Stress Reduction Factor for Unsupported Lengths

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The AISC Specification[1] allows $F_b = 0.66F_y$ for structural shapes meeting the compact criteria of Sect. 1.5.1.4.1, when loaded in the plane of their minor axis. A reduced bending stress, $F_b = 0.6F_y$, is allowed for shapes failing to meet the compactness criteria. The above allowable values require that the compression flange be braced at a maximum interval of lengths as determined by the smaller value obtained from the expressions

$$76\, b_f/\sqrt{F_y} \text{ and } 20{,}000/(d/A_f)F_y \text{ for } F_b = 0.66F_y$$

and

$$l/r_T \leq \sqrt{(102 \times 10^3 \times C_b)/F_y} \text{ for } F_b = 0.6F_y.$$

These lengths, designated as L_c and L_u, respectively, are listed in the AISC *Manual of Steel Construction*[2] for all shapes. Beyond these unsupported lengths, the allowable compression in flexure is gradually reduced as per Sect. 1.5.1.4.5 with the increase in unsupported lengths. This paper develops a chart to determine a length-related stress reduction factor, S_r, as a multiplier to maximum $F_b = 0.66F_y$.

Adequate lateral support for maximum allowable F_b cannot be provided in all cases of beam design, due to functional requirements of the structure. The inadequate support allows lateral buckling of the compression flange before it yields. This requires evaluation of the torsional and warping stresses in addition to the strong axis bending. Mathematical expressions for calculation of buckling stresses, to say the least, are complex and time consuming for everyday design office use. AISC Specification Sect. 1.5.1.4.5(2) provides a simple solution of reducing the allowable bending stress F_b as a function of lateral support spacing, which relieves the designer from torsion and warping stress analysis. It should be understood, though, that AISC Formulas (1.5-6a), (1.5-6b), and (1.5-7) are reasonable approximations of the exact condition in a convenient form. The limitations of these equations relative to the lateral bending and torsional resistance provisions should be understood by the designer.

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Section 1.5.1.4.5(2) states the the allowable bending stress F_b may exceed neither the larger values given by the following formulas nor $0.60F_y$:

When $\sqrt{(102 \times 10^3 C_b)/F_y} \leq l/r_T \leq \sqrt{(510) \times 10^3 C_b)/F_y}$:

$$F_b = [2/3 - F_y(l/r_T)^2/(1530 \times 10^3 C_b)]F_y \quad (1.5\text{-}6a)$$

When $l/r_T \leq \sqrt{(510 \times 10^3 C_b)/F_y}$:

$$F_b = 170 \times 10^3 C_b/(l/r_T)^2 \quad (1.5\text{-}6b)$$

Or, when the compression flange is solid and approximately rectangular in cross section and its area not less than that of tension flange:

$$F_b = 12 \times 10^3 C_b/(ld/A_f) \quad (1.5\text{-}7)$$

The graphical solution of these equations assumes $C_b = 1$ and provides the values for steel with $F_y = 36$ ksi and $F_y = 50$ ksi. The stress reduction factor is designated as $S_r = F_b/0.66F_y$.

For $F_y = 36$:

When $53 \leq l/r_T \leq 119$:

$$F_b = 0.66F_y - F_y{}^2(l/r_T)^2/(1530 \times 10^3)$$

or

$$S_r = 1 - F_y(l/r_T)^2/(1530 \times 10^3 \times 0.66)$$
$$= 1 - 3.565 \times 10^{-5}\,(l/r_T)^2$$

When $l/r_T \geq 119$:

$$F_b = 170 \times 10^3/(l/r_T)^2$$

or

$$S_r = 170 \times 10^3/[(l/r_T)^2 \times 0.66F_y]$$
$$= 7154.88/(l/r_T)^2$$

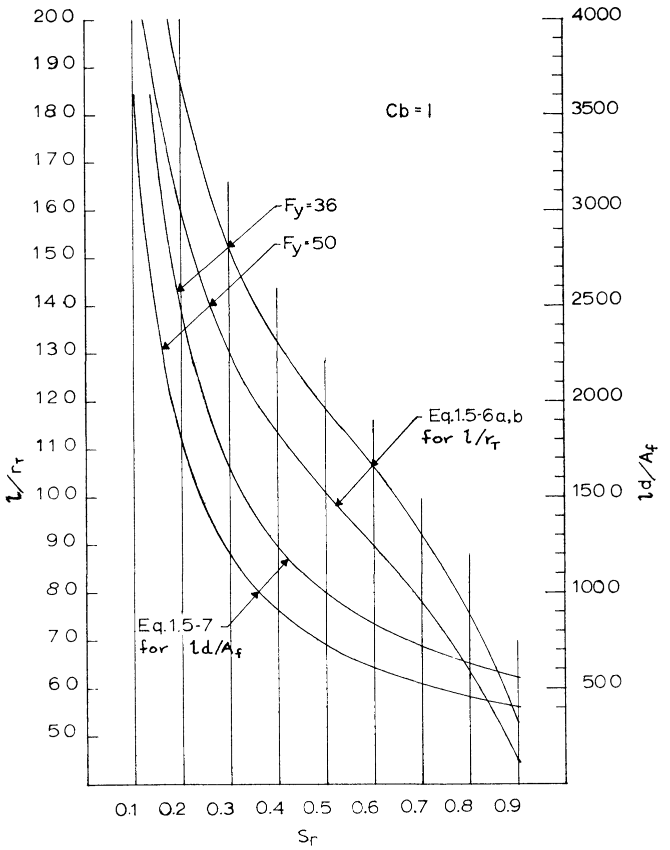

Fig. 1. Chart for stress reduction factor ($C_b = 1$)

Equation (1.5-7):

$$F_b = 12 \times 10^3/(ld/A_f)$$

or

$$S_r = 12 \times 10^3/[(ld/A_f) \times 0.66F_y]$$

$$= 505/(ld/A_f)$$

Similar results for $F_y = 50$ ksi are:

When $45 \leq l/r_T \leq 101$:

$$S_r = 1 - 4.951 \times 10^{-5}\ (l/r_T)^2$$

When $l/r_T \geq 101$:

$$S_r = 5151.51/(l/r_T)^2$$

Equation (1.5-7):

$$S_r = 363.63/(ld/A_f)$$

The chart is developed with S_r as abscissa, l/r_T the left ordinate, and ld/A_f the right ordinate. Values of S_r can be read for l/r_T and ld/A_f. Use the larger value of S_r in design, as follows:

$$M_{capacity} = S_r \times M_R \text{ (listed in the AISC Manual}^2\text{)}$$

$$F_b = S_r \times 0.66F_y$$

Example 1

Given: W16 x 36; $F_y = 50$ ksi; $M_R = 155$ kip-ft
$r_T = 1.79$; $d/A_f = 5.28$
Unbraced length $l = 120$ in.

Solution: $l/r_T = 67$ $ld/A_f = 633$
From chart: $S_r = 0.78$ (l/r_T controls)
$M_{capacity} = 0.78 \times 155 = 121$ kip-ft
$F_b = 0.79 \times 33 = 26$ ksi

Example 2

Given: W14 x 22; $F_y = 36$ ksi; $M_R = 58$ kip-in.
$r_T = 1.25$; $d/A_f = 8.20$
Unbraced length $l = 96$ in.

Solution: $l/r_T = 76$; $ld/A_f = 78$
From chart: $S_r = 0.8$ (l/r_T controls)
$M_{capacity} = 0.8 \times 58 = 46$ kip-ft
$F_b = 0.8 \times 24 = 19$ ksi

REFERENCE

1. Specification for the Design, Fabrication and Erection of Structural Steel for Buildings *American Institute of Steel Construction, Chicago, Ill., Nov. 1, 1978.*
2. Manual of Steel Construction *Eighth Edition, American Institute of Steel Construction, Chicago, Ill., 1980.*