Stability of Metal Structures—A World View

STRUCTURAL STABILITY RESEARCH COUNCIL
EUROPEAN CONVENTION FOR CONSTRUCTIONAL STEELWORK
COLUMN RESEARCH COMMITTEE OF JAPAN
COUNCIL OF MUTUAL ECONOMIC ASSISTANCE

PART B. APPROACHES AND DESIGN PROCEDURES

CHAPTER		CONTRIBUTORS			
No.	Title	Japan	North America	West Europe	East Europe
1	Compression Members	M. Wakabayashi	R. Zandonini, L. Tall	G. W. Schulz	O. Halasz, M. Ivanyi
2	Built-up Members	T. Suzuki	B. G. Johnston	L. Finzi	O. Halasz, M. Ivanyi
3	Beams	Y. Fukumoto	T. V. Galambos	J. Lindner	O. Halasz, M. Ivanyi
4	Plate and Box Girders	F. Nishino	A. Ostapenko	C. Massonnet	O. Halasz, M. Ivanyi
5	Beam-Columns	H. Akiyama	W. F. Chen	J. Strating	O. Halasz, M. Ivanyi
6	Frames	S. Morino	L. W. Lu	U. Vogel, C. Massonnet	O. Halasz, M. Ivanyi
7	Triangulated Structures	T. Nakamura	T. V. Galambos	P. Dubas, C. Urbano	O. Halasz, M. Ivanyi
8	Shells	S. Kobayashi	A. Chajes	D. Vandepitte	O. Halasz, M. Ivanyi
9	Composite Members	M. Wakabayashi	G. Winter	P. J. Dowling	—
10	Cold-formed steel	—	S. J. Errera	J. W. Stark	O. Halasz, M. Ivanyi

This comprehensive comparison of the current state of the art in research and design for structural stability, as viewed in four major regions of the world, is being published in serial form in the AISC Engineering Journal. It is expected eventually to be published in book form. This installment contains Chapters 2, 3 and 4 of Part B. The first installment, published in the 3rd Quarter 1981 issue, contained Part A and Chapter 1 of Part B, along with a List of Abbreviations and a Glossary of Terms covering the complete report.

Coordinating Editors:
Duiliu Sfintesco, CTICM, France
Lynn S. Beedle, Lehigh University, USA
Gerald W. Schulz, University of Innsbruck, Austria
Riccardo Zandonini, Technical University of Milan, Italy

Regional Editors:
Japan: Ben Kato, University of Tokyo
North America: T. V. Galambos, Washington University in St. Louis
West Europe: Duiliu Sfintesco, CTICM
East Europe: Otto Halasz, Technical University of Budapest

Chapter 2. Built-Up Members

a. REGIONAL RECOMMENDATIONS

JAPAN

(There are no regional recommendations in Japan.)

NORTH AMERICA

(Key Document: SSRC Guide[1])

Two chapters in the SSRC Guide (3rd Ed.) deal with built-up columns:

Chapter 12. Columns with Lacing, Battens, or Perforated Cover Plates

Chapter 13. Mill-Building Columns

Chapter 12 provides formulas (with varying degrees of empiricism) for evaluating the required reduction in allowable column stress to account for shear deformation in built-up laced or battened columns. The reduction in typical columns is only a few percent and is ignored in U.S. bridge and building specifications.

Chapter 12 also considers the limiting case of the battened column, where the battens act only as spacers and are ineffective in resisting column shear. Without end tie plates, the critical load for such a member is no greater than the sum of the pinned-end strength of the individual component elements. But with adequate end tie plates, the longitudinal components are forced to buckle in a modified second mode with a critical load that may approach four times that of the pinned-end mode. Charts and formulas are available for the calculation of the strength of such spaced columns.

The concept of the spaced column is relevant to mill-building columns, which are treated in Chapter 13 of the Guide. The column segment supporting the crane runway girders in a stepped mill-building column may be attached to the lower part of the building column by lacing or by ties which may be the equivalent of battens, or may simply act as spacers that force the two segments to deflect in the same shape. The importance of adequate diaphragms or tie plates at the top and bottom of such columns has been demonstrated by the spaced column studies. Chapter 13 also provides a detailed procedure for the overall design of a stepped mill-building column by means of an adaptation of the AISC specification for buildings.

WEST EUROPE

(Key Document: ECCS Recommendations[24])

Rules are given for centrally compressed built-up members made up of two equally cross-sectioned and parallel chords interconnected either by lacing or by batten plates. (See Fig. WE2.1.)

The stress analysis is made both for chords and connecting members, taking into account f_{max} (the geometrical second order effects.)

$$f_{max} = f_o/(1 - F^t/F_{cr}) \qquad \text{(WE2.1)}$$

$$f_o = f'_o + f''_o \qquad \text{(WE2.2)}$$

where f'_0 is the initial standard out-of-straightness (= $0.001L$) and f''_o takes care of mechanical imperfections (recent research indicates $f''_o = 0.001L$.)

$$N^t = F^t \qquad \text{(WE2.3)}$$

$$M^t = F^t \cdot f_{max} \qquad \text{(WE2.4)}$$

$$V^t = F^t \, \pi \, f_{max}/L \qquad \text{(WE2.5)}$$

where N^t is the design axial load, M^t is the design bending moment, and V^t is the design shear force. Each chord or lacing member has to be checked as a pin-ended simple strut whose length is the distance between the joints. Batten plates must be connected to the chords with not less than two rivets or bolts (turned or friction-grip).

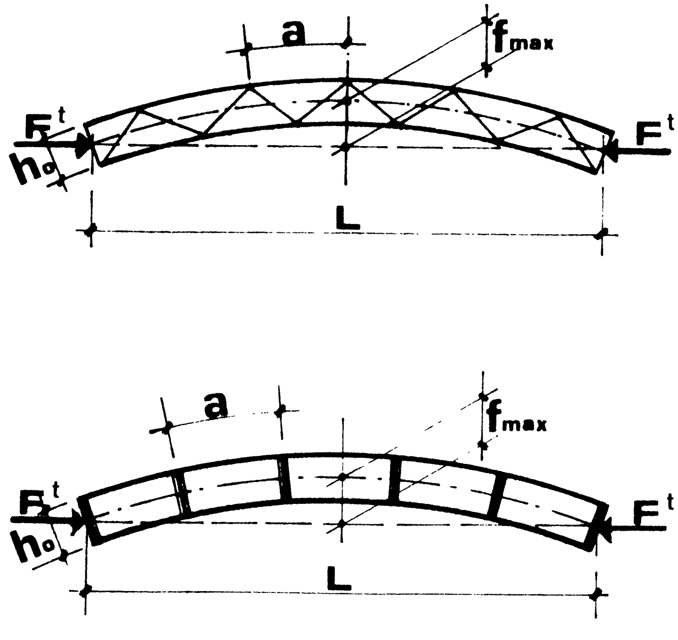

Figure WE2.1

EAST EUROPE

(Key Document: INCERC Design Guidelines for Steel)

Columns with batten plates:

$$\lambda_{eff} = \sqrt{\lambda_y{}^2 + \lambda_1{}^2} \quad ; \quad \lambda_1 = a_o/r_1 \qquad \text{(EE2.1)}$$

Formula is valid if

$$a_o/r_1 < 40 \quad \text{and} \quad J_{b2}\, a_o/J_1 b_o \geq 5$$

where J_1 and r_1 are the moment of inertia and radius of gyration of a single chord, respectively, and J_{b2} is the moment of inertia of batten plates around axis 2-2 (see Fig. EE2.1.)

Laced columns:

$$\lambda_{eff} = \sqrt{\lambda_y{}^2 + \frac{\pi^2 A/A_1}{2 \sin \alpha \cos^2\alpha}} \qquad \text{(EE2.2)}$$

where A and A_1 are the cross-sectional area of the built-up member and diagonal, respectively.

Design shear force:

$$V = 0.012A\ \sigma_y/\gamma_m$$

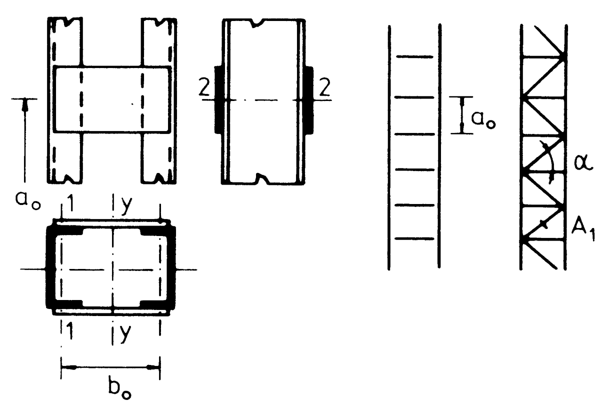

Figure EE2.1

b. SPECIFICATIONS AND CODES

JAPAN

AIJ SPECIFICATIONS

Slenderness Ratio of Built-Up Compression Members

1. Slenderness ratio about the solid web axis of built-up compression members shall be computed on the same basis as prescribed for compression members composed of single elements.
2. The computation of buckling of built-up compression members about their open web axis shall be made by use of an effective slenderness ratio that may be approximated by use of Eq. (J1) or Eq. (J2), as appropriate.

For $\lambda_1 > 20$:

$$\lambda_{ye} = \sqrt{\lambda_y{}^2 + \frac{m}{2}\lambda_1{}^2} \qquad (J1)$$

For $\lambda_1 \leq 20$:

$$\lambda_{ye} = \lambda_y \qquad (J2)$$

where

λ_y = slenderness ratio of built-up members assumed to resist as an integrated member
λ_{ye} = effective slenderness ratio
m = number of components or groups of components connected by lacing or tie plates

The value of λ_1 shall be computed by one of the following formulas as appropriate to the type of built-up members.

a. Members with Separators or Tie Plates:

$$\lambda_1 = l_1/i_1 \qquad (J3)$$

where

l_1 = spacing of separators or tie plates
i_1 = least radius of gyration of component

b. Latticed Members:

$$\lambda_1 = \pi\sqrt{\frac{A}{nA_d}\frac{l_d{}^3}{l_2 e^2}} \qquad (J4)$$

where

l_2 = length of lacing projecting on a plane parallel to the built-up member axis
l_d = length of lacing element
e = distance between axis of gravity of components
A = sum of sectional areas of column components
A_d = sectional area of lacing element, or sum of a pair of lacing elements in the case of double lacing

c. Members with Cover Plates Having Holes:

$$\lambda_1 = 1.7\sqrt{\frac{l_1}{p}}\cdot\frac{l_1}{i_1} \qquad (J5)$$

where

l_1 = length of hole
p = spacing of hole
i_1 = least radius of gyration of component

NORTH AMERICA

BRIDGE SPECIFICATIONS (AASHTO AND AREA)

Permitted types of steel have yield points ranging from 248 to 690 MPa.

Sides of members made up of batten plates are not permitted for compression members.

No reduction in allowable axial stress is specified to allow for reduced column strength caused by shear deformation in members with lacing or perforated cover plates.

Lacing bars are to be designed for the compression induced by the shearing force normal to the axis of the member.

In addition to the shear due to weight of member or external forces, a shear force is to be added to account for accidental load eccentricity and deflection as given by:

$$Q = \frac{F}{100}\left(\frac{100}{L/r + 10} + \frac{(L/r)\sigma_y}{22{,}754}\right)$$

where

Q = normal shear force, Newtons
F = allowable compressive axial load, Newtons
L = length of member, meters
r = radius of gyration about the axis perpendicular to the planes of lacing, meters

σ_y = specified minimum yield point of type of steel, MPa.

The bridge specifications provide explicit and detailed provisions governing the design and proportions of perforated cover plates and lacing bars in compression members.

CSA-S6: DESIGN OF HIGHWAY BRIDGES (1978)

Members with lacing bars, perforated cover plates, or battens are permitted. Battened members are allowed only when the member does not carry calculated bending moments in the plane of the battens. Detailed provisions for proportioning members with lacing or perforated cover plates are similar to those of the AASHTO Specification. The shear force to be added to that due to external forces and weight of member in the design of lacing is taken as 2.5% of the axial force in the member at working load.

Battens are required to resist simultaneously a longitudinal shear force

$$q = Qd/na$$

and a moment

$$M = Qd/2n$$

where

d = longitudinal distance center-to-center of battens

a = minimum distance between centroids of groups of fasteners or welds

Q = 2.5% of the total axial force in the member at working load

n = number of parallel planes of battens

BUILDING SPECIFICATIONS (AISC)

Requirements for built-up compression members are similar to those for bridge specifications and the same range of steel yield points are permitted (248 to 690 MPa). Major differences in comparison with bridge design include a lesser requirement for added shear force for design of lacing at 2% of the actual axial load and a less conservative limit for L/r of main member components between lacing bar connections, which may be as great as the governing slenderness ratio of the member instead of ⅔ of that amount. Members with battens are not permitted.

CSA-S16.1: STEEL STRUCTURES FOR BUILDINGS—LIMIT STATES DESIGN (1974)

Members with lacing bars, perforated cover plates, or battens are permitted. Requirements for proportioning perforated cover plates and lacing are similar to those of the AISC specifications for building design. Requirements for battens are similar to those of CSA-S6 (1978).

WEST EUROPE

Presently most of the National Codes ask for an elastic evaluation of the buckling load, by reference to an effective slenderness ratio:

$$\lambda_{eff} = (\lambda^2 + \lambda_1^2)^{1/2} \qquad \text{(WE2.6)}$$

where λ_1 is an auxiliary slenderness which takes care of the shear deformation.

Some Codes ask that the slenderness ratio of the individual sub-members do not go beyond a fixed value.

A conventional shear force of a few percent of the design axial force in the strut must be considered for checking the fasteners. New Codes are nevertheless being edited in Germany, Italy and Yugoslavia, largely in compliance with the ECCS Recommendations.[24]

An initial deflection $f_o = 0.002L$ is adopted for the second order theory approach.

EAST EUROPE

All specifications give similar design formulas, with slight differences in magnitude of design shear force V, mostly based on the formula

$$V = A \frac{\pi}{\lambda_{eff}} (\sigma_{u1} - \sigma_u) \qquad \text{(EE2.3)}$$

where σ_{u1} and σ_u are the limit buckling stresses of a single chord and the built-up member, respectively.

The **GDR** Specification requires:

$$V = N \frac{\pi}{\lambda_{eff}} \frac{\mu + \delta}{\mu - 1} e_o \qquad \text{(EE2.4)}$$

where $(\mu + \delta)/(\mu - 1)$ is the amplification factor [see Chap. 5., Eq. (EE5.24)] and e_o is the initial eccentricity [see Chap. 1., Eqs. (EE1.6) through (EE1.11).]

c. COLLOQUIUM CONTRIBUTIONS

JAPAN

No Japanese contribution was made at the Colloquium.

NORTH AMERICA

No North American contribution was made at the Colloquium.

WEST EUROPE

W. UHLMANN (LIEGE)[87]

When using ECCS rules, the value $f_o = 0.002L$ is verified as safe if compared with experimental results. A factor $\eta \leq 1$ is required to reduce the contribution to the total of the moments of inertia $2I_f$ of the chords when interconnected by batten-plates. The check of the single panels can be done according to plastic design rules.

G. BALLIO, L. FINZI, AND R. ZANDONINI (LIEGE)[88]

A numerical method for simulating the behavior of centrally compressed built-up struts is presented. The results for compact built-up members are compared with those obtained following ECCS rules. Collapse loads of the strut are in good agreement, while shear forces in the intermediate fasteners are sometimes overestimated.

J. SHORT (LIEGE)[89]

Two angles stitch-bolted together were tested in specially built equipment. To comply with the experimental results, an empirical formula is suggested to take care of the effect of the spacing between stitch-bolts.

$$\lambda_{eff} = (\lambda^2 + 0.5\,\lambda_1{}^2)^{1/2} \qquad \text{(WE2.7)}$$

with a conventional standard gap of 0.8 cm. The paper seems to be strictly related to the case of transmission towers.

R. ZANDONINI (LIEGE)[90]

For compact built-up members, a formula to evaluate the shear forces acting through the connectors is suggested and checked:

$$\begin{aligned} V^t{}_{tot} &= \sum_{i=1}^{n/2} V^t{}_i \\ &= 0.25\,\frac{h_o}{\rho_{tot}}\,\frac{f_o}{L}\,\frac{1}{1 - F^t/F_{cr}} \end{aligned} \qquad \text{(WE2.8)}$$

EAST EUROPE

J. MELCHER (BUDAPEST)[91]

Paper reports on more than 100 tests carried out to gain reliable data for the analysis of built-up columns consisting of two interconnected angles in torsion, buckling, and torsional-flexural buckling.

d. CURRENT STUDIES

JAPAN

The lateral stability of single span gabled frames and roof girders composed of built-up members subject to wind force or snow load is currently being investigated by Suzuki[92] and Kato.[93]

NORTH AMERICA

A limit states code is under development in Ontario, Canada, for the design of highway bridges.

A specification for Load and Resistance Factor Design (Limit States Design) is under development at Washington University, under direction of Dr. T. V. Galambos and sponsorship by AISI.

WEST EUROPE

Most of the research under way concerns built-up struts with batten plates (C.S.C.M. at the Politecnico of Milano, Italy and Technische Hochschule Darmstadt, Germany).

The evauation of the shear forces acting through the connectors still is somewhat of an open problem, especially for the ultimate design of compact built-up struts. Moreover, the present design rules lead to a safe design but not to the optimum one, as they require rather stiff connections. In some cases, it may be more economical to design the primary members at less than ultimate capacity, in order to permit lighter, less costly connections.

Because bolted fasteners with the standard 1 or 2 mm gap between bolts and holes are economical, the bolt slip effect on the bearing capacity of the built-up struts needs to be studied and design rules for such members are needed.

EAST EUROPE

Specifications are under development based on actual test results.

Chapter 3. Beams

a. REGIONAL RECOMMENDATIONS

JAPAN

(There are no regional recommendations in Japan.)

NORTH AMERICA

(Key Document: SSRC Guide[1])

Status of theory, research and specification practice is presented in Chapter 6 of the SSRC Guide (3rd Ed.). Beams are subject to the following stability limit states: (1) lateral torsional buckling, (2) flange local buckling, (3) web local buckling. This discussion covers lateral-torsional buckling.

WEST EUROPE

(Key Document: ECCS Recommendations[24])

STEEL STRUCTURES

The maximum bending stress σ due to the action of the applied design loads multiplied by the load enhancement factor must not exceed the limiting stress σ_D. See Fig WE3.1 and Eq. (WE3.1).

$$\sigma \leq \sigma_D \qquad \text{(WE3.1)}$$

where σ_D is obtained from

$$\sigma_D = \delta_r \cdot \alpha \cdot \sigma_r \qquad \text{(WE3.2)}$$

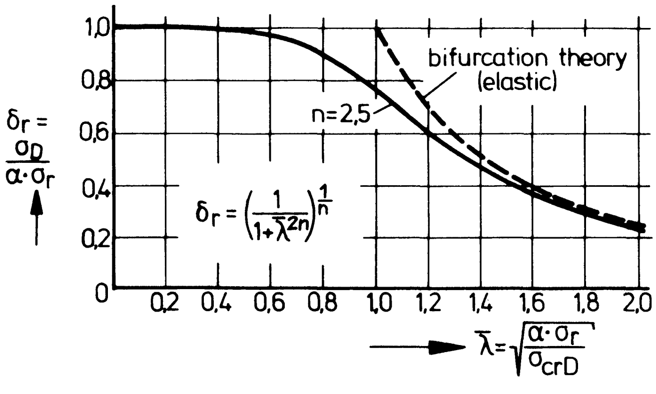

Figure WE3.1

and

δ_r = reduction factor
α = shape factor for major axis bending
σ_r = material yield stress

The reduction factor δ_r is obtained from Eq. (WE3.3).

$$\delta_r = \left(\frac{1}{1 + \bar{\lambda}^{2n}}\right)^{1/n} \qquad \text{(WE3.3)}$$

where n is the system factor; currently

$n = 2.5$
$\bar{\lambda}$ = a modified slenderness parameter
$= \sqrt{\alpha\sigma_r/\sigma_{crD}}$

Commentary—Equation (WE3.3) is based on the interpretation of theoretical studies in which allowance has been made for inelastic material behavior, residual stresses and initial geometrical imperfections, as well as test data. In establishing this curve, the following limitations were assumed:

1. The cross section is of I-shape and doubly symmetrical.
2. All eccentricities are accidental.
3. Distortion and local buckling are prevented.

The most important parameter is the system factor n. The choice of $n = 2.5$ is intended to ensure that the design curve corresponds to a mean value for the available test results, rather than a lower bound. At present no distinction is made between rolled and welded sections.

ALUMINUM STRUCTURES

The ECCS concept for Steel Structures is used. However, examination of the test results for aluminum beams has suggested that the system factor n be changed to $n = 2.0$.

EAST EUROPE

(Key Documents: INCERC Design Guidelines for Steel; CMEA Recommendations for Aluminum Structures)

STEEL

1. In case of laterally supported beams, check of lateral buckling is required if spacing l_s of lateral supports is

$$l_s > 40 r_y \sqrt{C_e}\, \eta \qquad \text{(EE3.1)}$$

where

r_y = radius of gyration of the compressed flange
$\eta = \sqrt{240/\sigma_y}$
σ_y = yield stress in MPa
$C_e = 1.75 - 1.05\beta + 0.3\beta^2$
= modification factor for non-uniform bending moment diagram (Fig. EE3.1)

Limit value σ_{ul} of the extreme fiber compression stress

$$\sigma_{ul} = M/W \qquad \text{(EE3.2)}$$

is taken equal to that of a centrally compressed column (column curve B) with slenderness ratio

$$\lambda_l = l_s / r_y \sqrt{C_e} \qquad \text{(EE3.3)}$$

2. In case of laterally unsupported beams, the analysis of lateral buckling is reduced to that of column buckling, using modified slenderness ratio

$$\bar{\lambda}_l = \sqrt{\sigma_y W / M_{cr}} \qquad \text{(EE3.4)}$$

where M_{cr} is the elastic critical moment of a perfect beam, depending on beam geometry, cross-sectional properties, restraints, and level of load application given by simple formulae based on Vlasov's theory.[94] Supposing the effect of plastic action and initial imperfections to be in the same range as centrally compressed elements, σ_{ul} is to be taken from column curves as a function of $\bar{\lambda}_l$.

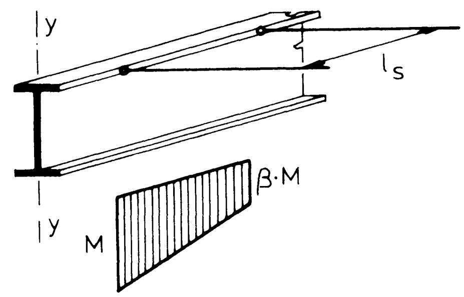

Figure EE3.1

3. In plastic design of continuous beams:

For $0.625 \le M_1/M_p \le 1$:

$l_s \le 35 r_y \eta$

For $-0.625 \le M_1/M_p < 0.625$:

$l_s \le (60 - 40\, M_1/M_p)\, r_y \eta$

For $-1 \le M_1/M_p < -0.625$:

$l_s \le 85 r_y \eta$

is required, where M_p and M_1 stand for bending moment at the cross section of plastic hinges and neighboring lateral supports, respectively.

ALUMINUM

Similar principles and formulae as for steel structures are adopted.

b. SPECIFICATIONS AND CODES

JAPAN

JRA SPECIFICATIONS FOR HIGHWAY BRIDGES (SEE FIG. J3.1)

Allowable stress design approach, in which the equation giving the allowable stress in the compression flange is:

$$\sigma_b = \frac{\sigma_y}{1.7}[1 - 0.412(\alpha - 0.2)] \qquad \text{for } \alpha > 0.2$$

where

$$\alpha = \frac{2}{\pi} K \frac{L}{b} \sqrt{\frac{\sigma_y}{E}}$$

$$K = \sqrt{3 + \frac{A_w}{2A_c}} \qquad \text{if } \frac{A_w}{A_c} > 2$$

$$= 2 \qquad \text{if } \frac{A_w}{A_c} \leq 2$$

but L/b is not greater than 30. L is the unsupported flange length (cm), b is the compression flange width (cm), A_w is the gross area of web and A_c is the gross area of compression flange.

Commentary—This formula is a modification of lateral buckling strength of simply supported I-beams under uniform moment. For unequal end moments, bending stress σ_b is increased by M/M_{eq}, with not greater than maximum allowable compression stress in bending, in which M is the bending moment in a designed section and $M_{eq} = 0.6M_1 + 0.4M_2$ but not less than $0.4M_1$, where $M_1 \geq M_2$.

JSCE SPECIFICATION FOR STEEL RAILWAY BRIDGES (SEE FIG. J3.2)

Allowable stress design approach, in which the equation giving the allowable stress in the compression flange is:

$$\sigma_b = \sigma_y/1.9 \qquad \text{for } 0 \leq \bar{\lambda}_e \leq 0.3$$

$$= (\sigma_y/F.S.)[1 - 0.449(\bar{\lambda}_e - 0.3)] \qquad \text{for } 0.3 \leq \bar{\lambda}_e \leq \sqrt{2}$$

$$= \frac{1}{2.8} \cdot \frac{\pi^2 E}{(L/r)_e{}^2} \qquad \text{for } \bar{\lambda}_e \geq \sqrt{2}$$

where

$$\bar{\lambda}_e = (1/\pi) \cdot \sqrt{\sigma_y/E} \cdot (L/r)_e$$

$$(L/r)_e = F(L/b)$$

$$F = \sqrt{12 + 2\beta/\alpha} \qquad \text{(for I-beam)}$$

$$= 1.3\sqrt{3\alpha + \beta} \cdot \sqrt{b/L} \qquad \text{if } \alpha \leq 2 \text{ (for box)}$$

$$= 1.3\sqrt{6 + \beta} \cdot \sqrt{b/L} \qquad \text{if } \alpha > 2 \text{ (for box)}$$

α = (flange thickness)/(web thickness)

β = (beam height)/b

L = unsupported length of compression flange (cm)

b = flange width (cm)

Commentary—This formula is a modification of lateral buckling strength of simply supported beams under uniform moment. The factor of safety varies from 1.9 to 2.8 between $\bar{\lambda} = 0.3$ and $\sqrt{2}$.

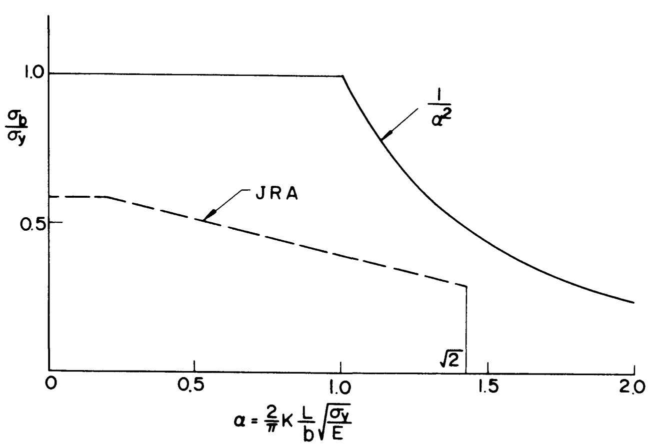

Figure J3.1

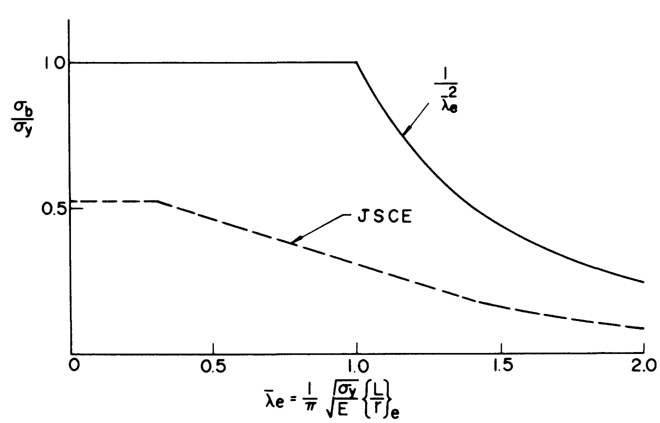

Figure J3.2

AIJ STANDARD FOR STEEL STRUCTURES

Allowable Stress Design—Allowable stress for rolled shapes and plate girders with H-section subjected to strong-axis bending is the larger of the following two equations and also shall not exceed allowable tensile stress f_t.

$$f_b = \left[1 - 0.4 \frac{\lambda_b{}^2}{C\Lambda^2}\right] f_t \qquad \text{(J3.1)}$$

$$f_b = \frac{0.429E}{(l_b h)/A_f} \qquad \text{(J3.2)}$$

where

$f_t = \sigma_y/1.5$ (for long-term loading)
$= \sigma_y$ (for short-term loading)
σ_y = minimum specified yield point
E = modulus of elasticity
l_b = unsupported length of a compression flange
i = radius of gyration of T-section composed of compression flange and one-sixth of web about web axis
$\lambda_b = l_b/i$
$\Lambda = \sqrt{\pi^2 E/(0.6/\sigma_y)}$
h = height of section
A_f = sectional area of compression flange
$C = 1.7 - 1.05(M_2/M_1) + 0.3(M_2/M_1)^2 < 2.3$

M_1 and M_2 are end moments, the larger of which is M_1. (M_2/M_1) is taken positive under single curvature bending and negative under double curvature bending. When the intermediate moment is greater than M_1, C-value should be unity.

When the beam has no possibility of lateral buckling as in the case of an H-section beam bent about weak axis of the section, f_b shall be f_t.

Plastic Design—Moment capacity of rolled shapes and plate girders with H-section subjected to strong-axis bending is specified as:

For $0 \le l_b h \sigma_y/A_f E \le 0.342$:

$$M_{cr} = M_p/C_m$$

For $0.342 \le l_b h \sigma_y/A_f E \le 1.143$:

$$M_{cr} = \frac{M_p}{C_m}\left[1 - 0.625\left(\frac{l_b h \sigma_y}{A_f E} - 0.342\right)\right] \qquad \text{(J3.3)}$$

For $1.143 \le l_b h \sigma_y/A_f E$:

$$M_{cr} = \frac{M_p}{C_m}\frac{0.571 A_f E}{l_b h \sigma_y}$$

where

M_p = fully plastic moment
$C_m = 0.6 + 0.4(M_2/M_1) \ge 0.4$

Commentary—Equation (J3.1) represents flexural resistance of compression flange to lateral instability and takes almost the same expression as the column curve applicable to short columns stressed beyond the elastic limit.

Equation (J3.2) indicates torsional resistance of beams and practically applies to long beams. Both equations are originally applicable to uniformly bent beams and their use is extended to general cases by introducing modification factor C.

Equation (J3.3) is based on the elastic analyses and test data of uniformly bent beams.

NORTH AMERICA

AISI SPECIFICATION FOR THE DESIGN OF COLD-FORMED STEEL STRUCTURAL MEMBERS (SEE FIG. NA3.1)

For $\sqrt{M_y/M_e} \le 0.6$:

$$M_{cr}/M_y = 1.0$$

For $0.6 \le \sqrt{M_y/M_e} \le 1.342$:

$$M_{cr}/M_y = 1.1111\,(1 - 0.278 M_y/M_e)$$

For $\sqrt{M_y/M_e} \ge 1.342$:

$$M_{cr}/M_y = M_e/M_y = \frac{\pi^2 E d I_{yc} C_b}{L_b{}^2 S_x \sigma_y}$$

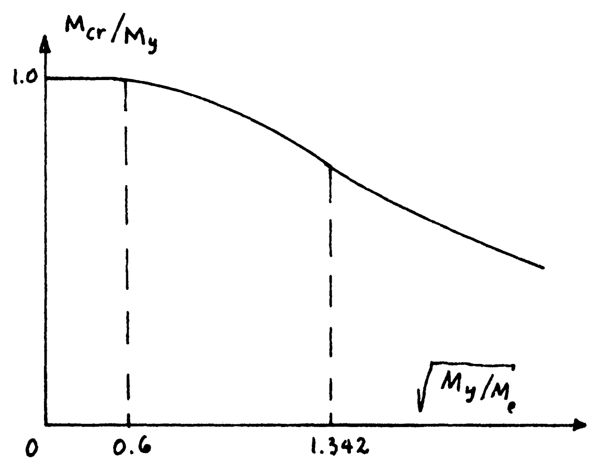

Figure NA3.1

CSA STANDARDS S16 AND S16.1: STEEL STRUCTURES FOR BUILDINGS

Compact Shapes:

For $\sqrt{M_p/M_e} \leq 0.683$:

$M_{cr}/M_p = 1.0$

For $0.683 \leq \sqrt{M_p/M_e} \leq 1.225$:

$M_{cr}/M_p = 1.15\,(1 - 0.28 M_p/M_e)$

For $\sqrt{M_p/M_e} \geq 1.225$:

$M_{cr}/M_p = M_e/M_p$

where

M_e = elastic lateral-torsional critical moment according to theory, as required by loading, geometry and end conditions.

Non-compact Shapes: Replace M_p by M_y .

AASHTO STANDARD SPECIFICATIONS FOR HIGHWAY BRIDGES (SEE FIG. NA3.2)

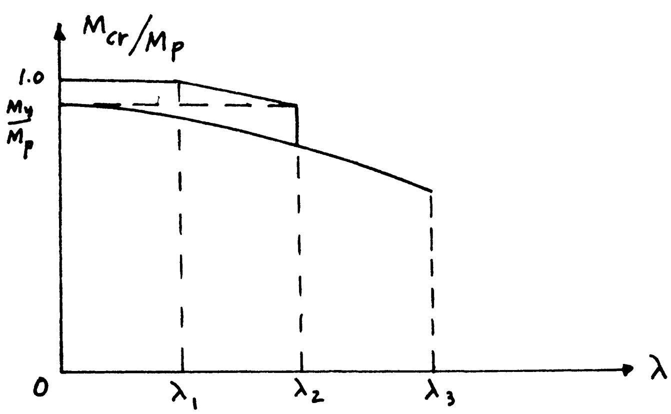

Figure NA3.2

For $\lambda \leq \lambda_1$:

$M_{cr}/M_y = 1 - \sigma_y/4\sigma_e$

except that for compact shapes

$M_{cr}/M_y = 1.0$ for $\lambda \leq \lambda_1$.

For $\lambda_1 \leq \lambda \leq \lambda_2$ there is a linear transition from $M_{cr} = M_p$ when $\lambda = \lambda_1$ to $M_{cr} = M_y$ when $\lambda = \lambda_2$.

$\lambda = L_b/(b/\sqrt{12})$
$\lambda_1 = 581 \sqrt{12}\, r_y/\sqrt{\sigma_y}\, b$ for uniform moment
$\lambda_2 = 996 \sqrt{12}\, r_y/\sqrt{\sigma_y} b$ for moment gradient
$\lambda_2 = 1.379 \times 10^8 \sqrt{12}\, t/\sigma_y d$
$\lambda_3 = 567 \sqrt{12}/\sqrt{\sigma_y}$
$\sigma_e = \pi^2 E/\lambda^2$

AISC SPECIFICATION FOR THE DESIGN, FABRICATION AND ERECTION OF STRUCTURAL STEEL FOR BUILDINGS (SEE FIG. NA3.3)

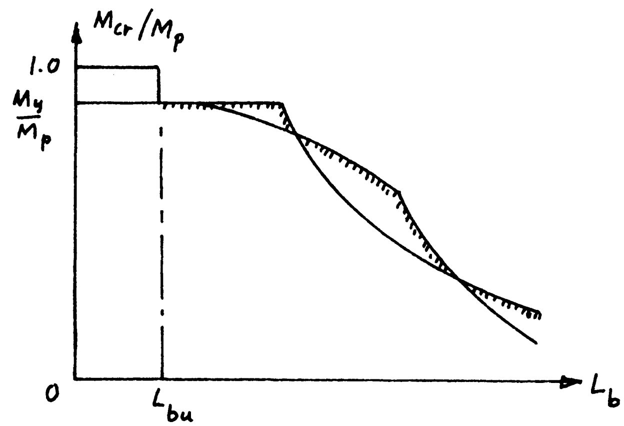

Figure NA3.3

M_{cr}/M_y is the larger value of:

1. For $L_b/r_T \leq 1875 \sqrt{c_b}/\sqrt{\sigma_y}$:

   $1.111\,[1 - 0.281\,(\sigma_y/\sigma_{e1})] \leq 1.0$

   For $L_b/r_T \geq 1875 \sqrt{c_b}/\sqrt{\sigma_y}$:

   σ_{e1}/σ_y

   where $\sigma_{e1} = \pi^2 E C_b/(L_b/r_T)^2$

2. $\sigma_{e2}/\sigma_y \leq 1.0$

   where $\sigma_{e2} = 0.690\ E C_b/(L_b d/bt)$ except that for compact shapes when $L_b \leq L_{bu}$:

   $M_{cr}/M_p = 1.0$

   where L_{bu} is the *smaller* of $200 b_f/\sqrt{\sigma_y}$ and $137{,}900 bt/d\sigma_y$.

AISC PROPOSED L.R.F.D. SPECIFICATION FOR BUILDINGS (SEE FIG. NA3.4)

For $\lambda < \lambda_u$: Plastic design permitted

For $\lambda \leq \lambda_1$: $M_{cr}/M_p = 1.0$

For $\lambda_1 \leq \lambda \leq \lambda_2$:

$$M_{cr}/M_p = C_b \left[1 - \left(1 - \frac{M_r}{M_p}\right)\left(\frac{\lambda - \lambda_1}{\lambda_2 - \lambda_1}\right)\right]$$

For $\lambda \geq \lambda_2$: $M_{cr}/M_p = M_e/M_p$

where

$\lambda = L_b/r_y$
$\lambda_1 = 788/\sqrt{\sigma_y}$

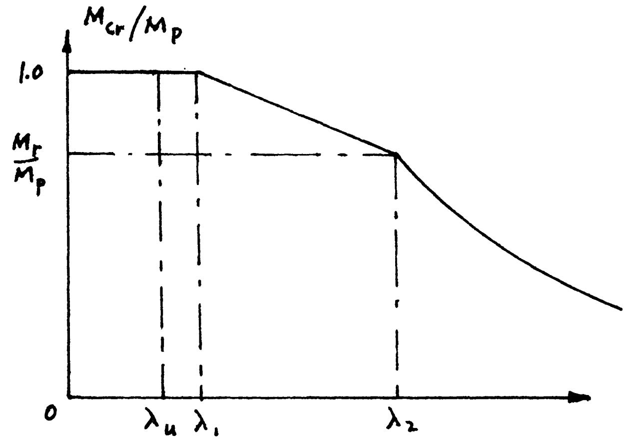

Figure NA3.4

λ_2 = the value of λ for which $M_e = M_r$ when $C_b = 1.0$

$\lambda_u = (24{,}800 + 15{,}200\, M_1/M_p)/\sigma_y$

$M_r = S_x\,(\sigma_y - \sigma_r)$

M_e = elastic lateral-torsional critical moment according to theory, as required by loading, geometry and end conditions

Commentary—North American design specifications for beams are based on the case of the simply supported beams, generally using simplified expressions. These formulae are usually conservative for other loading and boundary conditions.

WEST EUROPE

France
The French Recommendations use the ECCS concept, but modify the system factor n to:

Rolled sections: $n = 2.0$
Other beams: $n = 1.5$

Federal Republic of Germany (DIN 4114—Draft 1979)
The ECCS concept is used with $n = 2.5$ for both rolled and other sections. It is the intention to establish a different system factor n for monosymmetric sections.

Switzerland (SIA 161, New)
The ECCS concept is used, with $n = 2.25$. This system factor was chosen to be smaller than that given in the ECCS Recommendations because very deep beams, as used for bridge girders, are also to be designed by this method. For this case, further restrictions are placed on the elastic critical moment M_{crD}.

Great Britain (Draft 1977 to replace BS449)
The $\bar{\lambda}$ concept is used, but with a design curve based on a different type of formula. Generally this gives lower results than Eq. (WE3.3). A distinction is made between rolled and welded sections. Simplified methods for determining $\bar{\lambda}$, e.g., for monosymmetric sections, are included.

Yugoslavia (JUS.U.E7.101, 1978)
The ECCS concept is directly used.

Norway (NS3472E, 1973)
The $\bar{\lambda}$ concept is used, but the reduction factor δ_r shall be used for buckling on the y-axis.

EAST EUROPE

LATERALLY SUPPORTED BEAMS

Design rules in the **Rumanian, Hungarian,** and **German Democratic Republic** specifications are similar to those in the INCERC Guidelines, except that:

In the Hungarian specifications: $\sigma_{ul} = 1.1\sigma_u$

In the GDR specifications: $\sigma_{wl} = 1.13\sigma_w$

where σ_{ul} and σ_{wl} are limit (allowable) stresses for lateral buckling, σ_u and σ_w are limit (allowable) stresses for column buckling. Compression flange is defined by the **Hungarian** and **GDR** specifications according to Figs. EE3.2a and EE3.2b, respectively.

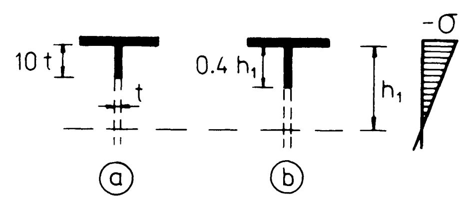

Figure EE3.2

LATERALLY UNSUPPORTED BEAMS

Design rules in **Rumanian** and **Hungarian** specifications are the same as described in the Regional Recommendations, except that the Hungarian specifications apply a factor of 1.1 to column limit stress, resulting in interaction formula (EE3.5):

$$\frac{\sigma_{ul}}{\sigma_y} + \frac{1.18\gamma_m}{1.625}\frac{\sigma_{ul}}{\sigma_y}\frac{\sigma_{ul}}{\sigma_E} + 1.3\frac{\sigma_u}{\sigma_E} = \frac{1.1}{\gamma_m} \qquad \text{(EE3.5)}$$

where σ_E = Euler stress and σ_y/γ_m = design stress.

On the other hand, **Soviet, Czechoslovakian** and **Polish** specifications define:

$$\sigma_{ul} = (\sigma^c_{cr}/\sigma_y) \cdot (\sigma_y/\gamma_m) \qquad \text{(EE3.6)}$$

where σ^c_{cr} is the critical stress of a perfect column, considering the effect of initial imperfections to be of minor importance as compared to centrally compressed members.

Reduction due to plasticity is taken into account according to Fig. EE3.3.

The **Soviet** specification applies reduction factor $\sqrt{E_t/E}$ and derives the tangent modulus E_t from the Tetmâjer formula:

$$\sigma^c_{cr}/\sigma_y = 1.2 - 0.32\bar{\lambda} \qquad \text{(EE3.7)}$$

The **Czechoslovakian** specification is based on the stress-strain relation

$$\sigma = \sigma_p + (\sigma_y - \sigma_p)\tanh\frac{E\epsilon - \sigma_p}{\sigma_y - \sigma_p};$$

$$\sigma_p = 0.8\sigma_y \qquad \text{(EE3.8)}$$

with reduction factor E_t/E, resulting in

$$\sigma^c_{cr}/\sigma_y = 0.8 + 0.02\bar{\lambda}^{-2} + \sqrt{(0.8 - 0.02\bar{\lambda}^2)^2 - 0.6} \qquad \text{(EE3.9)}$$

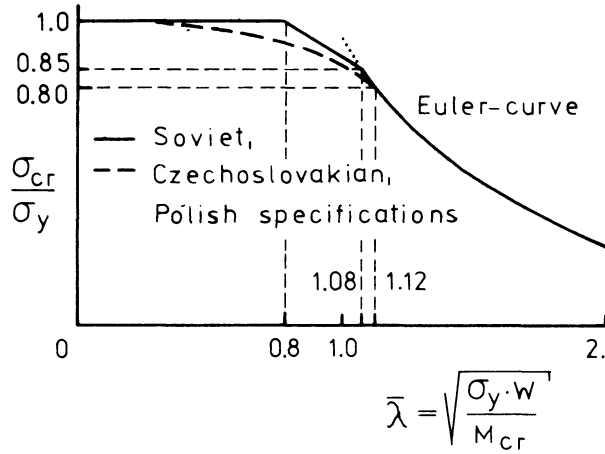

Figure EE3.3

Similarly, the **GDR** specification gives an allowable stress:

$$\sigma_{wl} = \sigma^c_{cr}/1.71$$

where σ^c_{cr} is calculated using the stress-strain relation of Eq. (EE3.8) and the reduced modulus theory.

Several simplified formulae for the elastic critical moment M_{cr} are given in the specifications.

Interaction formulae similar to those in the ECCS Recommendations are proposed in the drafts of new **Yugoslavian** and **GDR** specifications. The latter gives

$$\left(\frac{n\,\sigma_{wl}}{\sigma_y}\right)^a + \left(\frac{n\,\sigma_{wl}}{\sigma^c_{cr,e}}\right)^a = 1 \qquad \text{(EE3.10)}$$

$\sigma^c_{cr,e}$ being the elastic critical stress of a perfect beam. Safety factor n = 1.5, and a = 3.1 is suggested.

c. COLLOQUIUM CONTRIBUTIONS (COMMENTARIES)

JAPAN

Y. FUKUMOTO AND M. KUBO (LIEGE)[19]

An extensive survey has been made of the experimental studies on lateral buckling of beams. The form of the proposed design formula given in the Introductory Report (Liege) can well explain the test points for rolled and welded beams using the following n-values of system factor:

Rolled beams: n = 2.5 for mean and 1.5 for mean minus 2 × standard deviation ($m - 2s$)

Welded beams: n = 2.0 for mean and 1.0 for $m - 2s$

H. YOSHIDA (LIEGE)[95]

The following equation provides the ultimate bending moment of laterally unsupported welded beams:

$$\frac{M_u}{M_p} = \left[\frac{1}{1 + \lambda^{1/2n} + \lambda^{2n}}\right]^{1/n}$$

where

$\lambda = \sqrt{M_p/M_E}$

M_E = theoretical elastic lateral buckling moment

n = system factor

The inelastic lateral buckling strength of I-sections containing welded type residual stresses are examined. Discussed items are: yield stress level, loading and support condition, and cross-sectional dimensions. The buckling curves are proposed in the figure for welded beams of universal mill plates.

Y. FUKUMOTO AND M. KUBO (WASHINGTON)[20]

This review is intended to collect information concerning the experimental strength of laterally supported and unsupported beams and girders. Main emphasis is on review of the Japanese papers that have been published in this area. The surveyed results are summarized in tables in which all the reviewed test beams are listed and the corresponding reference moments are calculated unless they were given in the literature. The surveyed test points are also plotted on the existing proposed moment-slenderness coordinates for comparison with the proposed design formulas.

H. AKIYAMA AND B. KATO (TOKYO, LIEGE)[13,14]

The critical fibre stress for H-section beams subjected to strong-axis bending is given as

$$\sigma_{cr} = C(\lambda) \qquad (J3.4)$$

where

$\lambda = \lambda_f/\sqrt{\alpha c f}$
C = column curve function
$f = \left\{1 + \left[2\left(\frac{t_f}{h}\right)^2 + \frac{h}{B}\left(\frac{t_w}{h}\right)^2\left(\frac{t_w}{t_f}\right)^2\left(\frac{2G}{3E}\right)\right]\right\}^{1/2}$
$c = Bt_f/(Bt_f + ht_w/6)$
λ_f = slenderness of flange $= l/(B/2\sqrt{3})$
l = unbraced length of flange
B = width of flange
h = height of section
t_f = thickness of flange
t_w = thickness of web
G = shear modulus
α = magnification factor $= {}_e\alpha_{cr}/{}_e\alpha_{cro}$
${}_e\sigma_{cro}$ = elastic buckling stress of simply supported beam under uniform bending
${}_e\sigma_{cr}$ = elastic buckling stress under practical conditions

The lateral buckling of an H-section was recognized as the flexural buckling of compression flange modified more or less by the torsional resistance of the beam and Eq. (J3.4) is similar to the column curve adopted by the AIJ Specification, in which magnification factor is introduced based on numerical analyses for elastic beams under various practical conditions.

T. SUZUKI, T. ONO, AND I. KUBODERA (LIEGE)[96]

The critical stress of simply supported H-section beams under uniform moment is expressed as

$$\sigma_{cr} = C(\lambda) \qquad (J3.5)$$

where

$\lambda = \lambda_1/\{1 + [0.2t_f\, l/(Bh)]\}$
$\lambda_1 = l/i$
= slenderness of T-section composed of compression flange and one-sixth of web about web axis
C = column curve function
h = height of section
B = width of flange
t_f = thickness of flange
l = unbraced length of flange

Column analogy was shown to be applicable to lateral instability of beams. Equation (J3.5) is similar to the column curve adopted in the AIJ Specification, in which the slenderness of column is replaced by the equivalent slenderness of compression flange.

NORTH AMERICA

G. F. FOX (TOKYO)[97]

This is essentially an abbreviated restatement of the paper by Galambos (see below).

T. V. GALAMBOS (LIEGE)[98]

This contribution presents an update of the status of research beyond that reported in Chapter 6 of the SSRC Guide, with 47 new references cited. The paper ends with a list of research needs.

D. NIXON AND P. F. ADAMS (LIEGE)[99]

A simple formula is derived for the unbraced length of beams in industrial buildings with cantilever overhangs. This paper is useful for the designer.

A. J. REIS AND J. ROORDA (LIEGE)[100]

Elastic stability of imperfect thin-walled beams subject to interaction between lateral-torsional and local buckling is studied using a generalized Rayleigh-Ritz analysis along the imperfect equilibrium path. Numerical and experimental results are presented.

A. HASEGAWA (LIEGE)[101]

A justified criticism of North American beam design practice is presented. It is demonstrated that the simplistic approaches lead to contradictory criteria.

D. P. DU PLESSIS (WASHINGTON)[102]

Experimental and analytical results are presented on the problem of lateral stability of beams with end notches. This is a welcome contribution on a subject in which very little was known.

S. T. WANG AND R. S. WRIGHT (WASHINGTON)[103]

This is another excellent paper on the torsional-flexural buckling of locally buckled beams, using the finite-element method.

Commentary—The congress contributions presented (a) some solutions to problems often encountered in practice, and (b) a good start on the practical solution to the problems of combined local and lateral-torsional buckling.

WEST EUROPE

D. A. NETHERCOT AND J. C. TAYLOR (LIEGE)[104]

The usefulness of a modified slenderness $\bar{\lambda}_{LT}$ as it is used in the ECCS concept, was discussed.

J. LINDNER AND D. BAMM (LIEGE)[105]

Theoretical results were presented which showed that the influence of variations in yield stress over the cross section may be neglected. See Fig. WE3.2.

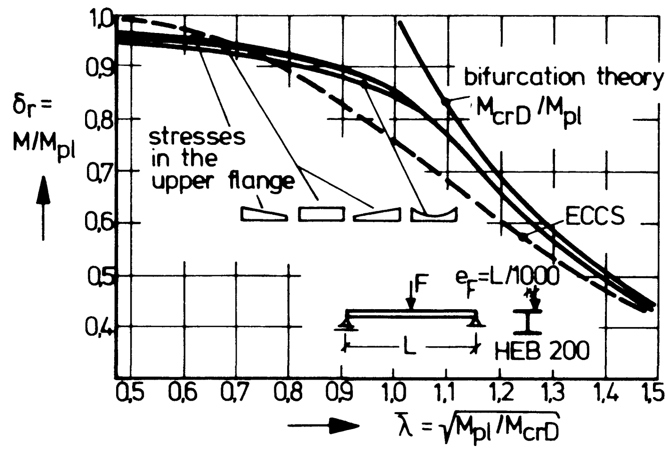

Figure WE3.2

J. OXFORT (LIEGE)[106]

On the basis of theoretical studies of diaphragm-braced beams, he showed that uniform moment is not necessarily the worst case. The presence of certain favorable effects which are not normally allowed for was mentioned.

J. LINDNER (WASHINGTON)[107]

Test results were presented which revealed no noticeable difference between rolled and annealed beams. The tests also suggested that, for the case of concentrated load at midspan, the ECCS curve with $n = 2.5$ seems to be conservative. See Fig. WE3.3.

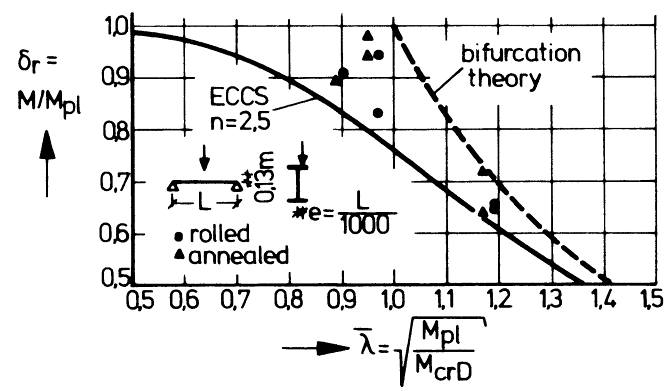

Figure WE3.3

B. W. YOUNG AND F. T. JARNOT (LIEGE)[108]

The authors presented the basis of an ultimate load method of analysis for beams with imperfections.

EAST EUROPE

H. E. GOEBEN (BUDAPEST)[109]

Paper deals with the lateral buckling of beams carrying cranes. According to GDR design rules, in addition to check of bifurcation of equilibrium under vertical load, a second-order elastic stress analysis is to be carried through under the effect of both vertical and horizontal forces. Differential equation of the problem is derived and computer solution and diagrams are presented for various cross sections and end restraints.

R. SOCHOR (BUDAPEST)[110]

Lateral buckling of beams (purlins) with restrained axis is dealt with, and diagrams are presented for I-beams loaded on the tension and compression flange, respectively, in case of free and prevented warping.

J. TUMA (BUDAPEST)[111]

Lateral buckling of laterally supported beams is investigated. The fictitious slenderness ratio

$$\lambda_l = \gamma \frac{l_s}{r_y}$$

is introduced (Fig. EE3.2) and the effect of non-uniform bending moment, continuity, and different types of lateral bracing is summarized in coefficient γ given by diagrams for I and T beams. Experiments with 8 different I-shapes showed good agreement with theory.

F. GYOKOS (BUDAPEST)[112]

Brief account of experimental investigation of lateral buckling of girders with tubular flanges and thin unstiffened webs is given.

d. CURRENT STUDIES

JAPAN

Stability problems of girders and beams for bridge structures are under investigation mainly at Nagoya University. They are: elastic lateral buckling of I-beams due to their own weight during lifting;[113] elastic and inelastic lateral buckling of U-shaped beams, where the combined effect of local and lateral buckling and the restraining effect of transverse beams are taken into account;[114] the effect of longitudinal stiffeners upon the lateral instability of plate girders;[115] the lateral buckling strength of parallel beams of highway bridges connected by cross beams.[116]

The effort in developing simplified design formulae of beams subject to lateral buckling under various loading and supporting conditions is being made by Wakabayashi[117] and Aoshima.[118]

NORTH AMERICA

Current studies tend to focus on the analysis of combined local and lateral-torsional buckling. This is especially important for cold-formed light gage beams, and it must be solidly supported by careful experiments. A need also exists for careful and thorough experiments to properly define the role of residual stresses and initial crookedness on inelastic lateral-torsional buckling; the magnitude and distribution of residual stresses in rolled and welded beam-type shapes needs to be statistically defined; and further tests on continuous beams are needed to help formulate realistic and accurate assumptions for the theoretical modeling of their behavior.

WEST EUROPE

Since several aspects of the lateral buckling problem remain unresolved, theoretical and experimental research is continuing.

Theoretical Studies

SEDLACEK/ECKSTEIN (Germany, Aachen): Influence of real construction practices on assumption of an isolated beam.
YOUNG/JARNOT (Great Britain, Brighton): Development of an ultimate strength method with particular emphasis on continuous beams.
GACHON/ENSAM (France, Paris): Calculation of the lateral buckling load of a beam in a structure, taking into account the real end conditions.

Experimental Studies

YOUNG/JARNOT (Great Britain, Brighton): A few tests for beams loaded by end moments.
LINDNER/KURTH (Germany, Berlin): Tests (~100) on single-span channel and C-sections, loaded by concentrated loads; major effect of deliberate load eccentricity.
LINDNER/GIETZELT (Germany, Berlin): Tests (~20) on monosymmetric I-sections loaded by concentrated loads and end moments.
LINDNER/SCHMIDT (Germany, Berlin): Tests (~50) on beams loaded by cross beams to study the effect of back twisting moments.
NETHERCOT (Great Britain, Sheffield): Tests (~30) on welded girders to study the effect of residual stresses and distortions.
DUBAS (Switzerland, Zürich): Tests on IPE-profiles to study the influence of unbraced length.

EAST EUROPE

Studies on lateral buckling of continuous beams under various loading conditions, on the effect of residual stresses, and on the response of special structures (as tapered beams) are in progress or planned for the near future.

Chapter 4. Plate and Box Girders

a. REGIONAL RECOMMENDATIONS

JAPAN

(There are no regional recommendations in Japan.)

NORTH AMERICA

(Key Document: SSRC Guide[1])

There are no unified recommendations specifically made for the North American region. The SSRC Guide gives a review of the current theories pertaining to the design of plate girders, but it does not consider box girders. The basic ultimate strength theory proposed by Basler[119] is reviewed there and the subsequent developments made by other researchers in various countries are briefly described. Some specific formulas, characteristic of a particular method, are presented, but no comprehensive value judgment of the methods against each other is made.

Since the greater accuracy and generality of the later proposed methods is achieved at the expense of greater complexity, the specification writing organizations in North America have all decided to adopt the simple, albeit not in all cases consistently conservative, method originally proposed by Basler. First, it was incorporated into the 1961 edition of the AISC Specification for buildings. In this approach, the shear strength is assumed to be a sum of the buckling and the diagonal tension field contributions. In the buckling strength evaluation the web plate is assumed to be simply supported at all four edges. Rigidity of the flanges is not taken into account.

At present, design of plate girders, both for buildings and highway bridges, is specified to be made for the ultimate strength rather than for the criteria of linear plate buckling. Only railroad bridges are designed for buckling.

WEST EUROPE

(Key Document: ECCS Recommendations[24])

Because Task Group VIII/3 was formed much later than the other ones, it has not yet been able to produce a whole set of recommendations. It must be recalled that the first goal assigned by the Task Group to its work was to prepare ultimate strength models for box girders as well as for plate girders. The Introductory Report to the Second International Colloquium on Stability gives an exhaustive description of these models and a comparison of the latter with respect to a series of appreciation criteria. A critical study of all the available models is now underway and a final conclusion on these topics may be expected around the end of the present year.

Nevertheless, in order to allow, in the meantime, calculations according to classical linear buckling theory, or design of constructional cases which cannot yet be covered by the ultimate strength models, the Task Group prepared provisional rules which are included as Annex to the second edition of the ECCS Recommendations; it must be kept in mind that such rules might disappear when more elaborate ultimate strength models, covering a sufficiently general domain, will be ready.

The only ultimate strength problem which can be considered as solved presently is the minimum width-to-thickness ratio b/t of structural elements subjected to axial compression, in order to prevent local buckling and thus any interaction between local and overall buckling. The minimum ratio is obtained from the condition

$$\sqrt{\frac{(\sigma_K \sigma_r)^{1/2}}{\sigma_{cr}}} < 0.8 \qquad \text{(WE4.1)}$$

The coefficient 0.8 is chosen in order to increase the wall thickness slightly above the value given by the "naive" theory of buckling, according to which the critical plate buckling stress is equated to the Euler column buckling stress. In the above equation, σ_K is the limiting stress, σ_r the yield stress, and σ_{cr} the critical stress, given by the linear plate buckling theory, is equal to

$$\sigma_{cr} = \alpha \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 \qquad \text{(WE4.2)}$$

Elementary transformations lead to

$$\frac{b}{t} \leq 0.760 \sqrt{\alpha} \sqrt{\frac{E}{(\sigma_K \sigma_r)^{1/2}}} \qquad \text{(WE4.3)}$$

This formula gives following results:

For one free edge, the other simply supported (angles), $\alpha = 0.426$:

$$\frac{b}{t} \leq 0.49 \sqrt{\frac{E}{(\sigma_K \sigma_r)^{1/2}}} \qquad \text{(WE4.4)}$$

For both edges simply supported, $\alpha = 4$:

$$\frac{b}{t} \leq 1.52 \sqrt{\frac{E}{(\sigma_K \sigma_r)^{1/2}}} \qquad \text{(WE4.5)}$$

When the value of the b/t ratio exceeds the recommended limits, the carrying capacity must be established by a scientific method.

EAST EUROPE

(Key Documents: INCERC Design Guidelines for Steel; CMEA Recommendations for Aluminum Structures)

STEEL

Two alternative methods are given for plate girder design:

1. Design based on linear buckling theory and follows the methods of the Soviet specifications (see "Specifications and Codes" below.)
2. An ultimate load design taking account both of the effect of initial imperfections and post-buckled behavior, following the pattern of the Czechoslovakian Specification (see below).

In the case of plastic design, maximum width-to-thickness ratios are given (Fig. EE4.1):

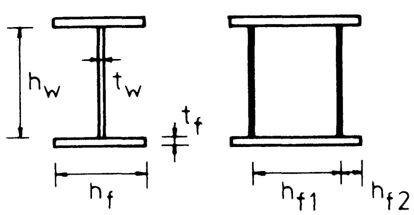

Figure EE4.1

For flanges of I-sections:

$h_f/t_f \leq 20\,\eta$

For flanges of closed sections:

$h_{f1}/t_f = 32\,\eta; \quad h_{f2} \leq 10\,\eta$

For webs:

If $0 \leq N/N_p \leq 0.75 A_w/A$:

$h_w/t_w \leq [67 - 33\,(A/A_w)(N/N_p)]\eta$

If $0.75 A_w/A \leq N/N_p \leq 1.0$:

$h_w/t_w \leq 42\,\eta$

where

N = axial load
$N_p = A\sigma_y/\gamma_m$
A = cross-sectional area
A_w = web area
$\eta = \sqrt{240/\sigma_y}$, with σ_y in MPa

ALUMINUM STRUCTURES

Analysis of web buckling is based on the classical linear buckling theory, using interaction formula for combination of compression (σ_A), bending (σ_B), and shear (τ):

$$\frac{\sigma_A}{\sigma_{Acr}} + \left(\frac{\sigma_B}{\sigma_{Bcr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2 = 1 \qquad \text{(EE4.2)}$$

Plastic reduction is taken into account by the fictitious slenderness ratio based on the equality

$$\frac{\pi^2 E}{\lambda_0^2} = k_{red} \cdot \sigma_{Ep} = k_{red} \frac{\pi^2 E}{12(1-\nu^2)} \frac{t^2}{h^2}$$

$$\lambda_0 = \frac{3.3}{\sqrt{k_{red}}} \frac{h}{t}$$

where k_{red} is derived from Eq. (EE4.2):

$$k_{red} \cdot \sigma_{Ep} = \frac{\sqrt{(\sigma_A + \sigma_B)^2 + 3\tau^2}}{\frac{\sigma_A}{2\sigma_{Acr}} \sqrt{\left(\frac{\sigma_A}{2\sigma_{Acr}}\right)^2 + \left(\frac{\sigma_B}{\sigma_{Bcr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2}}$$

and

$$(\sigma_A + \sigma_B)^2 + 3\tau^2 \leq \sigma_{up} \qquad \text{(EE4.3)}$$

is required.

Limit stress σ_{up} is given by the solid lines in the diagram in Fig. EE4.2, thus allowing a 15% reduction in safety in the elastic range.

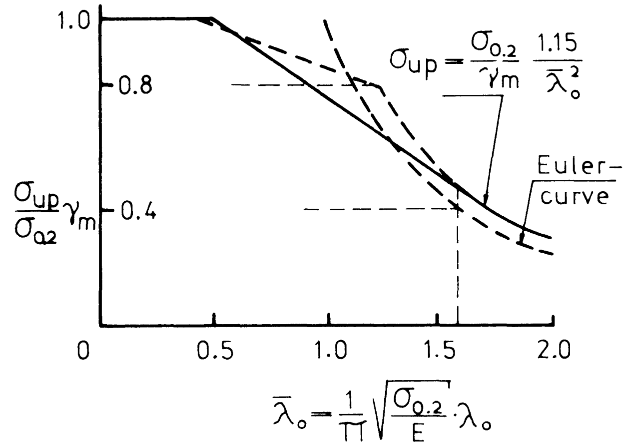

Figure EE4.2

b. SPECIFICATIONS AND CODES

JAPAN

JRA SPECIFICATIONS FOR HIGHWAY BRIDGES

Allowable stress design approach in which the equation giving the allowable stress for compression in extreme fibers is as follows:

For I and U cross sections:

$$\sigma_w = \frac{\sigma_r}{1.7} \quad \text{for } \lambda \leq 0.2 \qquad \text{(J4.1)}$$

$$\sigma_w = \frac{\sigma_r}{1.7}[1 - 0.412(\lambda - 0.2)] \quad \text{for } 0.2 < \lambda \leq \frac{4}{3} \qquad \text{(J4.2)}$$

where

$$\lambda = \frac{2}{\pi} k \frac{l}{b} \sqrt{\frac{\sigma_r}{E}} \qquad \text{(J4.3)}$$

$$k = 2 \quad \text{for } \frac{A_w}{A_c} \leq 2$$

$$= \sqrt{3 + \frac{A_w}{A_c}} \quad \text{for } \frac{A_w}{A_c} > 2$$

When moment distribution is not uniform within the supporting distance l, the allowable stress of Eq. (J4.2) can be increased by multiplying M/M_{eq}, where M is the moment at the design section and $M_{eq} = 0.6M_1 + 0.4M_2$, but not less than $0.4M_1$, M_1 and M_2 being moments at supporting points ($M_1 \geq M_2$).

For π and box sections:

$$\sigma_w = \frac{\sigma_r}{1.7} \qquad \text{(J4.4)}$$

Local buckling of component plates is prevented by limiting width-thickness ratio. Width-thickness ratio limitation for flange plates in compression is

$$\lambda \leq 0.7 \qquad \text{(J4.5)}$$

and for flange plates in compression stiffened with $(n - 1)$ equally spaced longitudinal stiffeners

$$\lambda \leq 0.7n \qquad \text{(J4.6)}$$

The moment of inertia of the longitudinal stiffener is

$$I_L = \frac{bt^3}{11}\gamma \qquad \text{(J4.7)}$$

where

$$\gamma = 4\alpha^2 n \left(\frac{t_0}{t}\right)^2 (1 + n\delta) - \frac{(\alpha^2 + 1)^2}{n} \qquad \text{(J4.8)}$$

when transverse stiffener of sufficient rigidity, I_Q, is provided within a_0. Otherwise,

$$\gamma = \frac{1}{n}\left\{\left[2n^2 \left(\frac{t_0}{t}\right)(1 + n\delta) - 1\right]^2 - 1\right\} \qquad \text{(J4.9)}$$

in which t = thickness of plate; t_0 = thickness determined by Eq. (J4.6); $\alpha = a/b$; a = distance between transverse stiffeners, b = width, $\delta = A_s/bt$, A_s = cross-sectional area of a longitudinal stiffener, and

$$I_Q = \frac{bt^3/11 + nI_L}{4\alpha^3} \qquad \text{(J4.10)}$$

$$a_0 = b\sqrt{2n^2\left(\frac{t_0}{t}\right)(1 + n\delta) - 1} \qquad \text{(J4.11)}$$

Table J4.1

No. of Horizontal Stiffeners \ σ_r(Pa)	240	320	360	460
0	$d/152$	$d/130$	$d/123$	$d/110$
1	$d/256$	$d/220$	$d/209$	$d/188$
2	$d/310$	$d/310$	$d/294$	$d/262$

Width-thickness ratio limitations for web plates are as shown in Table J4.1.

Intermediate vertical stiffeners shall be located to satisfy

$$\left(\frac{\sigma}{\sigma_{cr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2 \leq \left(\frac{1}{1.25}\right)^2 \qquad (\text{J4.12})$$

where

$$\sigma_{cr} = k_\sigma \sigma_e \qquad (\text{J4.13})$$

$$\tau_{cr} = k_\tau \sigma_e \qquad (\text{J4.14})$$

$$\sigma_e = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{d}\right)^2 \qquad (\text{J4.15})$$

$$k_\sigma = 23.9 \qquad (\text{J4.16})$$

$$k_\tau = 5.34 + \frac{4.00}{\alpha^2} \quad \text{when } \alpha > 1 \qquad (\text{J4.17a})$$

$$= 4.00 + \frac{5.34}{\alpha^2} \quad \text{when } \alpha \leq 1 \qquad (\text{J4.17b})$$

and $\alpha = a/d$, d being the distance between tensile and compression flanges. For webs with horizontal stiffeners, d is the distance between the tensile flange and the stiffener furthest from the compression flange. σ is the compressive stress at the location from which d is defined.

Moment of inertia of stiffeners is proportioned to satisfy Eq. (J4.7), where stiffness γ is, for intermediate vertical stiffeners,

$$\gamma = \frac{8}{\alpha^2} \qquad (\text{J4.18})$$

and, for horizontal stiffeners

$$\gamma = 30\alpha \qquad (\text{J4.19})$$

When the calculated stress is very small as compared with the allowable stress, the limitations of Eqs. (J4.5), (J4.6), (J4.11) and Table J4.1 can be increased by the following ratio:

$$(\text{allowable stress/calculated stress})^{1/2} \leq 1.2$$

Commentary—Equation (J4.2) is a lower bound fitting of the results of experimental studies for lateral torsional failure of welded built-up I-section beams. The parameter λ is derived from the formula for elastic lateral torsional buckling of a beam by neglecting St. Venant's torsional resistance.

Equation (J4.5) is specified based on experimental results on uniformly compressed plates of various boundary conditions, grade of steels, and of various manufacturing processes, and under the condition that plates will not lose stability until working stress reaches yield stress. Equation (J4.6) is an extension of Eq. (J4.5) for plate elements between longitudinal stiffeners of plates stiffened with sufficient rigidity and strength, which is specified in Eqs. (J4.7) through (J4.11). The moments of inertia of longitudinal and transverse stiffeners are specified under the condition that elastic overall buckling strength of stiffened plates become equal to elastic local buckling strength of component plates.

The width-thickness limitations of Table J4.1 for web plates are specified based on linear buckling analyses with variable factor of safety to adjust for post-buckling strength and with reduction of limiting values to adjust for the influence of residual stress and out-of-flatness. The factor of safety employed is the linear function of stress pattern with the values of 1.4 for pure bending and 1.7 for pure compression. The width-thickness ratios determined by the elastic buckling analysis is reduced, depending on the stress pattern, by multiplying a reduction factor. As this factor, a linear function of stress pattern is employed, with the values of 1.0 for pure bending and 0.8 for pure compression. The width-thickness limitations for webs with no horizontal stiffener are specified such that stability limit is reached when stress at compression flange reaches yield stress. The limitations for webs with horizontal stiffeners are specified under the additional condition that each component plate between stiffener and flange, or between stiffeners, simultaneously reaches the stability limit.

Stability of web plates under the action of normal and shear stresses are checked by an approximate interaction formula of Eq. (J4.12), with reduced factor of safety of 1.25 from the basic value of 1.7 to account for the larger amount of post-buckling strength. The stiffnesses as specified in Eqs. (J4.18) and (J4.19) are approximations of the values obtained by linear elastic analysis enforcing the stiffeners as nodes.

AIJ STANDARD FOR STEEL STRUCTURES

Width-thickness ratios for webs, and for parts of webs partitioned by vertical and horizontal stiffeners, of girders subjected to bending and compression are limited to satisfy:

$$\left(\frac{\sigma}{\sigma_{cr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right) \leq 1 \qquad (\text{J4.20})$$

where:

For $d/t \geq 56/C_1$:

$$\sigma_{cr} = \frac{1900}{(C_1\, d/t)^2} f_t \qquad (\text{J4.21a})$$

For $d/t < 56/C_1$:

$$\sigma_{cr} = (1.78 - 0.021 C_1\, d/t) f_t \leq f_t \qquad (\text{J4.21b})$$

For $d/t \leq 210/\sqrt{F/10}$:

$$\sigma_{cr} = f_t \qquad (\text{J4.21c})$$

under pure bending, and where:

For $d/t \geq 74/C_2$:

$$\tau_{cr} = \frac{3300}{(C_2\, d/t)} f_s \qquad (\text{J4.22a})$$

For $d/t < 74/C_2$:

$$\tau_{cr} = (1.74 - 0.0154\, C_2\, d/t) f_s \qquad (\text{J4.22b})$$

Other notations are as follows:

$C_1 = \sqrt{F/(10k_1)}$
$k_1 = (1 + \psi/6)(\psi^3 + 3\psi^2 + 4)$
$\psi = 1 - (\sigma_t/\sigma_c)$
σ_t = tensile stress at the edge
σ_c = compressive stress at the edge

where σ_t and σ_c are tensile and compressive stresses at the edges, resepctively; thus $\alpha = 0$ for uniform compression and $\alpha = 2$ for pure bending.

$C_2 = \sqrt{F/(10k_2)}$

where $k_2 = k_\tau$ of Eq. (J4.17) for webs with no horizontal stiffeners:

For $\alpha < 1$:

$$k_2 = 4.00 + \frac{5.34}{\alpha^2} + \frac{(n+1)^2\eta}{\alpha}\sqrt{\frac{8\mu}{3\alpha}} \qquad (\text{J4.23a})$$

For $\alpha \geq 1$:

$$k_2 = 5.34 + \frac{4.00}{\alpha^2} + \frac{(n+1)^2\eta}{\alpha}\sqrt{\frac{8}{3}\frac{\mu}{\alpha}} \qquad (\text{J4.23b})$$

$\eta = d_1/d$, where d_1 = smaller distance of width or depth of parts partitioned by vertical and horizontal stiffners; $\mu = 10.9\, I_L/dt^3$; I_L = moment of inertia of horizontal stiffener; and n = number of horizontal stiffeners, 1 or 2.

Moments of inertia of intermediate vertical stiffeners for webs with no horizontal stiffener are:

For $\alpha < 1$:

$$I \geq 1.1\, dt^3 \left(\frac{1}{\alpha^2} - 0.5\right) \qquad (\text{J4.24a})$$

For $\alpha \geq 1$:

$$I \geq 0.55\, dt^3 \qquad (\text{J4.24b})$$

and for webs with horizontal stiffeners:

For $\alpha' < 1$:

$$I \geq 1.1\, dt^3 \left[\frac{1}{(\alpha')^2} - 0.5\right] \qquad (\text{J4.25a})$$

For $\alpha' \geq 1$:

$$I \geq 0.5\, dt^3 \qquad (\text{J4.25b})$$

where α' is determined solving:

For $\alpha < 1$:

$$\frac{5.34}{(\alpha')^2} = \frac{5.34}{\alpha^2} + \frac{(n+1)^2\eta}{\alpha}\sqrt{\frac{8\mu}{3\alpha}} \qquad (\text{J4.26a})$$

For $\alpha \geq 1$:

$$\frac{4.00}{(\alpha')^2} = \frac{4.00}{\alpha^2} + \frac{(n+1)^2\eta}{\alpha}\sqrt{\frac{8\mu}{3\alpha}} \qquad (\text{J4.26b})$$

Radii of gyration, i, of horizontal stiffeners are found from:

$$i/t \geq C_m\, [135\,(0.5 - \eta)^3 + 3]\alpha^{2/3} \qquad (\text{J4.27})$$

$$C_m = 0.7 \frac{1}{200(n+1)} \frac{i}{t} \frac{1}{\delta} \qquad (\text{J4.28})$$

where

$$0.2 \leq \eta < 0.5 \text{ when } n = 1 \qquad (\text{J4.29a})$$

$$0.15 < \eta < 0.3 \text{ when } n = 2 \qquad (\text{J4.29b})$$

Radii of gyration of longitudinal stiffeners of webs subjected to bending and compression are found from:

For $\alpha < \left(\frac{2 + n^2}{1 + n}\right)$:

$$i/t \geq C_n\,(1 - 0.1\psi^2)(2 + 2n)\alpha^{2/3} \qquad (\text{J4.30a})$$

For $\alpha \geq \left(\frac{2 + n^2}{1 + n}\right)$:

$$i/t = C_n(1 - 0.1\psi^2)(4 + 2n^2) \qquad (\text{J4.30b})$$

where

$$C_n = 0.7 + \frac{1}{100(n+1)(1 + \psi/2)} \frac{i}{t} \frac{1}{\delta} \qquad (\text{J4.31})$$

Commentary—The width-thickness limitations to be checked by Eq. (J4.20) are based on linear buckling analysis. Equations (J4.21a) and (J4.22a) are approximate expressions of elastic buckling strengths applicable to buckling strength equal to or less than 60 percent of yield stress with factor of safety of 1.5. Equations (J4.21b) and (J4.22b) are straight line approximations of buckling strength to take into account inelastic action.

Moments of inertia for stiffeners are specified based on linear elastic buckling analysis, such that the stiffeners remain as nodes at the inception of buckling.

Although the principal method used in the regional specifications for design of plate girders and, to some extent, of box girders is the method proposed by Basler,[119] there are often variations in presentation and in some criteria.

PLATE GIRDERS

The basic dimensions of a typical plate girder panel are shown in Fig. NA4.1.

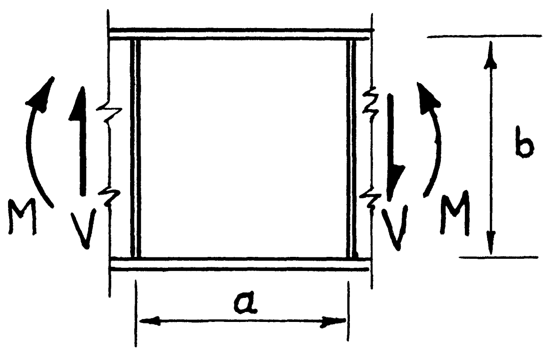

Figure NA4.1

t = web plate thickness
α = aspect ratio, a/b
β = slenderness ratio, b/t

AISC Specification for the Design, Fabrication and Erection of Steel for Buildings (1978)

This specification is for buildings, and only the allowable stress approach is used. However, the allowable stresses were obtained from the ultimate strength divided by a factor of safety.

Allowable shear stress:

$$\tau_a = \frac{\sigma_y}{2.89}\left(C + \frac{1 - C}{1.15\sqrt{1 + \alpha^2}}\right) \leq 0.4\,\sigma_y \qquad \text{(NA4.1)}$$

where

$$\left.\begin{aligned} C &= \frac{310{,}275}{\sigma_y \beta^2} k \quad \text{when } C < 0.8 \\ &= \frac{499}{\beta}\sqrt{\frac{k}{\sigma_y}} \quad \text{when } C > 0.8 \end{aligned}\right] \qquad \text{(NA4.2)}$$

$$\left.\begin{aligned} k &= 4.00 + \frac{5.34}{\alpha^2} \quad \text{when } \alpha < 1.0 \\ &= 5.34 + \frac{4.00}{\alpha^2} \quad \text{when } \alpha > 1.0 \end{aligned}\right] \qquad \text{(NA4.3)}$$

The end panels are designed for buckling only:

$$\tau_a = (\sigma_y / 2.89)\, C \qquad \text{(NA4.4)}$$

Allowable bending stress in the compression flange:

$$\sigma'_b = \sigma_b \left[1.0 - 0.005 \frac{A_w}{A_f}\left(\beta - \frac{1996}{\sqrt{\sigma_b}}\right)\right] \leq 0.6\,\sigma_y \qquad \text{(NA4.5)}$$

where

σ_b = basic allowable bending stress in tension, $0.6F_y$
A_w = area of the web, bt
A_f = area of one flange (the section is assumed to be symmetrical)

Required gross area of an intermediate transverse stiffener:

$$A_{st} = \frac{1 - C}{2}\left[\alpha - \frac{\alpha^2}{\sqrt{1 + \alpha^2}}\right]\frac{\sigma_{yw}}{\sigma_{yf}} D A_w \qquad \text{(NA4.6)}$$

where

σ_{yw} = yield stress of web
σ_{yst} = yield stress of stiffener
D = 1.0 for stiffeners in pairs
= 1.8 for single angle stiffener
= 2.4 for single plate stiffener

Depth slenderness limitation:

For girders without transverse stiffeners:

$$\beta_{max} = 5{,}252/\sqrt{\sigma_{yf}} \qquad \text{(NA4.7)}$$

When transverse stiffeners are used:

$$\beta_{max} = 96{,}530/\sqrt{\sigma_{yf}(\sigma_{yf} + \sigma_{rc})} \qquad \text{(NA4.8)}$$

where

σ_{yf} = yield stress of flange
σ_{rc} = residual compressive stress in flange

The interaction between shear and moment is defined by Fig. NA4.2, where τ_a and σ'_b are the allowable shear and bending stresses as given by Eqs. (NA4.1) and (NA4.5) for pure shear or bending, τ and σ are the allowable shear and bending stresses when the girder is subjected to combined action.

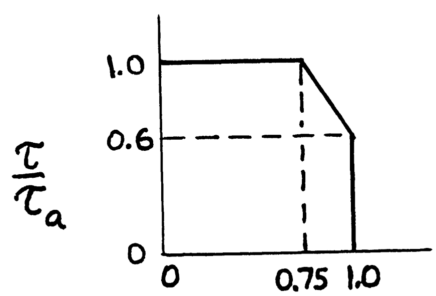

Figure NA4.2

Commentary—The above formulas were derived by Basler[119] and at first adopted only by the AISC Specification in its 1961 edition. Factor 2.89 in Eq. (NA4.1) transforms σ_y into the shear yield stress as well as incorporates a factor of safety of 1.67. Equation (NA4.5) takes into account the redistribution of bending stresses caused by web buckling. The depth limitation stipulated by Eq. (NA4.8) precludes the buckling of the web under the in-plane forces exerted by the flanges due to curvature bending.

AASHTO Standard Specifications for Highway Bridges (1978 Interim)

The AASHTO Specification offers both the allowable stress and the ultimate strength (load factor design) approaches. However, except for a few minor differences, the allowable stress approach represents simply a conversion from the ultimate strength to allowable stresses by using a constant factor of safety. Thus, only the ultimate strength formulation is discussed here.

Ultimate shear capacity of plate girders:

$$V_u = V_p \left[C + \frac{0.87(1 - C)}{\sqrt{1 + \alpha^2}}\right] \leq V_p \qquad \text{(NA4.10)}$$

where

$$V_p = 0.58\, \sigma_y A_w \qquad \text{(NA4.11)}$$

$$C = \frac{1490}{\beta} \sqrt{\frac{1 + 1/\alpha^2}{\sigma_y}} - 0.3 \leq 1.0 \qquad \text{(NA4.12)}$$

The ultimate moment capacity is limited by the yielding of a flange or the lateral-torsional buckling.

The interaction between moment and shear is provided by the relationship of Fig. NA4.2.

The maximum girder depth of symmetrical and unsymmetrical girders is controlled by:

$$\beta_c = \frac{b_c}{t} \leq \frac{1515}{\sqrt{\sigma_y}} \qquad \text{(NA4.13)}$$

where b_c is the portion of the web on the compression side of the centroidal axis. The girder slenderness ratio may be doubled by introducing a longitudinal stiffener at $0.4b_c$ from the compression flange. However, the increase in the shear strength due to the longitudinal stiffener is not taken into account.

An intermediate transverse stiffener shall have a gross area of at least

$$A = \left[0.15\, DA_w\, (1 - C) \frac{V}{V_u} - 18t^2\right] \frac{\sigma_{yw}}{\sigma_{yst}} \qquad \text{(NA4.14)}$$

where D, σ_{yw}, and σ_{yst} are defined as for Eq. (NA4.6) and the moment of inertia is not less than

$$I = at^3 J \qquad \text{(NA4.15)}$$

where

$$J = 2.5/\alpha^2 \leq 0.5 \qquad \text{(NA4.16)}$$

For longitudinally stiffened girders, b, as used in α and C of Eqs. (NA4.14) to (NA4.16) shall be the depth of the larger subpanel.

The longitudinal stiffener shall have the moment of inertia

$$I \geq A_w t^2\, (2.4\, \alpha^2 - 0.13) \qquad \text{(NA4.17)}$$

and the slenderness ratio

$$\frac{a}{r} \leq \frac{1909}{\sqrt{\sigma_y}} \qquad \text{(NA4.18)}$$

where r is the radius of gyration.

The shear in the end panel shall be:

$$V \leq 8275 \times 10^8\, (1 + 1/\alpha^2) \frac{t^3}{b} \leq V_p \qquad \text{(NA4.19)}$$

with $\alpha \leq 0.5$. (NA4.20)

Commentary—The ultimate strength method for shear computations has the same basis as the AISC Specification. Computation of the shear strength [Eqs. (NA4.10) to (NA4.12)] was made simpler than in the AISC Specification [Eqs. (NA4.1) to (NA4.3)] by replacing the two buckling coefficient formulas [Eq. (NA4.3)] with a single formula:

$$k = 5(1 + 1/\alpha^2) \qquad \text{(NA4.21)}$$

and incorporating it into other equations. Also, the more stringent web slenderness limitation, aimed at preventing the development of fatigue cracks due to the "breathing" of the web under repeated load application, made it reasonable to neglect the effect of the redistribution of bending stresses [Eq. (NA4.5)].

Canadian Specifications for Plate Girders

The Canadian plate girder specifications for buildings (CSA Standard S16.1-1974) and for highway bridges (CAN 3-S6-M78 and 1979 Revision of the Ontario Highway Bridge Code) are fundamentally the same as the AISC Specification. As in the AASHTO Specification, the web slenderness of bridge girders is limited to prevent fatigue cracks due to web "breathing." However, the redistribution of the bending stresses is not neglected.

AREA Specification (1976)

The AREA Specification applies to railroad bridge girders and the post-buckling web strength is neglected.

The shear strength is controlled by specifying the distance between transverse stiffeners to be:

$$a/t \leq 872/\sqrt{\tau} \qquad \text{(NA4.22)}$$

when

$$\beta = b/t > 60 \qquad \text{(NA4.23)}$$

where τ is the shear stress in the panel.

The maximum web slenderness is:

$$\beta_{max} = (1/170)\sqrt{\sigma_{ba}/\sigma} \qquad \text{(NA4.24)}$$

where

σ_{ba} = allowable bending stress
σ = actual bending stress

There are no specific area or moment of inertia requirements for intermediate transverse stiffeners given.

The bending strength is controlled by the lateral-torsional buckling requirements:

$$\sigma_{ba} = 138 - 0.00276\,(L/r_v)^2 \qquad \text{(NA4.25)}$$

where

L = unbraced length of compression flange
r_v = radius of gyration of the compression portion of the cross section about the web

Interaction between bending and shearing stresses is accomplished by keeping the combined diagonal tensile stress below the allowable stress for tension.

Commentary—The AREA Specification is the only holdout in the region in maintaining the buckling criteria for design of plate girder webs.

BOX GIRDERS

Current specifications of the region cover only the multiple-box, single-cell girders with a concrete or orthotropic deck. Since the provisions of the American (AASHTO) and the Canadian (CAN3) specifications differ only in the form of presentation and in the degree of rounding off some constants, the Canadian specification is discussed here as the latest version in the region.

CAN-3-S36-M78 (Highway Bridges)

Design of the webs is to be according to the rules for plate girders. The concrete deck requires no stability considerations and, thus, is not discussed here. The orthotropic deck is to be designed so that the "slenderness ratio of longitudinal ribs shall be adequate to ensure that overall buckling of the deck will not occur as a result of compression induced by bending of the longitudinal girders." No other stability criteria, except for the general local buckling width-thickness limitations, are imposed on the orthotropic deck.

The bottom flange is to be designed according to the following rules:

Unstiffened Compression Flanges:

a. When $w/t \leq 510/\sqrt{\sigma_y}$:

$$M_u = \sigma_y S \qquad \text{(NA4.26)}$$

where

M_u = ultimate moment capacity
w = flange width
S = section modulus

b. When $510/\sqrt{\sigma_y} < w/t \leq 1100/\sqrt{\sigma_y}$:

$$M_u = \sigma_{cr} S \qquad \text{(NA4.27)}$$

where

$$\sigma_{cr} = 0.592\,\sigma_y\left(1 + 0.687 \sin\frac{\pi c}{2}\right) \qquad \text{(NA4.28)}$$

$$c = (1100 - w/t\,\sqrt{\sigma_y})/590 \qquad \text{(NA4.29)}$$

c. When $w/t > 1100/\sqrt{\sigma_y}$:

$$M_u = \sigma_{cr} S \qquad \text{(NA4.30)}$$

where $\sigma_{cr} = 724{,}000/(w/t)^2$ (NA4.31)

Compression Flanges Stiffened Laterally:

a. When $w_s/t \leq 225/\sqrt{\sigma_y/k_1}$: (NA4.32)

$$M_u = \sigma_y S$$

where

w_s = plate width between longitudinal stiffeners
k_1 = buckling coefficient ≤ 4.0

b. When $255/\sqrt{\sigma_y/k_1} < w_s/t \leq 550/\sqrt{\sigma_y/k_1}$:
[Use Eqs. (NA4.27) and (NA4.28), but with

$$c = [550 - (w_s/t)\sqrt{\sigma_y/k_c}]/295 \qquad \text{(NA4.33)}$$

where $k_c = k_1$

c. When $w_s/t > 550\sqrt{\sigma_y/k_1}$:
Use Eq. (NA4.27), with

$$\sigma_{cr} = 180{,}000\,k_1/(w_s/t)^2 \qquad \text{(NA4.34)}$$

Each longitudinal stiffener shall have a moment of inertia of

$$I_s \geq z t^3 w_s \qquad \text{(NA4.35)}$$

where

$$z = 0.125\,k_1^3 \quad \text{for } n = 1 \qquad \text{(NA4.36)}$$
$$= 0.07 k_1^3 n^4 \quad \text{for } n > 1 \qquad \text{(NA4.37)}$$

n = no. of longitudinal stiffeners

A transverse stiffener of the size of the longitudinal stiffeners shall be placed near the dead load inflection points.

Compression Flanges Stiffened Longitudinally and Transversely:

a. When $w_s/t \leq 225/\sqrt{\sigma_y/k_2}$:

$$M_u = \sigma_y S \tag{NA4.38}$$

b. When $225/\sqrt{\sigma_y/k_2} < w_s/t \leq 550/\sqrt{\sigma_y/k_2}$:

$$M_u = \sigma_{cr} S \tag{NA4.39}$$

where σ_{cr} is from Eq. (NA4.28), with c from Eq. (NA4.33), and $k_c = k_2$.

c. When $w_s/t > 550/\sqrt{\sigma_y/k_2}$:

Use Eq. (NA4.39), with

$$\sigma_{cr} = 180{,}000\, k_2/(w_s/t)^2 \tag{NA4.40}$$

where

$$k_2 = \frac{[1 + (a/w)^2]^2 + 87.3}{(n + 1)^2(a/w)^2\,[1 + 0.1(n + 1)]} \leq 4.0 \tag{NA4.41}$$

with $a/w \leq 3.0$

The moment of inertia requirement for the longitudinal stiffeners is

$$I_s \geq 8t^3 w_s \tag{NA4.42}$$

and for the transverse stiffeners,

$$I_t \geq 0.055\,(n + 1)^3\, w_s{}^3 \frac{\sigma_{cr} A_f}{Ea} \tag{NA4.43}$$

where

E = modulus of elasticity
A_f = total area of the bottom flange

Bottom Tension Flange:

Shall have the effective width not greater than 0.2 of the span or of the distance between the inflection points.

Commentary—No provision is made by the specifications of the region for the use of multiple longitudinal web stiffeners, as would be desirable for very deep girders. Only the Canadian specification alludes to this possibility by stating that in the case of more than one longitudinal web stiffener "an approved method of analysis" should be employed.

WEST EUROPE

Most of the national specifications and codes in Western Europe do not yet refer to ultimate strength design models, except perhaps in Sweden and in Switzerland for some specifications; in several codes, almost nothing is said about the whole field of plate buckling.

A complete revision of two codes, which often serve as complementary (or reference) codes for foreign countries, is now under way, e.g., in the United Kingdom and in the Federal Republic of Germany. A first issue of the new British standard is expected for the end of the present year, and probably a little later for the German code.

French recommendations published in the journal *Construction Métallique* (1969) refer to an ultimate strength design model for plate buckling. A recommendation on the effects of concentrated forces acting in the plane of a web is under preparation.

The Belgian code for steelworks has adopted in its entirety the rules of Appendix C of the ECCS Recommendations, while awaiting a sufficiently general ultimate strength model.

Thus, it is too soon to make a useful comparison of the contents of the regional recommendations.

EAST EUROPE

SOVIET UNION (SEE REF. 120)

Linear buckling theory is adopted, in general cases, with formal safety ~15% lower than in usual strength analysis. Crane girders require 10% additional safety. No plastic reduction above limit of proportionality is included. Effect of elastic restraint due to torsional rigidity (GJ_t) of flanges is taken into account, depending on ratio GJ_t/hD, D being plate stiffness.

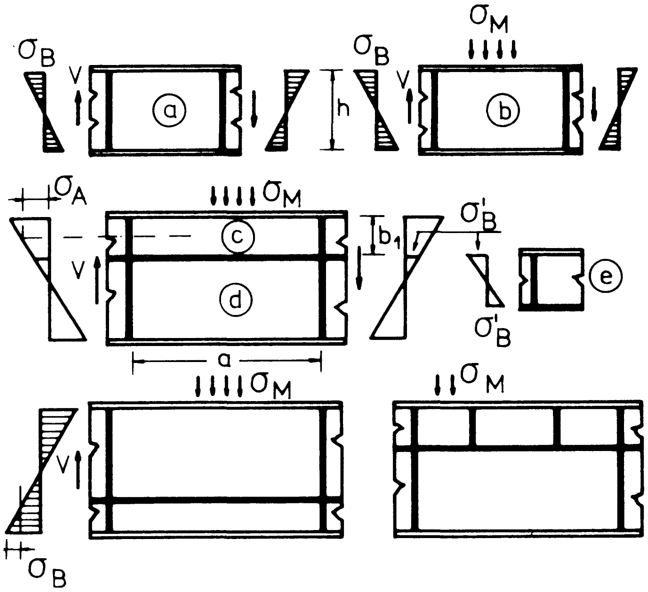

Figure EE4.3

Design formulae for cases represented in Fig. EE4.3 are given with interaction formulae:

a. $$\sqrt{\left(\frac{\sigma_B}{\sigma_{Bcr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2} \leq 1 \qquad \text{(EE4.4)}$$

b. $$\sqrt{\left(\frac{\sigma_B}{\sigma_{Bcr}} + \frac{\sigma_M}{\sigma_{Mcr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2} \leq 1 \qquad \text{(EE4.5)}$$

c. $$\frac{\sigma_A}{\sigma_{Acr}} + \frac{\sigma_M}{\sigma_{Mcr}} + \left(\frac{\tau}{\tau_{cr}}\right)^2 \leq 1 \qquad \text{(EE4.6)}$$

d. $$\sqrt{\left(\frac{\sigma'_B}{\sigma'_{Bcr}} + \frac{0.4\sigma_M}{\sigma_{Mcr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2} \leq 1 \qquad \text{(EE4.7)}$$

where σ'_{Bcr} is computed according to Fig. EE4.3e.

Required stiffness of longitudinal stiffeners is $J_s = 0.1ht^3\gamma^x$, where γ^x is a stiffness factor. The appropriate values of γ^x are:

For $b_1/h = 0.2$:

$$\gamma^x = \left(25 - 5\frac{a}{h}\right)\frac{a^2}{h^2} \qquad \text{(EE4.8)}$$

For $b_1/h = 0.25$:

$$\gamma^x = \left(15 - 4\frac{a}{h}\right)\frac{a^2}{h^2} \qquad \text{(EE4.9)}$$

For $b_1/h = 0.3$: $\gamma^x = 15$ (EE4.10)

For vertical stiffeners, $\gamma^x = 30$.

Similar design rules are given in **Rumanian** specifications.

SOVIET UNION: BRIDGES (SEE REF. 64)

In design of webs, limited benefit on post-buckled behavior is allowed for, replacing Eq. (EE4.5) by:

$$\sqrt{\left(\frac{\sigma_B}{\omega \cdot \sigma_{up}} + \frac{\sigma_M}{\sigma_{uM}}\right)^2 + \left(\frac{0.9\tau}{\omega_1\tau_u}\right)^2} \leq 1 \qquad \text{(EE4.11)}$$

where

$$\omega = 1 + 0.1\,\frac{\sigma_B - \sigma_{B2}}{\sigma_B} \qquad \text{(EE4.12)}$$

σ_{B2} = denotes lower edge bending stress

$$\omega_1 = 1 + 0.5\left(\frac{h}{200t} - 0.5\right) \text{ for } h/t > 100 \qquad \text{(EE4.13)}$$

and in σ_{up} and τ_u plastic reduction and effect of residual stresses is accounted for, according to Fig. EE4.4.

In the design of compressed plates with longitudinal and transverse ribs (Fig. EE4.5) the former (with adjacent parts of the sheet) are regarded as struts with effective length a and slenderness ratio

$$\lambda_e = a/r_e \qquad \text{(EE4.14)}$$

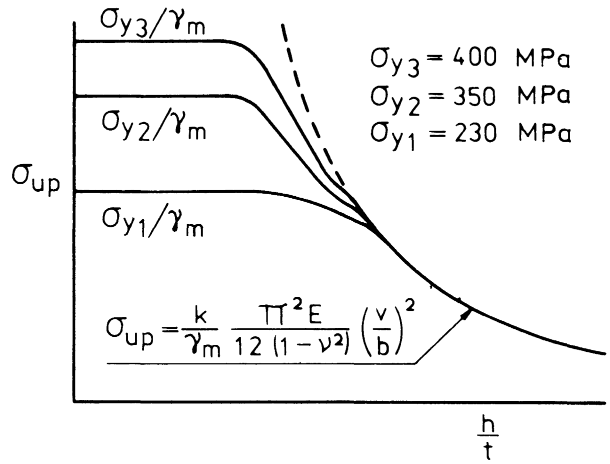

Figure EE4.4

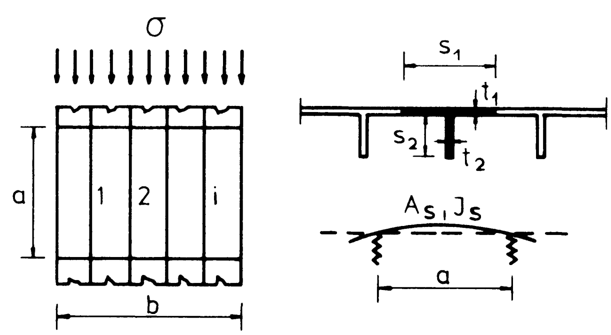

Figure EE4.5

where

$$r_e = \sqrt{J_e/A_s}$$

$$J_e = J_s + \frac{s_1 t_1^3}{12(1-\nu^2)}\left(2\frac{a^2}{b^2} + \frac{a^4}{b^4}\right)$$

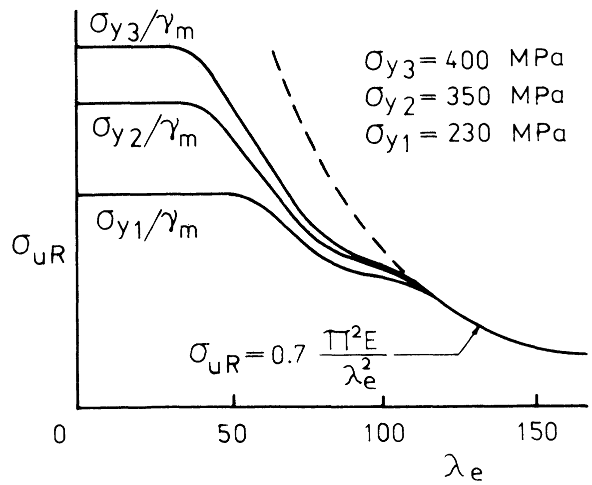

Figure EE4.6

using special column curves (Fig. EE4.6) including effects of both initial crookedness and residual stresses. (In case of tee-stiffeners, additional check of flexural-torsional buckling around the restrained axis is to be carried out.)

Ratios s_1/t_1 and s_2/t_2 are to be checked requiring $\sigma_{uR} = \sigma_{up}$, taking σ_{up} from curves similar to those given in Fig. EE4.4. Finally, as longitudinal ribs are continuous girders supported elastically by transverse stiffeners with flexural rigidity EJ_k (Fig. EE4.5), J_k is to be chosen to comply with requirement $l_e = a$, resulting in

$$J_k \geq 0.2(i+1)\left(\frac{a}{b}\right)^3 J \qquad \text{(EE4.15)}$$

CZECHOSLOVAKIA

Ultimate load design of webs, taking account both of initial imperfections and post-buckled behavior.

Introducing $\eta = \sqrt{240/\sigma_y}$ (σ_y in MPa), limit values of axial force (N_u), bending moment (M_u), shear force (V_u), and edge load (F_u) carried by the web are given (Fig. EE4.7):

$$N_u = \varphi_A A_w \sigma_y/\gamma_m \qquad \text{(EE4.16)}$$

$$M_u = \varphi_B W_w \sigma_y/\gamma_m \qquad \text{(EE4.17)}$$

$$V_u = 0.6\varphi_\tau A_w \sigma_y/\gamma_m \qquad \text{(EE4.18)}$$

$$F_u = \varphi_F A_p \sigma_y/\gamma_m \qquad \text{(EE4.19)}$$

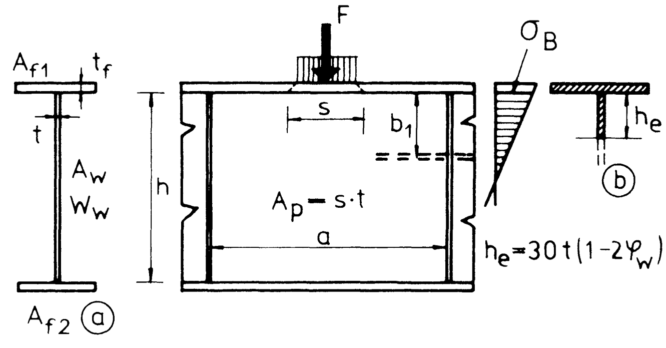

Figure EE4.7

where

$$\varphi_A = 40\eta\frac{t}{h} \leq 1.0 \qquad \text{(EE4.20)}$$

$$\varphi_B = 145\eta\frac{t}{h} \qquad \text{(EE4.21)}$$

$$\varphi_\tau = \varphi_w + \varphi_f;\quad \varphi_f = \varphi'_f \text{ or } \varphi_f = \varphi''_f \qquad \text{(EE4.22)}$$

$$\varphi_F = \left(1.46 + 24.4\frac{s}{h}\right)\frac{\sqrt{t\cdot t_f}}{s}\sqrt{1-\left(\frac{\sigma_B}{\sigma_y/\gamma_m}\right)^2}\cdot\eta \qquad \text{(EE4.23)}$$

$$\varphi_w = \left(6.4 - \frac{26}{\alpha}\right)\eta\frac{t}{h} \qquad \text{(EE4.24)}$$

$$\varphi'_f = \frac{1-\varphi_w}{1 - C\alpha A_w/A_f}$$

$$\varphi''_f = \sqrt{\frac{48Z_f}{aA_w}\frac{\sigma_{yf}}{\sigma_y}\cdot\frac{1-\varphi_w}{\alpha + 1/\alpha}} \qquad \text{(EE4.25)}$$

$Z_f\sigma_{yf}$ denotes ultimate bending moment of "effective flange" section according to Fig. EE4.7b. φ'_f describes the beneficial effect of the flanges upon the ultimate load performance, if limit state is defined by the onset of yielding ($C = 10$ for end panels, $C = 2 \sim 3$ for inner panels). In case of edge loading, $\varphi'_f = 0$.[121] φ''_f has the same role, if limit state is defined by the formation of failure mechanism.[122] In this case, additional check of flanges is required. For combined loading, interaction formula

$$\left(\frac{N}{N_u} + \frac{M}{M_u} + \frac{F}{F_u}\right)^2 + \left(\frac{V}{V_u}\right)^2 = 1 + 3\left(\frac{N}{N_y} + \frac{M}{M_y}\right)\left(\frac{F}{F_y}\right) \qquad \text{(EE4.26)}$$

is used, where N_y, M_y, F_y are defined by Eqs. (EE4.16), (EE4.17) and (EE4.19), respectively, taking $\varphi_A = \varphi_B = \varphi_F = 1.0$.

Required moment of inertia of stiffeners is given as:

$$J_s \geq 0.1ht^3m\gamma^x \qquad \text{(EE4.27)}$$

γ^x being the linear-buckling-theory optimum rigidity, and

$$m = 1 + (m_s - 1)\left(\frac{h}{t \cdot \bar{\eta}} - 1\right) \qquad \text{(EE4.28)}$$

where $\bar{\eta} = 40\eta$ (compression), $\bar{\eta} = 145\eta$ (bending) and $\bar{\eta} = 90\eta$ (shear), respectively, and m_s varies between 3 and 7 depending on the location (value b_1/h) of the longitudinal stiffener.

In combined loading

$$m\gamma^x = \sqrt{\left(\frac{m_A\gamma_A{}^x\sigma_A}{\varphi_A\sigma_y/\gamma_m} + \frac{m_B\gamma_B{}^x\sigma_B}{\varphi_B\sigma_y/\gamma_m}\right)^2 + \left(\frac{m_\tau\gamma_\tau{}^x\tau}{\varphi_\tau 0.6\sigma_y/\gamma_m}\right)^2} \qquad \text{(EE4.29)}$$

where indices refer to individual kinds of loading.

Ultimate load of compressed plates with longitudinal ribs (compression flanges of box-girder bridges) is calculated as the sum of (a) the buckling loads of the longitudinal ribs as compressed struts (their cross section consisting of rib and effective portion of adjacent sheet panel) and (b) the loads sustained by the two effective strips at the longitudinal edges.

GERMAN DEMOCRATIC REPUBLIC

Design of plates can be based on both (a) linear buckling theory and (b) postcritical behavior.

a. Elastic critical stress (σ_{1cr} and τ_{cr}) are computed assuming simply supported edges (Fig. EE4.8a). Interaction formula (EE4.30) is used:

$$\frac{1+\psi}{2}\frac{\sigma_1}{\sigma_{1cr}} + \frac{\sigma_M}{\sigma_{Mcr}} + \frac{1-\psi}{2}\left(\frac{\sigma_1}{\sigma_{1cr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2 = 1 \qquad \text{(EE4.30)}$$

Plastic reduction is taken into account by computing fictitious critical stress

$$\sigma_{ecr} = \sqrt{\sigma_1{}^2 + \sigma_M{}^2 - \sigma_1\sigma_M + 3\tau^2} \times \left[\frac{1+\psi}{2}\frac{\sigma_1}{\sigma_{1cr}} + \frac{\sigma_M}{2\sigma_{Mcr}} + \sqrt{\left(\frac{3-\psi}{4}\frac{\sigma_1}{\sigma_{1cr}} + \frac{\sigma_M}{2\sigma_{Mcr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2}\right]^{-1} \qquad \text{(EE4.31)}$$

and applying the same reduction, as with centrally compressed ideal columns, based on stress-strain relation Eq. (EE3.10) and reduced modulus theory (Fig. EE4.8b). If $\sigma_{ecr} > 1.5\sigma_y$, reduction is decreased by applying a factor

$$\frac{1}{0.9 + 0.1\left(\dfrac{1.5\sigma_y}{\sigma_{ecr}}\right)^2} \qquad \text{(EE4.32)}$$

Thus, safety factor in allowable stress design is:

$$n_b = \frac{\sigma_{erc,red}}{\sqrt{\sigma_1{}^2 + \sigma_M{}^2 - \sigma_1\sigma_M + 3\tau^2}} \qquad \text{(EE4.33)}$$

and its required value (in the so-called "main loading case") is:

For σ_M and τ: $n_b = 1.35$

For σ_1:

$$n_b = 1.5 - 0.075(1-\psi) \qquad \text{(EE4.34)}$$

In case of plates reinforced by longitudinal ribs (of number i):

For σ_M and τ, $i > 2$: $n_b = 1.5$

For σ_1, $i < 3$:

$$n_b = 1.71 - 0.21(1-\psi) \qquad \text{(EE4.35)}$$

For σ_1, $i > 3$ and $a/h < 0.9$:

Longitudinal stiffeners (with effective portion of the sheet) are to be analyzed as individual beam-columns.

Optimum rigidity of stiffeners calculated by linear buckling theory is to be multiplied by 2 or 3 depending on the number and cross sectional properties of the stiffeners.

b. Webs can be analyzed with regard to post-buckled behavior. In case of Basler bending theory, in case of shear, a tension-field theory can be applied and interaction formula

$$\left(\frac{n_bM}{M_u}\right)^2 + \left(\frac{n_bV}{V_u}\right)^2 \leq 1 \qquad \text{(EE4.36)}$$

is suggested.

To avoid intolerable out-of-plane deflection, ratio t/h is limited.

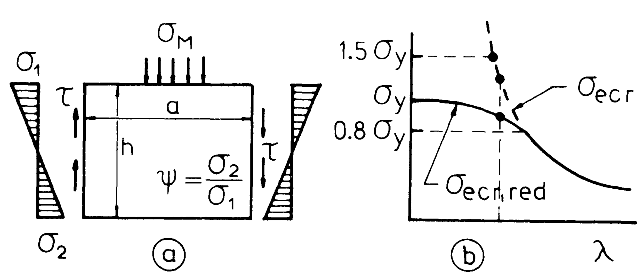

Figure EE4.8

POLAND

Linear buckling theory is adopted. Plastic reduction is taken into account for the individual stress components (Fig. EE4.8), applying the same reduction as with ideal compressed columns (see Fig. EE3.3). In combined loading these reduced critical stresses (σ_{ul} and τ_u) should satisfy condition

$$\frac{1+\psi}{4}(1-\bar{\delta})\frac{\sigma_1}{\sigma_{1u}} + \sqrt{\left[1-\frac{1+\psi}{4}(1-\bar{\delta})\frac{\sigma_1}{\sigma_{u1}}\right]^2 + 3\left(\frac{\tau}{\tau_u}\right)^2} \leq 1 \qquad \text{(EE4.37)}$$

where $\bar{\delta} = 5 - 4\,\overline{m} \geq 0$ and

$$\overline{m} = \frac{\sigma_y}{\gamma_m \sigma_{u1}} \text{ or } \overline{m} = \frac{\tau_y}{\gamma_m \tau_u}$$

whichever is less.

HUNGARY

Design formulae (EE4.2) and (EE4.3) are given; σ_{up} in the inelastic range is defined by a dashed line in Fig. EE4.2. Optimum rigidity γ^x is to be multiplied by 2 to 4, depending on the location and cross section of the stiffeners.

Compressed plates with longitudinal ribs are designed as described in the summary of the **Czechoslovakian** Specification; check of torsional buckling of tee-ribs around their restrained axis is required.

Design rules for plastic design in national specifications are similar to Eq. (EE4.1) with slight modifications.

c. COLLOQUIUM CONTRIBUTIONS (COMMENTARIES)

JAPAN

Y. FUJITA (TOKYO)[17]

The paper presents state-of-the-art reports on the influence of imperfections on the compressive strength of plates available and known in the Japanese shipbuilding industry. Discussions are presented on the "Japanese Shipbuilding Quality Standard" for tolerance in fabrication, actual measured results of initial imperfection, and results of theoretical and experimental studies on the compressive strength of plates with initial imperfections and residual stresses. See Fig. J4.1.

F. NISHINO (TOKYO)[18]

The theories and experiments reported in numerous literature on the ultimate bending and shear strengths of transversely stiffened symmetrical and homogeneous plate girders are reviewed and examined with particular emphasis on the failure due to the instability of web plates.

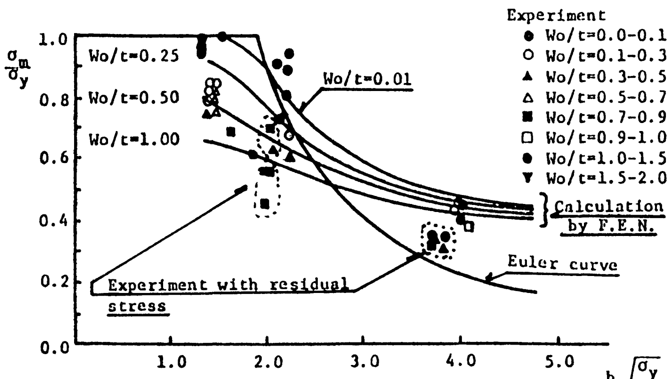

Fig. J4.1. Ultimate strength of square plates subjected to compression

NORTH AMERICA

J. L. DURKEE (TOKYO)[123]

A structural designer's point of view is expressed that the ultimate strength theories developed up to present are more than adequate to fulfill the needs of designers. What is desirable, if at all, are not some further theoretical refinements, but a formulation of practical specifications based on the already available research, which would take into account the variability of field conditions in actual structures rather than the abstractions of research institutions. Judging by the insigificant number of contributions made at the colloquia on plate and box girders from the North American region, the opinion expressed by the author apparently reflects the result of the satisfactory use of the ultimate strength method for plate girder design for almost twenty years in this region.

Y. SAWADA, G. C. LEE, AND M. ITO (LIEGE)[124]

Finite element method (CST) was used to analyze four plate girder specimens. The postbuckling contribution was obtained by considering that in this load range the elements were anisotropic with non-zero rigidity only in the direction of the panel diagonal up to full yielding. The computed ultimate loads agreed better with the test results than the Basler method, but not as well as some other methods, such as Rockey-Skaloud or Chern-Ostapenko. The analysis indicated that the Basler method underestimates the forces in the transverse stiffeners.

WEST EUROPE

P. DUBAS (TOKYO)[125]

In the discussion on plate and box girders, Dubas presents some remarks regarding the Japanese viewpoints on these fields and compares them with the European one. He comments on the influence of geometrical and structural imperfections and on the behavior of transversely stiffened webs with the emphasis on the postcritical effects.

C. A. CARLSEN, T. H. SOREIDE, AND N. T. NORDSVE (LIEGE)[126]

The paper deals with the effect of shear lag on the collapse of compression flanges. A finite element large deflection elasto-plastic analysis is used to determine approximately the redistribution capacity of a stiffened plate subjected to non-uniform displacements which are incremented to collapse.

W. C. FOK AND A. C. WALKER (LIEGE)[127]

The paper "The Inelastic Ultimate Load of Stiffened Plates with Stiffener Failure" goes some way towards providing answers to the question of stiffener outstand failure in a stiffened compression plate. Although the title infers that the analysis might be inelastic, the method presented is, in fact, an elastic analysis.

B. ROUVE (LIEGE)[128]

The paper deals with the "Non-linear Behaviour of Compression Plates Stiffened with Trapezoidal Stiffeners". Using the linear theory of buckling, the author confirms the well-known fact that (above) the optimum value of stiffener inertia the critical stress remains constant. For the non-linear elastic analysis, the finite element method is used. In the range between local critical buckling stress and overall critical buckling stress, the author suggests that the efficiency is greater than that given by von Karman's or Winter's formula. He attributes this partly to the fact that the plate over each stiffener is fully efficient and that the plate panels between stiffeners are partially restrained. A formula for the efficiency of such stiffened panels is suggested and the author concludes that an inertia greater than the optimum given by classical buckling theory must be provided to avoid a drop in efficiency caused by instability of the stiffener.

A. BERGFELT (LIEGE)[129]

The author considers the "Influence of Web Inclination on Compression Flange Buckling". The first part of his paper is devoted to some useful general theoretical considerations in relation to plate buckling, although there is little reference to the subject matter of the title. The main parts of the paper deal with sloping webs. Pilot tests on 12 very small scale box girders are reported.

K. S. CHAN, C. L. LAW, AND D. W. SMITH (LIEGE)[130]

Realistically scaled models of typical bridge decks have been tested under combined loading such as may occur in a typical orthotropic steel deck bridge. The aim was to assess the influence of the wheel loads on the collapse strength of the deck in compression.

K. C. ROCKEY, H. R. EVANS, AND D. M. PORTER (LIEGE)[131]

In their paper "Tests on Longitudinally Reinforced Plate Girders Subjected to Shear" the authors review the design method which they have developed over the past few years and reported upon in several papers. The results of 10 new tests are presented in this paper. The method is shown to give excellent agreement with tests, providing the stiffener remains straight right up to the ultimate load. The relative simplicity of the approach adopted should recommend the method to designers, but there are several aspects which remain to be clarified, not least of which related to the design of the longitudinal stiffening itself.

P. J. DOWLING, P. A. FRIEZE, AND J. E. HARDING (LIEGE)[132]

Most engineers will recognize the importance of the "Imperfection Sensitivity of Steel Plates under Complex Edge Loading". The paper sheds some light on the problem of the tolerance levels which should be specified for plate structures such as bridges.

C. D. BRADFIELD (LIEGE)[133]

In his paper "Collapse of Rectangular Outstands Loaded in Compression", the author tackles a similar problem to that considered by Fok and Walker,[127] except that he uses a more sophisticated elasto-plastic large deflection analysis and takes account not only of the rotational restraint afforded by the plate to the supported edge, but also that provided by a compact lip or bulb at the free edge. He uses a finite difference solution technique and the single layer approach described by Crisfield and based on Ilyushin's yield criterion. The effects of both initial distortion and residual stresses are considered.

F. FREY AND R. ANSLIJN (LIEGE)[134]

The authors report tests on four unstiffened plate girders of the type commonly used in Sweden in their paper "Shear Tests on Unstiffened Plate Girders". The results are compared with Höglund's curves, and show good agreement. A comparison with ECCS curves which are based on experimental results also show good agreement.

J. BROZZETTI (LIEGE)[135]

Investigations connected with unserviceability limit load are described in the paper "Experimental Behavior of Two Slender Girders—A Criterion for a 'Serviceability' Limit Load". The author defines a "reversibility" limit state as one such that, beyond this limit, permanent deformations or stresses will occur when unloading. Good agreement was obtained between test results and the theory of Basler and Thürlimann after making due allowance for the effects of lateral buckling. The author ends by recommending that longitudinal stiffeners should have a rigidity of between 4 to 7 times the optimal rigidity given by classical buckling theory to be effective. He also provides formulae to be used to estimate their "reversibility" limit state.

A. PLUMIER (LIEGE)[136]

The author suggests that "The Improvement of the Load Carrying Capacity of Webs by Means of Appropriate Residual Stresses" is possible. An experimental investigation is described and it is concluded that increases in ultimate load ranging from 5 to 15 percent can be brought about by the judicious choice of sequence and orientation of weld deposits.

P. J. DOWLING (WASHINGTON)[137]

A review of the main research developed in Britain and an outline of the recent developments on the inelastic analysis and design of plate and box girders. Extensive work and parametric studies have been concerned with imperfect unstiffened plates, stiffened plates, and assemblages of plates, the results of which are applied to the design of box girder compression flanges, of webs of plate and box girders, and of box girder diaphragms. The detailed results of this work are said to be used to form the basis for the relevant clauses of the British bridge code, the preparation of which is still underway.

M. EL GAALY (WASHINGTON)[138]

It is established that plates uniformly stiffened in one or two orthagonal directions can be treated as orthotropic plates. The author shows that a correlation exists between the buckling of stiffened plates and the buckling of isotropic plates. Simple formulas are given for this purpose and a comparison of buckling values obtained using these formulas and the corresponding available exact values proves to be satisfactory.

J. B. DWIGHT AND K. E. MOXHAM (WASHINGTON)[139]

The work described is concerned with the long-time researches performed in Cambridge by Dwight and his staff, on the compressive strength of welded plates. The authors discuss experimental results dealing with the load-shortening curve, conveniently regarded as an equivalent stress-strain curve of the plate, and the ultimate strength. The results have been summarized and useful proposals are made for plate strength curves.

R. MAQUOI AND J. RONDAL (LIEGE, BUDAPEST)[140,141]

The economical use of welded hybrid plate girders instead of homogeneous ones is enhanced by optimizing the cross-sectional dimensions and the web and flange steel grades by means of non-linear programming. Operational constraints which are related to strength, stability, and deflection requirements are taken into account.

A. PLUMIER (BUDAPEST)[142]

Tests performed on comparable unstiffened plane web and ondulated web girders, under combined shear, bending, and direct transverse loading, show that the ondulated web girder presents an appreciably higher strength, namely when the girder is likely to undergo web crippling or web buckling. However, the problem of ondulated web girders still need thorough theoretical as well as experimental studies.

EAST EUROPE

P. JUHAS (BUDAPEST)[143]

A new method, based on results of 25 experiments, for the prediction of the load-carrying-capacity of steel beams subjected to bending is presented, defining limit state by the onset of a limit strain in the extreme fiber of the web.

I. LHOTAKOVA AND M. SKALOUD (BUDAPEST)[144]

An experimental investigation of 12 large-scale steel box girders is described. The results show that the classical design concept, based on the linear-buckling theory optimum rigidity γ^x, is unable to ensure that the longitudinal stiffeners of the compression flange will remain effective in the post-buckled range. In order to achieve this objective, it is necessary to increase the stiffener rigidity so that it equals $4\gamma^x \sim 5\gamma^x$.

The main results of a series of measurements of initial imperfections on a new 425 meter long motorway box girder bridge are also presented.

A. LUTTEROTH AND I. KRETZSCHMAR (BUDAPEST)[145]

Twelve tests on compressed longitudinally stiffened plate panels are reported, indicating that the load-carrying capacity was 2 to 21% lower than buckling load according to classical linear theory. On the other hand, the column-buckling analogy proposed by F. Faltus and M. Škaloud in 1953 proved to be safe and in satisfactory agreement with experimental results.

H. STEUP (BUDAPEST)[146]

Longitudinally stiffened webs subjected to transverse edge loading are analyzed by "smearing up" the stiffeners, thus treating the stiffened web as an anisotropic plate. The optimum rigidity γ^x of the ribs is determined so as to ensure that the critical load of the whole stiffened web is equal to that of the most loaded plate panel.

W. HOYER (BUDAPEST)[147]

Rules for design of vertical stiffeners by the method of "drift forces" (applied in the Merrison report) are given for plates in compression, bending, and shear, comparing the results to those gained by linear buckling theory.

I. SZATMARI (BUDAPEST)[148]

Results of 16 tests on plate girders with flat and tubular flanges and unstiffened web are reported, and compared to Höglund's theory, finding good correlation in case of flat flanges. Tubular flanges of great flexural rigidity are stated to increase ultimate load substantially. A new computational method is alluded to.

J. DJUBEK AND I. BALAZ (BUDAPEST)[149]

Longitudinally stiffened compression flanges are analyzed by the non-linear theory of large deflections, "smearing up" the ribs and applying the orthotropic plate approach. Using a more complex assumption for the buckled surface and another definition of limit state, the authors obtained lower limit loads than those given in the 1971 paper by Maquoi and Massonnet.[150]

V. U. MOISEEV (BUDAPEST)[151]

The elastic-plastic stability of plate elements is analyzed, based on the plastic theory of small deformations, using Stowell's concept. Results for various boundary conditions are summed up by correction factors to elastic critical load in order to include plastic reduction.

Z. SADOVSKY (BUDAPEST)[152]

Dealing with the non-linear theory of large deflections of plates, special care is given to the formulation of the two types of boundary conditions governing the in-plane behavior. Numerical results regarding a web in pure bending are presented.

A. SCHINDLER (BUDAPEST)[153]

The shear-lag problem of wide flanges of box girders is studied, comparing results of Moffet, Dowling, Dishinger, and Ramberger, concluding very good agreement. It is also shown that when calculating shear lag, merely the effect of longitudinal ribs is to be taken into account, while that of transverse stiffeners can be disregarded.

J. FARKAS (BUDAPEST)[154]

The effect of residual stresses upon plate buckling is studied, giving simple formulas for the evaluation of welding residual stresses and verifying them for some particular cases by comparing them to experimental measurements. A modified Faulkner's formula for the effective width of compressed plates, able to take account of residual stresses, is suggested.

J. PECHAR (BUDAPEST)[155]

Two alternatives of the design of transverse stiffeners of longitudinally stiffened compression flanges are studied. The first is based on stiffness criteria, requiring provision of sufficient support for longitudinal ribs. The second is based on requiring bending stresses in transverse stiffeners to remain within allowable limits.

A. SCHINDLER (BUDAPEST)[156]

A brief comparison of the actual stress state in a box girder, evaluated by means of the folded plate theory, with the results obtained if the shear lag phenomenon is taken into account by using effective width concept is given.

M. SKALOUD (BUDAPEST)[157]

The folded plate theory is applied to investigate the effect of (a) the stiffener shape and (b) the position of the stiffening element with respect to the flange sheet on the buckling strength of longitudinally stiffened compression flanges. Outstanding efficiency of trapezoidal closed-section ribs in stiffening the compression flanges is concluded.

Z. SADOVSKY (BUDAPEST)[158]

Buckling of thin, unstiffened webs of asymmetrical plate girders is studied by means of a non-linear analysis assuming large deflections. Optimum distribution of the material between the upper and lower flange is found in case of laterally supported compression flange.

B. VERÖCI (BUDAPEST)[159]

Experimental investigation into ultimate load behavior of four plate girders with unstiffened webs subjected to point loading is described, three of them having single-sided web-to-flange fillet welds, while one had the conventional double-sided welded connection. No marked difference in ultimate load due to the eccentric connection could be observed.

M. DRDACKY (BUDAPEST)[160]

Weight and cost optimization of compressed longitudinally stiffened steel plates is presented, defining limit state by means of a column-buckling analogy. According to the results, price gives a more objective criterion for optimization.

J. FARKAS (BUDAPEST)[161]

Comparison of optimum design of beams based on elastic and plastic design concepts is given with strength and stability constraints. Results are illustrated by numerical examples.

M. DRDACKY (BUDAPEST)[162]

Test techniques for measuring out-of-plane displacements and stress pattern of webs using photogrammetric and photoelastic methods are described.

L. KIS PAPP, M. IVANYI, I. SZATMARI, AND B. VERÖCI (BUDAPEST)[163]

A description of photogrammetric measuring techniques with application in different stability tests (lateral buckling, plate buckling) are given.

d. CURRENT STUDIES

JAPAN

S. KOMATSU AND M. USHIO[164]

The paper presents the results of parametric study of buckling strength analysis of stiffened plates by finite strip method. Influence of residual stress is considered. Modifications of current JRC specification is proposed. One of the proposed modifications is that allowable stress for stiffened plates in compression should be limited to 80 per cent of $\sigma_r/1.7$ for λ between 0.3 and 0.7.

S. KOMATSU, Y. MORIWAKI, AND M. FUJINO[165]

A discussion is presented on tolerance of initial out-of-flatness of web plates for design in accordance with JRC specifications.

M. FUJINO[166]

A report of an experimental study of plate girders with initial imperfections. Equations are proposed, based on experimental results obtained on large size girders fabricated following current fabrication procedures in Japan.

Buckling strength of plate girders under combined shear and bending is approximated by:

$$(M_y/M_{cro})^2(M_{cr}/M_y)^2 + (Q_p/Q_{cro})^2(Q_{cr}/Q_p)^2 = 1 \qquad (J4.32)$$

Ultimate strength is approximated by:

$$Q_u/Q_p = (Q_{uo}/Q_p)\{1 - 0.145(M/M_y)/(Q/Q_p)\} \qquad (J4.33a)$$

when $Q_u/Q_{uo} > 1 - 0.145(M/M_y)/(Q/Q_p)$ and

$$M_u/M_y = M_{uo}/M_y \qquad (J4.33b)$$

when $Q_u/Q_{uo} \leq 1 - 0.145(M/M_y)/(Q/Q_p)$.

In Eq. (J4.32), when $Q_{cr}/Q_p \geq Q_u/Q_p$,

$$Q_{cr}/Q_p = Q_u/Q_f \qquad (J4.34a)$$

and, when $M_{cr}/M_y \geq M_u/M_y$,

$$M_{cr}/M_y = M_u/M_y \qquad (J4.34b)$$

where Q_{cr} = shear force, Q_{uo} = ultimate shear strength, Q_{cro} = shear buckling strength, Q_p = plastic shear strength and the meaning of the subscripts are the same for M (bending).

H. YONEZAWA, I. MIKAMI, M. DOGAKI, AND H. UNO[167]

Shear strength of plate girders with diagonally stiffened web is discussed and correlated with the experimental results.

Y. MORIWAKI AND M. FUJINO[168]

Formulas are proposed based on experimental results of plate girders with initial imperfections which are present in practical girders used for bridge construction.

Ultimate strength of plate girders is expressed:

1. For failure due to horizontal buckling of the compression flange:

When $1.22 < \lambda_L$:

$$_LM_{uo}/M_y = [2.42 + \{5.85 - 9.86 \times (0.62 - 1/\lambda_L{}^2)\}^{1/2}]/4.93 \qquad (J4.35a)$$

When $0.50 < \lambda_L \leq 1.22$:

$$_LM_{uo}/M_y = 0.085/\lambda_L{}^2 + 0.95 \qquad (J4.35b)$$

2. For torsional failure:

When $0.48 < \lambda_T$:

$$_TM_{uo}/M_y = [12 + \{145 - 52(3.3 - 1/\lambda_T{}^2)\}^{1/2}]/26 \qquad (J4.36a)$$

When $0.35 < \lambda_T \leq 0.48$:

$$_TM_{uo}/M_y = 0.040/\lambda_T{}^2 + 0.82 \qquad (J4.36b)$$

A. HASEGAWA, F. NISHINO, AND T. OKUMURA[169]

Ultimate shear strength is discussed in terms of stiffness of horizontal stiffeners, boundary conditions for web plates, initial imperfections based on experimental results on longitudinally stiffened plate girders.

A. HASEGAWA, F. NISHINO, AND T. OKUMURA[170]

Ultimate bending strength is discussed in terms of stiffness of horizontal stiffeners, residual stresses, and eccentricity of longitudinal stiffeners based on experimental results on longitudinally stiffened plate girders.

Y. MORIWAKI AND M. FUJINO[171]

Formulas are proposed based on experimental results of plate girders with initial imperfections which are present in practical girders used for bridge construction. Buckling strength of plate girders in shear is expressed by

$$Q_{cro}/Q_p = (0.14\psi - 1.04)\sigma_w{}^2 + (3.0/\psi - 0.27)\sigma_w \qquad (J4.37)$$

and ultimate strength by

$$Q_{uo}/Q_p = 2.0/\psi + 0.53 \qquad (J4.38)$$

NORTH AMERICA

As reflected by the insignificant number of the contributions made at the colloquia on plate and box girders from the North American region, only a few essentially uncoordinated, low intensity studies by individual researchers are currently conducted.

Vernarr and Ostapenko (Lehigh University)

Comparison of present methods for transversely stiffened plate girders and further improvements of the most versatile ones.

Ostapenko (Lehigh University)

Development of a "manageable" yet accurate method for longitudinally stiffened webs.

Galambos (Washington University)

Streamlining and adaptation of the present AISC and AASHTO methods to the Load and Resistance Factor Design (LRFD) approach.

Wolchuk and Mayrbaurl (New York, ASCE-AASHTO-FHWA Project)

This is probably the most intensive effort in this region on the development of design recommendations for steel box girder bridges. The aim is to formulate design specifications based on the most recent research results. In the process, many gaps in present knowledge have been brought to light and had to be urgently filled, in many instances in a very approximate and hopefully conservative manner.

Research Needed

- Variable depth
- Multiple longitudinal stiffeners
- Composite plate and box girders with deep webs
- Girders with longitudinal stiffeners but without transverse stiffeners
- Deep box girders with wide stiffened flanges and with webs having multiple longitudinal stiffeners
- Effect of high axial load on plate and box girder panels
- Improved design methods for transverse and longitudinal stiffeners
- Moment and shear redistribution characteristics of continuous plate and box girders
- Fatigue under combined loads (secondary web stresses) and more rational limitation criteria on web slenderness

WEST EUROPE

UNITED KINGDOM

Strength of Plates in Biaxial Compression:
Imperial College London (Dowling) and Cambridge University (Dwight).

Effect of Shear Lag on Inelastic Buckling of Wide Flanges:
Imperial College London (Dowling).

Strength of Multistiffened Web Plates and Web Plates with Openings:
University College Cardiff (Rockey).

Strength of Box Girder Diaphragm:
Imperial College London (Dowling) and Transport and Road Research Laboratories, Crowthorne (Crisfield).

Strength of Stiffened Plates under Combined Compression and Lateral Loading:
Imperial College London (Dowling).

Inelastic Behavior and Design of Web Stiffeners:
University of Manchester (Horne).

Strength of Composite Box Girders Under Combined Bending, Shear and Torsion:
Imperial College London (Dowling).

FRANCE

Effects of Concentrated Forces Acting in the Plane of a Stiffened Plate:
Ecole Nationale Supérieure des Arts et Métiers (Gachon).

SWITZERLAND

Tests on Two Box Girders Models with Closed Section Stiffeners:
Eidgenössiche Technische Hochschule Zürich (Dubas).

BELGIUM

Investigations of the Behavior of Web Plates Under Combined Shear, Bending and Transverse Loading:
Centre de Recherches de l'Industrie des Fabrications Métalliques (C.R.I.F.), Université de Liège (Anslijn).

EAST EUROPE

(No report on current studies in East Europe.)