

Generalized Design of Columns Subjected to Combined Axial Load and Bending Moment

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The design of wide-flange steel columns subjected to combined axial compression load and bending moment according to the AISC *Manual of Steel Construction*¹ involves a trial-and-error procedure. In this method the applied bending moment is converted to an equivalent axial load by using a bending factor. This allows the selection of a trial section from the AISC Column Tables. Then, using the modified AISC interaction formulas, the required axial tabular load can be determined. From this load, another trial section is selected, and the procedure is repeated until a satisfactory section is obtained. The selected section must be checked for compliance with the AISC Formulas (1.6-1a), (1.6-1b), and (1.6-2). This process is often tedious and time consuming and is not efficient for usage in the design office.

Several methods have been presented which tend to simplify the AISC design procedure for columns subjected to combined axial load and bending moment. These methods either modify the AISC column interaction equations to minimize the number of trials used in selecting the final section, or present design charts, graphs, and tables.

Rubinsky² presented two methods for the design of beam-columns. The first method converts the axial load and moment about the weak axis ($Y-Y$) into an equivalent moment about the strong axis ($X-X$). This reduces the problem to that of selection of a beam with bending about the strong axis. Selection of a member then could be obtained by using the AISC Beam Tables or Charts. His second method is similar to the AISC design procedure. Phang and Celenza³ presented tables and graphs which incorporate member properties, design parameters, and the AISC Specification, thus eliminating the need for referring to the AISC *Manual*. Burgett⁴ developed a modified interaction equation where the moments about the strong ($X-X$) and the weak ($Y-Y$) axes are converted into an

equivalent axial load through bending factors provided for each wide-flange column section. The equivalent axial load is used to select a trial section from the AISC Column Tables, and the final section is arrived at by successive approximation and trial-and-error procedure. Phang and Babyak⁵ provided charts which give values of the modified bending amplification factor in AISC Formula (1.6-1a). The use of these charts may reduce the time and effort put forth by the designer in selecting a beam-column section.

The methods indicated above are usually limited in application, and do not cover a wide range of column sections and material properties. Also, the designer, for most practical cases, has to interpolate the values between graphs or charts to select the proper column section, and usually the trial-and-error procedure is unavoidable. Thus the effectiveness of these methods is often reduced.

PROBLEM STATEMENT

This paper presents a technique for the design of wide-flange steel columns (column sections only) subjected to an axial compression load and bending moment about the strong axis ($X-X$). This technique permits a rapid preliminary selection for a member by using a nomograph and without the use of the AISC Column Tables. This selection is accomplished without using a trial section, and only the variables involved in designing of a wide-flange steel column subjected to combined loading are required. A study by Monasa⁶ for the design of reinforced concrete flat slabs has shown that such a technique is possible.

The selected section from the nomograph may be used for preliminary design without further checking by the column interaction equations. However, the members of the final structural system should be checked for compliance with the AISC Formulas (1.6-1a), (1.6-1b), and (1.6-2).

DEVELOPMENT OF THE METHOD

Empirical equations which relate the variables involved in the design of beam-column members are developed. These equations may replace the AISC interaction formulas. However, the selection of a member is facilitated by using a nomograph. To develop the empirical equations, the dimensional analysis technique is used to formulate a general

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qualitative expression. Dimensional analysis gives qualitative rather than quantitative relationships. However, when combined with experimental procedures or design data, it may give quantitative results and accurate prediction equations which may be used in structural design.

Dimensional analysis deals with the dimensions in which the variables are expressed. It bases results solely on the relationships that exist between variables because of their dimensions, not some physical or natural law.

The variables affecting the design of beam-columns have been chosen to cover a wide range of practical design conditions. The dimensional analysis technique is used to combine the variables to form a functional equation of dimensionless parameters. The nature of the functional equation is determined from a statistical analysis of data obtained from beam-column design problems.

The Functional Equation—The functional equation involves the dependent variable, the section modulus, S_x , and the variables which affect the design of beam-columns. These variables are:

<i>Symbol</i>	<i>Variable</i>	<i>Dimensions</i>
F_y	Yield strength of specified steel	FL^{-2}
P	Axial compression force	F
M	Bending moment about strong axis	FL
$K_x L_x$	Effective length, x -direction (strong axis)	L
$K_y L_y$	Effective length, y -direction (weak axis)	L
S_x	Section modulus about strong axis	L^3

where F and L represent force and length dimensions, respectively. The ratio $K_x L_x / K_y L_y$ is expressed as the symbol R , thus eliminating the variable $K_x L_x$ or $K_y L_y$.

It is desired that the section modulus be the unknown in the prediction equations. Therefore, S_x is expressed as a function of all other variables as follows:

$$S_x = f_1(F_y, K_y L_y, R, P, M) \quad (1)$$

Equation (1) contains six variables expressed in two dimensions, force and length. Using dimensional analysis, the variables in Eq. (1) can be combined in several ways to form sets of functional equations of dimensionless parameters. The minimum number of dimensionless parameters required to express the functional equation can be determined by using the Buckingham Pi Theorem.⁷ Henceforth, these parameters are called π terms. The Buckingham Pi Theorem states that the number of dimensionless and independent quantities required to express a relationship among variables is equal to the number of these variables minus the number of dimensions involved. This can be expressed as follows:

$$s = n - b$$

in which

- s = number of π terms
- n = total number of variables involved
- b = number of basic dimensions involved

Therefore, from Eq. (1), $s = 6 - 2 = 4$; hence, a minimum of four dimensionless quantities are required to express the relationship between the variables in Eq. (1). These variables may be combined in several ways to form numerous sets of dimensionless parameters. One such set is

$$\frac{(K_y L_y)^3}{S_x} = f_2 \left[\frac{P(K_y L_y)}{M}, R, \frac{F_y (K_y L_y)^2}{P} \right] \quad (2)$$

The above equation is derived in Appendix A; it may be written as follows:

$$\pi_1 = f_2(\pi_2, \pi_3, \pi_4) \quad (3)$$

where each π term in Eq. (3) represents the corresponding dimensionless parameter in Eq. (2). The above π terms are dimensionless and basically independent, i.e., none of them can be derived from the others.

To determine the nature of the unknown function f_2 in Eq. (2) or (3), data from beam-column design problems have to be obtained.

Design Data and Range of Variables—A wide range of the dimensionless parameters in the functional equation and values of independent variables were used in this study. The following represent this range:

$$\begin{aligned} \pi_2 &= 1.0 - 300.0 \\ \pi_3 &= 1.0, 1.25, 1.5, 1.75, 2.0 \\ \pi_4 &= 10.0, 20.0, 25.0, 30.0, 40.0, 50.0 \\ \pi_5 &= 10.0, 12.0, 14.0, 16.0, 18.0, 20.0 \end{aligned}$$

and

$$\begin{aligned} P &= 10.0 - 4500.0 \text{ kips} \\ M &= 10.0 - 3000.0 \text{ kip-ft} \\ F_y &= 36 \text{ and } 50 \text{ ksi} \\ K_y L_y &= 10 - 20 \text{ ft} \end{aligned}$$

Several combinations of these variables were prepared to obtain the design data according to the AISC Specification, where each design is based on the selection of the lightest available column section.

For a specific wide-flange section, and for given values of P , $(K_y L_y)$, R , and F_y , the moment capacity was determined. This was done by setting AISC Formulas (1.6-1a) and (1.6-1b) equal to unity and solving for the allowable moment, which is the minimum value calculated from the above formulas. It is assumed that both the axial compression load and bending moment are significant, so that neither could be ignored in the process of design. Only column sections, i.e., W14's, W12's, W10's, and W8's are considered in preparing the design data. Therefore, designs governed by AISC Formula (1.6-2) are not included. This formula controls when the ratio f_a / F_a is less than 0.15, and wide-flange sections other than column sections might be more effective. Coefficient C_m is considered to be equal to 0.85 in all designs.

The Prediction Equations—The functional equation, Eq. (2) or (3), involves dimensionless parameters, the π terms, and an unknown function, f_2 . In order to determine the nature of the function f_2 and to develop the prediction equations which would evaluate the required section modulus given the other design variables, component equations must be obtained. The component equations are found by arranging the π terms in the functional equation in such a way that all the independent π terms, except one, remain constant. Then by varying that one π term with respect to π_1 , the dependent π term, and using curve fitting techniques, a component equation relating π_1 and one independent π term is obtained. This procedure is performed for each independent π term to obtain the remaining component equations. The validity test⁷ is then used to determine the proper combination of the component equations which form the general prediction equation.

It was observed, when plotting π_1 versus π_2 while all other π terms remained constants, that the data did not plot as a smooth curve; however, data for the same $K_y L_y$ did plot smoothly. Therefore, the dimensionless parameter ($K_y L_y / 1.0$ ft) was added to the functional equation, Eq. (2), as the fifth π term. Thus, Eqs. (2) and (3) can be written as follows:

$$\frac{(K_y L_y)^3}{S_x} = f_3 \left[\frac{P(K_y L_y)}{M}, R, \frac{F_y (K_y L_y)^2}{P}, \frac{K_y L_y}{1.0} \right] \quad (4)$$

and

$$\pi_1 = f_3(\pi_2, \pi_3, \pi_4, \pi_5) \quad (5)$$

Prediction equations, to replace the AISC Formulas (1.6-1a) and (1.6-1b), are derived using two distinct sets of design data, where each set of data was governed by one of the above formulas.

From design data governed by AISC Formula (1.6-1b), it has been determined that the π term, π_3 or R , could be eliminated from the functional equation, Eq. (4). It was noted that when varying π_3 and holding all other independent π terms constant, the value of π_1 remained unchanged. This is true because AISC Formula (1.6-1b) governs when yielding controls. Therefore, Eq. (5), in this case, is written as follows:

$$\pi_1 = f_4(\pi_2, \pi_4, \pi_5) \quad (6)$$

Eq. (6) involves four π terms, therefore, three component equations can be obtained. These equations are combined to formulate the general prediction equation for beam-column designs governed by AISC Formula (1.6-1b).

The Component Equations—These equations are obtained by using the design data and range of variables of beam-column design problems. To obtain the component equation relating π_1 and π_2 , π_4 and π_5 are held constant, equal to 20.0 and 14.0, respectively. Using the graphs of Fig. 1 or available design data, several values of π_1 corresponding to different values of π_2 can be obtained. By

performing a least squares statistical analysis on π_1 and π_2 , the relationship between them is found to be logarithmic, i.e.,

$$\pi_1 = a \pi_2^b \quad (7)$$

and the values of a and b are found to be equal to 1.3287 and 0.7811, respectively. Therefore, Eq. (7) yields:

$$\pi_1 = 1.3287 \pi_2^{0.7811} \quad (8)$$

In a similar manner, letting $\pi_2 = 8.0$ and $\pi_5 = 14.0$ and using Fig. 1 or design data from available tables, values of π_1 corresponding to different values of π_4 can be obtained. This relationship can be expressed as:

$$\pi_1 = 0.3275 \pi_4^{1.0109} \quad (9)$$

Letting $\pi_2 = 8.0$ and $\pi_4 = 20.0$ and using Fig. 2, the relationship between π_1 and π_5 is expressed as:

$$\pi_1 = 4.6858 \pi_5^{0.1266} \quad (10)$$

Using the validity test,⁷ the functional equation Eq. (4) was found to exist as a product and can be expressed as follows:

$$\pi_1 = \frac{f(\pi_2, \bar{\pi}_4, \bar{\pi}_5) f(\bar{\pi}_2, \pi_4, \bar{\pi}_5) f(\bar{\pi}_2, \bar{\pi}_4, \pi_5)}{f(\bar{\pi}_2, \bar{\pi}, \bar{\pi}_5)^{s-2}} \quad (11)$$

where s is the number of dimensionless quantities and $f(\bar{\pi}_2, \bar{\pi}_4, \bar{\pi}_5)$ is the value of π_1 when $\pi_2 = 8.0$, $\pi_4 = 20.0$, and $\pi_5 = 14.0$. From Fig. 1 it is shown that

$$f(\bar{\pi}_2, \bar{\pi}_4, \bar{\pi}_5) = 6.7432$$

Substituting the component equations, Eqs. (8), (9), and (10), and the value of $f(\bar{\pi}_2, \bar{\pi}_4, \bar{\pi}_5)$ into Eq. (11) yields a prediction equation which corresponds to AISC Formula (1.6-1b), which may be written as follows:

$$\pi_1 = 0.04484 \pi_2^{0.7811} \pi_4^{1.0109} \pi_5^{0.1266} \quad (12)$$

The same approach was used to develop the prediction equations governing the AISC Formula (1.6-1a). Two prediction equations were derived to comply with the AISC Formula (1.6-1a) requirements. The first equation covers the range of π_2 between 0.0 and 50.0 and the second equation covers values of $\pi_2 > 50.0$. Also, it was observed that when plotting π_1 versus π_2 , and holding constant all other π terms in the functional equation, Eq. (4), that the data did not plot as a smooth curve, but data for individual values of F_y did. Therefore, curves for individual values of F_y (36 and 50 ksi) were plotted separately. The prediction equations for F_y equal to 36 and 50 ksi, respectively, can be expressed as

$$\pi_1 = 0.08936 \pi_2^{0.4482} \pi_3^{-0.1563} \pi_4^{0.9548} \pi_5^{0.2817} \quad (13)$$

and

$$\pi_1 = 0.09130 \pi_2^{0.4478} \pi_3^{-0.1587} \pi_4^{0.9535} \pi_5^{0.2811} \quad (14)$$

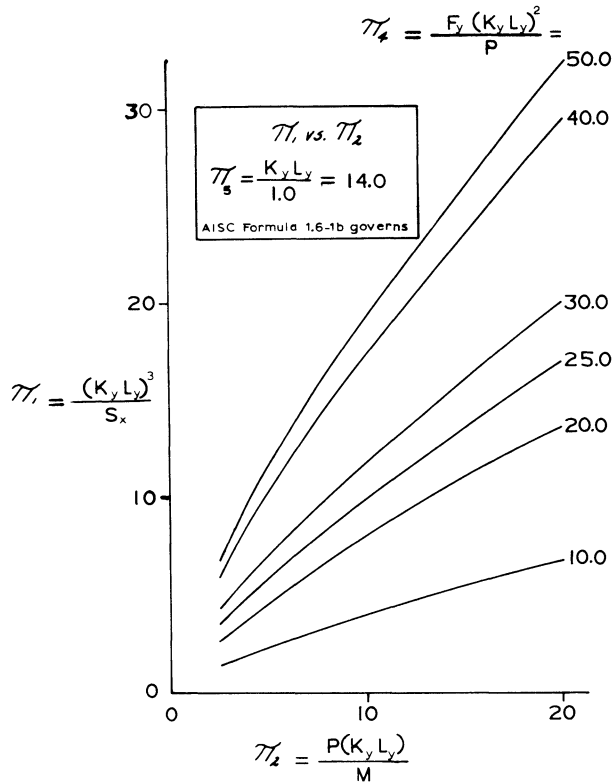


Fig. 1. π_1 versus π_2 ; π_4 variable

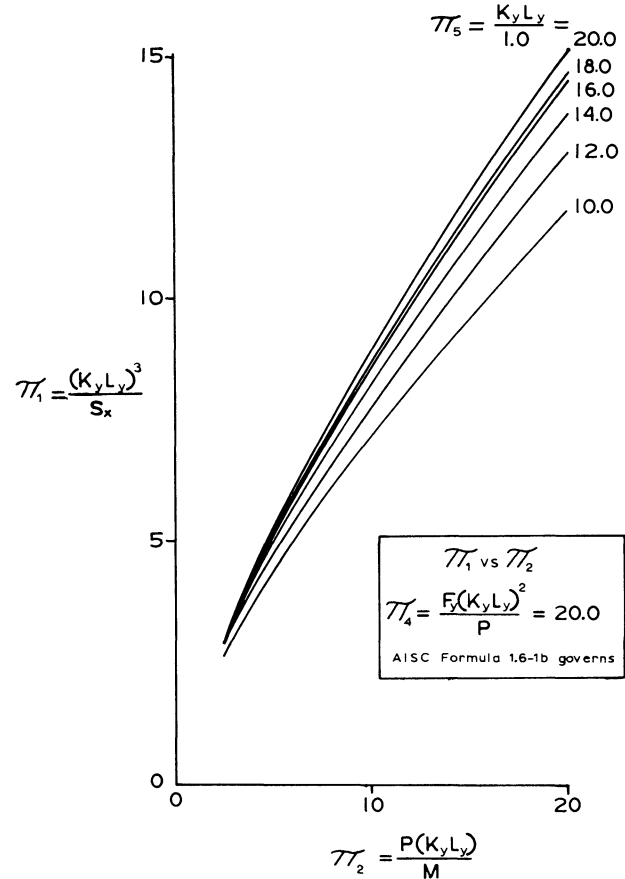


Fig. 2. π_1 versus π_2 ; π_5 variable

In order to use the same exponents in Eqs. (13) and (14), the exponents of Eq. (14) were replaced by the ones in Eq. (13) and, by adjusting the coefficient of Eq. (14), this equation yielded the following:

$$\pi_1 = 0.08349 \pi_2^{0.4482} \pi_3^{-0.1563} \pi_4^{0.9548} \pi_5^{0.2817} \quad (15)$$

The relationship between the coefficients in Eqs. (13) and (15), and F_y is logarithmic, and it can be expressed as

$$K = 0.1869 F_y^{-0.2060} \quad (16)$$

where K is the coefficient equal to 0.08936 and 0.08749 in Eqs. (13) and (15), respectively.

By substituting the value of K from Eq. (16) into Eqs. (13) and (15), the prediction equation for values of π_2 between 0.0 and 50.0 and any value of F_y is given as

$$\pi_1 = 0.1869 F_y^{-0.2060} \pi_2^{0.4482} \pi_3^{-0.1563} \pi_4^{0.9548} \pi_5^{0.2817} \quad (17)$$

In a similar manner the prediction equation for values of π_2 larger than 50.0 and any value of F_y is developed. This equation can be written as follows:

$$\pi_1 = 0.2552 F_y^{-0.2060} \pi_2^{0.2184} \pi_3^{-0.2032} \pi_4^{0.9659} \pi_5^{0.4977} \quad (18)$$

Equations (12), (17), and (18) constitute the final solution relating the variables in steel wide-flange beam-column design. These equations can be written where S_x , the dependent variable, is expressed as a function of all other variables. Therefore, these equations are, respectively:

$$S_x = 22.3025 \frac{P^{0.2298} M^{0.7812} (K_y L_y)^{0.0704}}{F_y^{1.0109}} \quad (19)$$

$$S_x = 5.3494 \frac{P^{0.5066} M^{0.4482} R^{0.1563} (K_y L_y)^{0.3605}}{F_y^{0.7488}} \quad (20)$$

$$S_x = 3.9192 \frac{P^{0.7475} M^{0.2184} R^{0.2032} (K_y L_y)^{0.3623}}{F_y^{0.7599}} \quad (21)$$

To simplify the design process and eliminate the use of the above prediction equations, Nomographs 1, 2, and 3 are constructed from Eqs. (19), (20), and (21), respectively. These nomographs are shown in Figs. 3, 4, and 5. For a comprehensive discussion on nomograph construction, the reader is referred to Refs. 8 and 9.

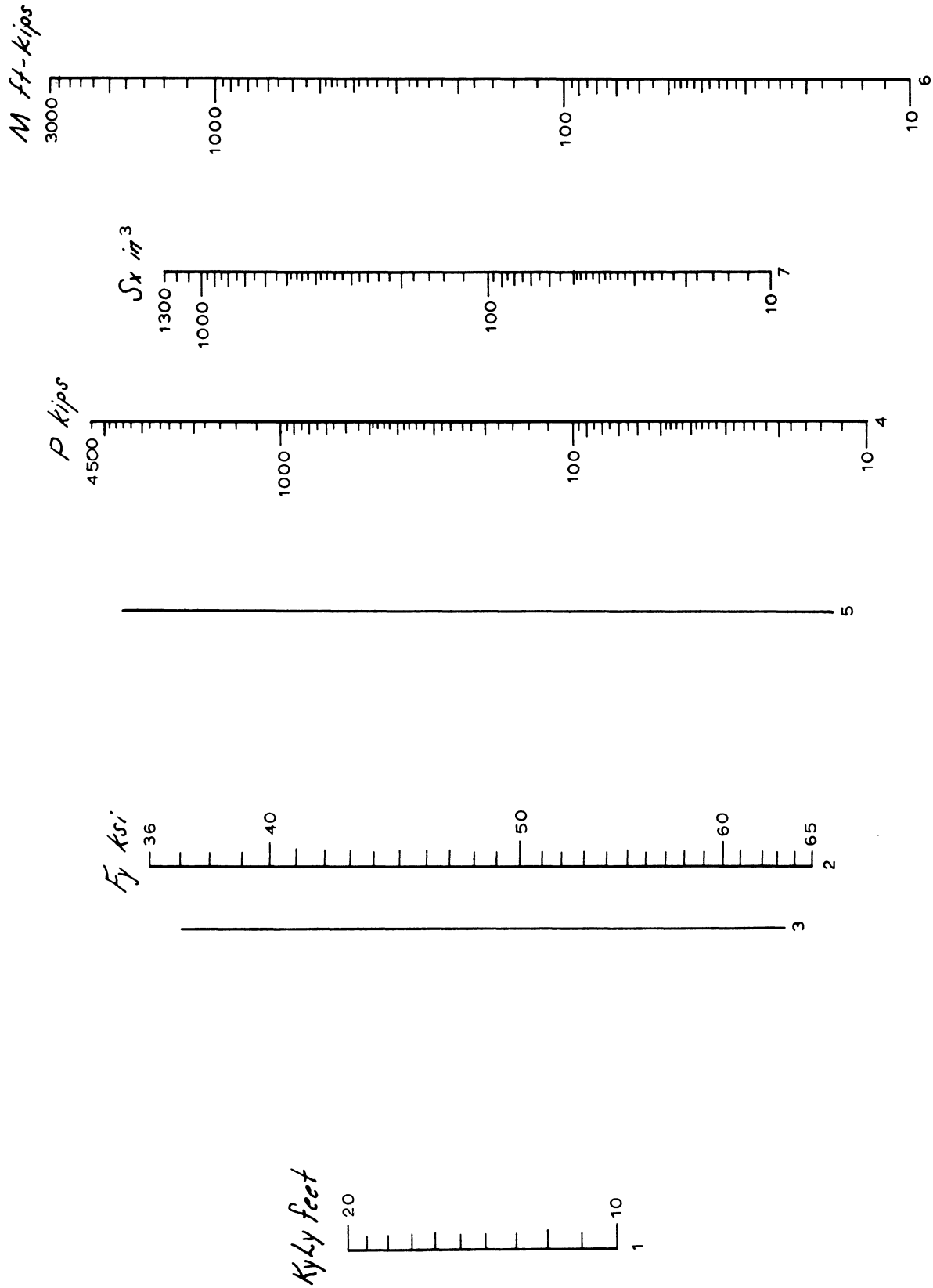


Fig. 3. Nomograph 1. Range of π_2 : 0-30

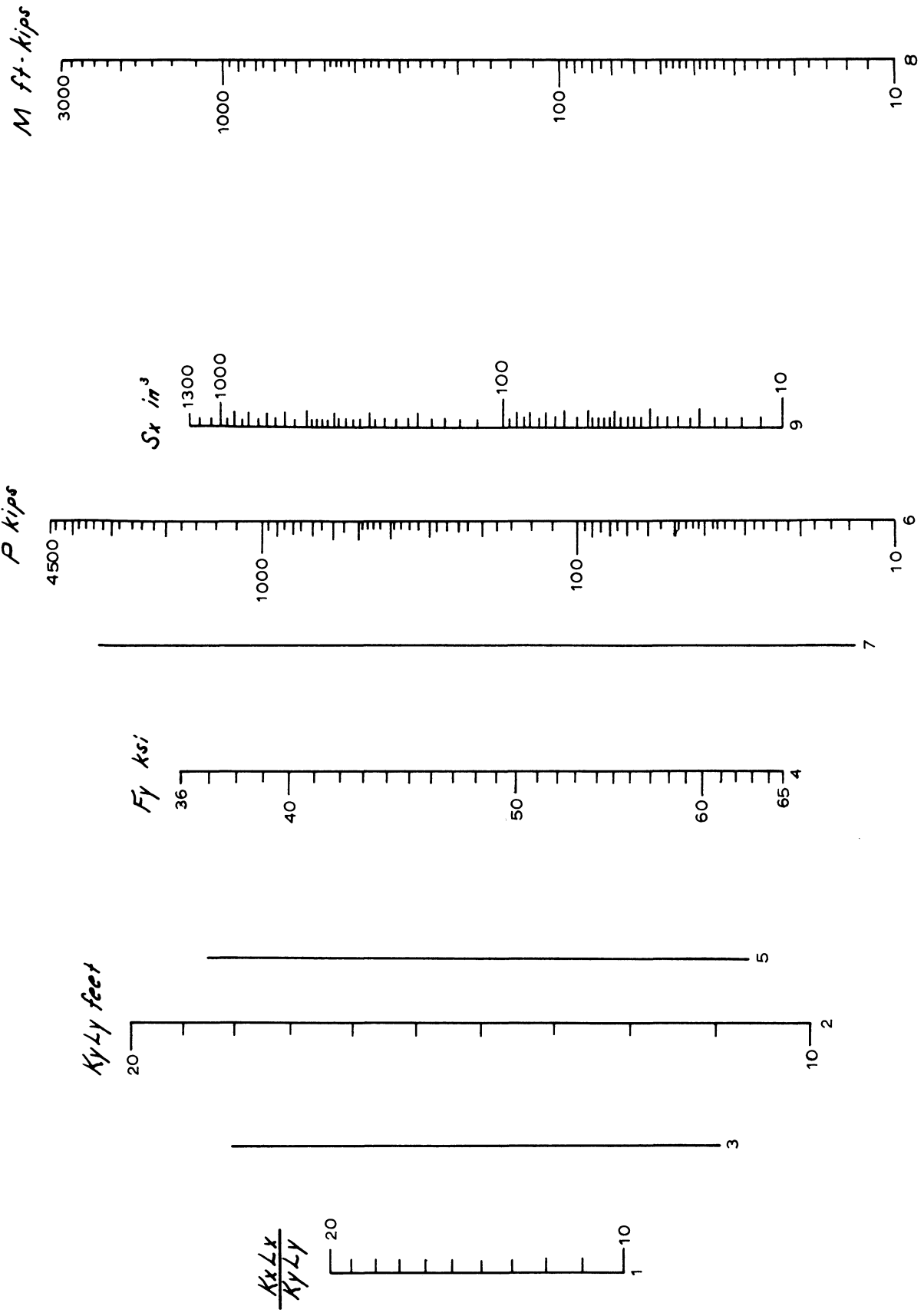


Fig. 4. Nomograph 2. Range of π_2 : 10-70

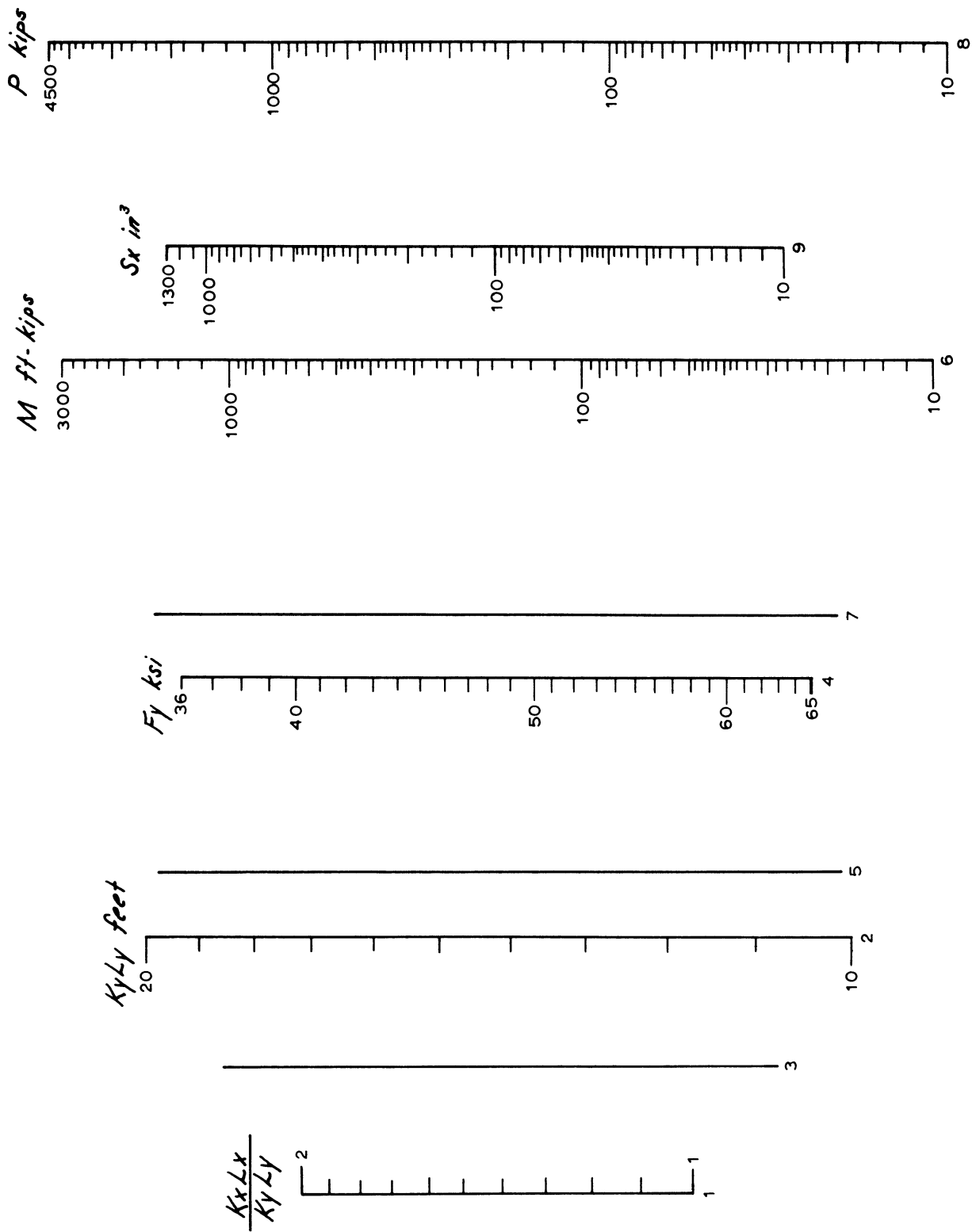


Fig. 5. Nomograph 3. Range of π_2 : 30 and above

DISCUSSION OF RESULTS

The calculated values of S_x from Eqs. (19), (20), and (21) or from Nomographs 1, 2, and 3, respectively, compared well with data used in the analysis and other arbitrary data. The average deviation for all three equations was approximately three percent. Poor results are generally obtained from Eq. (21) or Nomograph 3 when values of π_2 become greater than 300, because a value of π_2 greater than 300 gives a very small bending moment compared with the axial compression load.

By generating and observing data for many arbitrary designs and calculated values of S_x from Eqs. (19), (20), and (21), or Nomographs 1, 2, and 3, the following working ranges were determined:

Equation	Nomograph	Range of π_2
(19)	1	0-30
(20)	2	10-70
(21)	3	30 and above

There is some overlap in the working ranges of the equations. When this occurs, both equations must be tried. The larger calculated value of S_x must be used in the design.

The following are the basic steps to design steel wide-flange beam-column sections when using Eqs. (19), (20), and (21) or Nomographs 1, 2, and 3:

1. Determine the known variables in the problem: P , M , F_y , K_yL_y , and K_xL_x .
2. Calculate the value of π_2 , $P(K_yL_y)/M$, and determine the prediction equations or nomographs which are to be used.
3. Calculate the value of S_x from the governing prediction equations or nomographs, and use the larger value to select a cross section from the AISC *Manual* (column sections only). Select a section which has a section modulus close to the calculated value of S_x .
4. Check the selected section for compliance with AISC Formulas (1.6-1a), (1.6-1b), and (1.6-2). If selection does not comply with the AISC formulas, try either a larger or smaller section in order to obtain an optimum solution.

NUMERICAL EXAMPLES

The following three examples illustrate the design technique presented in this study.

Example 1

Given: $P = 200$ kips
 $M = 100$ kip-ft
 $K_yL_y = 13$ ft
 $R = 1.5$
 $F_y = 36$ ksi

Solution:

1. Calculate π_2 :

$$\pi_2 = \frac{P(K_yL_y)}{M} = \frac{200(13)}{100} = 26.0$$

Therefore, use Nomographs 1 and 2 to calculate S_x .

2. Calculate S_x and select a section:

$$S_x = 92 \text{ in.}^3 \text{ from Nomograph 1, Fig. 6}$$

$$S_x = 114 \text{ in.}^3 \text{ from Nomograph 2, Fig. 7}$$

Select a W14×74: $S_x = 112 \text{ in.}^3$

3. Check for compliance with AISC Formulas (1.6-1a), (1.6-1b), and (1.6-2):

$$A = 21.8 \text{ in.}^2$$

$$f_a = 200/21.8 = 9.17 \text{ ksi}$$

$$r_x = 6.05 \text{ in.}$$

$$K_xL_x/r_x = 1.5(13)(12)/6.05 = 38.68$$

$$r_y = 2.48 \text{ in.}$$

$$K_yL_y/r_y = 13(12)/2.48 = 62.9$$

$$KL/r = 62.9$$

$$F_a = 17.14 \text{ ksi (from AISC Specification, Table 1-36)}$$

$$f_b = 100(12)/112 = 10.71 \text{ ksi}$$

$$L_u = 25.8 \text{ ft} > 13 \text{ ft (AISC Column Tables)}$$

$$F_b = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

$$F_e' = 99.8 \text{ ksi}$$

$$f_a/F_a = 9.17/17.14 = 0.48 > 0.15 \text{ [AISC Formula (1.6-2)]}$$

\therefore Formula (1.6-2) does not govern

From AISC Formula (1.6-1a):

$$\frac{9.17}{17.14} + \frac{0.85(10.71)}{[1 - 9.17/99.8] 21.6} = 0.99 < 1.0$$

From AISC Formula (1.6-1b):

$$\frac{9.17}{0.6(36)} + \frac{10.71}{21.6} = 0.92 < 1.0$$

Selection is o.k.

Example 2

Given: $P = 400$ kips
 $M = 90$ kip-ft
 $K_yL_y = 18$ ft
 $R = 2.0$
 $F_y = 50$ ksi

Solution:

1. Calculate π_2 :

$$\pi_2 = 400(18)/90 = 80$$

Therefore, use Nomograph 3 to calculate S_x .

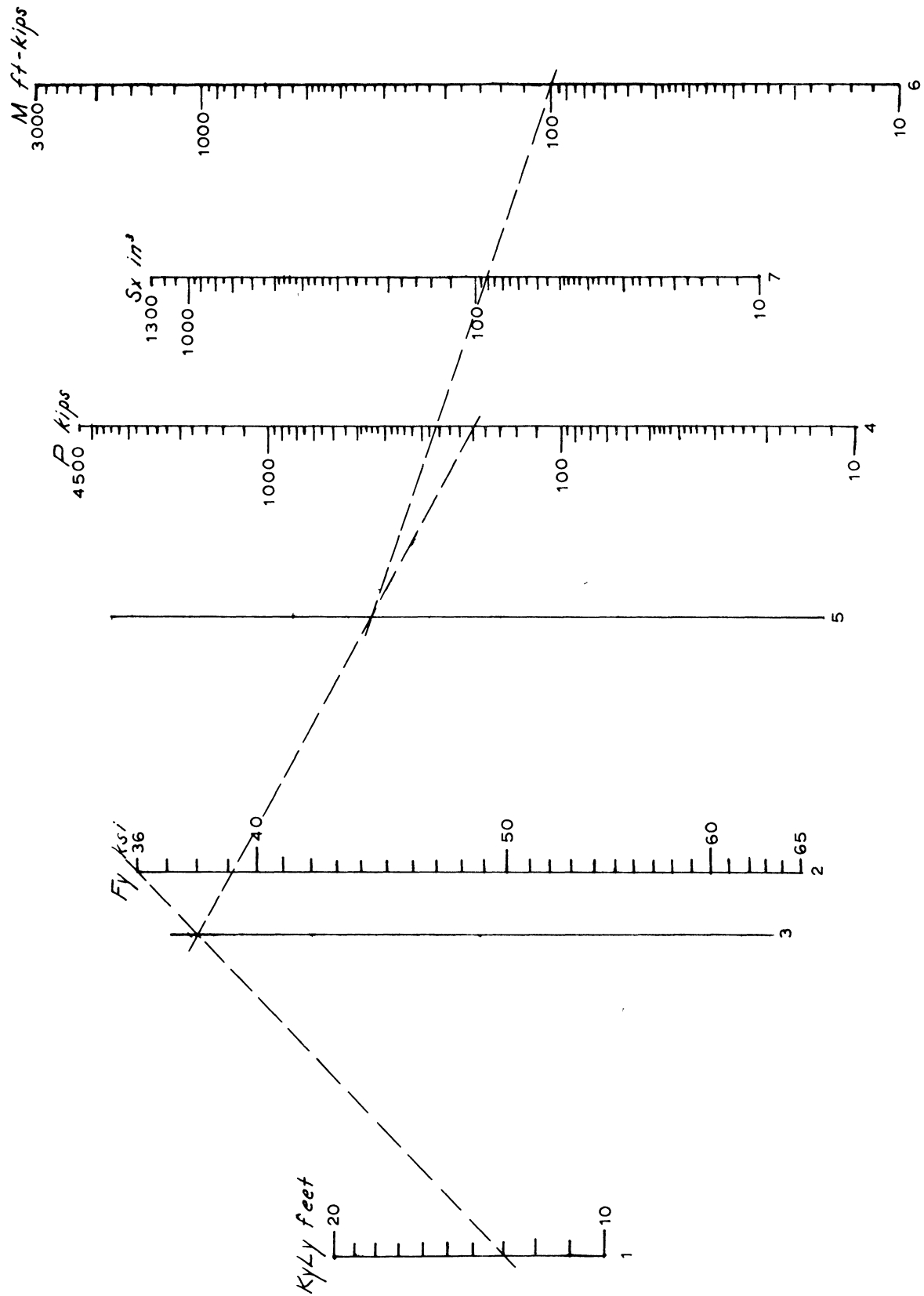


Fig. 6. Example 1. Nomograph 1

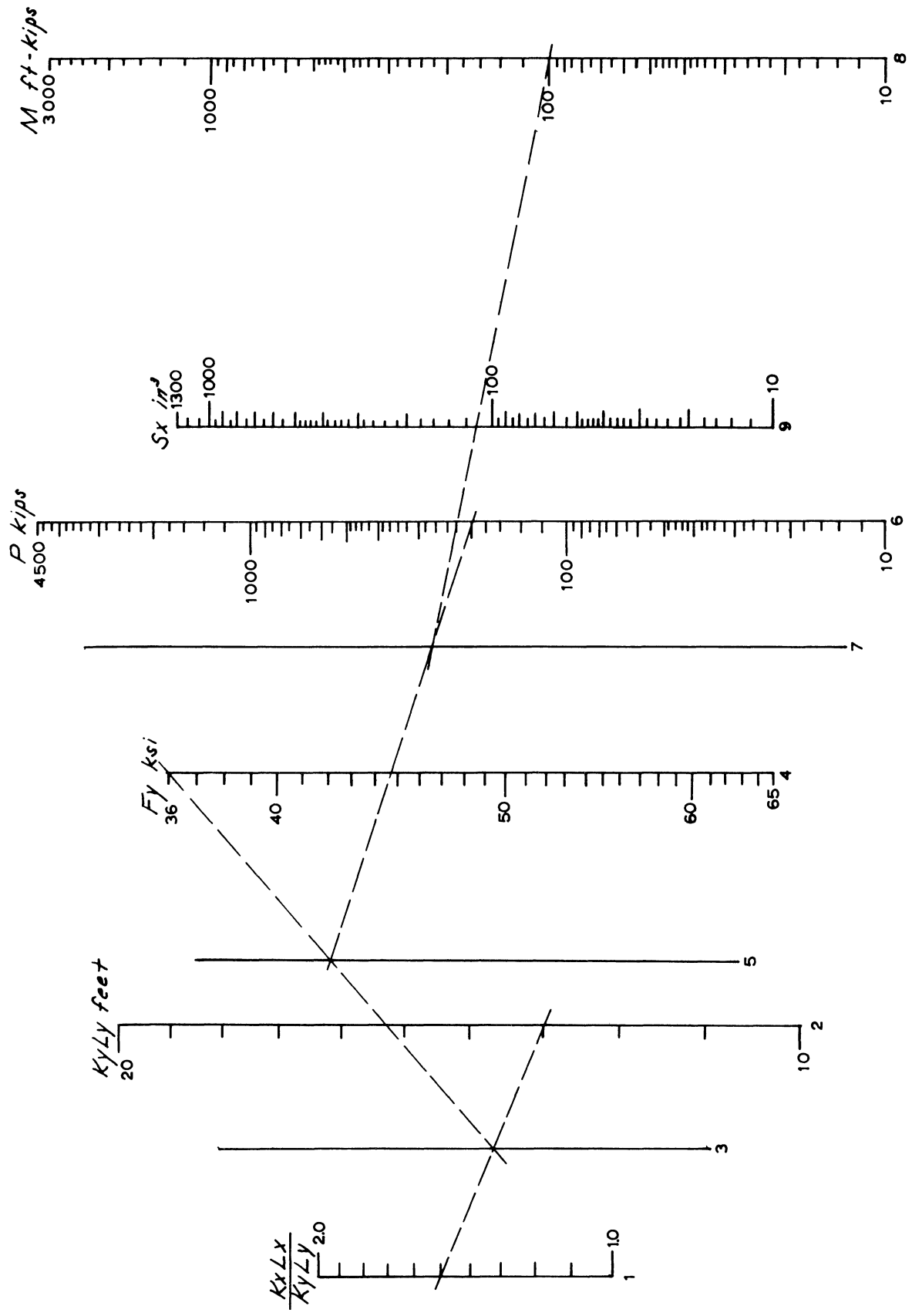


Fig. 7. Example 1. Nomograph 2

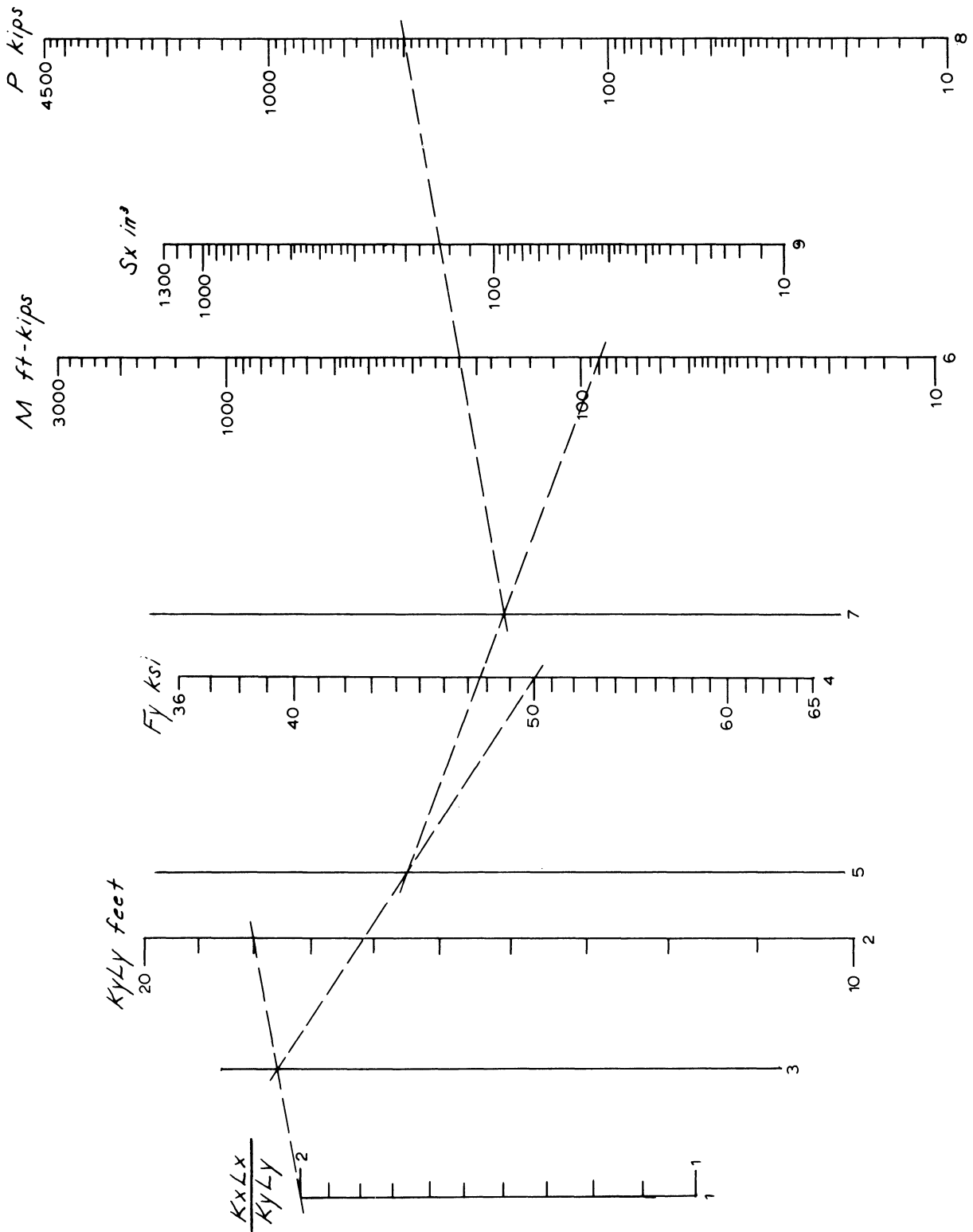


Fig. 8. Example 2. Nomograph 3

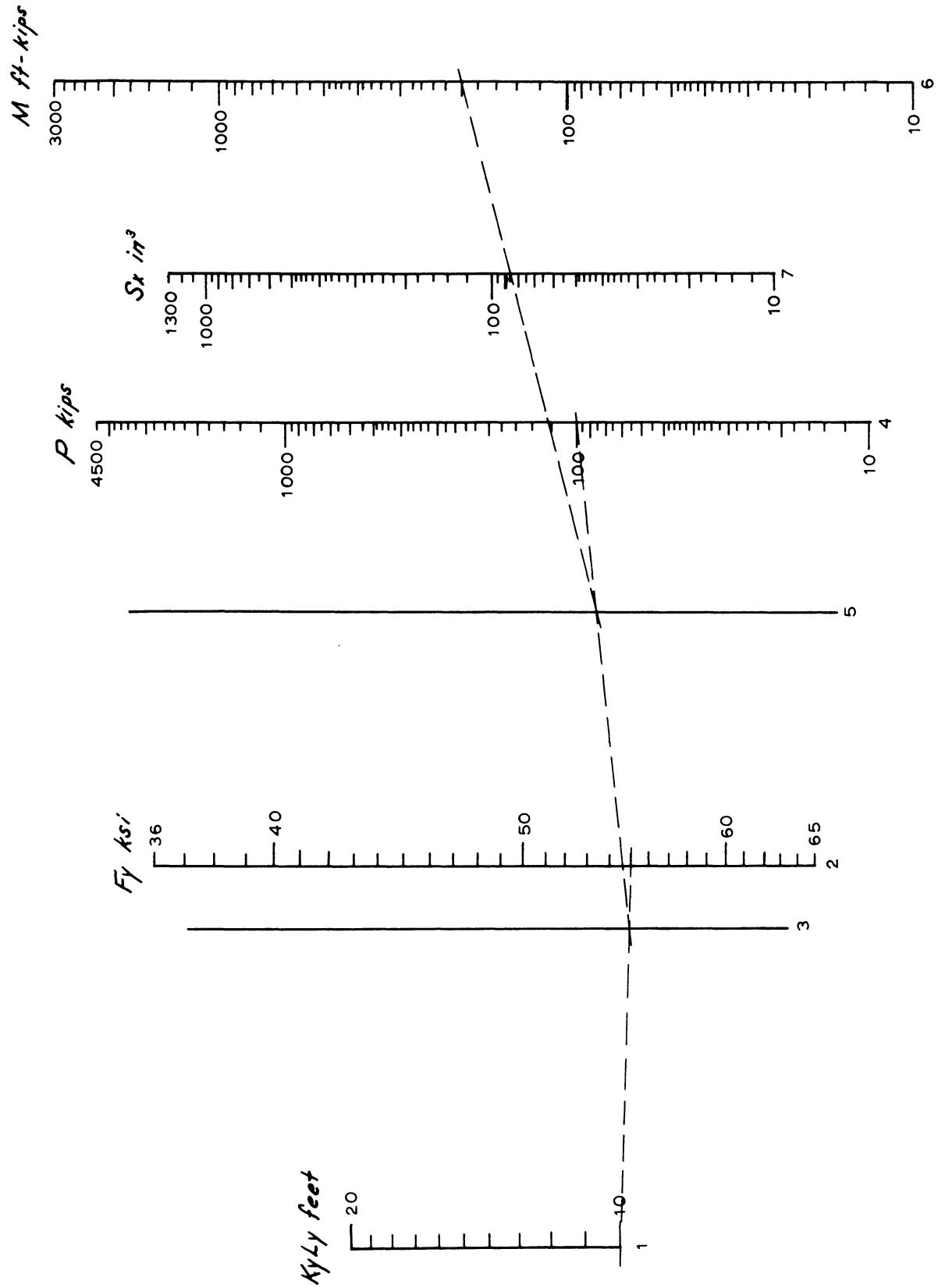


Fig. 9. Example 3. Nomograph 1.

2. Calculate S_x and select a section:
 $S_x = 154 \text{ in.}^3$ from Nomograph 3, Fig. 8
 Select a W14×103: $S_x = 164 \text{ in.}^3$
3. Check for compliance with AISC Formulas (1.6-1a), (1.6-1b), and (1.6-2):
 $A = 30.3 \text{ in.}^2$
 $f_a = 400/30.3 = 13.2 \text{ ksi}$
 $r_x = 6.21 \text{ in.}$
 $K_x L_x / r_x = 2.0(18)(12)/6.21 = 69.56$
 $r_y = 3.72 \text{ in.}$
 $K_y L_y / r_y = 18(12)/3.72 = 58.10$
 $KL/r = 69.56$
 $F_a = 21.0 \text{ ksi}$ (from AISC Specifications, Table 1-50)
 $f_b = 90(12)/164 = 6.58 \text{ ksi}$
 $L_u = 38.6 \text{ ft} > 18 \text{ ft}$ (AISC Column Tables)
 $F_b = 0.6(50) = 30.0 \text{ ksi}$
 $F_e' = 30.86 \text{ ksi}$
 $f_a/F_a = 13.2/21.0 = 0.63 > 0.15$
 [AISC Formula (1.6-2)]
 \therefore Formula (1.6-2) does not govern

From AISC Formula (1.6-1a):

$$\frac{13.2}{21.0} + \frac{0.85(6.58)}{[1 - (13.22/30.86)]30.0} = 0.96 < 1.0$$

From AISC Formula (1.6-1b):

$$\frac{13.32}{30.0} + \frac{6.58}{30.0} = 0.66 < 1.0$$

Selection is **o.k.**

Example 3

Given: $P = 100 \text{ kips}$
 $M = 200 \text{ kip-ft}$
 $K_y L_y = 10 \text{ ft}$
 $R = 1.0$
 $F_y = 55 \text{ ksi}$

Solution:

1. Calculate π_2 :
 $\pi_2 = 100(10)/200 = 5$
 Therefore, use Nomograph 1 to calculate S_x
2. Calculate S_x and select a section:
 $S_x = 85 \text{ in.}^3$ from Nomograph 1, Fig. 9
 Select a W14×61: $S_x = 92.2 \text{ in.}^3$
4. Check for compliance with AISC Formulas (1.6-1a), (1.6-1b), and (1.6-2):
 $A = 17.9 \text{ in.}^2$
 $f_a = 100/17.9 = 5.59 \text{ ksi}$

$$r_x = 5.98 \text{ in.}$$

$$K_x L_x / r_x = 10(12)/5.98 = 20.10$$

$$r_y = 2.45 \text{ in.}$$

$$K_y L_y / r_y = 10(12)/2.45 = 49.0$$

$$KL/r = 49.0$$

$$F_a = 26.55 \text{ ksi}$$
 (AISC Specification, Table 1-55)
$$f_b = 200(12)/92.2 = 26.03 \text{ ksi}$$

$$L_c = 10.6 \text{ ft} > 10 \text{ ft}$$
 (AISC Column Tables)
$$F_b = 0.66(55) = 36.3 \text{ ksi}$$

$$F_e' = 62.19 \text{ ksi}$$

$$f_a/F_a = 5.59/26.55 = 0.21 > 0.15$$
 [AISC Formula (1.6-2)]
 \therefore Formula (1.6-2) does not govern

From AISC Formula (1.6-1a):

$$\frac{5.59}{26.55} + \frac{0.85(26.03)}{[1 - (5.59/62.19)] 36.3} = 0.88 < 1.0$$

From AISC Formula (1.6-1b):

$$\frac{5.59}{33.0} + \frac{26.03}{36.3} = 0.96 < 1.0$$

Selection is **o.k.**

SUMMARY AND CONCLUSIONS

This paper provides a rapid selection of steel wide-flange columns subjected to combined axial compression load and bending moment about the strong axis of the member, in compliance with the AISC Specification.

Three nomographs have been developed which cover a wide application of design of columns subjected to combined loading. These nomographs would be very useful to the designer, where a rapid selection of columns subjected to combined loading can be achieved. These nomographs are constructed using data for steel wide-flange column sections only, i.e., W14's, W12's, W10's, and W8's. Coefficient C_m is assumed to be equal to 0.85.

Three examples to demonstrate the design of beam-columns using nomographs are given in this paper.

A similar technique can be developed to construct nomographs to be used in the design of columns subjected to axial load and biaxial bending, and for any value of C_m .

REFERENCES

1. Manual of Steel Construction *Seventh Edition*, American Institute of Steel Construction, New York, N.Y., 1971.
2. Rubinsky, M. A. Rapid Selection of Beam-Columns *Engineering Journal, AISC, Vol. 5, No. 3, pp. 100-122, July 1968.*
3. Phang, M. K. S. and E. Celenza Direct Design of Columns Subjected to Combined Loading *Engineering Journal, AISC, Vol. 9, No. 4, pp. 142-153, Oct. 1972.*

4. Burgett, L. B. Selection of a "Trial" Column Section *Engineering Journal, AISC, Vol. 10, No. 2, pp. 56-59, Second Quarter, 1973.*
5. Phang, M. K. S. and R. C. Babyak Simplified Solution to Interaction Equation *Engineering Journal, AISC, Vol. 12, No. 1, pp. 30-33, First Quarter, 1975.*
6. Monasa, F. Generalized Design of Reinforced Concrete Flat Slabs *M.S. Thesis, Virginia Polytechnic Institute, Blacksburg, Va., July 1964.*
7. Murphy, G. Similitude in Engineering *The Ronald Press Company, New York, 1950.*
8. Davis, D. Empirical Equations and Nomography *McGraw-Hill Book Company Inc., New York, 1943.*
9. Levens, A. S. Nomography 2nd Ed., *John Wiley and Sons Inc., New York, 1959.*
10. Snyder, J. Generalized Design of Columns Subjected to Combined Axial Load and Bending Moment *M.S. Report, Michigan Technological University, Houghton, Mich., Sept., 1979.*

APPENDIX A

The following is the derivation of the functional equation, Eq. (2). Given Eq. (1):

$$S_x = f_1(F_y, K_y L_y, R, P, M)$$

This equation can be expressed as:

$$S_x = f_2(F_y^{C_1})(K_y L_y^{C_2})(R^{C_3})(P^{C_4})(M^{C_5}) \quad (A-1)$$

The basic dimensions of the variables are expressed in terms of force, F , and length, L . Therefore, the corresponding dimensional equation is:

$$L^3 = (FL^{-2})^{C_1}(L)^{C_2}(LL^{-1})^{C_3}(F)^{C_4}(FL)^{C_5} \quad (A-2)$$

From Eq. (A-2), two auxiliary equations may be written:

$$L: -2C_1 + C_2 + C_5 = 3 \quad (A-3)$$

$$F: C_1 + C_4 + C_5 = 0 \quad (A-4)$$

Therefore,

$$3 + 2C_1 - C_5 = C_2 \quad (A-5)$$

$$-C_1 - C_4 = C_5 \quad (A-6)$$

Substituting Eq. (A-6) into Eq. (A-5) yields:

$$3 + 3C_1 + C_4 = C_2 \quad (A-7)$$

Substituting Eqs. (A-6) and (A-7) into Eq. (A-2) yields:

$$S_x = f_2(F_y)^{C_1}(K_y L_y)^{3+3C_1+C_4}(R)^{C_3}(P)^{C_4}(M)^{-C_1-C_4} \quad (A-8)$$

or

$$\frac{S_x}{(K_y L_y)^3} = f_3 \left[\frac{P(K_y L_y)}{M}, R, \frac{F_y(K_y L_y)^3}{M} \right] \quad (A-9)$$

Equation (A-9) is a functional equation of dimensionless parameters, expressing a relationship between the variables. Other dimensionless parameters can be obtained from combinations of the ones in Eq. (A-9). For example, dividing the fourth π term by the second yields $F_y(K_y L_y)^2/P$ which may replace the fourth π term. Then by inverting the first π term in Eq. (A-9), the functional equation can be expressed as:

$$\frac{(K_y L_y)^3}{S_x} = f_4 \left[\frac{P(K_y L_y)}{M}, R, \frac{F_y(K_y L_y)^2}{P} \right]$$