A Graphical Method for Design of Cantilevered and Suspended Span Beams

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Cantilevered and suspended span beams are widely used in industrial building roof construction, particularly in single story structures, where joist spacing and beam spans are equal. This type of construction can save a considerable amount of structural steel and reduce erection time.

According to the National Building Code of Canada,[1] beams should be designed for loading conditions which produce maximum stresses.* For continuous beam framing alternate span loading produces maximum moments.

To minimize work in determining maximum beam moments and length of cantilevers, the author has prepared charts which, for specific loading conditions, can give coefficients for moment and cantilever length.

If one span of a cantilevered beam is fully loaded with dead load plus live load (total load) and the adjacent span with dead load or dead load plus partial live load, then for the ratio

$$R_o = \frac{\text{Dead Load}}{\text{Total Load}}$$

or

$$R_o = \frac{\text{Dead Load + Partial Live Load}}{\text{Total Load}}$$

a coefficient, C, can be selected for each beam moment and cantilever length.

Maximum moment is obtained by multiplying coefficient C by the total point load and the span length. Similarly, cantilever length is equal to coefficient C multiplied by the span length. Once the maximum moments are obtained, the beam can easily be designed.

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* *In the U.S., the AISC Specification stipulates that "any probable arrangement of loads resulting in the highest stresses" is to be used in the design (Sect. 1.3.2).*

The effect of continuity in beams of more than one span is to reduce the magnitude of the maximum bending moment. Cantilever and suspended span construction has the same advantage. If a beam of span L is loaded with point loads P of equal magnitude and spacing, as shown in Fig. 1a, the bending moment diagram for this loading will be as in Fig. 1b. Similarly, the loading shown in Fig. 2a will generate the bending moment diagram as shown in Fig. 2b. Suppose the span shown in Fig. 2a is loaded with full dead load and live load and the adjacent spans are loaded with dead load or dead load plus partial live load. The resulting negative moment at the supports will be reduced from M_1 to M_1R_o, as shown in Fig. 2c. For certain lengths of cantilever the positive moment can be balanced with moment M_1. The same reasoning can be applied to Figs. 3a and 3b.

This balanced condition will provide the maximum moment for which the beam should be designed. Based on the above facts, all the cantilever lengths a, b, c, d and moments M_1 through M_5 are established mathematically and charts plotted.

USE OF CHARTS (Figs. 4 and 5)

Select the alternate span loading and calculate ratio R_o. At the calculated value of R_o, proceed vertically up to the intersection with the moment curve or the cantilever curve. From that point, draw a horizontal line to intersect the vertical axis of the chart and read the value of coefficient C.

$$\text{Maximum moment} = \text{coefficient } C \times \text{Span } L \times P_{total}$$

$$\text{Cantilever length} = \text{Coefficient } C \times \text{Span } L$$

Limitations

1. Charts are applicable only to beams of equal spans and equally spaced point loads (either 3 or 4 point loads per span).
2. All point loads shall be equal in magnitude and direction.

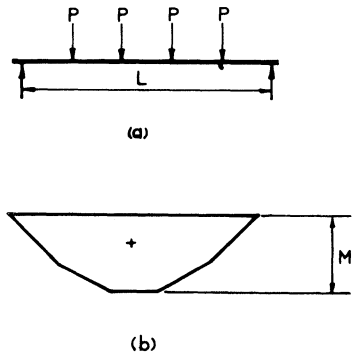


Figure 1

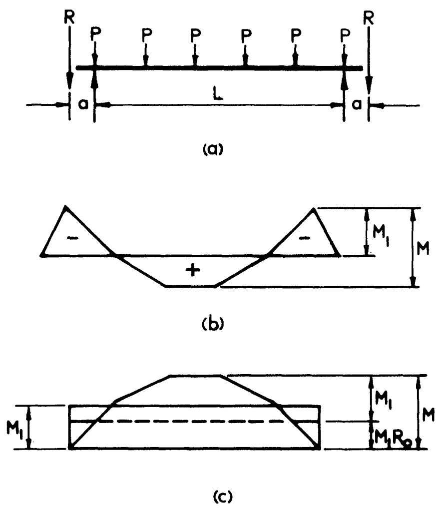


Figure 2

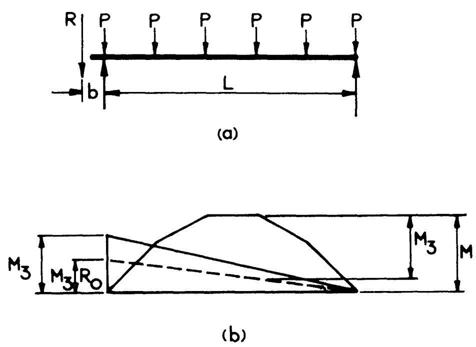


Figure 3

Example

Given:
4-point load per span, equally spaced.
Span of beam: L = 25 ft-0 in.
Point Load: P (dead load) = 2 kips
P (live load) = 3 kips
P_{total} (D.L. + L.L.) = 5 kips

Solution:
Ratio $R_o = P$ (dead load)$/P_{total} = 2/5 = 0.4$
From Fig. 5 (4-point load/span), for $R_o = 0.4$:
Select C-values for maximum moments and cantilever lengths:

Moment	C	Length	C
M_1	0.43	a	0.23
M_2	0.17	b	0.29
M_3	0.515	c	0.25
M_4	0.135	d	0.175
M_5	0.43		

Max. Moment = $C \times L \times P_{total}$:
$M_1 = 0.43 \times 25 \times 5 = 53.75$ kip-ft
$M_2 = 0.17 \times 25 \times 5 = 21.25$ kip-ft
$M_3 = 0.515 \times 25 \times 5 = 64.37$ kip-ft
$M_4 = 0.135 \times 25 \times 5 = 16.87$ kip-ft
$M_5 = 0.43 \times 25 \times 5 = 53.75$ kip-ft

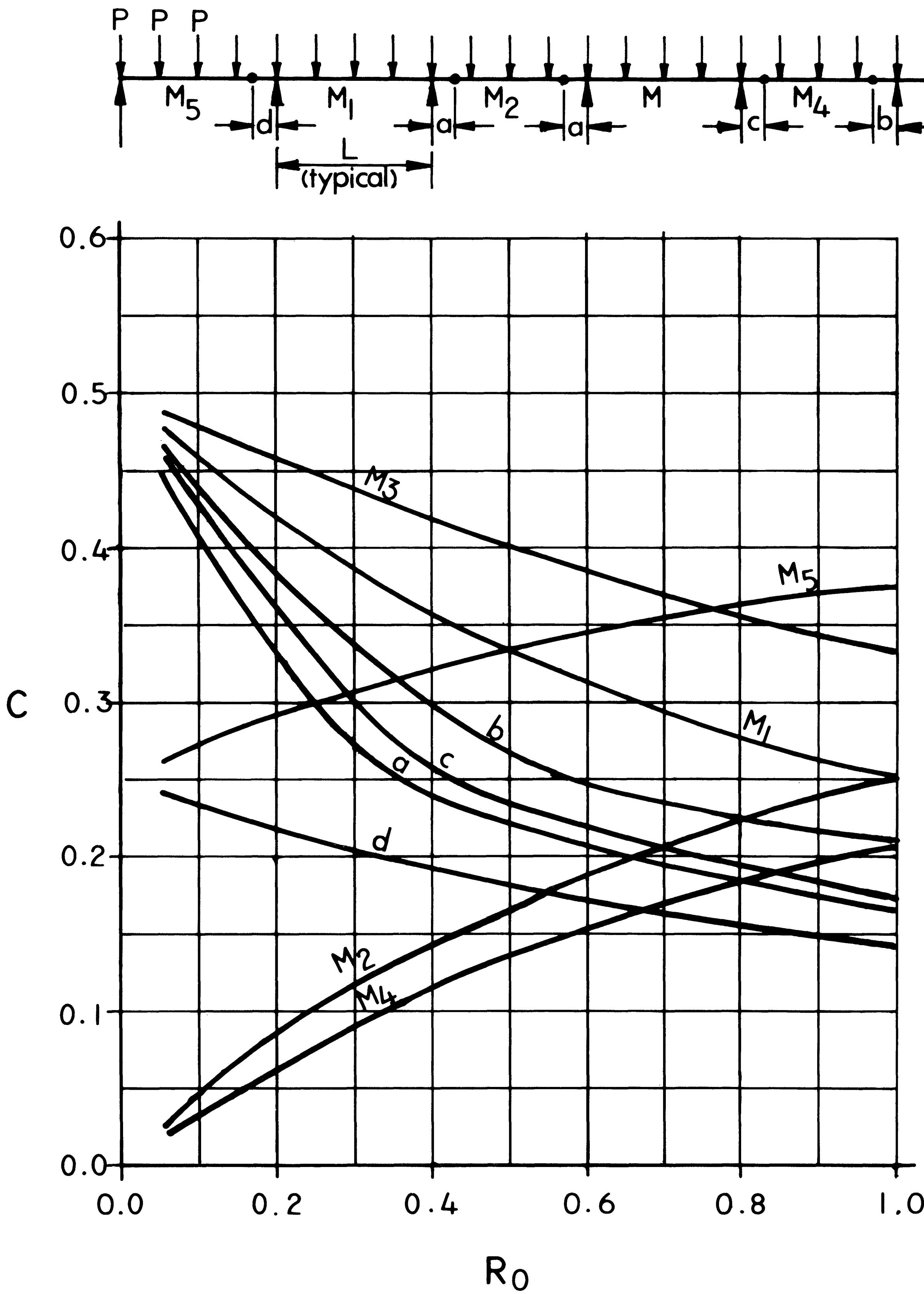
Cantilever length = $C \times L$
$a = 0.23 \times 25 = 5.75$ ft
$b = 0.28 \times 25 = 7.0$ ft
$c = 0.25 \times 25 = 6.25$ ft
$d = 0.175 \times 25 = 4.37$ ft

Selection of sections (all steel ASTM A36):*
For M_1: W14 × 22 $M_R(L_u = 6.0$ ft$) = 57.2$ kip-ft
M_2: W12 × 14 $M_R(L_b = 5.0$ ft$) = 28.2$ kip-ft
M_3: W12 × 27 $M_R(L_u = 8.4$ ft$) = 67.5$ kip-ft
M_4: W12 × 14 $M_R(L_b = 5.0$ ft$) = 28.2$ kip-ft
M_5: W14 × 22 $M_R(L_u = 6.0$ ft$) = 57.2$ kip-ft

REFERENCES

1. National Building Code of Canada 1977 *National Research Council of Canada, Ottawa, 1977.*
2. Handbook of Steel Construction *2nd Edition, Canadian Institute of Steel Construction, Toronto, 1976.*

* *Values of L_u are from Ref. 1 (CISC).*



3-POINT LOAD / SPAN

Figure 4

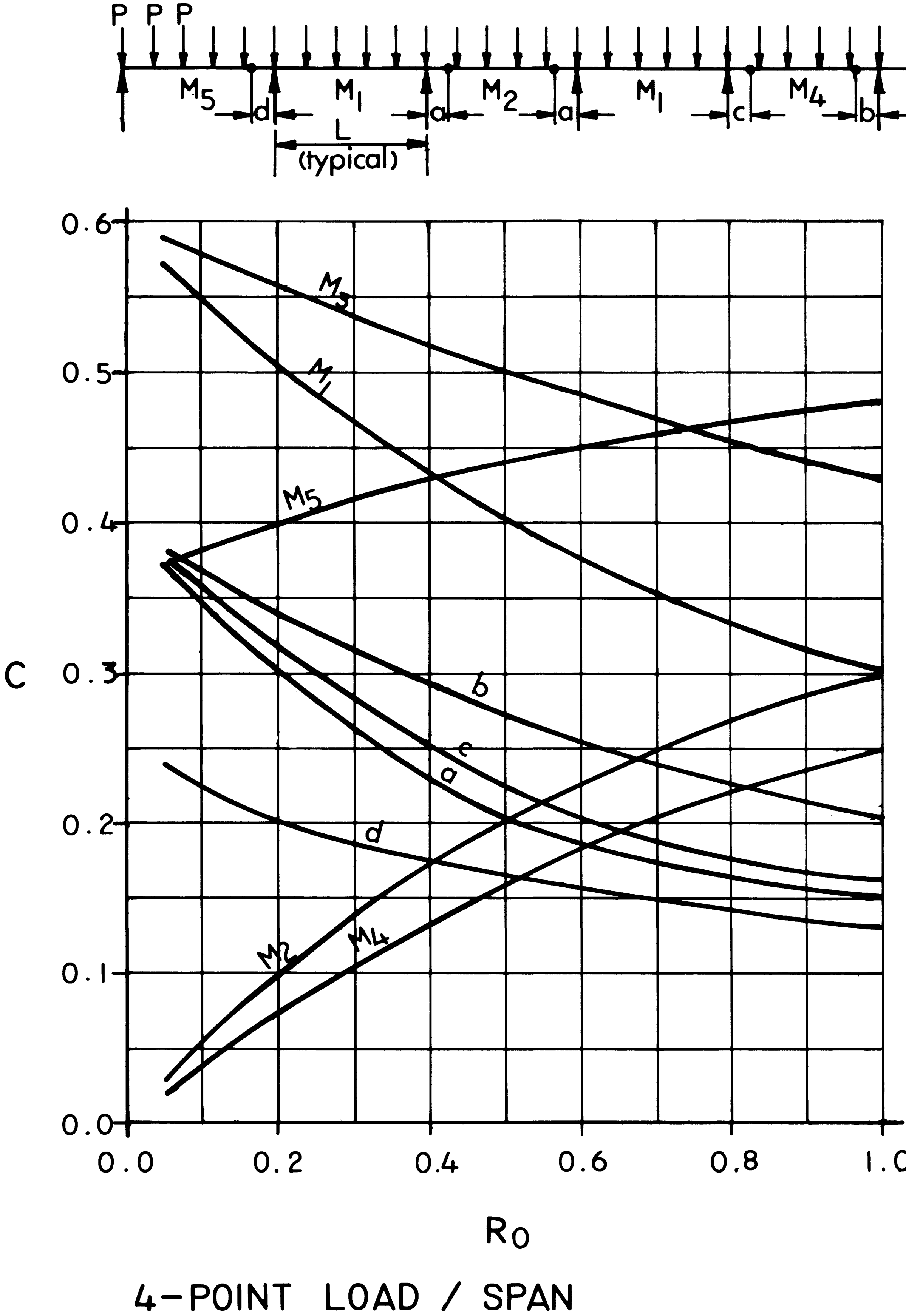

P P P
M5
d
M1
L
(typical)
a
M2
a
M1
c
M4
b
M3
C
0.6
0.5
0.4
0.3
0.2
0.1
0.0
M3
M1
M5
b
c
a
d
M2
M4
0.0
0.2
0.4
0.6
0.8
1.0
Ro
4-POINT LOAD / SPAN

Figure 5