Method of Combining Mechanisms in Plastic Analysis

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GIVEN THE COLLAPSE load for the independent mechanisms of a structure, the failure load of the combined mechnisms can be obtained by superposition if hinge cancellations are noted. Although this method has been presented previously, a modified type of solution is presented herein and an example is given. This simplified method gives a better understanding of the actual process of combining independent mechanisms and the resulting combined mechanism thereof. Superposition is valid for any number of component independent mechanisms. In the simplification presented, an equation is written for each location of desired plastic hinge cancellation in the combined mechanism. In this "consistent deformation" type of solution, each independent mechanism will be multiplied by a suitable factor which is obtained from the equations. The collapse load of a combined mechanism composed of four independent mechanisms is found as an illustration of the method.

INTRODUCTION

The method of combining mechanisms to find the ultimate load of complex mechanisms was presented by Neal,[1,2] Symonds,[3] Lothers,[4] Hodge,[5] Beedle,[6] and others. A simplified approach to the method is presented herein. It has been recognized that superposition of basic independent mechanisms is valid and that if hinge cancellations occur in the process, a lower collapse load will be obtained. Since, in general, the collapse load of the basic independent mechanisms is found by the mechanism method, any new combined collapse load will be a smaller upper bound when one or more hinges are cancelled. When two or more independent mechanisms are superposed to cancel two or more hinges, the value of the plastic hinge rotation of each component mechanism (generally denoted by θ) at the different locations must be considered and sometimes modified so that the desired hinge cancellations result. An equation for each cancelled hinge is written and modifying factors for each independent mechanism are found. These correction factors are applied to the deflected configurations and to the individual work equations. The cancellation for each mechanism is then subtracted from each modified work equation. It has been found convenient to place a sign on each plastic hinge rotation, positive or negative according to whether the original angle increases or decreases in the structure when the plastic rotation takes place.

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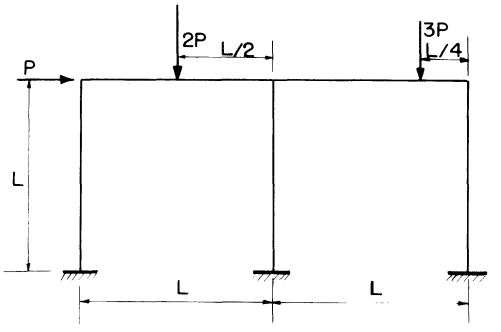

Fig. 1. Two-bay rectangular portal frame

PROCEDURE

The steps in finding the collapse load of a combined mechanism from the basic independent mechanisms are as follows:

1. Write the work equation for each independent mechanism. The rotation at each plastic hinge is noted. A sketch is very helpful.
2. For each plastic hinge cancelled in the combined mechanism, a "consistent deformation" equation is written. All but one of the independent mechanisms rotations will be multiplied by a factor.
3. From these simultaneous equations, the multiplying factors are found.
4. Each work equation is multiplied by its corresponding factor and cancelled hinges result in subtracted internal energy.
5. Adding the modified equations gives the collapse load of the combined mechanism.

ANALYSIS OF A SPECIFIC TWO-BAY RECTANGULAR PORTAL FRAME

An illustration of the method of combined mechanisms where four independent mechanisms are superposed is given for the structure shown in Fig. 1. The monotonic system of loads, i.e., proportionately increasing loads, consists of two vertical loads, $2P$ and $3P$, at the center of the left beam and off-center on the right beam, respectively. A lateral load P representing wind effects acts at the top of the left column. The two spans are equal, the plastic strength of the members, M_p, is constant and the three column bases are fixed.

Figures 2(a), (b), (c) and (d) show the basic independent mechanisms. The corresponding work equations are:

Mechanism No. 1:

$$P\theta L = 4M_p\theta \tag{1}$$

Mechanism No. 2:

$$\frac{3P\theta L}{4} = \frac{8M_p\theta}{3} \tag{2}$$

Mechanism No. 3:

$$P\theta L = 6M_p\theta \tag{3}$$

Mechanism No. 4:

$$0 = 3M_p\theta \tag{4}$$

These equations correspond to the left beam, right beam, sway and joint mechanisms respectively. Where three members intersect, as at the top of the center column, the hinge cancellations occur at the *ends* of individual members, such as at the top of the center column and at the left end of the right beam. For example, the collapse load of combination 1, 2, 3 and 4 (see Fig. 3) results due to cancellation of plastic hinges at the left corner of the frame, at the top of the center column, and at the ends of the beams. The work equations and configurations of mechanisms 1, 2 and 3 are multiplied by factors m, n and p, respectively, so that upon adding the modified mechanisms 1, 2, and 3 with 4, the plastic hinges are cancelled at the locations mentioned. Referring to Fig. 2, these equations are:

Left corner:

$$m(-\theta) + p(+\theta) = 0 \tag{5}$$

Top of center column:

$$p(-\theta) + \theta = 0 \tag{6}$$

Left end of right beam:

$$n\left(-\frac{\theta}{3}\right) + \theta = 0 \tag{7}$$

from which

$$m = 1 \qquad n = 3 \qquad p = 1$$

The modified work equations are:

$$P\theta L = 4M_p\theta - M_p\theta \tag{8}$$

$$\frac{9P\theta L}{4} = 8M_p\theta - M_p\theta \tag{9}$$

$$P\theta L = 6M_p\theta - 2M_p\theta \tag{10}$$

$$0 = 3M_p\theta - 2M_p\theta \tag{11}$$

The terms that subtract are due to hinges cancelled in each independent mechanism. On adding Equations (8) through (11), the collapse load of the combined mechanism shown in Fig. 3 results:

$$P_{1234} = \frac{60}{17}\frac{M_p}{L} \tag{12}$$

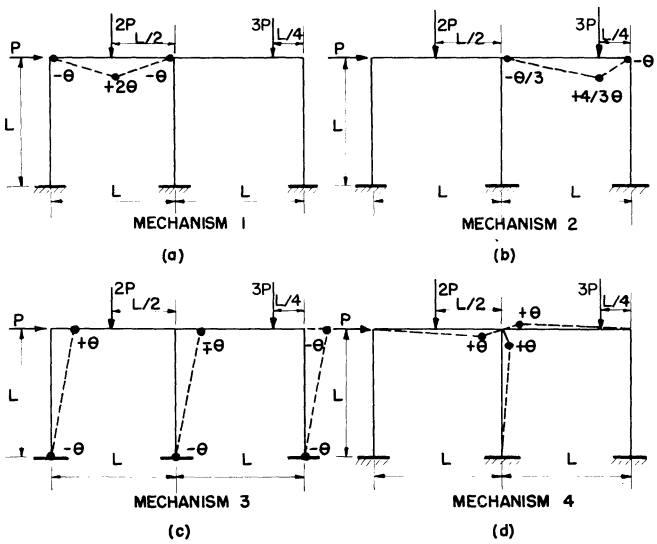

Fig. 2. Independent mechanisms

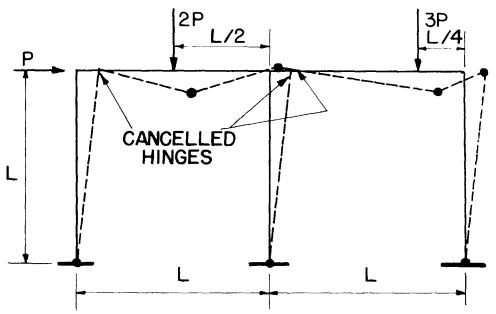

Fig. 3. Combined mechanism

CONCLUSIONS

1. The collapse load of combined mechanisms may be found by superposition if the plastic hinge rotations in the component basic mechanisms are such that a hinge cancellation takes place on superposition. Often, factors that multiply both sides of the basic work equations must be used. These factors apply to both the geometric changes and to the work equations. The effect of the cancelled hinges must be included in each of the final work equations used to obtain the collapse load of the combined mechanism.
2. It is convenient to place an algebraic sign on each plastic hinge rotation to facilitate computation of the combined collapse loads.
3. The collapse load of a combined mechanism due to superposition of four independent mechanisms has been found and is used as an illustration. The example consists of a two-bay rectangular portal frame with fixed bases under a monotonic system of loads consisting of a vertical load at the center and off-center on each rafter and a horizontal load at the top of one column.
4. The superposition method is general and may be applied to any frame when the work equations of the basic mechanisms are known. In rectangular frames, these basic mechanisms are beam mechanisms, panel mechanisms (sidesway), gable mechanisms and joint mechanisms (in multi-bay and multi-story rigid frames). The collapse load of combinations of two, three, four, etc., basic mechanisms can be obtained with relative ease as illustrated by the example using the simplified superposition concept presented.

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