal force N. This force will act along the elastic centroidal axis of the tee-section (axis x–x) if the shear at center of hole, V, is equal to zero. For high shear, the position of N is along the plastic centroidal axis (axis x'–x'). For low shear, the position of N lies between the elastic and plastic centroidal axes.
2. Shearing force, equal to $V/2$ at each end.
3. A plastic moment, M_p , at each end.

In the present analysis (as in all previous works dealing with this problem), it is assumed that the shearing force in the tee-beam is taken by web only.

Considering the equilibrium of the part of the beam (tee-section) at the vicinity of the hole, it is found that:

$$V(L - a) = Ny_{ct} - 2M_p \quad (3)$$

$$V(L + a) = Ny_{ct} + 2M_p \quad (4)$$

$$\therefore VL = M = Ny_{ct} \quad (5)$$

in which y_{ct} = lever arm between the top and bottom normal forces N.

$$Va = 2M_p$$

hence,

$$M_p = Va/2 \quad (6)$$

It is also assumed that the normal stress σ in the web at yielding is governed by the von Mises yield criterion,

$$\sigma = \sqrt{\sigma_y{}^2 - 3\tau^2}$$

or in other words

$$\sigma = \sqrt{\sigma_y{}^2 - \frac{3}{4}\frac{V^2}{s^2 t_w{}^2}} \quad (7)$$

where τ = shear stress in the web at yielding = $V/2st_w$. The assumed stress distributions for the top tee at sections 1 and 2 for low values of shearing force is shown in Fig. 4.[6] The stress distributions for high shear are shown in Fig. 5.[6] For the bottom tee, the stress distributions are identical with the top tee, except that the normal stresses have opposite signs.

It can be shown that the assumed stress distributions achieve equilibrium in every respect; therefore they can be regarded as lower bounds.[6]

Case of Low Shearing Force—Consider the case when the shear is low (Fig. 4). In this case, it is assumed that the normal stresses causing the moment M_p is at a distance k_2t_f from the outside of the flange, and k_1s above the tip of the web, where s = distance from tip of web to bottom of the flange.

Considering the equilibrium of the tee-section,

$$bk_2t_f\sigma_y = t_w k_1 s\sigma \qquad (k_1 < 1, k_2 < 1)$$

$$\therefore k_1 s = \frac{b\sigma_y}{t_w\sigma}k_2t_f \quad (8)$$

$$M_p = bk_2t_f\sigma_y\left(s + t_f - \frac{k_2t_f}{2} - \frac{k_1s}{2}\right) \quad (9)$$

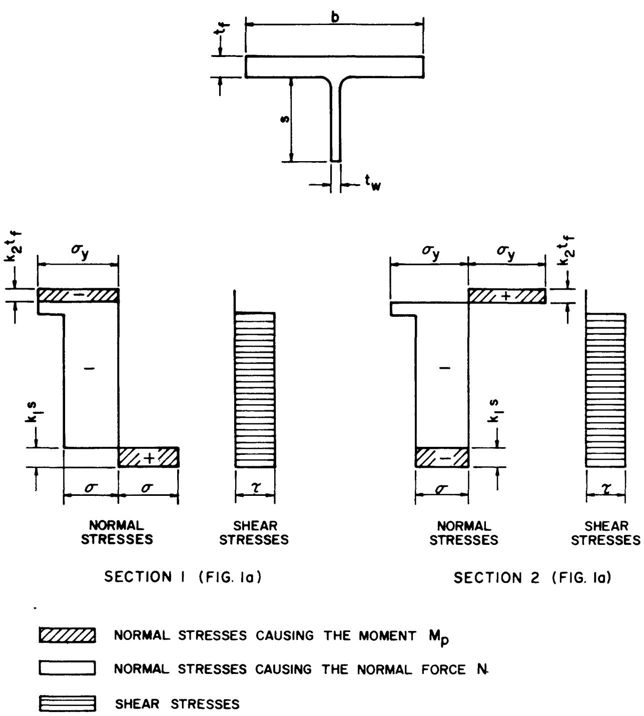


Fig. 4. Stress distribution at sections 1 and 2 (Fig. 1a); low shear

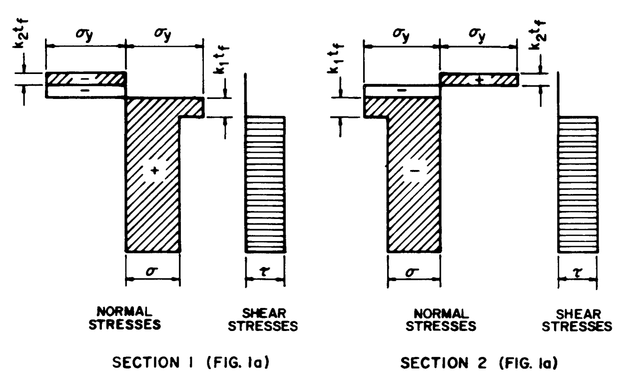


Fig. 5. Stress distribution at sections 1 and 2 (Fig. 1a); high shear

Substituting Eqs. (6) and (8) into Eq. (9) and rearranging, a value of k_2t is obtained as follows:

$$k_2t_f = \frac{C_1}{b\sigma_y\,(s+t)}\sqrt{1-\frac{aV}{C_1}} \tag{10}$$

where

$$C_1 = \frac{bt_w(s+t_f)^2\sigma\sigma_y}{b\sigma_y + t_w\sigma} \tag{11}$$

The value of the normal force, N, is equal to:

$$N = bt_f\sigma_y + st_w\sigma - 2C_1\sqrt{1-\frac{aV}{C_1}} \tag{12}$$

The position of the normal force, N for low shearing force (where $k_1s < s$) is at a distance y_o from the outside of the flange, where:

$$y_o = k_2t_f + \frac{b\sigma_y\dfrac{A_1{}^2}{2} + t_w\sigma A_2\left(\dfrac{A_2}{2}+A_1\right)}{b\sigma_yA_1 + t_w\sigma A_2} \tag{13}$$

$$A_1 = t_f - k_2t_f$$

$$A_2 = s - \frac{k_2t_fb\sigma_y}{t_w\sigma} \tag{14}$$

and the lever arm y_{ct} is equal to

$$y_{ct} = d - 2y_o \tag{15}$$

Therefore, the moment at the mid-length of the hole, M is found from

$$M = N(d - 2y_o) \tag{16}$$

Case of High Shearing Force—For high shearing force the distributions of the stresses are as shown in Fig. 5. The distributions of normal stresses causing M_p are at a distance k_2t_f from the outside of the flange and k_1t_f from the inside of the flange. Considering the equilibrium of the tee-section:

$$bk_2t_f\sigma_y = st_w\sigma + bk_1t_f\sigma_y \tag{17}$$

from which

$$k_1t_f = k_2t_f - \left(\frac{t_w\sigma}{b\sigma_y}\right)s \tag{18}$$

Also,

$$M_p = M_{po} - \frac{b\sigma_y}{4}(t_f - k_2t_f - k_1t_f)^2 \tag{19a}$$

where M_{po} = plastic hinge moment at the corner of the hole when $N = 0$.

$$M_p = b\sigma_yC_2\left(\frac{C_2}{2} + C_3\right) \tag{19b}$$

$$C_2 = \frac{bt_f\sigma_y + st_w\sigma}{2b\sigma_y} \tag{19c}$$

$$C_3 = \frac{0.5b(t_f - C_2)^2\sigma_y + st_w\left(t_f + \dfrac{s}{2} - C_2\right)\sigma}{b(t_f - C_2)\sigma_y + st_w\sigma} \tag{19d}$$

Substituting Eqs. (6), (18), and (19b) into Eq. (19a), the following equation for k_2t_f is obtained:

$$k_2t_f = C_2 - \sqrt{C_4} \tag{20}$$

in which,

$$C_4 = C_2\left(\frac{C_2}{2} + C_3\right) - \frac{aV}{2b\sigma_y} \tag{21}$$

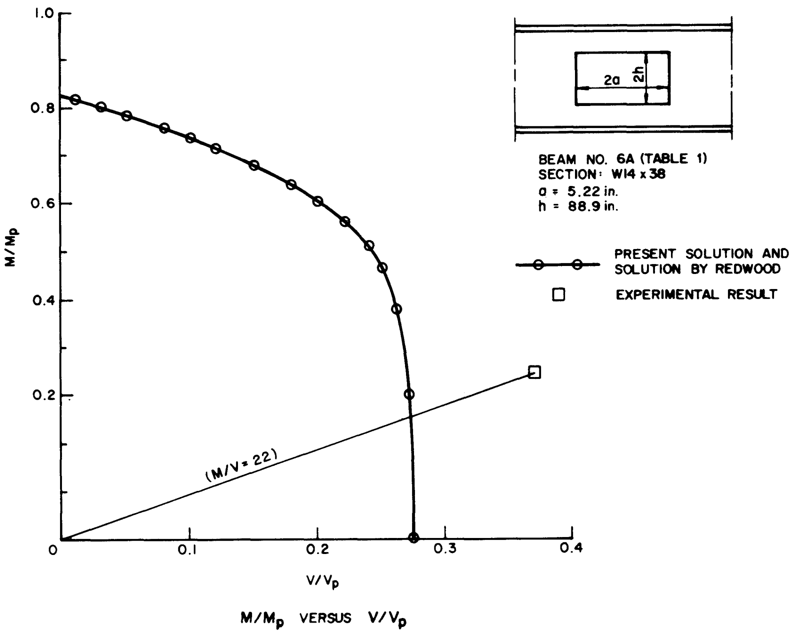


Fig. 6. Interaction diagram, beam 6A

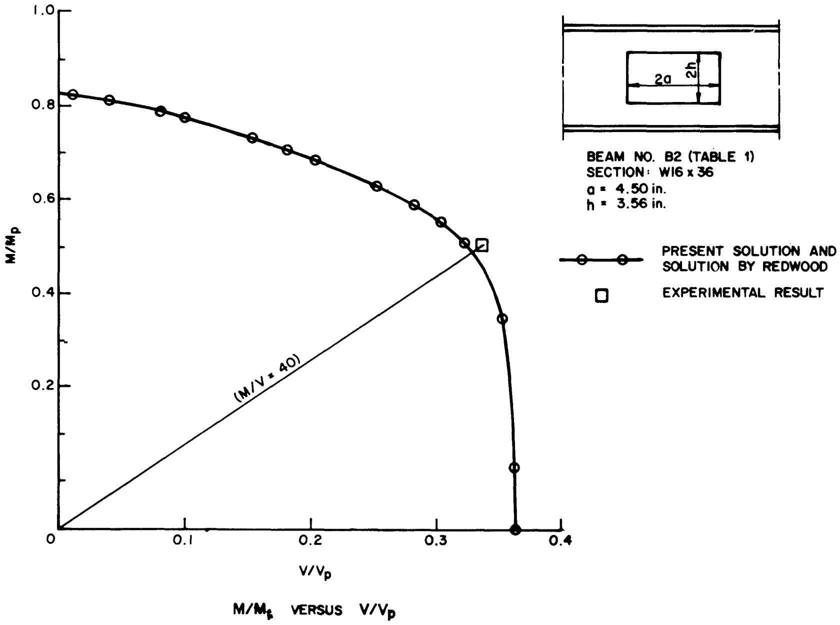


Fig. 7. Interaction diagram, beam B2

The value of the normal force N is equal to:

$$N = bt_f\sigma_y + st_w\sigma - 2bk_2t_f\sigma_y \qquad (22)$$

and in this case the normal force acts at the plastic centroidal axis, the position of which is at a distance C_2 from the outside of the flange. Consequently the bending moment, M, at the mid-length of the hole is equal to:

$$M = N(d - 2C_2) \qquad (23)$$

To obtain a solution, i.e., the interaction diagram relating V with M, a value of V is assumed, and by substituting in Eqs. (7) to (16) (case of low shear), one point in the interaction diagram is obtained providing that the value of $k_1s \leq s$.

If $k_1s > s$, the value assumed for the shearing force, V, and any other greater value will bring the problem to the case of high shear. In such case, the value of M is obtained by using Eqs. (17) to (23).

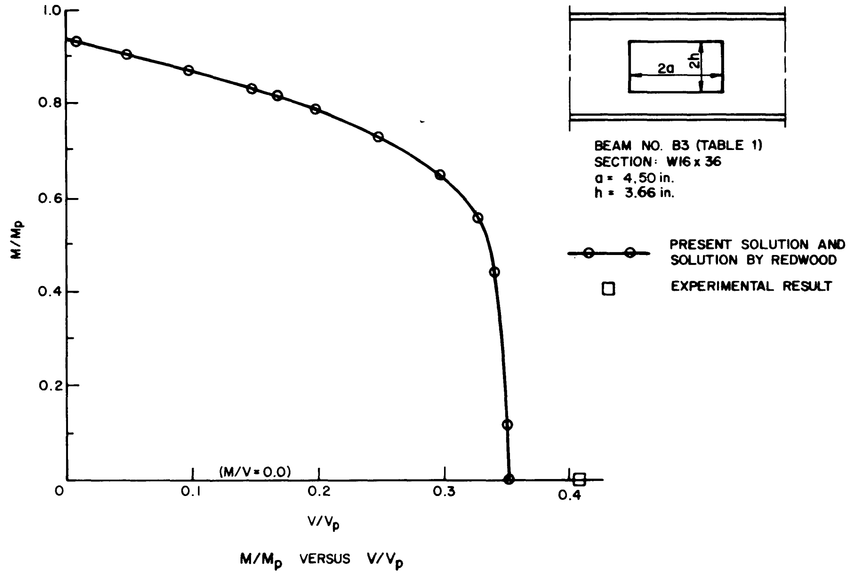


Fig. 8. Interaction diagram, beam B3

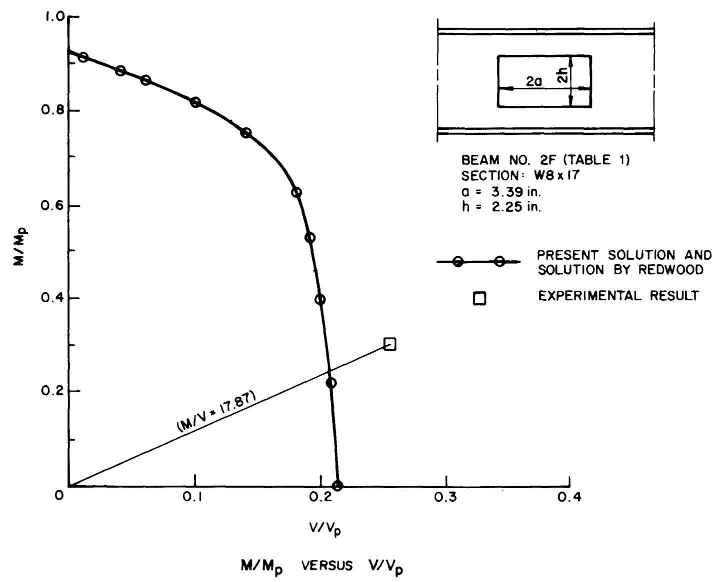


Fig. 9. Interaction diagram, beam 2F

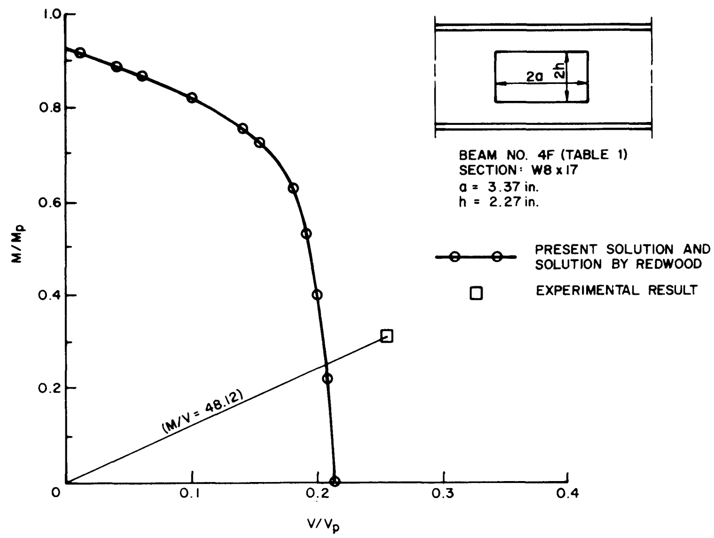
COMPARISON WITH PREVIOUS EXPERIMENTAL AND THEORETICAL WORKS

Typical interaction diagrams for a number of steel beams with mid-depth web holes are shown in Figs. 6 to 10. The beam dimensions and the sizes of the holes are chosen in such a way as to be compared with available experimental data (Refs. 1, 3, and 7). A summary of the dimensions of beams, sizes of holes and properties of material is shown in Table 1.

Comparison between the proposed method and the experimental data available shows that the method appears satisfactory for low shear values.

For the case of high shear, the proposed method somewhat on the conservative side. The discrepancies b tween the theoretical and experimental results are main due to strain-hardening of the steel. The effect of tl strain-hardening has been analyzed in a previous resear report.[1] It has been shown that its effect is significant f the case of high shear in which the values of the plasl hinge moments at the corners of the hole are high.

The theoretical results from the proposed analysis a also compared with a previous accurate method present by Redwood.[6] The results of both methods are identic However, the method given by Redwood requires consi



M/Mp VERSUS V/Vp

Fig. 10. Interaction diagram, beam 4F

Table 1

Beam (1)	Section Nominal Size (2)	b (in.) (3)	d (in.) (4)	t_f (in.) (5)	t_w (in.) (6)	a (in.) (7)	h (in.) (8)	Flange σ_y (ksi) (9)	Web σ_y (ksi) (10)	M/V (in.) (11)	$\frac{V^*}{V_p}$ (12)	$\frac{V^{**}}{V_p}$ (13)	$\frac{12-13}{13}$ × 100 (14)	Ref. (15)
6A	W14 × 38	6.68	14.22	0.49	0.31	5.22	3.50	44.6	55.8	0.87	0.346	0.276	+25	1
B2	W16 × 36	7.12	15.81	0.42	0.30	4.50	3.56	35.6	40.2	1.57	0.335	0.328	+2	3
B3	W16 × 36	7.09	15.88	0.43	0.31	4.50	3.66	33.7	37.7	0.00	0.405	0.353	+14.7	3
2F	W8 × 17	5.25	8.06	0.30	0.24	3.39	2.25	43.5	45.6	0.70	0.256	0.21	+21.9	7
4F	W8 × 17	5.25	8.12	0.32	0.25	3.37	2.27	46.1	54.1	1.89	0.182	0.176	3.4	7

* Experimental.
** Analytical.

erable computer time, whereas the authors' method can be solved with hand calculations or use of a small hand calculator.

CONCLUSIONS

A method of deriving an interaction relationship between bending moments and shearing forces at the mid-length of an unreinforced hole in a steel beam is presented. The method is based on the formation of four plastic hinges at the corner of the hole and gives a lower bound solution, since equilibrium and yielding are satisfied everywhere. The present method is simpler than a previous theoretical method reported by Redwood[6] and gives the same results.

The authors' proposed method can be extended to analyze steel beams with single or multiple holes, as in the case of castellated beams.

NOTATIONS

The following notations are used in this paper, unless otherwise stated:

a = half-length of hole
b = flange width
d = total depth of beam
h = half-depth of hole
k_1, k_2 = coefficients used in stress diagrams (see Figs. 4 and 5)
L = distance from the centerline of the hole to the simple support (Fig. 1)
M = bending moment at the center of the hole
M_p = plastic moment at the corners of the hole
M_{po} = plastic moment at the corner of the hole when $N = 0$
N = normal force acting on the tee-section (Fig. 3)

s = distance from top of web to bottom of the flange
t_f = flange thickness
t_w = web thickness
V = shearing force at the center of the hole
y_{ct} = lever arm
y_o = position of the normal force N from the outside of the flange (case of low shear)
Z_p = plastic modulus of section
σ_y = yield stress
σ = normal stress in the web of the tee-section at yielding
τ = shear stress in the web of the tee-section at yielding

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