Stability of Metal Structures—A World View

STRUCTURAL STABILITY RESEARCH COUNCIL
EUROPEAN CONVENTION FOR CONSTRUCTIONAL STEELWORK
COLUMN RESEARCH COMMITTEE OF JAPAN
COUNCIL OF MUTUAL ECONOMIC ASSISTANCE

PART B. APPROACHES AND DESIGN PROCEDURES

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This comprehensive comparison of the current state of the art in research and design for structural stability, as viewed in four major regions of the world, is being published in serial form in the AISC Engineering Journal. It is expected eventually to be published in book form. This installment contains Chapters 5, 6 and 7 of Part B. The first installment, published in the 3rd Quarter 1981 issue, contained Part A and Chapter 1 of Part B, along with a List of Abbreviations and a Glossary of Terms covering the complete report. The second installment, published in the 4th Quarter 1981 issue, contained Chapters 2, 3 and 4.

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Chapter 5. Beam-Columns

a. REGIONAL RECOMMENDATIONS

JAPAN

(There are no regional recommendations in Japan.)

NORTH AMERICA

(Key Document: SSRC Guide[1])

NONLINEAR INTERACTION

The following nonlinear equation was suggested by SSRC:

$$\left(\frac{M_x}{M_{ucx}}\right)^{exponent} + \left(\frac{M_y}{M_{ucy}}\right)^{exponent} \leq 1.0$$

By deriving the correct exponents, an interaction curve could be defined which would fit the actual strength curve of a biaxially loaded beam-column of a particular cross section, such as a wide-flange shape.

Chen and his team[172] determined empirical interaction curves for wide-flange beam-columns at various levels of axial stress. Exponent expressions giving a good fit to these curves have been developed.

At a braced location, the following equation should be satisfied:

$$\left(\frac{M_x}{M_{pcx}}\right)^{\alpha} + \left(\frac{M_y}{M_{pcy}}\right)^{\alpha} \leq 1.0$$

For wide-flange shapes having a flange-width-to-web-depth ratio from 0.5 to 1.0:

$$\alpha = 1.6 - \frac{P/P_y}{2 \ln (P/P_y)}$$

where ln indicates the natural logarithm.

For a square box column, the exponent expression

$$\alpha = 1.7 - \frac{P/P_y}{\ln (P/P_y)}$$

is suggested by Chen.[173]

To check stability between braced points, the following equation should be satisfied:

$$\left(\frac{C_{mx}M_x}{M_{ucx}}\right)^{\beta} + \left(\frac{C_{my}M_y}{M_{ucy}}\right)^{\beta} \leq 1.0$$

in which

C_{mx}, C_{my} = equivalent moment factors used in the AISC Specification interaction formula

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4$$

where M_1/M_2 is the ratio of the smaller to larger moments at the ends of that portion of the member unbraced in the plane of bending under consideration. M_1/M_2 is positive when the member is bent in single curvature and negative when it is bent in reverse curvature.

M_x, M_y = the greater of the moments applied at one or the other end of the beam-column

$\beta = 0.4 + P/P_y + B/D \geq 1.0$ when $B/D \geq 0.3$

$= 1.0$ when $B/D < 0.3$

B = flange width of W or I section

D = depth of W or I section

For the case of a square box column, the exponent expression

$$\beta = 1.3 + \frac{1000(P/P_y)}{(L/r)^2} \geq 1.4$$

is suggested by Chen.[173]

In the above equations, M_{ucx} and M_{ucy} may be determined as follows:

At braced points:

$$M_{ucx} = M_{pcx} = 1.18M_{px}\,[1 - (P/P_y)] \leq M_{px}$$
$$M_{ucy} = M_{pcy} = 1.19M_{py}\,[1 - (P/P_y)^2] \leq M_{py}$$

Considering stability:

$$M_{ucx} = M_{ux}[1 - (P/P_u)][1 - (P/P_{ex})]$$
$$M_{ucy} = M_{py}[1 - (P/P_u)][1 - (P/P_{ey})]$$

in the above equations.

M_{pcx}, M_{pcy} = plastic moment capacity about the x- or y-axis, respectively, reduced for the presence of axial load

M_{px}, M_{py} = plastic moment capacities

M_{ux} = plastic moment capacity about the x-axis, reduced for the presence of lateral torsional buckling, if necessary

$$= M_m$$
$$= M_{px}\left[1.07 - \frac{(L/r_y)\sqrt{F_y}}{3160}\right] \leq M_{px}$$

F_y = yield stress in simple tension test

P_{ex}, P_{ey} = Euler buckling loads

P_u = ultimate load capacity of axially loaded column

P_y = axial load at full yield condition

General Comments—The advantages of the design interaction equation suggested by Chen et al.[172] are that both strength and stability are covered by the same basic equation and the equation has universal applicability, for if the exponent for the particular section being designed has not been determined, it may conservatively be assumed to be unity.

The assumptions made in all the design procedures are:

1. The design is for an isolated member, not for a part of a framework in which the forces redistribute as ultimate load is approached.
2. The design method is valid for beam-columns in which the axial load and end moments are known, being determined by a first or second order elastic analysis as appropriate to braced or unbraced frames. The adoption of biaxial plastic design is not suggested.
3. The sections are compact, i.e., premature failure will not occur due to local buckling of flanges or webs prior to the member attaining its ultimate strength.
4. The beam-column is initially twisted and out-of-straight, contains residual stresses, and the material is elastic-perfectly plastic.

In using these methods, the determination of axial load capacity of the beam-column should be as realistic as possible, such as the use of multiple column curves described in the SSRC Guide,[1] rather than the unmodified tangent modulus critical load given by the SSRC column strength equation. The latter is based on the concept of buckling of a perfectly straight column using the tangent modulus dependent on residual stresses, whereas the fundamental analysis of beam-column is based on the load-deflection approach to the determination of ultimate strength considering geometrical and material imperfections.

In using these expressions, we have taken advantage of the redistribution of stress which occurs after the initiation of yielding. Consideration should thus be given to the serviceability of the structure once yielding has occurred under service load condition.

WEST EUROPE

(Key Document: ECCS Recommendations[24])

BEAM-COLUMNS IN BRACED FRAMES

For beam-columns in braced frames the general design formulae are:

$$\sigma + \frac{\mu_x}{\mu_x - 1}\left(\frac{\nu\beta_x M_x + N e_x{}^t}{W_x}\right) + \frac{\mu_y}{\mu_y - 1}\,\frac{\beta_y M_y}{W_y} \leq \sigma_r \tag{WE5.1}$$

$$\sigma + \nu\,\frac{\mu_x}{\mu_x - 1}\,\frac{\beta_x M_x}{W_x} + \frac{\mu_y}{\mu_y - 1}\left(\frac{\beta_y M_y + N e_y{}^t}{W_y}\right) \leq \sigma_r \tag{WE5.2}$$

$$\sigma + \frac{M_x}{W_x} + \frac{M_y}{W_y} \leq \sigma_r \tag{WE5.3}$$

where e^t is a representative parameter of imperfection. For plastic design, the same type of formula is used except that W_x and W_y are replaced by the plastic section modulus W_{pl}. As a provision against the formation of a plastic hinge at one end (or two), the following condition must be satisfied:

$$(M_x/M_{red,x})^\alpha + (M_y/M_{red,y})^\alpha < 1 \tag{WE5.4}$$

In general $\alpha = 1.0$. For compact H or I shapes with width-to-depth ratios not less than 0.30, a more refined value can be used:

$$\alpha = 1.6 - (N/N_{pl})/[2\ln(N/N_{pl})] \tag{WE5.5}$$

Formulae are also given for M_{red} and for $M_{x,y}$ as a function of N. For elastic design, the effective slenderness of the column may be used. For plastic design, λ_{kx} is determined for the actual column length. The slenderness about the weak axis λ_{ky} can be replaced by the effective slenderness, if the weak axis beams remain fully elastic. For this purpose design charts are given.

The imperfection parameter e^t can be approximated using the following table:

ECCS curve	e^t/h
a	1/500
b	1/250
c	1/200

where h = depth of the column.

BEAM-COLUMNS IN UNBRACED FRAMES

For elastic design, no general formulae are given in the recommendations. Instead it is stated that a full second-order analysis should be carried out using a proper initial imperfection (see Chapter 6.)

COMMENTARY

The main difference between the ECCS interaction formula for beam-columns and the conventional type of interaction formula is the introduction of a representative parameter of imperfection e^t. This factor is a function of the ultimate load N_k.

ALUMINUM STRUCTURES

The European Recommendations for Aluminum Structures have not adopted the e^t approach for columns in braced frames. The conventional σ_k formula is used instead. For columns in unbraced frames, a second order analysis is required. For welded members, the cross-sectional data must be evaluated by taking into account the heat-affected zones.

EAST EUROPE

(Key Documents: INCERC Design Guidelines for Steel; CMEA Recommendations for Aluminum Structures)

STEEL STRUCTURES

Design Formulae (Fig. EE5.1)

1. Disregarding lateral-torsional buckling:

$$\frac{N}{A\sigma_{ux}} + \frac{\mu_x}{\mu_x - 1} \frac{C_x M_{max}}{W_x \sigma_y/\gamma_m} \leq 1 \qquad \text{(EE5.1)}$$

2. Lateral-torsional buckling:

$$\frac{N}{A\sigma_{uy}} + \frac{\mu_x}{\mu_x - 1} \frac{C_x M_{max}}{W_x \sigma_{ul}} \leq 1 \qquad \text{(EE5.2)}$$

3. Biaxial bending:

$$\frac{N}{A\sigma_{umin}} + \frac{\mu_x}{\mu_x - 1} \frac{C_x M_{xmax}}{W_x \sigma_{ul}} + \frac{\mu_y}{\mu_y {}^{-1}} \frac{C_y M_{ymax}}{W_y \sigma_y/\gamma_m} \leq 1 \qquad \text{(EE5.3)}$$

where

σ_{ux} = limit stress for strong-axis column buckling
σ_{uy} = limit stress for weak-axis column buckling
σ_{ul} = limit stress for lateral buckling

$$\mu_x = (\pi^2 E/\lambda_x{}^2)(A/N) \qquad \text{(EE5.4a)}$$

$$\mu_y = (\pi^2 E/\lambda_y{}^2)(A/N) \qquad \text{(EE5.4b)}$$

$$C = \sqrt{0.3\,(1+\beta^2) + 0.4\beta} \qquad \text{(EE5.5)}$$

In case of plastic design W is replaced by the plastic section modulus.

ALUMINUM (see Ref. 174)

1, 3. Biaxial bending, disregarding lateral-torsional buckling:

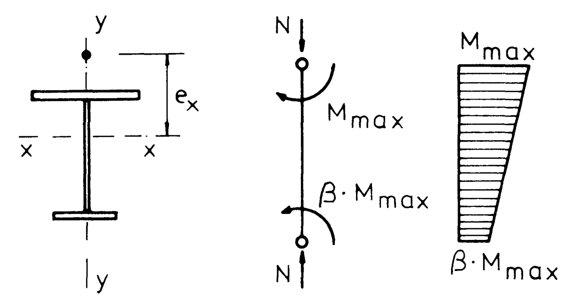


Figure EE5.1

$$\frac{N}{A} + \frac{\bar{\lambda}_c{}^2}{\bar{\lambda}_c{}^2 - \bar{\lambda}_x{}^2} \frac{M_x}{W_x} + \frac{\bar{\lambda}_c{}^2}{\bar{\lambda}_c{}^2 - \bar{\lambda}_y{}^2} \frac{M_y}{W_y} \leq \frac{\sigma_y}{\gamma_m} \qquad \text{(EE5.6)}$$

where

$$\bar{\lambda}_x = \frac{1}{\pi}\sqrt{\frac{E}{\sigma_{0.2}}} \cdot \lambda_x$$

$$\bar{\lambda}_y = \frac{1}{\pi}\sqrt{\frac{E}{\sigma_{0.2}}} \cdot \lambda_y$$

$$\bar{\lambda}_c = \frac{1}{\pi}\sqrt{\frac{E}{\sigma_{0.2}}} \cdot \lambda_c$$

λ_c = the slenderness ratio of a fictitious column, for which $\sigma_u\,(\lambda_c) = N/A$

The amplification factors (replacing those adopted in case of steel structures) are intended to include the effect of non-linearity of the stress-strain diagram.

$$\frac{N}{A} + \frac{M_x}{W_x} \leq \sigma_u(\lambda'_{eff}) \qquad \text{(EE5.7)}$$

Explanation of $\sigma_u(\lambda'_{eff})$ is given in the section on "Specifications and Codes" for Hungary.

b. SPECIFICATIONS AND CODES

JAPAN

AIJ STANDARD FOR STEEL STRUCTURES

1. Allowable Stress Design:

Stresses of beam-columns subjected to combined axial compression and bending must satisfy Eqs. (J5.1) and (J5.2).

$$\frac{\sigma_c}{f_c} + \frac{{}_c\sigma_b}{f_b} \leq 1 \qquad \text{(J5.1)}$$

$$\frac{{}_t\sigma_b - \sigma_c}{f_t} \leq 1 \qquad \text{(J5.2)}$$

where

f_c = allowable compression stress
f_b = allowable bending stress
f_t = allowable tensile stress
= $\sigma_y/1.5$ for long-term loading
= σ_y for short-term loading
σ_c = mean compressive axial stress
${}_c\sigma_b$ = compressive bending stress
${}_t\sigma_b$ = tensile bending stress

Stresses of beam-columns subjected to combined axial tension and bending must satisfy Eqs. (J5.3) and (J5.4).

$$\frac{\sigma_t + {}_t\sigma_b}{f_t} \leq 1 \qquad \text{(J5.3)}$$

$$\frac{{}_c\sigma_b - \sigma_t}{f_b} \leq 1 \qquad \text{(J5.4)}$$

where σ_t = mean tensile axial stress

2. Plastic Design:

Moment capacity of beam-columns subjected to combined axial compression and bending is obtained as

$$M_1 = \frac{M_{cro}}{C_m}\left(1 - \frac{N}{N_{cr}}\right)\left(1 - \frac{N}{N_e}\right) \leq M_{pc} \qquad \text{(J5.5)}$$

where

N = compressive axial force
N_e = Euler load in plane of bending
N_{cr} = ultimate buckling strength under pure compression
M_{pc} = fully plastic moment influenced by axial thrust
M_{cro} = moment capacity under uniform bending
C_m = $0.6 + 0.4(M_2/M_1) \geq 0.4$ for H-section members subjected to strong-axis bending
= $1 - 0.5[1 - (M_2/M_1)]\sqrt{N/N_e} \geq 0.25$ for members unaffected by lateral-torsional buckling.
M_1, M_2 = end moments, the larger of which is M_1. The ratio (M_2/M_1) is positive under single curvature bending and negative under double curvature bending.

Commentary—Eq. (J5.1) is an empirical formula extended from simple elastic limit criterion in which yield point stress is replaced by f_c and f_b. Elastic limit strength of the beam-column deformed in one plane under couple of equal end moments and axial compression is obtained as

$$M_1 = M_y\,[1 - (N/N_y)][1 - (N/N_e)]$$

where

$M_y = Z\sigma_y$
Z = section modulus
N_y = yield axial force

Eq. (J5.5) is also an empirical formula extended from the above equation.

JRA SPECIFICATIONS FOR HIGHWAY BRIDGES

Stresses of beam-columns which are designed on allowable stress design basis must satisfy the following criteria:

Under axial compression and bending:

$$\frac{\sigma_c}{\sigma_{ca}} + \frac{\sigma_{bc}}{\sigma_{ba}} \leq 1.0 \qquad \text{(J5.6)}$$

Under axial tension and bending:

$$\left.\begin{aligned} \sigma_t + \sigma_{bt} \leq \sigma_{ta} \\ -\sigma_t + \sigma_{bc} \leq \sigma_{ba} \end{aligned}\right] \qquad (J5.7)$$

where

σ_{ca} = allowable compressive stress about minor axis column buckling
σ_{ba} = allowable bending stress
σ_{ta} = allowable tensile stress
σ_c = mean compressive axial stress
σ_{bt} = tensile bending stress
σ_{bc} = compressive bending stress
σ_t = mean tensile axial stress

When non-uniform bending is applied, σ_{bc} can be decreased to $C_m \sigma_{bc}$. C_m is a reduction factor shown in Eq. (J5.5).

Commentary—The constitution of foregoing formulae is same as those of AIJ Specifications, except for specified values of allowable stress in those specifications.

NORTH AMERICA

GENERAL

Traditionally, beam-columns in North America have been designed based on linear interaction equations relating the effects of axial force P and bending moments M_x and M_y. For the case of uniaxial bending combined with axial compression, this procedure is expected to be the prevalent design method for some time to come. However, for the general case of biaxial bending, modifications of this to nonlinear interaction equations are forthcoming. In fact, the Canadian code S16.1-1974 allows for the use of certain refined nonlinear interaction equations in its Appendix.

In the conventional North American practice, using working stress method of design, a first-order linear elastic analysis is performed at the working load, neglecting any effect of instability, and the resulting bending moment and axial force distributions are then used to proportion the members.

LINEAR INTERACTION EQUATIONS

The expression governing the working stress design of beam-columns subject to biaxial bending has the form

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}\left(1 - \dfrac{f_a}{F'_{ex}}\right)} + \frac{f_{by}}{F_{by}\left(1 - \dfrac{f_a}{F'_{ey}}\right)} \leq 1.0$$

where

f_a = computed axial stresses
f_{bx}, f_{by} = computed compressive bending stresses at the point under consideration
F_a = axial stress that would be permitted for the column if axial force alone existed
F_{bx}, F_{by} = compressive bending stresses that would be permitted for the beam if bending moment alone existed. The stresses are determined considering all failure modes including plastic yielding, lateral torsional buckling, and local buckling.
F'_{ex}, F'_{ey} = (12/23) times the Euler buckling stresses in the plane of bending

The term F_a is based on the ultimate load-capacity of an axially loaded column P_u modified by a variable safety factor which increases from 5/3 = 1.67 to 23/12 = 1.92 as the slenderness ratio increases. In the current AISC Specification (1978) the value of P_u is determined from the basic SSRC Column Curve. The CSA Standard S16.1-1974 now adopts Column Strength Curve 2 as proposed in the current Third Edition of the SSRC Guide,[1] instead of previous standards using the basic SSRC Column Curve. The basic Column Curve is based on tangent-modulus theory which, while accounting for residual stresses directly, assumed initially perfectly straight columns. A variable factor of safety (1.67 to 1.92) was then applied by the AISC to the column curves to account for the increasing sensitivity of long columns to variations in the effective length factor K and the effects of initial crookedness. SSRC Column Strength Curve 2, on the other hand, is an empirical curve based on the maximum strength curves of a range of column shapes and types. Each individual curve was based on an actual measured residual stress distribution and an assumed initial out-of-straightness at mid-height of $L/1000$. A constant ϕ factor (factor of safety) has been applied by CSA to Column Strength Curve 2 in S16.1-1974. The term F'_e is based on the Euler buckling load modified by a constant value of safety factor (23/12) in the current AISC Specification (1978). The term F_{bx} is based on a 10 percent increase over the initial yield moment to recognize that the fully plastic moment capacity of compact sections is greater than the nominal initial yield moment, that is, the permissible strong axis bending stress F_{bx} is $1.1(0.6F_y) = 0.66F_y$. The term F_{by} is treated similarly to F_{bx}, but in wide-flange sections the shape factor about the weak axis is approximately 1.55 versus 1.12 about the strong axis. Therefore, when the moment about the weak axis of the wide-flange shape is significant, or for beam-columns subjected to only weak axis bending, there may be a marked difference between design by the AISC working strength method (Part 1 of the AISC Specification) and by the counterpart of the above expression for the ultimate strength method as given by the SSRC Guide:

$$\frac{P}{P_u}+\frac{M_x}{M_{ux}\left(1-\frac{P}{P_{ex}}\right)}+\frac{M_y}{M_{uy}\left(1-\frac{P}{P_{ey}}\right)}\le 1.0$$

where

M_{ux}, M_{uy} = ultimate bending moments of a beam about the x- and y-axis, respectively, reduced for the possible presence of lateral torsional buckling, if necessary
P_{ex}, P_{ey} = Euler buckling loads
P_u = ultimate load of axially loaded column

For wide-flange shapes, M_{uy} would be the plastic moment capacity of the section, M_{py}. M_{ux} would be the plastic moment capacity of the beam about the major axis reduced as may be necessitated by lateral-torsional buckling considerations $M_{ux} = M_m \le M_{px}$. Therefore, the conservatism mentioned previously with respect to the different shape factors about the strong and weak axes is eliminated in this ultimate strength expression, since correct ultimate moment capacity about each respective axis is used.

The two interaction expressions, stated above, are both straight line interaction of moments about the orthogonal axes, i.e., for a given ratio of axial load to axial capacity P/P_u, the variation in moment about the two orthogonal axes is linear. This implies that a simple linear interaction equation may be written in the following form:

$$\frac{M_x}{M_{ucx}}+\frac{M_y}{M_{ucy}}\le 1.0$$

where M_{ucx} and M_{ucy} are the ultimate symmetrical single curvature moment capacities of an axially loaded beam-column, about the x- or y-axis, respectively, when there is zero moment about the other axis. These denominator terms may be determined from the expressions given in the counterpart of the nonlinear interaction equations described above. For H sections, CSA S16.1-1974 adopts the SSRC Recommendations described earlier under "Regional Recommendations".

For square sections (tubes), CSA S16.1-1974 makes the following simple modification from the above linear interaction for H sections:

$$C\left(\frac{M_x}{M_{ucx}}+\frac{M_y}{M_{ucy}}\right)\le 1.0$$

where C is defined in terms of biaxial eccentricities e_x and e_y as

$$C=\frac{\sqrt{e_x{}^2+e_y{}^2}}{e_x+e_y}=\frac{\sqrt{M_x{}^2+M_y{}^2}}{M_x+M_y}$$

together with

$$\frac{M_x}{M_{px}}+0.5\frac{M_y}{M_{py}}\le 1.0$$

and

$$\frac{P}{P_y}+0.85\left(\frac{M_x}{M_{px}}+0.5\frac{M_y}{M_{py}}\right)\le 1.0$$

WEST EUROPE

FEDERAL REPUBLIC OF GERMANY

The revision of DIN 4114 is based on limit states design. The state of equilibrium of a column (or frame) is checked according to an exact ultimate strength analysis, a second order plastic hinge theory, or a second order elastic theory. A set of representative geometrical imperfections is given, which have been derived by the ECCS buckling curves.

ECCS buckling curve	Initial deflection	Initial out-of plumb
a	1/500	1/250
b	1/250	1/125
c	1/200	1/100
d	1/140	1/70

For a second-order elastic theory analysis, the imperfections may be reduced to 75%. For second-order plastic hinge theory, additional conditions apply. In unbraced systems the initial deflections and the initial out-of plumb must be considered simultaneously only if

$$1/\sqrt{N/EI} > 1.6 \qquad \text{(WE5.6)}$$

No interaction formulae are given in the German draft code. Design aids, however, have been prepared.

First-order theory may be used if

$$1/\sqrt{N/EI} \le 1.0 \qquad \text{(WE5.7)}$$

If lateral-torsional buckling has to be considered, the following interaction formula has to be satisfied

$$N/N_{ky} + (M_x/M_{Dx})^{\beta} \le 1 \qquad \text{(WE5.8)}$$

The exponent β is a function of the bending moment distribution; for instance, if the moment is constant along the length of the column $\beta = 1.1$. Members subjected to biaxial bending and compression, but not susceptible to lateral-torsional buckling, may be considered according to monoaxial design by considering both bending directions separately. If, however, lateral-torsional buckling has to be considered, an additional interaction formula has to be satisfied.

SWITZERLAND

The new Swiss code for steel structures (1979) has adopted the type of interaction formula for beam-columns given in the ECCS Recommendations.[24]

Some minor modifications have been introduced. Instead of the "exact" value of the parameter of imperfection e^t, a constant value 1/300 (elastic design) and 1/250 (plastic design) has been adopted. The design formulae in the code apply only when lateral-torsional buckling does not occur.

For elastic design:

$$\frac{N}{A} + \frac{\mu}{\mu - 1} \frac{\beta M_{max} + Ne^t}{W} \leq \sigma_r \qquad \text{(WE5.9)}$$

For plastic design:

$$\frac{N}{A} + \frac{\mu}{\mu - 1} \frac{\beta M_{max} + Ne^t}{W_{pl}\sigma_r} \leq 1 \qquad \text{(WE5.10)}$$

N and M_{max} are the largest forces determined according to first-order theory.

Commentary—The Swiss member of the ECCS stability committee has pointed out that the ECCS rules for bending about the strong axis ($M_y = 0$) are too conservative for columns with a high torsional stiffness and suggested revision of these regulations.

BELGIUM

The Belgian code (1977) has introduced the conventional interaction formula as established by Campus and Massonnet.[175] This formula is of the type

$$\frac{N}{N_k} + \frac{\mu}{\mu - 1} \frac{\beta M}{M_D} \leq 1.0 \qquad \text{(WE5.11)}$$

The same type of formula is used for braced and unbraced frames.

THE NETHERLANDS

The Dutch code for steel structures (1972) adopted the conventional interaction formula. For braced frames, the elastic effective length may be used; for unbraced frames, the elastic effective length has to be used.

FRANCE

The French "Recommendation" for the plastic design of steel structures (1975) uses the conventional interaction formula.

NORWAY

The Norwegian code (1972) is presently being revised. Publication is planned for 1979/1980. It can be expected that the ECCS approach will be introduced. The existing code uses the conventional interaction formula.

AUSTRIA

The interaction formula given in the Austrian code (rev. 1972) is of the same type as in the existing DIN 4114. A formula for the equivalent bending moment is also given.

SPAIN

The Spanish code, Norma MV-103 gives an interaction formula derived from DIN 4114.

YUGOSLAVIA

The new Yugoslav standard JUS.U.E7 096 (1978) has adopted the ECCS interaction formula for beam-columns.

GREAT BRITAIN

The British code BS 449, uses a conventional interaction curve.

EAST EUROPE

SOVIET UNION

1. Disregarding lateral-torsional buckling:

$$N/A \leq \sigma_{u,ecc} \qquad \text{(EE5.8)}$$

where $\sigma_{u,ecc}$ is the average stress at peak load $N_{u,ecc}$ (Fig. EE5.2), computed according to the stress-strain diagram in Fig. EE1.1 and tabulated as a function of λ_y and $e_x(A/W_x)$ for six groups of cross sections.

2. Lateral-torsional buckling:

   a. In the elastic range ($\lambda_y > \pi\sqrt{E/\sigma_y}$):

$$N/A \leq (1/\rho^2)\sigma_{uy} \qquad \text{(EE5.9)}$$

where $(1/\rho^2)\sigma_{uy}$ is the critical average stress computed by the interaction formula

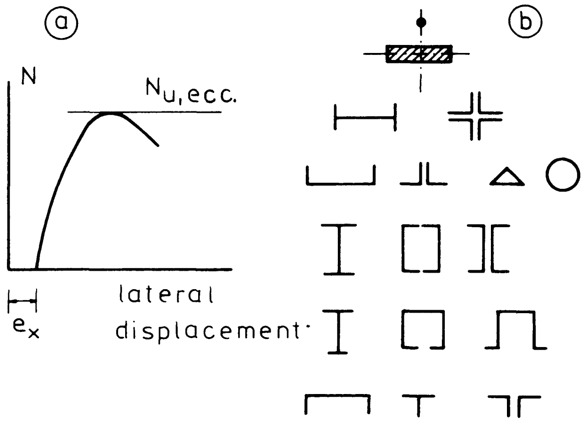


Figure EE5.2

$$\left.\begin{array}{c}\left(\dfrac{N \cdot e}{M_{cr}}\right)^2 + \left(1 - \dfrac{N}{N_{Ey}}\right)\left(1 - \dfrac{N}{N_w}\right) = 0 \\ \dfrac{1}{\rho^2}\dfrac{N}{N_{Ey}}\end{array}\right] \quad \text{(EE5.10)}$$

where M_{cr}, N_{Ey} and N_w are the critical bending moment, Euler-load and critical axial load causing torsional buckling, respectively.

b. In the inelastic range ($\lambda_y \leq \pi\sqrt{E/\sigma_y}$):

$$\frac{N}{A} + \alpha_0 \frac{M}{W_x} \leq \sigma_{uy} \qquad \text{(EE5.11)}$$

with correction factors $\alpha_o = 0.7 \sim 0.9$ (for I sections).

3. Biaxial bending:

$$N/A \leq \sigma_{u,ecc} \cdot \sqrt{c} \qquad \text{(EE5.12)}$$

where $\sqrt{c}$ is a modification factor depending on σ_{uy} and ρ.

For the case of moment gradient, formulae for equivalent moment are given, depending on β, $e(A/W)$ and λ.

CZECHOSLOVAKIA

1 and 2:

$$\frac{N}{A\sigma_{uy}} + \frac{C_x M_x}{W_x \sigma_{ul}} \leq 1 \qquad \text{(EE5.13)}$$

If sidesway is prevented:

$$C_x = 0.6 + 0.4\beta \geq 0.4 \qquad \text{(EE5.14)}$$

If sidesway is permitted:

$$\begin{aligned} C_x &= 1 \text{ for } \beta \geq 0 \\ &= 1 + 0.15\beta \text{ for } \beta \leq 0 \end{aligned} \qquad \text{(EE5.15)}$$

For check of extreme fiber tensile stress:

$$-\eta^x \left(\frac{N}{A}\right) + \frac{M}{W_{xt}} \leq \sigma_y/\gamma_m \qquad \text{(EE5.16)}$$

where

$$\eta^x = 1 - \lambda^{-2} \frac{(N/A) + (\sigma_y/\gamma_m)}{\sigma_y/\gamma_m}$$

POLAND

1.

$$\frac{N}{A \cdot \sigma_{ux}} + \frac{M}{W_x \sigma_y/\gamma_m} \leq 1.05 \qquad \text{(EE5.17)}$$

For check of extreme fiber tensile stress:

$$-\frac{N}{A\sigma_{ux}} + \frac{300 + 2\lambda_x}{W_{xt}\sigma_y/\gamma_m} \leq 1.0 \qquad \text{(EE5.18)}$$

2.

$$\frac{N}{A} \leq \sigma_u(\lambda_{eff}) \qquad \text{(EE5.19)}$$

where σ_u is the limit stress for column buckling due to the fictitious slenderness ratio (λ_{eff}) according to interaction formula, Eq. (EE5.10):

$$\lambda_{eff} = \rho \cdot \lambda_y \qquad \text{(EE5.20)}$$

3.

$$\frac{N}{A\sigma_{uy}} + \frac{M_x}{W_x \sigma_y/\gamma_m} + \frac{M_y}{W_x \sigma_y/\gamma_m} \leq 1.1 \qquad \text{(EE5.21)}$$

HUNGARY

1 and 2:

$$\frac{N}{A} + \frac{\mu_x}{\mu_x - 1}\frac{M}{W_x} \leq \sigma_u(\lambda'_{eff}) \qquad \text{(EE5.22)}$$

where σ_u is the limit for extreme fiber stress, that is, according to interaction formula Eq. (EE5.10), the limit stress of a fictitious centrally compressed member with slenderness ratio

$$\lambda'_{eff} = \rho \cdot \frac{1}{\sqrt{1 + \dfrac{M}{N} \cdot \dfrac{A}{W}}} \lambda_y \qquad \text{(EE5.23)}$$

By this method plastic reduction due to maximum (rather than to average) stress is introduced.

RUMANIA

See "Regional Recommendations."

1 and 3:

$$\frac{N}{A} + \frac{\mu + \delta}{\mu - 1}\frac{N}{A}e_o + \frac{\mu_x + \delta_x}{\mu_x - 1}\frac{M_x}{W_x} + \frac{\mu_y + \delta_y}{\mu_y - 1}\frac{M_y}{W_y} \leq \sigma_y/n \quad \text{(EE5.24)}$$

where

$e_o(W/A)$ = the supposed initial eccentricity of a centrally compressed member
$\delta, \delta_x, \delta_y$ = correction factors depending on the shape of the bending moment diagram
$\mu = \mu_x$ or μ_y
n = factor of safety = 1.5

Extreme fiber tensile stress can be checked by a similar formula.

2.

$$\frac{N}{A} \leq \sigma_w(\lambda_{eff}) \quad \text{(EE5.25)}$$

where

σ_w = the allowable column buckling stress at slenderness ratio $\lambda_{eff} = \rho\lambda_y$ according to interaction Eq. (EE5.10).

The draft of the new **Yugoslavia** specifications will adopt ECCS Recommendations; that of the **GDR** specifications is considering a general interaction formula for lateral-torsional buckling of beams and beam-columns (see Chapter 3):

$$\left(\frac{n\,\sigma_{wl}}{\sigma_l}\right)^a + \left(\frac{n\,\sigma_{wl}}{\sigma_{cr,e}{}^c}\right)^a = 1 \quad \text{(EE5.26)}$$

with a depending on ratio M/N.[176]

c. COLLOQUIUM CONTRIBUTIONS (COMMENTARIES)

JAPAN

B. KATO AND H. AKIYAMA (TOKYO)[177]

The design formulae were derived on the basis of ultimate strength of beam-columns subject to couple of end moments and axial thrust under the condition that lateral-torsional buckling and local buckling are prevented.

a. Bending about strong axis:

The larger value of Eqs. (J5.8) and Eq. (J5.9) gives ultimate moment capacity.

$$M_x = \Gamma_{1x}(1 - N/N_r)\,M_{rx} \quad \text{(J5.8)}$$

$$\left.\begin{aligned} &0 \leq N/N_r \leq 0.15: \quad M_x = 1.14\Gamma_{2x}M_{rx} \\ &0.15 < N/N_r \leq 1.0: \\ &\qquad M_x = 1.34\Gamma_{2x}(1 - N/N_r)M_{rx} \end{aligned}\right] \quad \text{(J5.9)}$$

where the following requirement is imposed:

$$N \leq N_k$$

In the foregoing,

$\cos \pi\gamma_x + \alpha \leq 0$: $\quad \Gamma_{1x} = \dfrac{\sin \pi\gamma_x}{\sqrt{1 + 2\alpha \cos \pi\gamma_x + \alpha^2}}$

$\cos \pi\gamma_x + \alpha > 0$: $\quad \Gamma_{1x} = 1$

$\cos \pi\gamma'_x + \alpha \leq 0$: $\quad \Gamma_{2x} = \dfrac{\sin \pi\gamma'_x}{\sqrt{1 + 2\alpha \cos \pi\gamma'_x + \alpha^2}}$

$\cos \pi\gamma'_x + \alpha < 0$: $\quad \Gamma_{2x} = 1$

$\gamma_x = \sqrt{N/N_{cr,x}}$

$\gamma'_x = 1.225\gamma_x$
$N_{cr,x}$ = Euler load about strong axis (x-axis)
N = axial load
$\alpha = M_{1x}/M_{2x}$ (where $-1 \leq \alpha \leq 1$), the ratio of the smaller to larger moments at column ends. M_{1x}/M_{2x} is positive under double curvature bending and is negative under single curvature bending.
$M_x = |M_{2x}|$
$N_r = A\sigma_r$, yield axial load
A = section area
σ_r = yield stress
M_{rx} = elastic limit moment about strong axis
N_k = ultimate buckling strength of column under pure compression

b. Bending about weak axis:

The larger value of Eq. (J5.10) and Eq. (J5.11) gives ultimate moment capacity.

$$M_y = \Gamma_{1y}(1 - N/N_r)M_{ry} \quad \text{(J5.10)}$$

$$\left.\begin{aligned} &0 \leq N/N_r \leq 0.4: \quad M_y = 1.5\Gamma_{2y}M_{ry} \\ &0.4 < N/N_r \leq 1: \\ &\qquad M_y = 1.78\Gamma_{2y}[1 - (N/N_r)^2]M_{ry} \end{aligned}\right] \quad \text{(J5.11)}$$

where the following requirement is imposed:

$$N \leq N_k$$

In the foregoing,

$$\cos \pi\gamma_y + \alpha \leq 0: \quad \Gamma_{1y} = \frac{\sin \pi\gamma_y}{\sqrt{1 + 2\alpha \cos \pi\gamma_y + \alpha^2}}$$

$$\cos \pi\gamma_y + \alpha < 0: \quad \Gamma_{1y} = 1$$

$$\cos \pi\gamma'_y + \alpha \leq 0: \quad \Gamma_{2y} = \frac{\sin \pi\gamma'_y}{\sqrt{1 + 2\alpha \cos \pi\gamma'_y + \alpha^2}}$$

$$\cos \pi\gamma'_y + \alpha < 0: \quad \Gamma_{2y} = 1$$

$$\gamma_y = \sqrt{N/N_{cr,y}}$$

$\gamma'_y = 1.414\,\gamma_y$
$N_{cr,y}$ = Euler load about weak axis
$\alpha = M_{1y}/M_{2y}$ (where $-1 \leq \alpha \leq 1$)
$M_y = |M_{2y}|$
M_{ry} = elastic limit moment about weak axis

The above formulae are derived from closed form of elastic solution for beam-columns extended to allow for the evaluation of ultimate strength. Inherent residual stresses of rolled shapes are taken into account.

NORTH AMERICA

W. F. CHEN (WASHINGTON)[178]

The paper provides an up-to-date review of the theory and design of beam-columns and offers a unified approach to the various analytical and numerical solutions with which the literature abounds. It covers the basic theoretical principles, methods of analysis in obtaining solutions of beam-column problems in elastic or plastic, in-plane or in-space, short or long, steel or concrete or composite material. It shows how the latest developments of theories of biaxially loaded beam-columns can be used in the solution of practical design problems. The general validity of the proposed interaction approach to design of biaxially loaded beam-columns is demonstrated by a statistical evaluation of the predicted against computed and tested collapse loads. Directions of further study are indicated.

P. G. STANFORD (WASHINGTON)[179]

The paper describes the CISC-CSICC Column Selection Program 3 (CSP3) showing various attributes of the program which aims squarely for the practicing engineer. Special emphasis has been made to point out the various program functions which are the result of either changes in design philosophy or advances in column and beam-column design procedures.

J. SPRINGFIELD (LIEGE)

The theme paper defines the beam-column problems from a practicing engineer's viewpoint and summarizes recent research results and design recommendations in North America. The research relevant to each class of problems described is closely related to the activities or contacts made by members of the SSRC Task Group 3 "Columns with Biaxial Bending."

W. F. CHEN (LIEGE)[173]

This paper extends the non-linear biaxial interaction equation to box columns. Different values for the exponent β as given above are proposed.

WEST EUROPE

K. ROIK AND R. BERGMANN (LIEGE)[180]

The paper presents an approach for beam-columns identical to the ECCS approach. No interaction formulae are given, however, but interaction diagrams which correspond to various column shapes and bending moment distributions. For the limiting case of the centrally loaded column, the ECCS buckling curves are used. The interaction diagrams are presented on a vertical axis normalized for N_k instead of N_{pl} which reduces the variability of the interaction curves considerably. An illustrative design example is given.

A. R. GENT AND T. K. SEN (LIEGE, WASHINGTON)[181,182]

The authors discuss the behavior of columns under high axial loads. Particular attention is given to the deformation capacity of the column as it influences the capacity to shed end-moments from adjacent members. Full scale tests were carried out on H-sections. The tests showed that the columns with slenderness ratios over 50 failed in the lateral-torsional mode. The authors concluded that the application of the ECCS design rules, using e^t, correctly described the failure behavior.

G. VALTINAT AND R. MULLER (LIEGE)[183]

The ultimate strength of beam columns made of aluminum alloys is discussed. One of the differences between steel and aluminum alloys is that the properties of aluminum alloys are affected by the welding heat input. The material is weakened around the weld seam in the heat affected zone. Numerical calculations were carried out using a procedure identical to the one presented by Beer and Schulz.[184] A design proposal is presented for aluminum alloys structures.

J. LINDNER AND G. WIECHERT (LIEGE)[185]

The paper gives boundary values for the slenderness ratios which enable the designer to quickly determine which failure mode governs his design, flexural buckling about the strong or weak axis or lateral-torsional buckling. The authors have calculated the boundary curves based on assumed imperfections. They find that the failure mode depends mainly on the slenderness ratio and on the relative eccentricity of the column, defined as e/h.

W. R. DE SITTER (LIEGE)[186]

The prepared discussion by de Sitter points out the fact that is is very important to consider the affinity between the first-order deformation and the assumed buckling modes at collapse. Particularly for the sway buckling mode, the differences between experiment and simple calculations become quite large.

D. A. NETHERCOT AND J. C. TAYLOR (LIEGE)[104]

This paper discusses a further simplification of the ECCS beam column formula by introducing the approximation

$$N_k = \sqrt{N_{cr}N_{pl}/2} \qquad \text{(WE5.12)}$$

which ultimately leads to

$$M/M_r = (1 - N/N_k)/(1 + \tfrac{1}{2}\,N/N_k) \qquad \text{(WE5.13)}$$

Equation (WE5.19) has been proposed for the revision of BS 449.

An even simpler linear approximation seems to be

$$M/M_r = 0.9 - (N/N_k) \qquad \text{(WE5.14)}$$

EAST EUROPE

F. FALTUS (BUDAPEST)[187]

Paper reports on experiments carried out to clear up the effect of the eccentricity of column-to-gusset plate welded connection on load carrying capacity of compressed members.

G. SZABO (BUDAPEST)[188]

Paper gives detailed report on experiments to demonstrate the manifold behavior of eccentrically compressed columns with open cross section failing because of lateral-torsional buckling.

d. CURRENT STUDIES

JAPAN

An effort is being made to develop a simple design formula of beam-columns, by Sakamoto, Miyamura, and Watanabe,[189] where an approximate analytical solution for elastic limit strength is modified so as to allow for estimation of ultimate strength.

NORTH AMERICA

EXPERIMENTAL AND THEORETICAL EVALUATION

In an extension of an earlier evaluation of the linear interaction equation by Galambos,[190] Springfield and Hegan[191] included the Birnstiel tests[192] and compared also the non-linear interaction equation. The latter equation was found to be superior in predicting capacities of sections with varying flange width to web depth. For the Birnstiel tests, the non-linear equation was found extremely reliable (Mean 1.01, S.D. 0.074) while the linear interaction equation was conservative (Mean 1.20, S.D. 0.085). A further verification of non-linear equation is its good agreement with Birnstiel's incremental analytical procedure. Aside from one result, in which the error was 7 percent conservative, all the other values agree to within 3 percent.

FURTHER WORK NEEDED

A general understanding of the behavior and strength of an isolated beam-column subjected to in-plane or biaxial loading has been achieved in recent years through many analytical and experimental investigations. The analytical methods available to date are quite reliable and they can be used to predict accurately the behavior and strength of any isolated beam-columns. These predictions have very good accuracy and little variability when applied to beam-column tests for which the required data are carefully determined, which include material properties, initial deflections, residual stresses, and loading and boundary conditions. As a result of this analysis capability, a considerable amount of data concerning the behavior and strength of isolated beam-columns has been obtained and critically studied. Against the background of this information, several design methods have been derived and developed. The general validity of these proposed methods has been demonstrated by comparisons of computed loads with limited test results. For further development in this aspect, a systematic *evaluation* of various proposed design methods by practicing engineers is essential. The evaluation should also include some comparative designs of actual structural frameworks. Only through this critical evaluation can a satisfactory method for practical design be recommended for general use.

In the present development, the equivalent moment concept (C_m-method) has been used for the case of unsymmetric loading conditions. Limited verification on this extension has been made. To show the general validity of the approach when applied to all combinations of biaxial bending (single curvature about one axis and double curvature about the other, etc.), extensive studies on various combinations are needed, using any one of the available analytical methods or computer models.

Since all the proposed design methods are based essentially on the information on hot-rolled wide flange sections and box sections, verification of the applicability of the axial and biaxial design proposals to the design of large built-up sections is required, such as those used for bridge truss chords and web members.

Since, in an actual structural framework, the beam-columns are restrained, additional theoretical and experimental studies on restrained beam-columns with biaxial loading and on sway and non-sway subassemblages are needed. A detailed study of some typical subassemblages will provide insight into the behavior of beam-columns as a part of the framework. Both analytical methods and numerical solutions along with well controlled experiments on typical simple subassemblages are needed. As a further development along this line, consideration of the effect on beam-columns of subassemblages with limited moment beam-to-column connections should be given, possibly using bolted, or welded and bolted combinations, as typical types of system having economic possibilities in the design of actual structural frameworks.

In a recent work, Kanchanalai and Lu[193] study the framed columns subjected to weak axis bending. Both sway and non-sway columns are included. Since the current uniaxial design procedures are based largely on the previous studies on beam-columns bent about the strong axis, the linear interaction for uniaxial weak axis bending case is found to give results which do not agree well with their theoretical solutions. Based on their results, a new set of beam-column design formulas are proposed. Further, they propose specific design provisions for sway columns considering the effect of frame instability.

WEST EUROPE

The beam-column seldom exists as an individual member, but is generally part of a structure. For a better understanding of the behavior of such a structural element, it will be necessary to study the structure as a system. This approach may also solve the effective length problem.

LESCOUARC'H AND GALEA (CTICM, FRANCE)

Minimum stiffness of the bracing system of a structure in order to consider the structure as braced when calculating the effective length of the columns. Effective length of a column in a "partially braced frame".

GALEA (ENSAM-CTICM, FRANCE)

Computer simulation of the behavior of beam-columns under a fire.

GACHON (ENSAM, FRANCE)

Elastic-plastic behavior of beam-columns. Load-deflection curves (tests).

JANSS (CRIF, BELGIUM)

An experimental program is being developed to test the behavior of asymmetrical cross sections. Computer simulation will also be done.

LINDNER AND KURTH (BERLIN, GERMANY)

Flexural-torsional buckling of high strength steel columns. Tests on hinged columns of varying slendernesses and load eccentricities.

EAST GERMANY

Structural behavior of beam-columns as part of a complete system is to be investigated, to clarify:

- the effect of beam-column response on the load-deflection diagram of the whole structure
- the effect of the actual (practical) lateral restraints (often giving incomplete torsional restraint)
- the response of structural elements with variable cross section and axial load
- the effect of continuity on the behavior of individual members

Full-scale tests with frame are being carried out concerning the problems mentioned above.

Chapter 6. Frames

a. REGIONAL RECOMMENDATIONS

JAPAN

(There are no regional recommendations in Japan.)

NORTH AMERICA

(Key Document: SSRC Guide[1])

ELASTIC DESIGN

Three procedures are available for designing columns in building frames: the effective length procedure, the P-Δ procedure and the procedure requiring story drift control. In all these procedures the columns are designed to satisfy the interaction formulas given in the AISC Specfication:

$$\frac{f_a}{F_a} + \frac{C_m f_b}{\left(1 - \frac{f_a}{F'_e}\right) F_b} \leq 1.0$$

$$\frac{f_a}{0.60\,F_y} + \frac{f_b}{F_b} \leq 1.0$$

In the first formula, the effective slenderness ratio KL/r is used in calculating F_a and F'_e with the effective length factor K determined either for the sway or non-sway mode of buckling. The P-Δ procedure is an iterative approach in which the effect of frame instability is considered by calculating the P-Δ moment in each story. The structure is analyzed by a first-order analysis for a set of amplified lateral loads. Both braced and unbraced frames may be designed by this procedure. The third approach has been developed primarily for unbraced frames and requires no consideration of the P-Δ effect if certain conditions are met, the most important of which is a limitation on story drift. The first-order story-drift index at working load is to be limited to

$$\frac{\Delta}{H} < \frac{1}{7}\frac{\Sigma V}{\Sigma P}$$

where

Δ = drift of story due to ΣV
H = story height
ΣV = total story shear due to lateral loads
ΣP = total gravity load on the story

If this limit is not satisfied, a design based on a second-order analysis is recommended.

PLASTIC DESIGN

The procedures described in ASCE Manual of Engineering Practice *Plastic Design in Steel—A Guide and Commentary* is recommended. In essence, these procedures all require the direct inclusion of the P-Δ moment in calculating the ultimate strength of frames. The calculations may be performed for the entire frame or for the individual one-story assemblages. Similar procedures have also been developed for braced frames.

WEST EUROPE

(Key Document: ECCS Recommendations[24])

ELASTIC DESIGN

Braced Frames—See Chapter 5, Beam-Columns (effective length concept, interaction formulae are used.)

Unbraced Frames—Design according to second-order theory; a representative initial imperfection must be adopted.

PLASTIC DESIGN

Single-story Frames—May be designed according to simple plastic theory if, due to horizontal loads applied to the top of each column and equal to 0.01 of the vertical load in the column at the required factored collapse load, the sway deflection f does not exceed 1.8/1000 of the height of the column. Otherwise second-order theory has to be applied.

Multi-story Braced Frames—The columns must be designed for the most unfavorable loading condition; interaction formulae, using the effective length concept as covered in Chapter 5 are used; whether elastic or plastic design is used depends on the design and framing system for the minor and major axis beams (for detailed information see ECCS-R5.9.2). The vertical bracing system must resist the effect of an assumed out-of-plumb equal to 1/200 of the height of each story.

Multi-story Unbraced Frames—With α_{fl} being the failure load factor, α_{pl} the rigid plastic load factor (hinge-mechanism), and α_{cr} the elastic critical load factor (for the calculation of which design aids including the possible effect of cladding are given), the stability check is made as follows:

If $\alpha_{cr}/\alpha_{pl} > 10$: $\quad \alpha_{fl} = \alpha_{pl}$

If $4 \leq \alpha_{cr}/\alpha_{pl} \leq 10$:

$\alpha_{fl} = \alpha_{pl}/(0.9 + \alpha_{pl}/\alpha_{cr})$

(Modified Rankine-Merchant formula, but no plastic hinges are allowed in the columns, except at the base, for the mechanism belonging to α_{pl}.)

If $\alpha_{cr}/\alpha_{pl} < 4$:

An elastic-plastic second-order method (P-Δ effect considered) must be used, including the effect of an initial out-of-plumb equal to 1/200 of the height of each story.

In order to take care of the danger of lateral-torsional buckling, of the local imperfections, and of the secondary moments $P\delta$ (where δ is the difference between the deflected shape and the chord of the column) in addition to the design methods mentioned above, the columns must be checked as beam-columns (plastic design method), with the effective length equal to the column height, in the strong as well as in the weak plane.

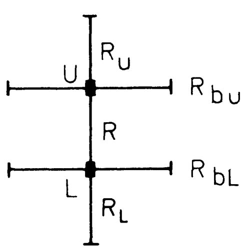
EAST EUROPE

(Key Document: INCERC Design Guidelines for Steel)

Using the concept of the effective length (Kl), frame analysis is reduced to member analysis; design formulae for beam-columns (Chapter 5) can be used.

Factor K is to be computed by elastic theory or can be taken from graphs and tables. For braced frames, the approximate analysis presented by Fig. EE6.1 is suggested, with $R = J/l$ and $c = 2$ for fixed far end and $c = 1.5$ for pinned far end members, respectively.

Plastic design procedure is given for frames with prevented or restricted sidesway only, supposing that plate thickness requirements, given in Chapter 4, and lateral bracing requirements, given in Chapter 3, are met. Beams and beam-columns are to be analyzed by formulae similar to Eqs. (EE5.1) through (EE5.5), replacing the elastic section modulus by the plastic one. In case of low axial loads ($N < 0.1N_u$), instability phenomena may be disregarded.



$$k_U = \frac{\sum c_i R_{bU}}{\sum c_i R_{bU} + c_U R_U + R}$$

$$k_L = \frac{\sum c_i R_{bL}}{\sum c_i R_{bL} + c_L R_L + R}$$

$$K = \frac{3 - 1.6\,(k_U + k_L) + 0.84\,k_U k_L}{3 - (k_U + k_L) + 0.28\,k_U k_L}$$

Figure EE6.1

Analyzing effective length factors, the presence of plastic hinges are to be taken into account.

b. SPECIFICATIONS AND CODES

JAPAN

AIJ STANDARD FOR STEEL STRUCTURES (1970)

ffective Length of Frame Columns—In braced frames, ıe effective length of columns shall be taken as the distance etween panel points unless the analysis shows that a naller value may be used. In unbraced frames, it shall be etermined by a rational method and shall not be less than ıe distance between panel points.

ommentary—Frames where the sidesway is not preented are designed to resist the loads due to earthquake r wind by the bending stiffness of beams and columns. In apan, stresses due to the earthquake load become comıaratively large, and hence the slenderness ratio of columns ı such frames does not become too large in most cases, and : is sufficient for design purposes to use approximate values f the slenderness ratio determined from formulae or lignment charts prepared for regular rectangular frames. The AIJ Standard recommends in the commentary that the lesigner refer to *Design Manual for H-Shape Steel* of ſSSC;[195] other references including the Commentary to he AISC Specification and the AIJ *Recommendations for Plastic Design of Steel Structures*[195] also provides alignment harts. Yokoo, Wakabayashi, and Sakamoto[197] investigated he accuracy of the beam-column formula in the AIJ Standard, analyzing a series of subassemblages subjected o combined vertical and horizontal loads. AIJ Standard uotes in the commentary the results as shown in Fig. J6.1, ınd concludes that the error is small.

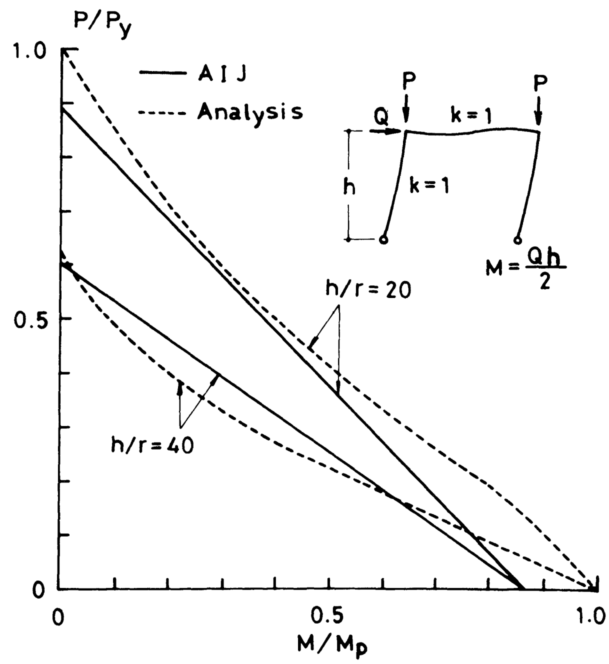


Figure J6.1

AIJ TECHNICAL GUIDE (1974)[198]

General Recommendation

1. Simple structural arrangement is desired.
2. Structural elements resisting horizontal loads should be distributed so as to avoid the torsional deformation.
3. Sufficient strength and ductility should be provided.
4. Deformation in the building should be limited to a certain amount in view of safety and serviceability.

Commentary—The basic requirement comprised in this guide is as follows. Under strong wind or relatively frequent strong earthquakes, the building should behave elastically, no cracks should occur in the walls, and secondary elements should follow the deformation of structural elements. Under

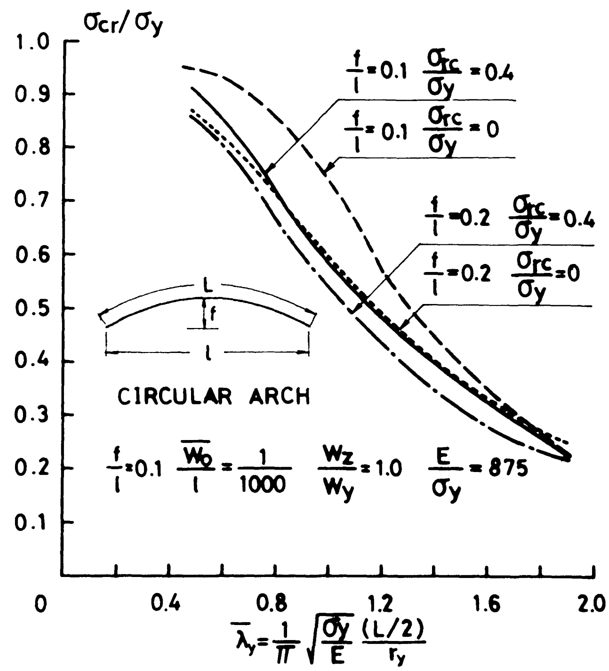


Figure J6.2

the rare extremely strong earthquakes, damages should be limited so that repair is possible. In order to achieve these requirements, it is needed to carry out a dynamic response analysis of the designed building frame based on a rational mathematical model which must accurately simulate the real behavior.

NORTH AMERICA

AISC SPECIFICATION

Elastic Design—Columns are to be designed for their effective lengths KL/r. In addition, the specification also requires that consideration be given to the significant instability effects resulting from the deflected shape of the structure or of individual elements of the lateral load resisting system, including the effects on beam, columns, bracing, connections, and shear wall.

New design recommendations are being proposed which will involve the use of amplified bending moments and axial forces to account for the instability effects.

Plastic Design—For one- or two-story frames, the maximum strength may be determined by a routine plastic analysis procedure and the effect of frame instability may be ignored. For braced multistory frames, provisions should be made to include this effect in the design of bracing system and frame members. For unbraced multistory frames, this effect should be included directly in the calculation for maximum strength. The frames shall be designed to be stable under (1) factored gravity loads ($L.F. = 1.70$) and (2) factored gravity loads plus factored lateral loads ($L.F. = 1.30$). In braced frames, the axial forces in the members of the vertical bracing system caused by factored gravity plus lateral loads shall not exceed $0.85P_y$. The axial forces in the columns of unbraced frames at the two factored load levels shall not exceed $0.75P_y$.

WEST EUROPE

Most of the national specifications and codes in West Europe (for example, **Belgium, Sweden, Switzerland**) use interaction formulae similar or equal to the ECCS Recommendations for the stability check of the columns in braced and unbraced frames. The effective-length concept is used to determine the amplification factor $1/(1 - 1/\alpha_{cr})$ for the bending moments. In some countries (**United Kingdom, Norway**) new codes are under preparation and will be more or less in compliance with the ECCS Recommendations.

France has adopted the following rules for the stability check of frames designed plastically:

If $1 < \alpha_{cr} < 5$: A second-order analysis is required.

If $\alpha_{cr} \geq 5$: A first-order or second-order analysis may be used, with the following additional requirements:

If a first-order analysis is used:

$$\Psi_r < 0.015\,(1 - 1/\alpha_{cr})$$

$$\alpha_{pl} > \alpha_{cr}/(\alpha_{cr} - 1)$$

If a second-order analysis is used:

$$\Psi_r < 0.015\ (= 1/67)$$

where Ψ_r = column slope in story r under factored load.

The **Federal Republic of Germany** has adopted the following rules for the stability check of frames:

Braced Frames:

In general, a second-order elastic or a second-order plastic hinge analysis, including the influence of geometric imperfections, is required. First-order theory may be used if the following conditions hold:

Using elastic theory:

$$\epsilon = l\sqrt{N/EI} \leq 1$$

Using plastic hinge theory:

$\epsilon \leq 0.25$ (in general)

$0.25 < \epsilon \leq 1$ (members with no plastic hinge, or the last plastic hinge only, within the field length)

where

l = member length
N = axial thrust under factored load
EI = bending stiffness

Unbraced Frames:

In general, a second-order elastic or second-order plastic hinge analysis is required, including the effect of an initial out-of-plumb

$\Psi_o = (1/150) r_1 r_2$

where

$r_1 = 0.5\,(1 + 1/n)$
$r_2 = \sqrt{10/h}$
n = number of columns in a story
h = height of building in m, if $h > 10\ m$

If an elastic analysis is performed, the following approximations may be used:

If $\alpha_{cr} \geq 10$:

First-order analysis without imperfections

If $4 \leq \alpha_{cr} < 10$:

First-order analysis may be used, but the story shear, including the effect of initial out-of-plumb, $(Q_r = H_r + V_r \Psi_o)$ has to be multiplied by amplification factor $\alpha_{cr}/(\alpha_{cr} - 1)$, where H_r = story shear due to external horizontal load and V_r = total vertical load in story r.

If a plastic hinge analysis is performed, first-order analysis may be used if

$|\Psi_r| \leq |Q_r/10\ V_r|$

EAST EUROPE

Generally specifications adopt the concept of effective length, and approximate formulae, graphs and tables are given for several practical cases (one-story and multi-story frames, frames in industrial buildings). The concept is extended to the cases of variable cross section and variable axial loads as well, using the corresponding elastic solution. The **GDR** specification requires as a rule a second-order elastic analysis which can be substituted by the effective length concept.

Few specifications give methods for plastic design (**Czechoslovakian, Hungarian, Polish**), restricting it mostly for the case of one-story and braced multi-story frames.

The **Hungarian** specification allows first-order plastic design of one- and two-story unbraced and multi-story braced frames, if for every column

$$\frac{N}{A(\sigma_y/\gamma_m)} \leq \frac{870\eta}{\lambda^2} \qquad \text{(EE6.1)}$$

where

λ = the effective slenderness of column in the plane of the frame
$\eta = \sqrt{240/\sigma_y}$, with σ_y in Mpa

If $\dfrac{870\eta}{\lambda^2} \leq \dfrac{N}{A(\sigma_y/\gamma_m)} \leq 1.5\left(\dfrac{870\eta}{\lambda^2}\right)$:

Limit stress (σ_y/γ_m) is to be reduced to:

$$\left(\frac{\sigma_y}{\gamma_m}\right)^1 = \frac{\sigma_y}{\gamma_m}\left[\frac{1}{0.9 + \dfrac{1.15}{\eta}\left(\dfrac{\lambda}{100}\right)^2}\right] \qquad \text{(EE6.2)}$$

If $\dfrac{N}{A(\sigma_y/\gamma_m)} > 1.5\left(\dfrac{870\eta}{\lambda^2}\right)$:

Second-order limit analysis is to be carried out. Equations (EE6.1) and (EE6.2) correspond to the modified Rankine-Merchant formula.

c. COLLOQUIUM CONTRIBUTIONS (COMMENTARIES)

JAPAN

T. SAKIMOTO AND S. KOMATSU (LIEGE)[199]

The paper presents the results of a parametric study on the effects of initial imperfections on the ultimate load carrying capacity of steel arches of box profile subject to uniformly distributed in-plane load. Parameters selected as variables are: compressive residual stress level σ_{rc}/σ_y, section modulus ratio W_z/W_y rise-to-span ratio f/l, amplitude of initial out-of-plane deflection assumed to be of a sine curve w_0/l, and normalized slenderness ratio $\bar{\lambda}$. It is concluded that the results show close agreement with the ECCS column curves b and c.

S. KURANISHI AND. T. YABUKI (LIEGE)[200]

The paper deals with a conventional arch bridge composed of two main arch ribs connected to each other by lateral and sway bracings, and having comparatively high out-of-plane rigidity on the whole.

In such a case, the main role of the lateral loads is considered to be in the reduction of the in-plane rigidity due to the increased stresses and early yielding. Figure J6.3 shows the maximum load carrying capacity of two-hinged arches subject to in-plane and out-of-plane distributed loads, where q_o denotes the value of uniformly distributed load which causes yielding at springings. In conclusion, for the usual values of S_w between 0.05 and 0.1, the influence of the lateral load is not substantial.

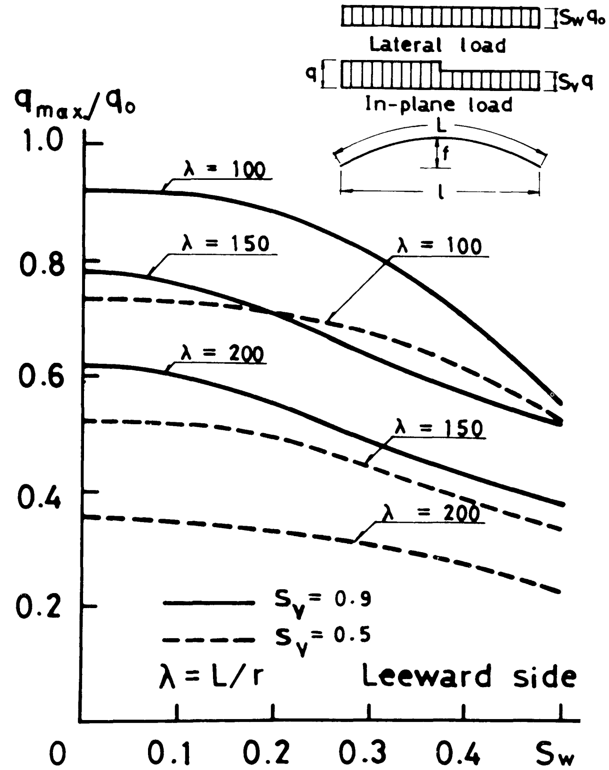


Figure J6.3

NORTH AMERICA

W. J. LEMESSURIER (WASHINGTON)[201]

The paper presents a method for performing elastic second-order analysis of simple rigid frames. Design examples are given to illustrate the application of the method. The author has previously published a paper dealing with pin-connected frames.[202]

D. BEAULIEU AND P. F. ADAMS (WASHINGTON)[203]

The authors applied the routine of probability and statistics to define the effects of column and wall out-of-plumbs on the overall stability of corebraced buildings. Different code requirements on stability related to out-of-plumbs are compared with new expressions derived from statistics. While some effects are found non-significant, others are and must be considered in design.

F. Y. CHENG (WASHINGTON)[204]

Buckling strength of three-dimensional building systems is studied with emphasis on effect of floor slabs. Numerical studies show that floor slab can significantly influence the buckling capacity of the higher modes of unbraced systems but affect all modes of braced systems.

G. KUHN AND H. LUNDGREN (WASHINGTON)[205]

The accuracy of the Julia-Lawrence column effective length alignment charts is examined, using results obtained frôm rigorous elastic frame buckling solutions. The findings are: (1) for non-symmetrical frames and loading, the effective length factors can be considerably in error, either conservatively or unconservatively, and (2) the greatest error occurs generally in the more slender columns.

F. CHEONG-SIAT-MOY (WASHINGTON)[206]

The paper points out some fallacies in using the effective length factors in the design of framed columns. A design procedure based on the story stiffness concept is proposed.

C. K. WANG AND C. H. TAY (WASHINGTON)[207]

Results of computer bifurcation analyses of three single-story frames, considering non-linear inelastic stress-strain characteristics of the columns, are presented. Extensive parameter studies are also presented. The major findings are (1) when a structure is analyzed as a whole, the consideration of inelastic action of columns does not always reduce the effective length ratios when compared to the elastic solution, (2) all columns in a story always interact with each others through the girders. This interaction results in load sharing by the columns which, as pointed out by the authors, is due more to the non-linear stress-strain characteristics than to a constant modulus of elasticity condition, and (3) elastic buckling solutions for structures whose columns are stressed into the inelastic range may be unconservative in some cases.

WEST EUROPE

M. R. HORNE AND L. J. MORRIS (LIEGE)[208]

The paper presents stability and shakedown considerations in multi-story sway frames. It shows that there is no significant aggravating effect of shakedown due to its combination with minimum weight design and change of geometry and stability effects for regular multi-story sway frames.

O. S. OKTEN (LIEGE)[209]

The paper presents a preliminary design method for unbraced steel frames, based on the sway subassemblage concept, similar to the work of Daniels and Lu[210] at Lehigh University. It enables the designer to analyze and to design a frame in a storywise manner in such a way that the load-deflection diagram of the story meets the specified design criteria.

Y. LESCOUARC'H (LIEGE)[211]

The paper presents simple formulae for the effective length of frame columns, taking into account in a realistic manner the effect of plastic hinges.

J. WITTEVEEN (LIEGE)[212]

The paper presents simple criteria which have to be fulfilled in order to meet the rotation capacity requirements for columns with plastic hinges.

C. MASSONNET (LIEGE)[213]

In the General Report and the Conclusions to Theme 9 "Frames" of the Liege Colloquium the author discusses all the other colloquium contributions, which cannot be covered here (due to lack of space), in a logical order. Some of these contributions deal with the justification of (or reservations against) the ECCS Recommendations for the stability of frames.

EAST EUROPE

D. KORONDI (BUDAPEST)[214]

Refined method of calculating effective length of columns is given, taking loading conditions of the neighboring columns into account. Based on results of a simple model (subassemblage), design charts are presented.

E. NEMESTOTHY (BUDAPEST)[215]

Paper presents charts for the effective length of elastically restrained columns.

K. ZALKA (BUDAPEST)[216]

Overall stability of regular frames is investigated by "smearing up" the beams and analyzing columns with continuous elastic restraint against rotation. Displacement between two floors is accounted for by a Dunkerley-type interaction formula.

J. KORDA (BUDAPEST)[217]

Inaccurate results gained by the effective length concept are pointed out by comparison to more accurate (and often not even more complicated) second-order procedures.

A. BARTLOVA (BUDAPEST)[218]

Paper proposes to calculate ultimate load of axially loaded frames by introducing initial imperfections similar in shape to the buckling figure and calculating onset of yielding.

K. RABOLDT (BUDAPEST)[219]

Simplified analysis of frames containing truss members is proposed by replacing them by an equivalent plate girder member and including the effect of shear deformation.

O. HALASZ (BUDAPEST)[220]

Simplified method for second-order elastic-plastic analysis of sturdy frames is presented, resulting in a modified version of the Rankine-Merchant formula capable of including the effect of initial imperfections.

E. DULCZ (BUDAPEST)[221]

Results of ultimate load test on small-scale frames are presented and the effect of strain-hardening is emphasized.

A. R. RZANICIN (BUDAPEST)[222]

Paper presents a concise matrix method for stability analysis of axially loaded elastic frames.

M. IVANYI (BUDAPEST)[223]

The outlines of a theoretical and experimental project to clear up limiting values of allowable width-to-thickness ratios of constituting plate-elements of beam-columns in plastic design are presented.

K. CZAPLINSKY AND S. CZEMPKOWSKI (BUDAPEST)[224]

Referring to the collapse of a steel skeleton, importance of stability check with due care for changing condition during erection is emphasized and demonstrated.

L. KOLLAR (BUDAPEST)[225]

Rigidity requirements concerning the stiffening core of a steel skeleton with hinged joints are investigated with special attention to the torsional buckling of the whole assembly. Simple design rules are given, including interaction of flexural and torsional buckling.

M. IVANYI (BUDAPEST)[226]

A project launched in connection with the preparation of the Hungarian Specification for Plastic Design of Steel Structures, including failure tests on a large series of full-scale one-bay one-story steel frames, is summarized, describing experimental techniques and results of pilot tests.

R. DABROWSKY (BUDAPEST)[227]

The outlines of a second-order theory for the members of spatial frames is presented.

d. CURRENT STUDIES

JAPAN

BRACED FRAMES

In 1976, the work on the behavior of braces and braced frames was reviewed by the AIJ Special Committee for Steel Structures. Since then, research is in progress on the following topics: derivation of mathematical models simulating hysteretic behavior of bracing, effect of change in axial thrust in columns adjacent to bracing, dynamic response analysis and tests of braced frames, and horizontal shear carried by bracing at the ultimate load stage.

PLASTIC DESIGN AND ANALYSIS

In 1973 a paper[228] was published by JSSC in which static and dynamic behavior of plastically designed frames are discussed and a new design method was introduced. Detailed surveys on the plastic analysis are given in the book *Analytical Methods of Framed Structures.*[229] Currently, work is proceeding on: sensitivity of the behavior of minimum weight plastic frames to perturbations in the plastic moment and lateral load distribution, collapse process of strain-hardening frames under abnormal vertical loads, and inelastic behavior of portal frames with tapered columns.

SPACE FRAMES

Current topics are as follows: development of accurate methods of analysis of the instability problem, elasto-plastic dynamic response analysis, and experiments on space frames under monotonic and repeated horizontal loads.

HYSTERETIC BEHAVIOR OF FRAMES

A number of frame tests under repeated horizontal load have been reported, including prototype frames with concrete slabs and high strength steel frames. The effect of local buckling on the frame instability has been extensively investigated, and a procedure of aseismic limit design of frames was recently proposed.

EFFECTIVE COLUMN LENGTH

Alignment charts for the effective length of framed columns including the effect of shear deformation was developed.

A study is in progress on the development of a method to determine effective length of columns in rather irregular frames, which is based on the alignment charts of AISC type.

NORTH AMERICA

Research is in progress to develop a rational procedure for use in allowable-stress design, using the concept of moment amplification. In this procedure, the design moment is determined by applying an amplification factor to the moment obtained from a first-order analysis. The actual column length is then used to check for the effect of member instability. The amplification factor is given by

$$A_f = \frac{1}{1 - \alpha \dfrac{\Sigma P \, \Delta}{\Sigma V H}}$$

in which α is an empirical coefficient provided to account for the non-linear response of the structure. Δ is based on first-order analysis.

Other active research in this area includes extensive studies in the effect of out-of-plumbs on frame strength, stability of building cores under combined bending moment, axial thrust and twist, dynamic response of three-dimensional building structures subjected to multicomponent earthquake ground motions.

WEST EUROPE

The stability problems of frames involve such a large area of research interest and unsolved questions that only a few of the current studies, known to the authors to be underway at various locations, are listed below:

Criteria for the possible use of first-order analysis in ultimate collapse load problems, simple approximations and design aids for the application of second-order analysis, statistical evaluation of imperfections, influence of lack-of-fit in connections, shake down and stability, out-of-plane buckling, stability of spatial frames, simple methods for preliminary design including instability effects, computer programs for inelastic buckling of planar and spatial frames, justification for second-order plastic hinge analysis (Karlsruhe, Darmstadt, Liege, Manchester, Lausanne, Milano).

EAST EUROPE

Research is needed (and partly completed) to find rational procedures for everyday design use in the field of second-order elastic and plastic analysis; to clear up the effects of various imperfections and the stabilizing effect of different bracing systems; on the contribution of diaphragm action of cladding to the stability of the structure. A bigger series of full-scale tests on one-bay frames are being carried out.

Chapter 7. Triangulated Structures

a. REGIONAL RECOMMENDATIONS

JAPAN

(There are no regional recommendations in Japan.)

NORTH AMERICA

(Key Document: SSRC Guide[1])

Triangulated structures are planar or space trusses of various types, used especially for bridges, roofs and transmission poles. No new North American regional recommendations have been formulated. Standard design practice consists of determining the forces acting on each individual element, using approximate force amplification due to second order effects where appropriate, and then to design the element as a beam, a column, or a beam-column, depending on the loads. Guidelines for obtaining the effective length are given in Sects. 3.7 and 12.7 of the SSRC Guide (3rd edition), and in the ASCE *Guide for the Design of Aluminum Transmission Towers.*[230]

WEST EUROPE

(Key Documents: ECCS Recommendations[24,58])

TRUSSES

The ECCS Recommendations contain no special indications for trusses, particularly no prescriptions for the effective length of compression chords or diagonals.

The *ECCS Manual on the Stability of Steel Structures*[2] indicates that members in laterally braced trusses can be designed using the ECCS Column Curves. Awaiting more general research results on the behavior of trusses at ultimate load level (see Section d, "Current Studies"), the use of effective length factors in limit states design, in particular for full span loading cases, so far is not recommended.

This simplified philosophy, however, allows the designer to employ in transmission towers thinner angles than in normal structures.

TRANSMISSION TOWERS (STEEL)

Buckling Curve—Nondimensional buckling curve a_o is adopted for the design of rolled or cold-formed angles in compression.

For thicknesses t up to 30 mm, the following yield stresses σ_r are adopted:

Fe 360: $\sigma_r = 240\ \text{Nmm}^{-2}$
Fe 430: $\sigma_r = 265\ \text{Nmm}^{-2}$
Fe 510: $\sigma_r = 360\ \text{Nmm}^{-2}$

Local and Torsional Buckling—Rules are given for determining the yield stress σ_r (actual or modified) which must be adopted for the transformation of the non-dimensional curve a_o.

Three different topics are considered:

1. Equal leg rolled angles
2. Equal leg cold-formed angles
3. Rolled or cold-formed unequal leg angles

Design of Members—The types of members considered in the ECCS Manual[2] are: legs and chords, compound members, and web members.

The carrying capacity is obtained from the basic buckling curve, taking into account a slenderness ratio which is determined according to different rules for the various members.

Design criteria are also given for redundants, nominally unstressed.

The influence of the detailing on the buckling strength is pointed out; particularly in the joints of the chords, care must be taken to avoid any possible eccentricity.

Commentary—Curve a_o, transformed in dimensional form with elastic limit of 360 Nmm^{-2} for Fe 510, is in good agreement with the results collected by utilities and power boards from tests on transmission towers. The influence of local and torsional buckling is taken into account by introducing a modified yield stress which depends on the thinness of the legs and on the manufacturing procedure.

TRANSMISSION TOWERS (ALUMINUM)

The buckling curves adopted in the European Recommendations for Aluminum Alloy Structures[58] also cover the shapes commonly used in transmission towers. The extrusion procedure enables variously shaped cross sections with bulbs, lips and stiffeners (impossible to form by rolling) which are particularly convenient for improving the buckling strength and simplifying the joint details.

EAST EUROPE

(Key Document: INCERC Design Guidelines for Steel)

TRUSSES

Effective length of compression chords for both in-plane and out-of-plane buckling is equal to the distance between supported points (Fig. EE7.1a).

Elastic restraint can be taken into account, in case of in-plane buckling of diagonals, by $l_e = 0.8l$.

In case of non-uniform axial force (Fig. EE7.1c):

$$l_e = l\left(0.75 + 0.25\frac{N_2}{N_1}\right) \qquad \text{(EE7.1)}$$

where $N_2 < N_1$.

For out-of-plane buckling of compressed diagonals in Fig. EE7.1d:

If diagonals are continuous:

Compression in both diagonals:

$l_e = l_1$

Tension in one diagonal:

$l_e = l$

No axial force in one diagonal:

$l_e = 0.7l_1$

If diagonal is non-continuous, but connection provides flexural rigidity:

Compression or no axial force in one diagonal:

$l_e = l_1$

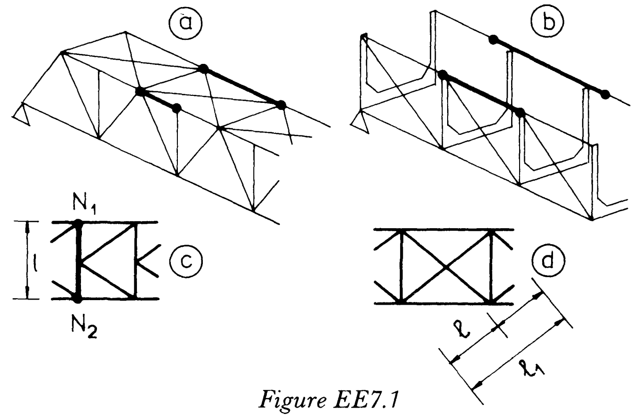


Figure EE7.1

Tension in one diagonal:

$$l_e = 0.7l_1 \qquad \text{(EE7.2)}$$

Similar rules for effective length are given in most specifications.

GDR and Czechoslovakian specifications give more precise formulas for the case in Fig. EE7.1d and for out-of-plane buckling of chords with elastic lateral supports (pony trusses, Fig. EE7.1b). Special approximate rules are applied for single angles.

TRANSMISSION TOWERS

Regional recommendations give no specific regulations for transmission towers.

b. SPECIFICATIONS AND CODES

JAPAN

TRUSSES

The following standards and specifications contain sections dealing with compression members in trusses or truss girders:

AIJ Design Standard for Steel Structures

JSCE Specifications for Steel Railway Structures

JRA Specifications for Highway Structures

The effective length of a compression member in a conventional design method without accurate buckling analysis of a truss as a whole is specified in the AIJ and JRA documents.

For the in-plane buckling of chord members and web members in trusses, the effective length should usually be taken to equal the distance between panel points, neglecting the restraining effect of adjacent members. The effective length could be reduced up to the distance between the centers of gravity of groups of rivets or bolts at the connections for web members (AIJ and JRA), 0.9 times the distance between panel points for lateral bracings or sway frame elements with no gusset plate (JRA), and 0.8 times the distance between panel points for the members connected to a chord member by a gusset plate (JRA), taking the restraining effect of adjacent members and connections into account.

For out-of-plane buckling, the effective lengths of a uniformly compressed chord member and a web member could be taken to equal the distance between laterally braced panel points and the distance between panel points, respectively. The conventional design method for a single-angle web member in a truss are given in the AIJ standard. The formulas for the reduction of the effective length of members subjected to unequal compressive forces or intersected by each other are shown in the AIJ and JRA specifications.

In the JSCE and JRA Specifications, box sections having higher torsional rigidity than open sections are recommended for chord members, columns in end frames, and diagonal members attached to intermediate support points of truss girders to prevent lateral-torsional buckling. In addition, limitations for proportioning of a cross section and the ratio of the slenderness ratio about the vertical axis to that about the horizontal axis are prescribed, to assure smooth and direct stress transfer through the gusset plate at the connection and to make the in-plane buckling dominant, respectively.

TRANSMISSION TOWERS

The following standards contain sections dealing with compression members in steel transmission towers:

AIJ Standard for Structural Calculation of Steel Tower Structures

JEC-127 Design Standards for Power Transmission Steel Towers

In the AIJ Tower Standard, design tables for the effective length of a chord member of steel tower structures are prepared on the basis of the bifurcation load analysis for the model of a chord member to which infinite number of web members with pin-ends are connected with a constant panel spacing. The effective length of a chord member could be taken to equal some value between 0.5 and 1.0 times the panel spacing, corresponding to the composition of a cross section and the arrangement of web members. The effective length of a web member could be taken to equal the distance between the centers of gravity of groups of rivets at the end connections might be reduced appropriately if the web is subjected to unequal compression or the members intersect each other.

The effective length of the member in transmission towers could usually be taken to equal 0.9 times the distance between panel points. For web members, it could be reduced up to 0.8 times the panel spacing, corresponding to the rigidity of end joints. Examples of the measure of the distance between panel points and the radius of gyration to calculate the slenderness ratio are given in the JEC Standard, showing drawings of the arrangement of members.

NORTH AMERICA

Major sections dealing with triangulated structures are contained in the following codes and specifications:

Standard Specifications for Open Web Steel Joists, Longspan Steel Joists and Deep Longspan Steel Joists (SJI, AISC)

Standard Specifications for Joist Girders (SJI)

Standard Specifications for Highway Bridges (AASHTO)

WEST EUROPE

TRUSSES

Effective Length of Members in Laterally Braced Trusses—Nearly all codes include prescriptions for the effective length of truss members. For buckling out of the plane, the effective length is usually kept equal to the total length between laterally braced nodes.

For buckling of the truss in the plane, to the contrary, a more or less important reduction is allowed. In the **Federal Republic of Germany** (DIN 4114, 6.2 or 6.3) the effective length is taken equal to the theoretical one for chord members, and the distance between centers of the bolted or welded connections for web members. DIN 4114 (draft 1978) allows for web members: effective length = $0.9l$. In the **Netherlands** the effective length factor K for web members is equal to

$$\left(0.7 + 0.3\,\frac{\Sigma EJ_i/l_i}{\Sigma EJ_k/l_k}\right)$$

where subscripts i and k represent compressed members and all members, respectively, at the end nodes. In **Belgium** (NBN B 51-001/1977) a K-factor of 0.8 is allowed for all truss members. The same regulation has been adopted in the previous **Swiss** code (SIA 161/1974), but in the newer edition (SIA 161/1979) the K-factor for the compression chords has been increased to 0.9, according to the results of investigations noted later in Section c, "Colloquium Contributions". Exactly the same regulations, i.e., $K = 0.8$ for web members and $K = 0.9$ for chord members, have been adopted in the **French** code CM 1966 (5.222).

Trusses with Laterally Unbraced Compression Chords (Pony Truss)—The design concept is generally based on the bifurcation solution given by well known Engesser formula.

Effect of "Secondary Stresses"—Existing codes give no precise indications relating to the effect of "secondary stresses" on the stability of a truss.

Limitations on Slenderness Ratio—In many codes limitations on the slenderness ratio are prescribed according to the kind of structure (bridges, buildings).

TRANSMISSION TOWERS

The design specifications for transmission towers in Belgium, Great Britain, Italy and Switzerland adopt simplified buckling curves for centrally loaded angles. Allowable stresses are given by linear functions in the range of low slenderness ratios; hyperbolic functions are introduced for higher slendernesses. In Germany, the buckling curves quoted in the DIN 4114 specifications are adopted for centrally loaded angles.

Eccentrically loaded web-members are considered as beam-columns and simplified interaction formulas are specified by the different codes.

In France, the Resal formula is adopted both for centrically and eccentrically loaded angles. The parameter for the Resal formula varies in the range 1 to 4, taking into account the eccentricity of the axial load, the end restraints, and the continuity of the member.

EAST EUROPE

TRUSSES

See previous comments in Section a, "Regional Recommendations."

TRANSMISSION TOWERS

For single-angle chords of laced towers, approximate effective slenderness formulae are given. For example:

GDR and Czechoslovakian Specifications

If compression prevails:

$\lambda = 0.8l/r_{min}$

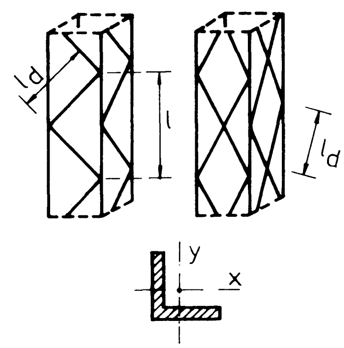
If bending prevails:

$\lambda = 0.7l/r_{min}$

Soviet Specification

$$\lambda = \bar{\mu}l/r_x \qquad \text{(EE7.3)}$$

where $\bar{\mu}$ is given in Fig. EE7.2.



$\frac{J \cdot l_d}{J_d \cdot l}$	$\bar{\mu}$
1.0	0.98
1.25	1.00
2.50	1.03
5.0	1.08
10.0	1.13

Figure EE7.2

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c. COLLOQUIUM CONTRIBUTIONS (COMMENTARIES)

JAPAN

S. MORINO, C. MATSUI AND Y. NAJIMA (LIEGE)[23]

The paper presents the results of the numerical analysis for lateral-torsional buckling of trusses with and without lateral bracing, and proposes design formulas and design charts to estimate the critical strength and effective slenderness ratio of the compression chord member in the truss, taking the effects of web members and tension chord members into account.

Critical stress of the compression chord member is given by Eq. (J7.1):

$$\sigma_{cr}/\sigma_{crs} = a(k)^b + 1 \qquad \text{(J7.1)}$$

where

σ_{cr} = critical stress, taking the effects of web members and a tension chord into account.

σ_{crs} = critical stress of a single member of slenderness ratio λ_B, obtained from the AIJ column curve.

a = value obtained from Fig. J7.1.

b = 0.5

k = ratio of the moment of inertia about the x-axis of web members to that of chord members.

λ_B = slenderness ratio corresponding to distance between panel points at which lateral deflections are prevented (see Fig. J7.1.)

λ_s = slenderness ratio corresponding to distance between panel points (see Fig. J7.1.)

Effective slenderness ratio of a chord member of a truss beam with lateral braces is obtained from Eq. (J7.2):

$$\bar{\lambda} = \lambda_s\,(\pi/z) \qquad \text{(J7.2)}$$

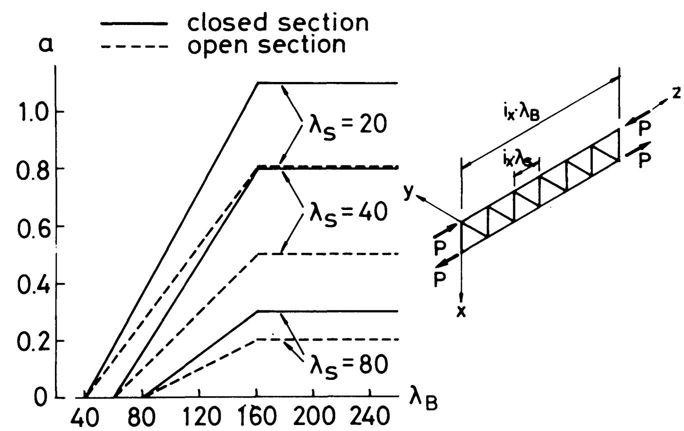

Figure J7.1

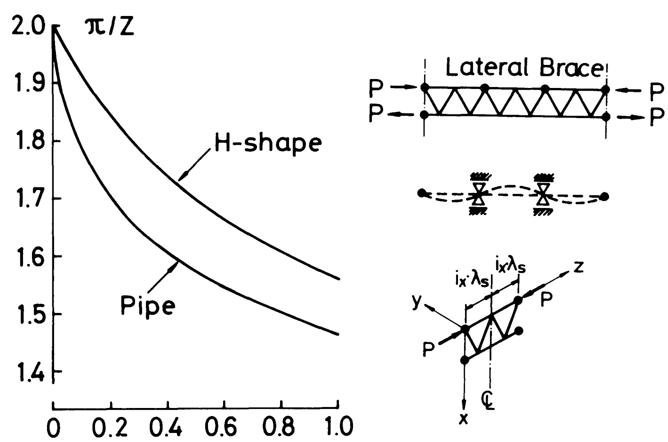

Figure J7.2

where

$\bar{\lambda}$ = effective slenderness ratio
π/z = value obtained from Fig. J7.2.

k = the ratio of the moment of inertia about the x-axis of web members to that of chord members.

NORTH AMERICA

(No colloquium contributions from North America.)

WEST EUROPE

P. DUBAS (LIEGE)[232]

The paper presents the results of numerical investigations concerning the ultimate load of the compression chord of a pony truss, taking into account the geometrical and the material imperfections. (See Fig. WE7.1.) The usual design procedure based on effective lengths given by the bifurcation theory seems to be very conservative, due to the fact that the stabilizing members remain elastic. An improved method using a buckling modulus $T = E \cdot \bar{\lambda} \cdot \sigma_u / \sigma_r$ is presented.

J. A. PACKER AND G. DAVIES (BUDAPEST)[233]

The paper presents an approximate approach to the theoretical analysis of local buckling failure in the strut member of a truss of rectangular hollow steel sections. By considering the overall behavior of the strut-to-chord connection, a large-deflection yield line theory and an approximate elastic load line have been used to predict the ultimate buckling strength of the strut member. Comparison between this approach and the results of 20 strut members gave satisfactory correlation.

J. MOUTY AND J. P. GRIMAULT (LIEGE)[234]

The paper presents the results of an experimental investigation on the influence of the torsional stiffness of hollow sections upon the stability of lattice girders without lateral supports. This paper is therefore connected with the one mentioned above. A method for determining the effective length of web members in such trusses is given.

P. DUBAS (LIEGE)[235]

The paper presents the first results of a numerical investigation on the ultimate strength of laterally braced trusses. The elastoplastic second order computations are based on the same assumptions as the ECCS Column Curves. The effective lengths given by the bifurcation theory for buckling in the plane lead to a satisfactory agreement with the "exact" one. However, it should be noted that the prescriptions for the effective length given by various codes may be on the unsafe side, especially for welded trusses (due to the influence of high residual stresses in the truss joints).

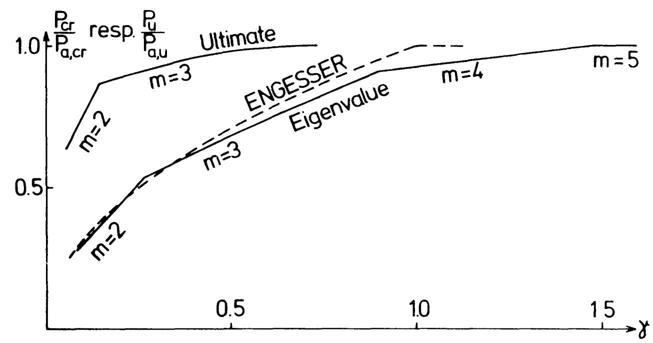

Figure WE7.1

G. W. SCHULTZ (LIEGE)[236]

The paper underlines the differences between the allowable stress design and the limit state design in regard to the influence of end restraints. At ultimate load level the restraining effect is quite reduced, so that chord and web members should, in principle, be designed as pin-ended struts.

B. A. CAUZILLO (LIEGE)[237]

The paper presents the results of a theoretical and experimental study on the buckling strength of angles and T-shapes in multiple lattices.

The analysis is carried out in the elastic range. It is shown that equal leg angles can assure stability of the panel under the design shear loads. Tests performed on real towers confirm the theoretical results.

J. P. CUILILIÉ AND M. LORIN (LIEGE)[238]

The paper presents the results of an experimental study of the influence of the connections on the buckling strength of tower web members.

A large number of test results are compared with the buckling curves adopted in different areas; comparisons are made on a statistical basis.

It is shown that the buckling curves in the ECCS Manual are in good agreement with the test results.

J. SHORT (LIEGE)[239]

The discussion presents a number of test results on eccentrically loaded equal leg angles.

When buckling occurs in the plane of minimum stiffness the results are in good agreement with the ECCS rules.

If buckling is forced in the plane parallel to the angle leg (with stays at intermediate points) the test results are considerably less than the buckling curves for web-members adopted in the ECCS Manual.

J. W. STARK (LIEGE)[240]

The discussion presents the result of tests performed on two models of legs for transmission towers.

The behavior of the web (redundants) members during failure is pointed out. It is also shown that in this case the design criteria of the ECCS Manual are on the safe side.

EAST EUROPE

Z. BYCHAWSKI (BUDAPEST)[241]

In a General Report on "Special Problems of Stability" comments are given on various contributions.

E. CHLADNY (BUDAPEST)[242]

Buckling of chords of pony trusses, that is, of trusses with elastic lateral support along their compression chord, is investigated supposing initial lateral displacements of the supports. Fictitious forces are introduced for checking the required characteristics of the supports in accordance with former design practice.

T. TARNAI (BUDAPEST)[243]

Lateral buckling of compression chord of a truss without lateral support within its span is investigated by means of a continuous model containing a fictitious web with no stiffness and hinged web-to-chord connection.

Z. GASPAR (BUDAPEST)[244]

The manifold aspects of the instability phenomena of a two-degree-of-freedom structure are demonstrated by means of the catastrophe theory.

A. DJUBEK (BUDAPEST)[245]

The critical load of angular truss members alternately supported in two planes is computed and tabulated, giving more conservative results than earlier authors.

d. CURRENT STUDIES

JAPAN

Out-of-plane buckling problems of a rigid plane truss subjected to in-plane loads are studied continuously by several researchers. Recently, a new design method and design charts were proposed for the estimation of the effective length of the chord member under compression, in which the effects of a tension chord and web members are taken into account.[246,247]

There is a preliminary investigation of the load-carrying capacity of single angle webs of a plane truss subjected to eccentrical compressive force through the gusset plate. In some cases, the conventional design method specified in the Design Standard shows an unsafe-side error.[248] Further detailed investigation is expected.

Several papers concern the buckling of three-dimensional trusses near the base of a transmission tower. Main interest is concentrated on the instability phenomenon of a truss as a whole prior to the buckling of an individual member between panel points. Bifurcation analyses, second-order elastic analysis taking into consideration the non-linearity due to deflections, and some experiments have been conducted.[249,250] Although some understanding of the problem has been achieved, a practical design method remains to be investigated in the future.

NORTH AMERICA

Washington University: Lateral stability of light roof truss systems during construction.

Model tests and analytical studies are in progress.

WEST EUROPE

The problem of the ultimate load of trusses, taking account of the imperfections, is being investigated at the University of Liège (Belgium) and at the Swiss Federal Institute of Technology, Zurich (elastoplastic computer program); corresponding tests are presently being carried out in various places.

Experimental and theoretical studies are investigating, in a more refined way, the buckling strength of continuous legs and chords with panels of different length and increasing axial load (symmetrical and staggered bracing is considered). The effect of the eccentricity of the loads transmitted by the bracing to the chords and the effect of the eccentricity of the joints of the chords is also being studied. In order to obtain simpler connections (and lower costs) of the web members to the chords, more information is needed about the end restraints which can be adopted for the design of the web members. Extensive tests in this field are now being carried out, particularly in France (EDF, Paris) and in Great Britain (CEGB, Guildford), on full scale towers and on simplified structures.

EAST EUROPE

Effective length considerations need further experimental verifications in the inelastic range. Some pilot tests with simple triangles have been carried out. Theoretical and experimental work concerning effective length factors is needed, and partly carried out.

APPENDIX

APPENDIX A. REFERENCES

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APPENDIX B. NOTATIONS

Japan

A = Cross-sectional area
A_g = Mean value of cross-sectional area; total cross-sectional area of a member
a = Coefficient used in the proposed design formula
b = Coefficient used in the proposed design formula; distance between axes of the arch ribs
C_m = Reduction factor
f = Initial crookedness; rise of the arch
f_b = Allowable bending stress
f_c = Allowable compressive stress
f_t = Allowable tensile stress
H = Horizontal force
h = Column height
I_A = Moment of inertia of the arch rib
I_{AC} = Moment of inertia of the arch rib at the crown
I_B = Moment of inertia of beam
I_C = Moment of inertia of column
I_G = Moment of inertia of stiffening girder
i_{min} = Radius of gyration around the weakest axis
k = Stiffness ratio; ratio of the moment of inertia about the x-axis for out-of-plane buckling of web members to that of chord members
L = Distance between panel points; span length; length of the arch axis
l = Effective length; distance between panel points; unsupported length of member; span length of the arch
$l', \bar{l}$ = Distance between panel points
l_k = Effective length of compression member
M = Bending moment
M_{cro} = Bending buckling strength
M_p = Plastic moment
M_{pc} = Fully plastic moment influenced by axial thrust
M_r = Elastic limit moment
M_y = Yield moment
M_1, M_2 = End moments, the larger of which is M_1
N = Axial load
N_{cr} = Euler load; ultimate buckling strength under pure compression
N_e = Euler load in plane of bending
N_k = Ultimate buckling strength of column under pure compression
N_r = Yield axial load ($A\sigma_r$)
N_y = Yield axial force
N_1 = Larger compressive force
N_2 = Smaller compressive force, negative in tension
P = Axial force
P_y = Axial load at full yield condition
Q = Horizontal load
q = Distributed load
q_o = Load intensity which causes yielding at springings
r = Radius of gyration; radius of gyration about the axis perpendicular to the supporting axis
r_d = Radius of gyration to calculate the slenderness ratio
r_{min} = Radius of gyration about the weakest axis
S = Half-length of the arch axis
W_y, W_z = Section modulus about y- or z-axis, respectively
Z = Section modulus
β_y, β_z = Effective length coefficients
γ = Ratio of the load carried by hangers or posts
$\bar{\lambda}$ = Normalized slenderness ratio; effective slenderness ratio
λ_B = Slenderness ratio corresponding to the distance between the panel points at which lateral deflections are prevented
λ_s = Slenderness ratio corresponding to the distance between two panel points
ΣP = Total gravity load on a story
ΣV = Total story shear due to lateral loads in frame
σ_{ba} = Allowable bending stress
σ_{bc} = Compressive bending stress
${}_c\sigma_b$ = Compressive bending stress
σ_{bt} = Tensile bending stress
${}_t\sigma_b$ = Tensile bending stress
σ_c = Mean compressive stress
σ_{ca} = Allowable compressive stress
σ_{cr} = Critical stress taking the effects of web members and a tension chord member into account
σ_{crs} = Critical stress of a single member of the slenderness ratio λ_B obtained from the AIJ column curve
σ_r = Yield stress
σ_{rc} = Compressive residual stress
σ_t = Mean tensile axial stress
σ_{ta} = Allowable tensile stress
σ_y = Yield stress

North America

B = Flange width of W or I section
C_m = Equivalent moment factors used in the AISC Specification interaction formula
D = Depth of W or I section
e_x, e_y = Axial eccentricity
F_a = Axial stress that would be permitted for the column if axial force alone existed
F_b = Compressive bending stress that would be permitted for the beam if bending moment alone existed
F'_e = 12/23 of the Euler buckling stress in the plane of bending
f_a = Computed axial stresses
f_b = Computed compressive bending stress at the point under consideration
H = Story height
K = Effective length factor
L = Unbraced length of column
M_p = Plastic moment; plastic moment capacity for flanges
M_{pc} = Plastic moment capacity, reduced for the presence of axial load
M_u = Ultimate bending moment of a beam, reduced for the possible presence of lateral torsional buckling, if necessary
M_{uc} = Ultimate symmetrical single curvature moment capacity of an axially loaded beam-column
M_x, M_y = Bending moments; the greater of the applied moments at an end of the beam-column
M_1/M_2 = Ratio of smaller to larger end moments on unbraced segment
P = Axial force
P_u = Ultimate load capacity of an axially loaded column
P_y = Axial load at full yield condition
r = Radius of gyration
α = Exponent in interaction equation for strength of biaxially loaded beam-column
β = Exponent in equation for stability of biaxially loaded beam-column
Δ = Story drift

West Europe

A = Cross-sectional area
E = Young's modulus; longitudinal modulus of elasticity
e = Eccentricity
e^t = Parameter of imperfection
H_r = Story shear in story r due to external horizontal design load
h = Depth of section
I = Moment of inertia
I_x, I_y = Second moment of inertia
J = Second moment of plane area
K = Effective length factor
l = Length, member length; column length
M_D = Lateral buckling moment
M_{max} = Absolute value of M
M_r = Yield moment
M_{red} = Reduced bending moment
M_x, M_y = Bending moments; the greater of the applied moments at an end of the beam-column
N = Axial load
N_{cr} = Critical axial load
N_k = Limiting axial load
N_{pl} = Full plastic axial load
n = Number of columns in a story
P_e = Euler buckling load
Q_r = Story shear, including the effect of initial out-of-plumb
T = Buckling modulus
t = Thickness
V_r = Total vertical load in story r due to design load
W_{pl} = Plastic section modulus
W_x, W_y = Section modulus about the x- or y-axis, respectively
α = Exponent depending on shape
α_{cr} = Elastic critical load factor
α_{fl} = Failure load factor
α_{pl} = Rigid plastic load factor
β_{mx} = Coefficient which depends on the bending moment distribution
β_x, β_y = Moment reduction factor
ϵ = Characteristic member value, $l\sqrt{N/EI}$
$\bar{\lambda}$ = Nondimensional modified slenderness ratio
λ_k = Effective slenderness of column
μ = Ratio of the critical stress to the actual stress
ν = Coefficient for lateral-torsional buckling
σ = Stress
σ_k = Limiting stress
σ_r = Yield stress (steel); material yield stress
σ_u = Ultimate buckling strength
ψ_o = Initial column slope ("out-of-plumb")
ψ_r = Column slope in story r under design load

East Europe

A = Cross-sectional area
C_x, C_y = Correction factors for the case of moment gradient
E = Young's modulus
e = Eccentricity
J = Moment of inertia
K = Effective length factor
l = Member length
l_e = Effective length
M = Bending moment
M_{cr} = Elastic critical moment for lateral buckling of a perfect beam

N = Axial load
N_{Ey} = Euler load for weak-axis buckling
N_u = Limit axial load
$N_{u,ecc}$ = Peak load on eccentrically loaded column
N_w = Critical axial load on centrally compressed column for torsional buckling
n = Safety factor
R = Rigidity factor (J/L)
W_x, W_y = Section modulus about the x- or y-axis, respectively
β = Ratio of moments
γ_m = Factor to yield stress
δ = Correction factors for amplification factor
η = Reduction factor $(\sqrt{240/\sigma_y})$
η^x = Modification factor for simplified second-order analysis of extreme-fiber tensile stress
λ = Slenderness ratio
$\bar{\lambda}$ = Reduced slenderness ratio
λ_{eff} = Equivalent slenderness for lateral-torsional buckling $(\rho\lambda_y)$
$\mu = \dfrac{\pi^2 E}{\lambda^2}\dfrac{A}{N}$
$\sigma_{cr,e}{}^c$ = Elastic critical stress for buckling of a perfect column
$\sigma_{u,ecc}$ = Average stress at peak load on eccentrically compressed column
σ_{ul} = Limit extreme-fiber stress for lateral buckling
σ_{ux}, σ_{uy} = Limit stresses for strong-axis and weak-axis column buckling, respectively
σ_w = Allowable stress for column buckling
σ_{wl} = Allowable extreme-fiber stress for lateral buckling
σ_y = Yield stress
σ_y/γ_m = Design strength

8th Edition Manual Errata

The following errata will be corrected in the next press run (Sixth Impression) of the 8th Edition *Manual of Steel Construction.* For other errata, corrected in the Second, Third and Fourth Impressions, refer to the 2nd and 3rd Quarter 1980 and 2nd Quarter 1981 issues of *Engineering Journal*, respectively.

Pg. 3–43: The allowable load in kips for 6 × 4 nominal size rectangular tubing, 3/8-in. thick, effective length 16 ft, should read "63" instead of "53".

Pg. 4–29: In Example (a), second line of the solution, change "230 kips" to "222 kips".

In Example (b), first line of the second paragraph of the solution, change "76.6 kips" to "72.7 kips".

Pg. 4–31: In the first column of Table III, headed "Capacity, kips" for Weld A, the last 3 tabulated values should read:

61.8
49.5
37.1

instead of

80.2
64.2
48.1

Note: This error appeared *only in the Fifth Impression.*

Pg. 4–36: In the first column (capacity—Weld A), the tabular value second from the bottom (1/4-in. weld size) should read "97.2" instead of "92.2" (as incorrectly printed in the 4th and 5th Impressions) or "105" (as shown in the 1st, 2nd and 3rd Impressions).