Analysis and Design of a Web Connection in Direct Tension

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Light bracing composed of a tension plate welded to the web of a column or beam section, as shown in Fig. 1, is employed quite often in the design of steel structures. Generally, the strength of a web in carrying the tensile load is of primary concern. There are two analysis procedures which have been suggested for the strength analysis of the web. The first is an elastic method suggested by Blodgett,[1] and the second, an ultimate strength yield line method proposed by Kapp.[2] Experimental tests[3] have indicated that the strength of these connections are, in general, much greater than the strength predicted by either the elastic or the yield line theories. Therefore, a study was undertaken to further investigate the behavior of this connection type in the plastic range by use of the finite element technique.[4]

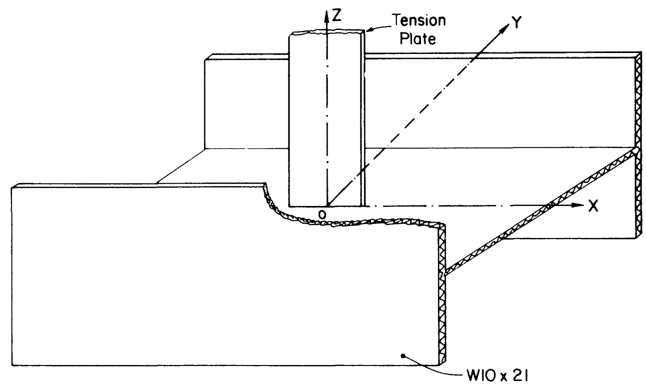
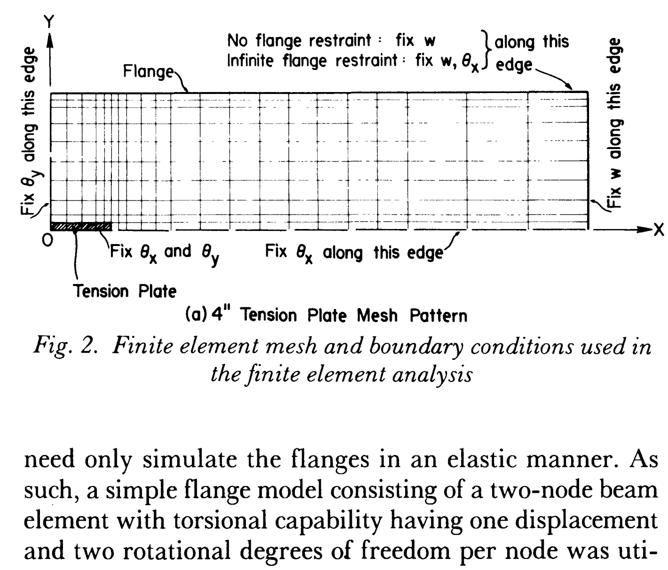


Fig. 1. A typical welded light bracing web connection

ELASTIC-PLASTIC FINITE ELEMENT SOLUTION

Description of the Computer Model—A connection system consisting of 4″ x 3/8″ and 8″ x 3/8″ tension plates welded to a W10x21 wide-flange section of 3-ft length was used for the computer analysis. The models conform to those that were used in the experimental tests described in Ref. 3. Due to symmetry, only one-fourth of the connection need be modelled. The element configuration employed for the study, shown in Fig. 2, consists of 210 rectangular plate bending elements interconnected by 242 nodes with three degrees of freedom per node.

The plate bending model developed for the study was capable of only planar analyses. A method whereby the out-of-plane flanges of a W10x21 section could be modelled in the analysis was necessary. Experimental evidence[3] has shown that, as the load is applied to the tension plate and the web deflects, the flanges remain plane and simply rotate in a rigid body motion. This indicates that plasticity is of minor concern in the flanges and that any flange model



(a) 4″ Tension Plate Mesh Pattern

Fig. 2. Finite element mesh and boundary conditions used in the finite element analysis

need only simulate the flanges in an elastic manner. As such, a simple flange model consisting of a two-node beam element with torsional capability having one displacement and two rotational degrees of freedom per node was utilized.

An elastic analysis using a W10x21 section and a 4-in. wide tension plate was performed. A comparison with experimental strain gage data at a load of 1000 lbs, as shown in Fig. 3, indicates a very close agreement between the measured and calculated strains in the elastic range, thereby assuring the accuracy of the proposed finite element model.

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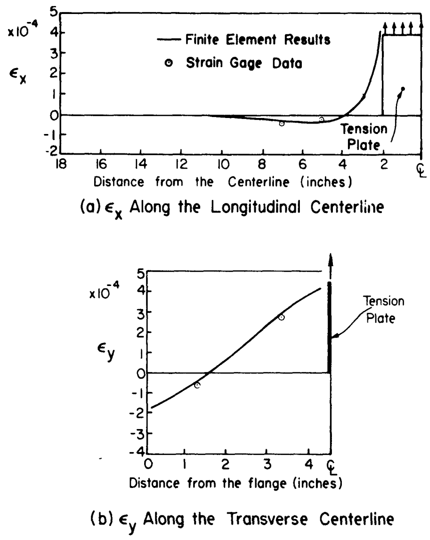

Fig. 3. Comparison of analytical and experimental strains with 4-in. tension plate

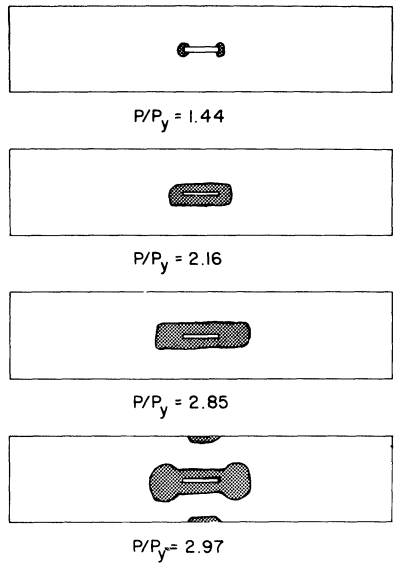

Fig. 5. Yield zones at various loads for a 4-in. tension plate with flange simulation (P_y = 1.82 kips)

Elastic-Plastic Analysis with Various Boundary Restraints—A series of six computer analyses were carried out using the 4-in. and 8-in. wide tension plates. Bounding solutions featuring no flange restraint and infinite flange restraint were obtained in addition to the more realistic solution using the flange element.

Load versus deflection at the center line of the tension plate for various flange restraints used in the model is shown in Fig. 4. It can be observed from these curves that throughout most of the loading range the deflections predicted with the flange simulation are much closer to those calculated without any flange restraint than to those assuming infinite flange restraint. Consequently, one could assume no flange restraint for a reasonable estimate of the load-deflection behavior.

The development of plastic regions in the web are shown in Figs. 5 and 6 for the cases of flange simulation. It is seen that the plastic regions develop progressively around the tension plate only. It should be noted that no elastic regions are contained within a closed plastic zone at the apparent collapse. It is further seen that plasticity does not develop along narrow lines, as is assumed in the yield line theory; rather, the plastic regions extend in broad areas.

Finally, a comparison of the load-deflection curves for the flange simulation cases indicates that a doubling of the tension plate length increases the collapse load by approximately 30%.

COMPARISON WITH BLODGETT'S ELASTIC METHOD

Blodgett[1] has suggested an elastic analysis, as outlined in Appendix A, in which it is assumed that the tensile load

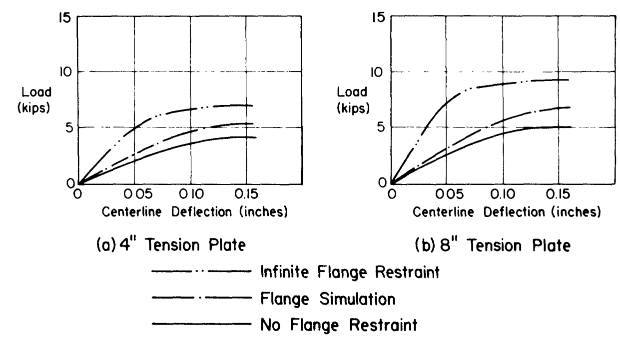

Fig. 4. Load-deflection curves for various flange restraints

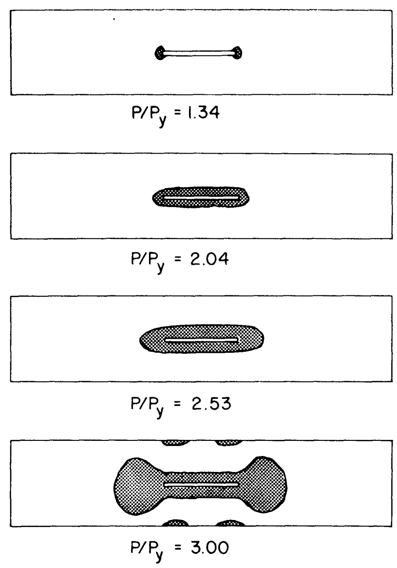

Fig. 6. Yield zones at various loads for an 8-in. tension plate with flange simulation (P_y = 2.37 kips)

Table 1. Comparison of Ultimate Loads Using Various Solution Techniques

Member	Tension Plate Width (in.)	Flange Restraint	Ultimate Load (kips) Computer Solution	Ultimate Load (kips) Yield Line Solution	Initial Yield Load (kips) Computer Solution	Initial Yield Load (kips) Blodgett's Elastic Solution
W10x21	4	None	4.1	7.0	1.5	1.0
W10x21	4	Simulation	5.4	—	—	—
W10x21	4	Infinite	7.0	10.5	2.3	2.0
W10x21	8	None	5.0	8.0	1.9	1.6
W10x21	8	Simulation	7.1	—	—	—
W10x21	8	Infinite	9.0	12.4	3.2	3.2

produces a uniform stress under the tension plate which dies out linearly from the ends of the tension plate.

This design procedure has two inherent shortcomings. The first concerns the assumed load distribution in the web. Previous finite element studies[3] have shown that large stress concentrations exist at the ends of the web-tension plate connection. This could be viewed as if concentrated loads are being applied to the web at these points and which could result in a reduction of the permissible load. However, because of the highly localized nature of these stress concentrations, rapid dissipation of large stresses occurs according to St. Venant's principle. Therefore, this phenomenon of stress concentration can be neglected especially in light of the inherent ductility of the connection materials.

The second shortcoming concerns the simplifying assumption that the web can be analyzed as a one-dimensional beam rather than as a two-dimensional plate. This is a conservative assumption. The beam solution approaches the two-dimensional plate solution as the width of the connection plate increases. An examination of the last two columns of Table 1 indicates that as the tension plate width increases, the initial yield load predicated by Blodgett's elastic solution approaches the more accurate computer solution. Due to an increase in the width of the plate, the load is distributed more uniformly to the web and the effect of stress concentrations at the ends of the plate is reduced. Additionally, an increase in the flange restraint along the edges of the web together with an increase in the plate length, reduces the amount of web curvature in the longitudinal direction. Consequently, Poisson's effect is reduced and beam, rather than plate, bending becomes more pronounced.

COMPARISON WITH KAPP'S YIELD LINE METHOD

The yield line method, as proposed by Kapp,[2] is an upper-bound limit analysis procedure that depends on a proper yield pattern assumption for its accuracy. Kapp's design procedure is reproduced in Appendix B for the reader's convenience.

The results obtained from the elasto-plastic computer solution for the two tension plate widths and various boundary restraints are shown in Table 1, along with the corresponding yield line solutions. It can be seen that the yield line theory predicts ultimate loads that are much greater than those given by the computer solution. The discrepancy between the yield line solution and the corresponding computer solution can be attributed to the errant assumption of the formation of narrow plastic zones along the yield lines in the former case. In fact, the plasticity in the web develops in broad areas, as can be seen in Figs. 5 and 6.

EXPERIMENTAL RESULTS

The discussion above indicates reasons for the discrepancy between the results of the yield line theory and the computer solution. However, the question of why the experimentally obtained values of the collapse loads for these connections are much greater than even the upper-bound ultimate loads predicated by the yield line theory remains unanswered.

Although the yield line theory assumes that the deflections remain small, application of design loads based upon a yield line analysis can lead to relatively large deflections (i.e., greater than one-third to one-half the plate thickness) which are known to induce tensile membrane stresses that significantly increase the strength of a plate made of a ductile material. As an example, the yield line load of 7.0 kips for a 4-in. tension plate, Table 1, results in a factored design load of 7.0/1.7 = 4.1 kips. For this design load, the resulting deflection from Fig. 4a is found to be approximately equal to 0.16-in., which is ⅔ the thickness of the web plate (0.24-in.). A consequence of this large deflection is that the web experiences an increase in load-carrying capability as a result of the induced membrane action, which is much more efficient in supporting loads than the bending action.

Based on a large number of laboratory tests, Csernak[3] has developed an empirical relationship which predicts the ultimate strength of this connection type. This relationship is given by

$$P_u = \sigma_o h(0.33L + 0.30d) \qquad (1)$$

where L is the depth of the wide-flange section, d is the width of the tension plate, h is the web thickness and σ_o is the yield stress of the connection materials. The interesting aspect of Eq. (1) concerns the linearity of the ultimate load with respect to the web thickness. This indicates that the membrane action is the principle load transfer mechanism at load levels corresponding to the collapse load. Csernak also observed large deflections at collapse loads in his experiments.

A comparison of the collapse loads predicted by Eq. (1) with the ultimate loads computed by the yield line method, Table 1, shows that the loads from Eq. (1) are approximately two to four times larger than those given by the yield line method, indicating the additional load carrying capacity at collapse due to the membrane action.

EVALUATION OF RESULTS

Based upon the experimental and analytical evidence presented in this study, it is event that the yield line theory is not applicable to this connection type. Computer results have shown that the assumption of narrow yield lines is incorrect. In addition, experimental data indicates that membrane rather than bending action prevails as the principal means of load transfer, which results in limit loads that are significantly higher than those predicted by the yield line theory.

Although the application of the yield line method is seen to lead to conservative estimates of the ultimate strength of these connections, use of the design loads determined by the yield line theory leads to excessively large deflections in the web. This behavior, in conjunction with the resulting large flange rotations, could cause local buckling failure of the web. Although web deflections can be reduced slightly by increasing the width of the tension plate, this reduction is insignificant. Therefore, for connections with unstiffened webs, the design loads must be kept at a fairly low level in order to avoid excessive deflections in the web.

Initial yield loads predicted by the Blodgett's elastic method provide reasonably accurate estimates when compared with the computer solution. These are relatively low in magnitude and web deflections due to these loads remain within the small deflection range. Consequently, loads obtained by the Blodgett method provide safe and conservative values for the design loads in welded light bracing connections of the type discussed here.

It should be emphasized, however, that for those connections where excessive deflections are of minor concern, design loads can be estimated based on the yield line theory. Although relatively large in magnitude compared to those calculated by the Blodgett elastic method, these design loads still provide a large margin of safety to ultimate loads determined experimentally. An estimate of the ultimate loads can be made by using the experimentally obtained empirical Eq. (1). It should also be noted that web deflections at these loads are extremely large, due to the ductility of the connection materials.

In utilizing any of the analysis procedures described here for the design of these connections, one has the option to assume that the flanges provide either infinite restraint or no restraint at the web-flange juncture. Although the actual edge restraint provided the web by the flanges lies somewhere between these two limiting conditions, it has been found that the assumption of no flange restraint renders a more reasonable estimate of the load-deflection behavior.

RECOMMENDATIONS FOR DESIGN

Based upon the findings of this study, the following recommendations can be made for the design of the welded tensile web connections:

1. Where the deflections must remain small, Blodgett's elastic method can be used to obtain safe and reliable design loads.
2. Where the deflections are of minor concern, Kapp's yield line method can be utilized to calculate design loads.
3. Design loads computed by either of recommendations 1 and 2 should utilize the assumption of no flange restraint rather than infinite flange restraint.
4. Large deflections at design loads computed by the yield line theory can be avoided by using stiffeners, doubler plates, or tee sections.
5. The actual ultimate loads can be estimated by using the empirical Eq. (1), which gives values that are two to four times larger than those obtained by the yield line theory. Therefore, a quick and conservative estimate of the yield line collapse load can be achieved by dividing the values obtained from Eq. (1) by a factor of 4.

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APPENDIX A: BLODGETT'S ELASTIC METHOD

As shown in Fig. A1b, it is assumed in this method that the tensile load, P, produces a uniform stress under the tension plate that dies out linearly from the ends of the tension plate to a distance of 12 times the web thickness. This produces a force per unit width of the web which may be easily computed and is given by

$$f = \frac{P}{d + 12h} \tag{A.1}$$

By assuming that the flanges provide no restraint, a unit width of the web can be analyzed as a simply supported beam subjected to a concentrated load at the mid-span. The maximum allowable force, f, can be determined from the maximum allowable moment that a unit width of the web cross-section can support. This is given by

$$f = \frac{4\sigma_a S}{L} \tag{A.2}$$

in which σ_a is the allowable stress and S is the section modulus for a unit width of the web. Equating Eqs. (A.1) and (A.2) leads to the value of the maximum allowable load, P, which is found to be

$$P = \frac{2\sigma_a h^2(d + 12h)}{3L} \tag{A.3}$$

The bounding solution for infinite flange restraint can be easily found by noting that the maximum bending mo-

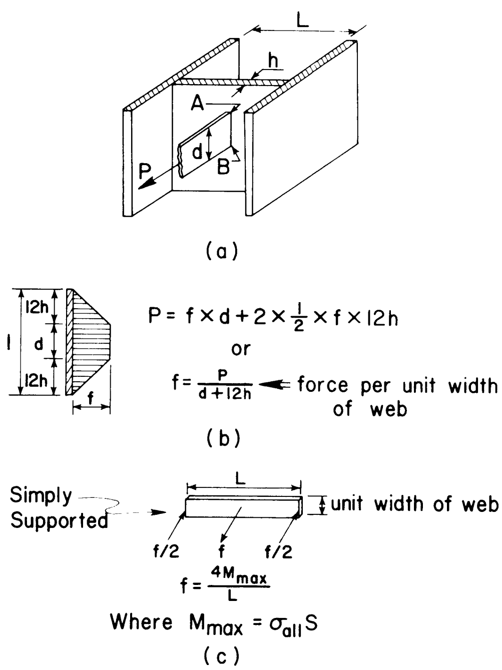

Fig. A1. Elastic analysis as proposed by Blodgett

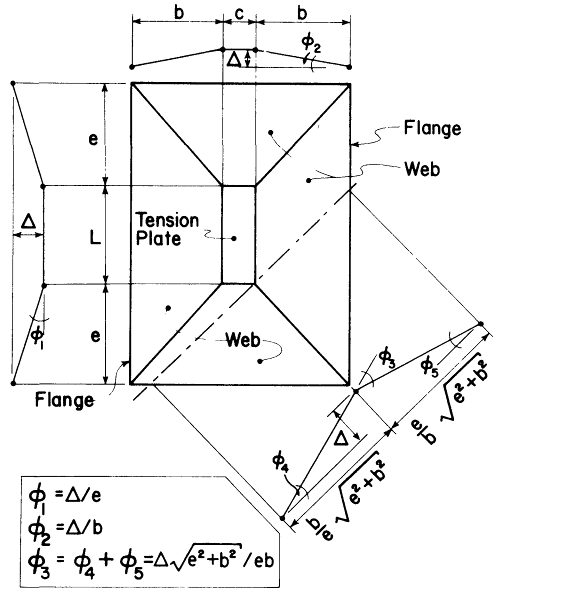

Fig. B1. Yield line pattern for the light bracing connection

ment for full-fixity is one-half that of the simply-supported case. Therefore, a simple doubling of Eq. (A.3) will give the allowable load for this connection assuming infinite flange restraint.

APPENDIX B: KAPP'S YIELD LINE METHOD

The yield line pattern assumed for this connection is shown in Fig. B1. By assuming that the parts of the web contained within the yield line rotate as rigid bodies, the internal work is exclusively done along the yield lines and is given for a unit length by the product of the uniform plastic moment, $M_p = \sigma_o h^2/4$, and the corresponding rotation, ϕ, measured normal to the yield lines. Using the virtual work principle, by summing the internal work and equating it to the external work done by the collapse load P_u on the deflection of the tension plate, results in an expression for the value of P_u in terms of the geometry of the yield pattern and the geometric and material properties of the web.

Referring to Fig. B1, and assuming that the flanges provide no restraint to the web, summation of the internal work along the yield lines results in

$$W_i = \frac{\psi}{4}[2(2b + 2c)\,\phi_1 + 2L\,\phi_2 + 4\sqrt{e^2 + b^2}\,\phi_3] \tag{B.1}$$

in which $\psi = \sigma_o h^2$. The corresponding external work performed by the ultimate load, P_u , is given by

$$W_e = P_u \Delta \tag{B.2}$$

In Eq. (B.1), the internal work is seen to be a function of a variable length, e, along the flanges, as shown in Fig. B1. By minimizing the work done by the plastic moment along the yield lines, a relationship can be obtained for the smallest load that will result in a failure mechanism of the web. Thus, differentiating Eq. (B.1) with respect to e and equating to zero leads to

$$e = b\sqrt{2 + c/b} \tag{B.3}$$

Equating internal and external work, given by Eqs. (B.1) and (B.2), respectively, and utilizing Eq. (B.3) results in

$$P_u = \psi L/2b + 2\psi\sqrt{2 + c/b} \tag{B.4}$$

By a similar procedure, the ultimate load corresponding to the bounding solution for infinite flange restraint can be shown to be given by

$$P_u = \psi L/b + 4\psi\sqrt{1 + c/2b} \tag{B.5}$$

Because this is an ultimate strength criteria, a reduction in the computed load is required to obtain the working load. Load reduction factors of 1.7 for live and dead loads, or 1.3 for these loads acting in conjunction with wind loads, are the recommended practice.