Plastic Design of Unbraced Multistory Steel Frames with Composite Beams

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A method for the plastic design of unbraced multistory steel frames under combined permanent gravity loads and horizontal accidental loads due to wind or earthquake effects is described in Ref. 1. The method permits the designer to plot horizontal load-lateral displacement curves (Q-Δ curves) for one or more stories, corresponding to assumed preliminary sizes of beams and columns, considering the formation of successive plastic hinges and taking into account the $P\Delta$ effect. The characteristics of these curves and the sequence of formation of plastic hinges indicate whether the sizes assumed for beams and columns are adequate or not, and furnish the designer with information about the individual members whose modification would more likely contribute to obtain a satisfactory behavior. Final shapes are selected keeping in mind the convenience of having all plastic hinges forming in the beams. The two basic equations to obtain the Q-Δ curve of a story are:[1]

$$\frac{\Delta}{h} = \frac{h}{12E\Sigma I_c} \Sigma M_v + \theta \tag{1}$$

$$Q = \frac{\Sigma M_v}{h} - \frac{P\Delta}{h} \tag{2}$$

where

h = story height
P = weight of all the stories above the one under consideration
Q = horizontal shear force at the story under consideration
Δ = relative horizontal displacement between the upper and lower levels of the story
θ = angle of rotation of the joints at the upper level (it is assumed to be the same for every joint in that level)
ΣM_v = sum of moments at both ends of all beams in the level, corresponding to the value of θ
ΣI_c = sum of moments of inertia of all the columns which contribute to the story's lateral stiffness

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The Q-Δ curve for the story is plotted through the following steps:

1. The total gravity loads, multiplied by the load factor corresponding to the combination of permanent and accidental loads, are applied on the structure from the beginning of the analysis; an increasing horizontal load is also applied.
2. The value of the rotation angle θ, corresponding to the first plastic hinge which appears in the structure under the combined action of constant vertical and increasing horizontal loads, both of them factorized, is determined.
3. The moments at the ends of each beam corresponding to the angle θ which rotate every story joint, as determined in step 2, are computed. The sum of the moments at the ends of all the beams is ΣM_v.
4. Equation (1) is employed to compute the horizontal displacement of the story, which is obtained as Δ/h.
5. Equation (2) is used to compute the horizontal load Q which produces the first plastic hinge and the corresponding horizontal displacement Δ, obtained in step 4.
6. The values of Δ/h and Q, as determined in steps 4 and 5, are the coordinates of the point corresponding to the formation of the first plastic hinge; the straight line connecting the origin with that point represents, with good accuracy, the first part of the Q-Δ curve.

Pairs of values of Q and Δ/h corresponding to the formation of each of the plastic hinges necessary to transform the story into a mechanism, under the combined action of constant vertical and increasing horizontal loads, are obtained through repeated applications of the process just described.

The additional angle θ, which every joint has to rotate for the next plastic hinge to form, is computed in each stage of the process; the increments in Δ/h and Q, which have to be added to the values at the end of the preceding stage to obtain their new values at the end of the stage under consideration, are also computed. As ΣI_c corresponds to all the columns that contribute to the lateral stiffness of the story in a given stage, the moments of inertia of the columns which have ceased to contribute to that stiffness, because of the formation of plastic hinges in the beam or beams which connect to them, are not included in the sum.

INFLUENCE OF COMPOSITE ACTION ON FRAME BEHAVIOR

The composite action between the steel beams and the concrete slab which form the floor system produces several effects that contribute to increase the strength and lateral stiffness of the structure:

1. The fixed-end moments, due to the vertical load acting on the beams, are smaller than those obtained when the loads are supported by the bare steel beams.
2. The ultimate resistant moment increases at the central portion of the beams and at the end with the concrete slab in compression, and remains constant at the other end (disregarding the small increment due to the steel reinforcement of the slab).
3. The stiffness of the beams increases because the moment of inertia of a composite section is noticeably greater than the moment of inertia of the steel shape alone.

The bending moments produced by the horizontal loads have to increase, due to the combined action of effects 1 and 2, to form plastic hinges in the central portion and at one end of the beams; the increase in stiffness (effect 3) originates a decrease in the corresponding θ angles.

Due to the composite action, the angle θ which corresponds to a given value of ΣM_v decreases; the story relative displacement Δ/h also decreases, Eq. (1), and the horizontal load Q increases, Eq. (2). Therefore, both stiffness and strength of the story under horizontal loads increase simultaneously. Besides, the increase in ultimate strength of the central zone and one end of the beams produces an increment in the value of the sum of moments (ΣM_v) which corresponds to the formation of each plastic hinge, increasing again the total lateral resistance of the story.

BEHAVIOR OF COMPOSITE BEAMS

In the following discussion, the ultimate moment of the composite section is considered only in zones of positive bending moment (concrete slab in compression). In negative bending moment areas the ultimate strength of the section is considered equal to the plastic moment of the bare steel beam, disregarding the small increment due to the longitudinal steel bars of the concrete slab.

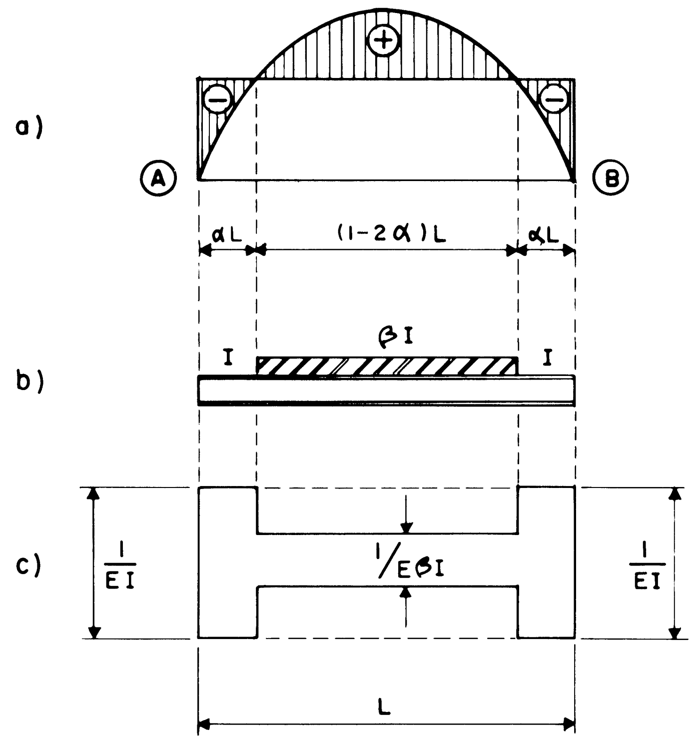

Fig. 1. Bending moment diagram due to uniform vertical load; corresponding composite beam and cross-section of the analogous column

Fixed-end Moments—Figure 1a shows the bending moment diagram of a fixed-end beam of variable inertia which sustains a uniform transverse load. Figure 1b shows the moments of inertia of the composite beam cross sections, based on the assumption that the slab does not contribute to the strength in the areas of negative bending moment. Figure 1c depicts the cross-section of the analogous column corresponding to the composite beam.[2,3] In Fig. 1, I is the moment of inertia of the steel beam, βI is the moment of inertia of the composite section, and αL is the length of the end segments of the beam, which are under negative bending moment.

Application of the column analogy[2,3] leads to Eq. (3) for computing the beam fixed-end moments:

$$M_{AB} = M_{BA} = WL^2 \frac{\alpha^2\beta\left(\frac{1}{2} - \frac{\alpha}{3}\right) + \frac{1}{12} - \frac{\alpha^2}{2} + \frac{\alpha^3}{3}}{2\alpha\beta + 1 - 2\alpha} \tag{3}$$

As the position of the points of contraflexure is not known beforehand, α is not known, and the fixed-end moments can not be computed by direct application of Eq. (3). Nevertheless, the problem can be solved for each value of β, through the following steps:

1. Assume a value for α.
2. Using Eq. (3), compute fixed-end moments corresponding to the value of α assumed in step 1.
3. Compute the determinate bending moment in section $x = \alpha L$, assuming the beam to be simply supported.
4. Compare bending moments computed in steps 2 and 3; if they have the same value, for a given degree of approximation, the moment determined in step 2 is the actual fixed-end moment. If not, modify α and repeat the computations. The results shown in Table 1 have been obtained with this procedure.*

Table 1. Fixed-end Moments of Composite Beams under Uniform Load

$\beta = \dfrac{I_{comp}}{I_{beam}}$	α	M/WL^2
1.00	0	0.0833 = 1/12.00
1.20	0.201	0.0804 = 1/12.44
1.40	0.193	0.0778 = 1/12.85
1.50	0.189	0.0767 = 1/13.04
1.60	0.186	0.0756 = 1/13.23
1.80	0.179	0.0736 = 1/13.59
2.00	0.174	0.0717 = 1/13.95
2.20	0.169	0.0701 = 1/14.27
2.40	0.164	0.0686 = 1/14.58
2.50	0.162	0.0679 = 1/14.73
2.60	0.160	0.0672 = 1/14.88
2.80	0.156	0.0659 = 1/15.17
3.00	0.153	0.0647 = 1/15.46
3.20	0.150	0.0636 = 1/15.72
3.40	0.147	0.0626 = 1/15.97
3.50	0.145	0.0621 = 1/16.10

Stiffnesses—The bending moment diagram corresponding to a fixed-end beam with uniform vertical load is shown in Fig. 2a. Figures 2c and 2e indicate the two diagrams which can be obtained by superimposing on that diagram (Fig. 2a) the effects of the horizontal loads when the beam is part of a rigid frame, assuming that those loads act from left to right.

When the bending moment diagram is as shown in Fig. 2a, which corresponds to vertical load alone, or as depicted in Fig. 2c, due to vertical load and small horizontal forces, there are segments under negative bending moment at both ends of the beam. Both segments have the same length in the first case (αL), but they are different (γL and δL) in the second. (Note that γ is always smaller than α, and δ may vary between α and 0.5, but never reaches either limit.)

* *Table 1, and the coefficients for computing stiffnesses presented later in Tables 2 and 3, were obtained using a programmable pocket calculator.*

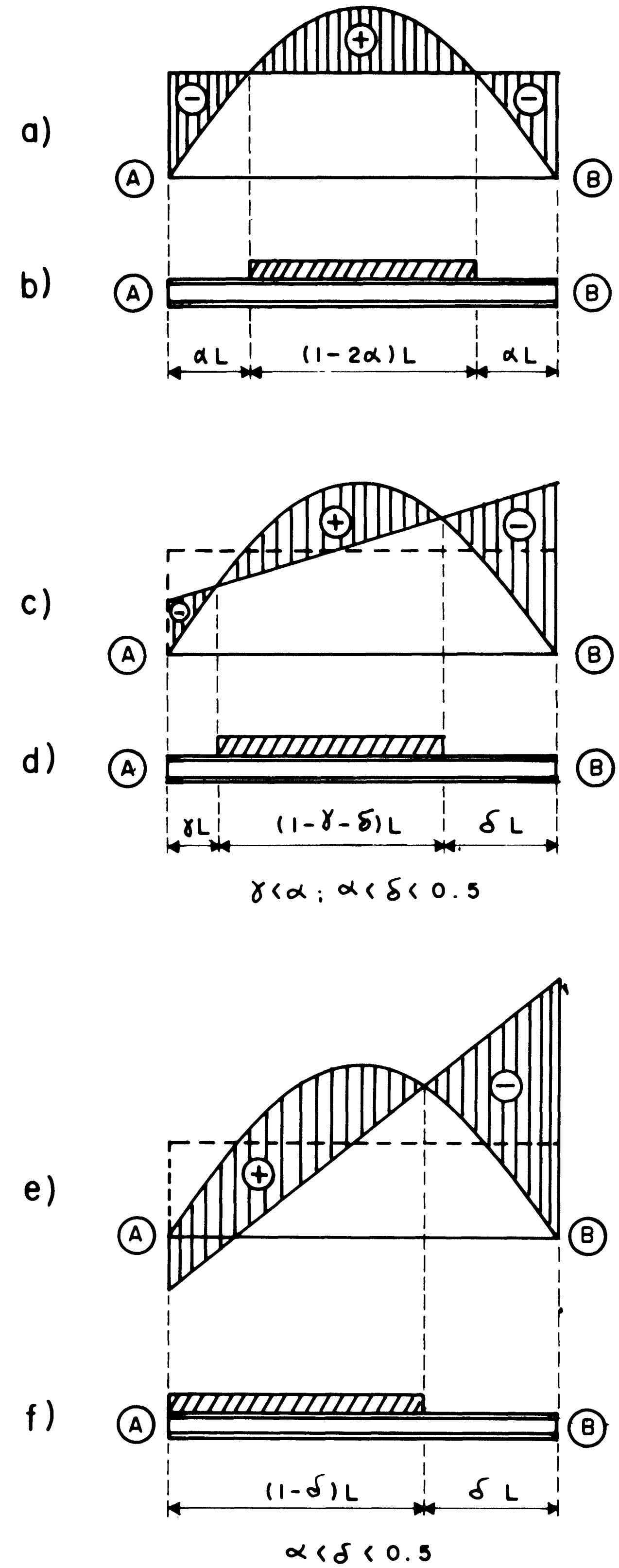

Fig. 2. Bending moment diagrams due to vertical load and simultaneous vertical and horizontal loads; corresponding composite beams

The left-hand end negative bending moment decreases, and eventually disappears (Fig. 2), as the horizontal load increases; simultaneously the length δL increases, but remains less than $L/2$.

Assuming that in areas of negative bending moment the concrete slab does not contribute to the moment of inertia of the cross section, the composite beams shown in Figs. 2b, 2d, and 2f are obtained. Only the segment of slab which is in compression has been shown in the figures.

If the bending moment diagram of a composite beam is as shown in Fig. 2e, Ref. 4 indicates that three beam segments of different resistance have to be considered (this is discussed in detail below), but in order to evaluate the beam stiffness the complete effective width of the concrete flange can be used in the positive moment region.[4,5] This criterion is employed here, and only two different moments of inertia are used for each beam in stiffness computations: the moment of inertia of the steel beam in the negative moment regions, and the moment of inertia of the composite section, corresponding to the complete effective width of the concrete flange, in regions of positive bending moment.

In stiffness computations, only the case shown in Fig. 2d is considered, as cases shown in Figs. 2b and 2f can be reduced to it. (In Fig. 2b, $\gamma = \delta = \alpha$; in Fig. 2f, $\gamma = 0$.) The stiffness at end A has to be evaluated for two support conditions, the first corresponding to equal rotations at both ends of the beam, valid from the beginning of the loading process to the formation of a plastic hinge at the right-hand end, and the second corresponding to end B hinged; at the right-hand end B, only the stiffness corresponding to equal rotations at both ends is of interest. The stiffness at end A for both support conditions is computed with Eqs. (4) and (5).[2]

For equal rotation at both ends:

$$r_A' = r_A(1 + t_{AB}) \tag{4}$$

For end B hinged:

$$r_A'' = r_A(1 - t_{AB}t_{BA}) \tag{5}$$

where

r_A = stiffness at end A of beam AB when end B is fixed

t_{AB} = carry-over factor from A to B when end B is fixed

t_{BA} = carry-over factor from B to A when end B is free to rotate and end A is fixed

Stiffness r_A and carry-over factor t_{AB} are computed using Eqs. (6) and (7), which are obtained applying the column analogy[2]; an equation similar to (7) is employed to compute the carry-over factor t_{BA}.

$$r_A = \frac{1}{A_e} + \frac{x_A^2}{I_{ye}} \tag{6}$$

$$t_{AB} = -\left(\frac{1}{A_e} + \frac{x_A x_B}{I_{ye}}\right) \Big/ r_A \tag{7}$$

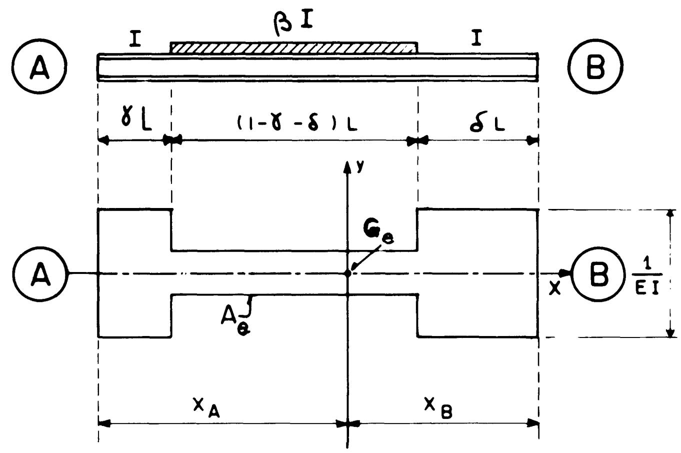

Fig. 3. Composite beam; general case and corresponding elastic area

In the above equations (see Fig. 3),

A_e = area of the cross-section of the analogous column (elastic area)

I_{ye} = moment of inertia of the elastic area with respect to the principal axis y through the centroid

x_A, x_B = distances from the centroid of the elastic area to the ends of the beam

G_e = centroid of the elastic area (Fig. 3)

Stiffnesses r_A' and r_A'' corresponding to two values of β, 2.5 and 3.0, and to coefficients γ and δ varying from 0 to 0.5, are shown in Tables 2 and 3. (The numerical values of the stiffnesses presented here are those required to solve the numerical example at the end of the paper; through the use of the program developed to compute them, it is not difficult to generate the additional information required to solve any other problem or to calculate a collection of tables corresponding to all values of β of practical interest.)

The stiffness r' at each end of the beam, corresponding to a known value of α (Fig. 2b), and the stiffness at the left-hand end A, corresponding to given values of γ and δ (Figs. 2d and 2f), are read directly from the tables; to obtain stiffness r' at end B the same tables are used, but γ and δ are interchanged. (For instance, if a beam is in the condition shown in Fig. 2f, r_A' and r_A'' are determined with $\gamma = 0$ and the value of δ taken from a bending moment diagram drawn to scale, and r_B' is read in the table, taking for γ the value of δ from the diagram and making δ equal to zero.)

Figure 4 shows the bending moment diagram for a simply supported uniformly loaded beam (determinate bending moment diagram) and three redundant moment diagrams: Diagram 0 corresponds to vertical load and fixed

Table 2. Composite Beam Stiffnesses (β = 2.5)

$\gamma \backslash \delta$	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.00	7.50	7.50	7.49	7.46	7.41	7.33	7.21	7.05	6.84	6.60	6.32
	15.00	**13.45**	**12.71**	**12.36**	**12.21**	**12.18**	**12.20**	**12.23**	**12.24**	**12.20**	**12.07**
0.05	6.18	6.18	6.17	6.15	6.12	6.06	5.98	5.87	5.73	5.55	5.35
	11.78	**10.66**	**10.12**	**9.85**	**9.74**	**9.71**	**9.73**	**9.76**	**9.78**	**9.77**	**9.71**
0.10	5.33	5.33	5.33	5.31	5.29	5.25	5.18	5.10	4.99	4.86	4.71
	10.04	**9.12**	**8.66**	**8.43**	**8.33**	**8.30**	**8.31**	**8.34**	**8.37**	**8.37**	**8.34**
0.15	4.75	4.75	4.75	4.74	4.72	4.68	4.63	4.56	4.48	4.37	4.25
	9.02	**8.19**	**7.77**	**7.56**	**7.46**	**7.43**	**7.43**	**7.46**	**7.48**	**7.50**	**7.48**
0.20	4.33	4.33	4.33	4.32	4.30	4.27	4.23	4.18	4.10	4.01	3.91
	8.41	**7.61**	**7.20**	**6.99**	**6.89**	**6.86**	**6.86**	**6.88**	**6.91**	**6.92**	**6.92**
0.25	4.02	4.02	4.01	4.01	3.99	3.97	3.93	3.88	3.82	3.74	3.65
	8.04	**7.25**	**6.85**	**6.63**	**6.53**	**6.49**	**6.48**	**6.50**	**6.52**	**6.54**	**6.54**
0.30	3.78	3.78	3.77	3.77	3.75	3.73	3.70	3.66	3.60	3.53	3.45
	7.84	**7.05**	**6.63**	**6.41**	**6.30**	**6.25**	**6.24**	**6.25**	**6.27**	**6.29**	**6.30**
0.35	3.59	3.59	3.59	3.58	3.57	3.55	3.52	3.48	3.43	3.37	3.30
	7.76	**6.94**	**6.51**	**6.28**	**6.16**	**6.10**	**6.09**	**6.10**	**6.12**	**6.14**	**6.14**
0.40	3.45	3.45	3.44	3.44	3.43	3.41	3.38	3.35	3.30	3.24	3.17
	7.74	**6.90**	**6.46**	**6.22**	**6.09**	**6.02**	**6.01**	**6.01**	**6.03**	**6.05**	**6.05**
0.45	3.33	3.33	3.33	3.33	3.32	3.30	3.27	3.24	3.20	3.14	3.08
	7.77	**6.91**	**6.45**	**6.20**	**6.06**	**5.99**	**5.97**	**5.97**	**5.99**	**6.00**	**6.01**
0.50	3.24	3.24	3.24	3.24	3.23	3.21	3.19	3.16	3.11	3.06	3.00
	7.81	**6.94**	**6.47**	**6.21**	**6.06**	**5.99**	**5.96**	**5.96**	**5.98**	**5.99**	**6.00**

Note: Light face rows show values of $(L/EI)r''_A$ (end B hinged); bold face rows indicate values of $(L/EI)r'_A$ (same rotation at both ends). See Fig. 3.

Table 3. Composite Beam Stiffnesses (β = 3.0)

$\gamma \backslash \delta$	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.00	9.00	9.00	8.98	8.94	8.86	8.73	8.54	8.29	7.98	7.61	7.20
	18.00	**15.69**	**14.71**	**14.29**	**14.13**	**14.12**	**14.16**	**14.19**	**14.18**	**14.07**	**13.85**
0.05	7.00	7.00	6.99	6.97	6.92	6.84	6.72	6.57	6.37	6.13	5.86
	13.20	**11.67**	**11.01**	**10.71**	**10.60**	**10.58**	**10.61**	**10.65**	**10.67**	**10.64**	**10.54**
0.10	5.84	5.84	5.83	5.81	5.78	5.72	5.64	5.53	5.39	5.22	5.02
	10.87	**9.66**	**9.11**	**8.86**	**8.76**	**8.74**	**8.76**	**8.80**	**8.83**	**8.83**	**8.78**
0.15	5.08	5.08	5.07	5.06	5.03	4.99	4.93	4.85	4.74	4.61	4.45
	9.60	**8.51**	**8.01**	**7.78**	**7.68**	**7.66**	**7.67**	**7.70**	**7.73**	**7.75**	**7.72**
0.20	4.55	4.55	4.55	4.54	4.52	4.48	4.43	4.37	4.28	4.17	4.04
	8.88	**7.83**	**7.35**	**7.11**	**7.01**	**6.98**	**6.99**	**7.01**	**7.04**	**7.06**	**7.05**
0.25	4.17	4.17	4.17	4.16	4.14	4.11	4.07	4.01	3.94	3.85	3.74
	8.47	**7.43**	**6.94**	**6.70**	**6.59**	**6.55**	**6.55**	**6.57**	**6.60**	**6.62**	**6.62**
0.30	3.89	3.89	3.89	3.88	3.86	3.84	3.80	3.75	3.69	3.61	3.51
	8.27	**7.21**	**6.70**	**6.45**	**6.32**	**6.27**	**6.27**	**6.29**	**6.31**	**6.33**	**6.34**
0.35	3.67	3.67	3.67	3.66	3.65	3.63	3.59	3.55	3.49	3.42	3.33
	8.20	**7.10**	**6.57**	**6.30**	**6.17**	**6.11**	**6.10**	**6.11**	**6.13**	**6.15**	**6.16**
0.40	3.50	3.50	3.50	3.50	3.48	3.46	3.43	3.39	3.34	3.27	3.19
	8.21	**7.07**	**6.52**	**6.24**	**6.09**	**6.02**	**6.00**	**6.01**	**6.03**	**6.05**	**6.06**
0.45	3.37	3.37	3.37	3.37	3.35	3.34	3.31	3.27	3.22	3.16	3.09
	8.26	**7.09**	**6.52**	**6.22**	**6.06**	**5.99**	**5.96**	**5.97**	**5.99**	**6.00**	**6.01**
0.50	3.27	3.27	3.27	3.26	3.25	3.24	3.21	3.17	3.13	3.07	3.00
	8.31	**7.13**	**6.55**	**6.24**	**6.07**	**5.99**	**5.96**	**5.96**	**5.97**	**5.99**	**6.00**

Note: Light face rows show values of $(L/EI)r''_A$ (end B hinged); bold face rows indicate valves of $(L/EI)r'_A$ (same rotation at both ends). See Fig. 3.

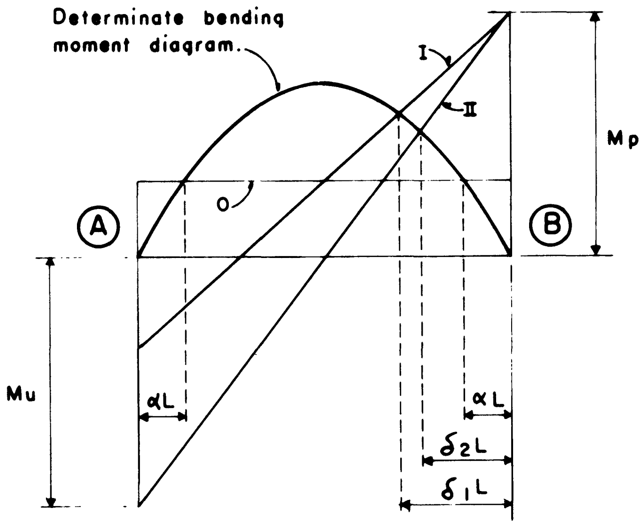

Fig. 4. Determinate and redundant bending moment diagrams

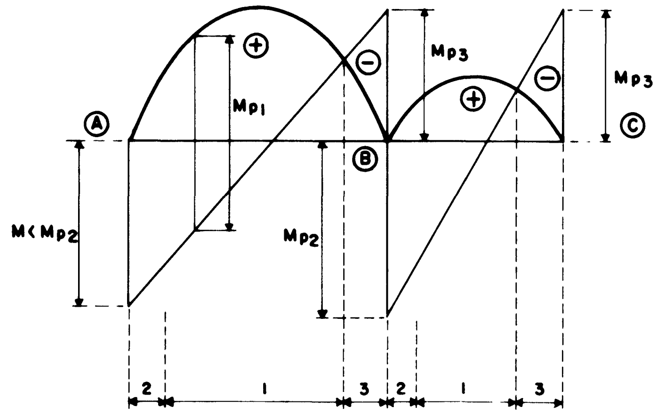

Fig. 5. Sections in which plastic hinges form, and regions to be considered in composite beams

ends. Diagram I corresponds to the formation of a plastic hinge at the right-hand end of the beam, when the bending moment at that end, due to a combination of vertical and horizontal loads, reaches the plastic moment M_p of the steel beam. Finally, diagram II corresponds to the formation of a second plastic hinge at the left-hand end A, when the moment in this section is M_u, the plastic moment of the composite section.

The initial beam stiffness, when the horizontal load starts acting upon the structure, is obtained considering that there are two lateral segments of the same length, αL, with no composite action (Fig. 2b). When the first plastic hinge forms, the stiffness has changed because the beam behaves as a composite section from the left-hand end to the point of intersection of diagrams 0 and I, and as a bare steel beam from that point to the right-hand end (Figs. 2f and 4).

As the horizontal load increases from zero to the load which causes the formation of the first plastic hinge, the beam stiffness changes continuously from the first to the second value. The situation is similar during later stages of the loading process.

Since it is not practical to make computations using stiffnesses which vary continuously with loads, the stiffness employed in each stage is the one determined with the bending moment diagram associated with the beginning of that stage.

Stiffnesses are, then, only approximate, and it is not necessary to obtain their values by interpolation in the tables, as enough precision, from a design point of view, is obtained using the values corresponding to the coefficient β which is closest to the one computed for the beam under study, and taking from the tables the values for α, or γ and δ, which are closest to those read to scale in the bending moment diagram at the beginning of the loading stage.

Strength—To transform a story of an unbraced frame under combined vertical and lateral loads into a mechanism, two plastic hinges have to appear in each beam, since it is convenient to make the design in such a way that no plastic hinges develop in the columns. Sections in which plastic hinges form can be one of the beam ends (the right-hand one, if the earthquake or wind loads act from left to right) and one intermediate, or both ends of the beam. The first case is frequent in upper and intermediate stories and the second one in lower stories, as the importance of horizontal loads increases from top to bottom of the frame, but it is also possible for both types of beam mechanisms to appear in the same level (Fig. 5).

Three different regions have to be considered in each beam, as bending moments are resisted in different ways in each one of them[4] (Fig. 5).

Region 1 is that portion of the beam, under positive bending moment, in which the complete effective width of the concrete slab contributes to the strength. Region 2, between the exterior face of the column and the section in which Region 1 begins, works also as a composite section, but the effective width of the concrete slab is conservatively taken equal to the width of the column flange. (This width can be increased using a wider plate of adequate thickness, welded to the column.) Finally, in Region 3, under negative bending moment, bending is resisted by the steel beam alone, with no slab contribution. (The effect of the reinforcing bars parallel to the beam has not been considered.)

Region 2 is assumed to end in the section in which the edges of the effective slab of Region 1 are intercepted by straight lines drawn from the column axis with a 45° inclination[4] (Fig. 6).

To draw the story's horizontal load-lateral displacement curve, the plastic moments of all sections in which plastic hinges will develop have to be known; it is, then, necessary

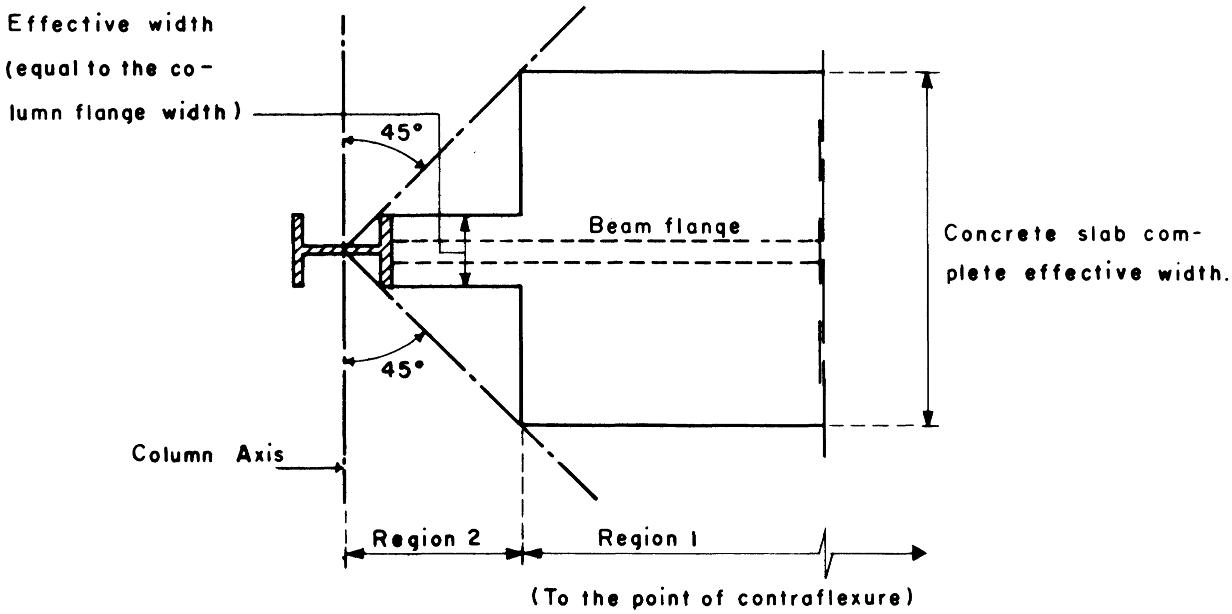

Fig. 6. Length of Region 2

to compute moments M_{p1} and M_{p2} (Fig. 5) corresponding to composite sections with concrete slab widths as mentioned above (Fig. 7). Moment M_{p3} is the plastic moment of the steel beam alone.

The effective width of the concrete flange in Region 1 (Fig. 7) shall be taken as not more than one-fourth of the span of the beam, and its effective projection beyond the edge of the beam shall not be taken as more than one-half the clear distance to the adjacent beam, nor more than eight times the slab thickness[6]. These dimensions change when the slab is present on only one side of the beam.

Plastic moments of the composite sections have been computed following the procedure described in Ref. 7, using a program for a pocket calculator which takes into account the position of the neutral axis across the concrete slab, the beam flange, or the beam web.

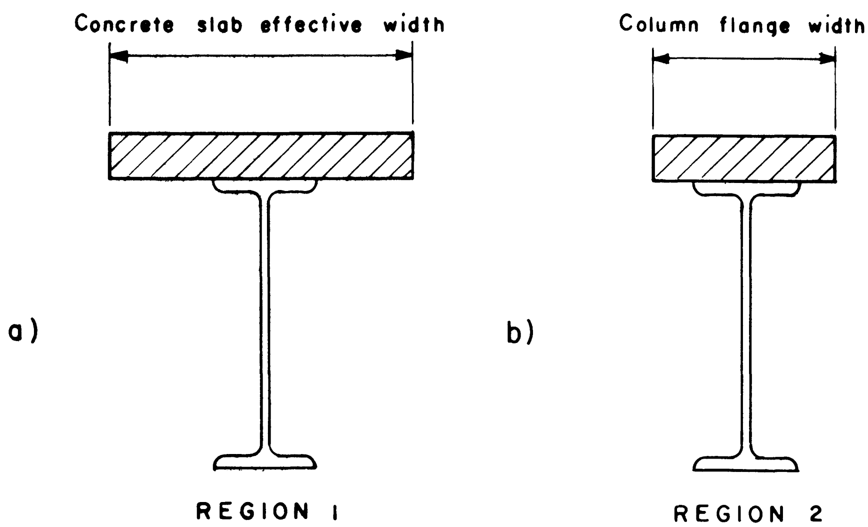

Fig. 7. Composite sections for Regions 1 and 2

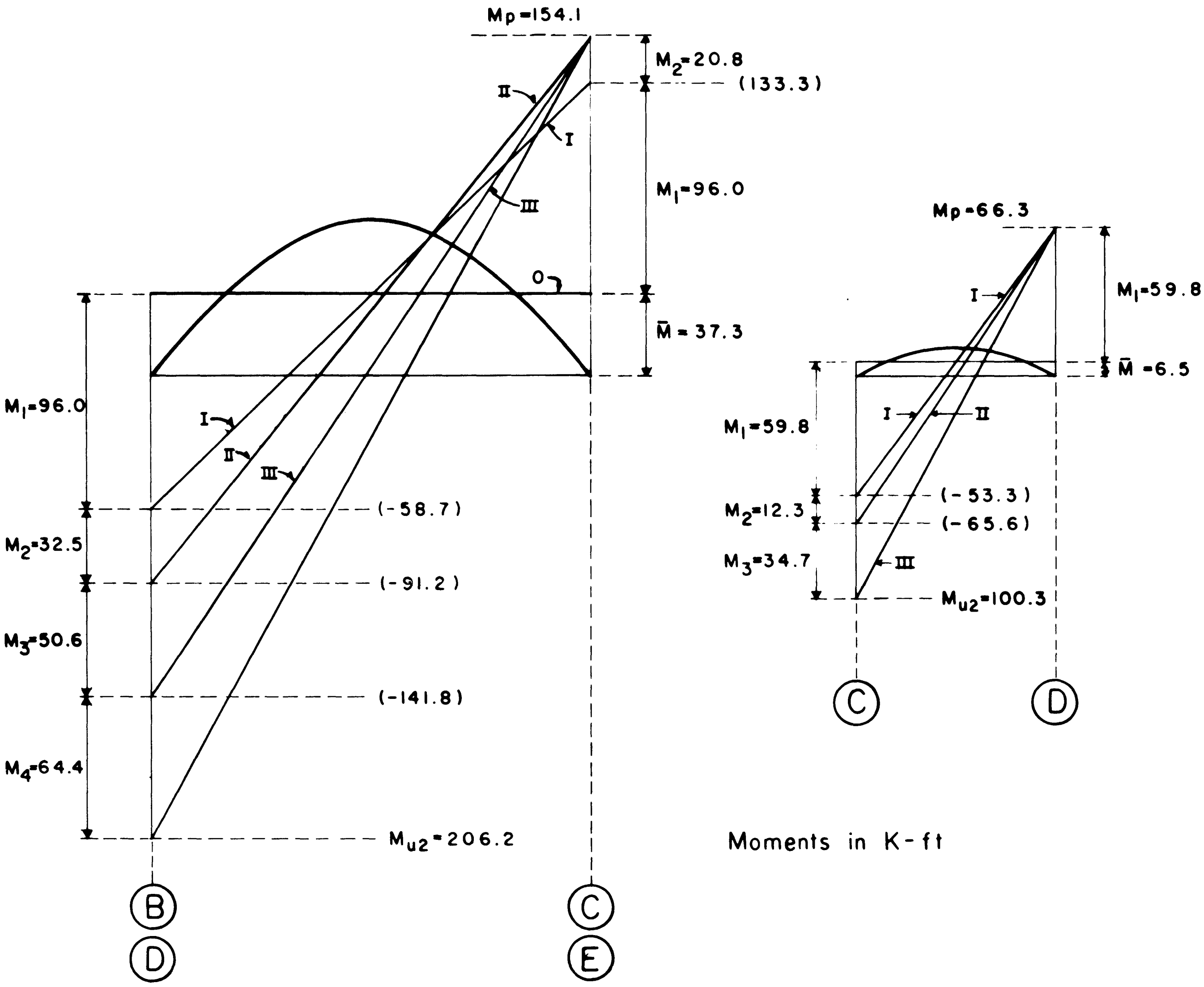

Fig. 8. Bending moment diagrams; illustrative example

ILLUSTRATIVE EXAMPLE

The results obtained considering the composite action of the beams in the story analyzed in Example 1 of Ref. 1 are shown in Figs. 8 and 9; only the bare steel beams were considered in that reference.

Figure 8 depicts the bending moments in the beams at the end of each stage. The fixed end moments, 37.3 kip-ft and 6.5 kip-ft, were computed using Table 1; coefficients α, γ, and δ, which are needed to read stiffnesses in Tables 2 and 3, were computed from measures taken to scale in the moment diagram corresponding to the beginning of each stage of loading. Values of the coefficient β (moment of inertia of the composite section divided by the moment of inertia of the steel beam) were 2.65 for beams BC and DE and 2.94 for beam CD.

The Q-Δ curve from Ref. 1 and the curve obtained by taking into account the composite behavior of the beams are compared in Fig. 9; the maximum horizontal load that can be resisted by the story increases from 63.48 to 81.26 kips (28 percent), and the drift index under working lateral load decreases from 0.0056 to 0.0045 (19.6 percent).

The bending moments applied by the beams to the columns at the formation of the collapse mechanism increase when the composite action is considered; as a consequence, it may be necessary to reinforce the columns.

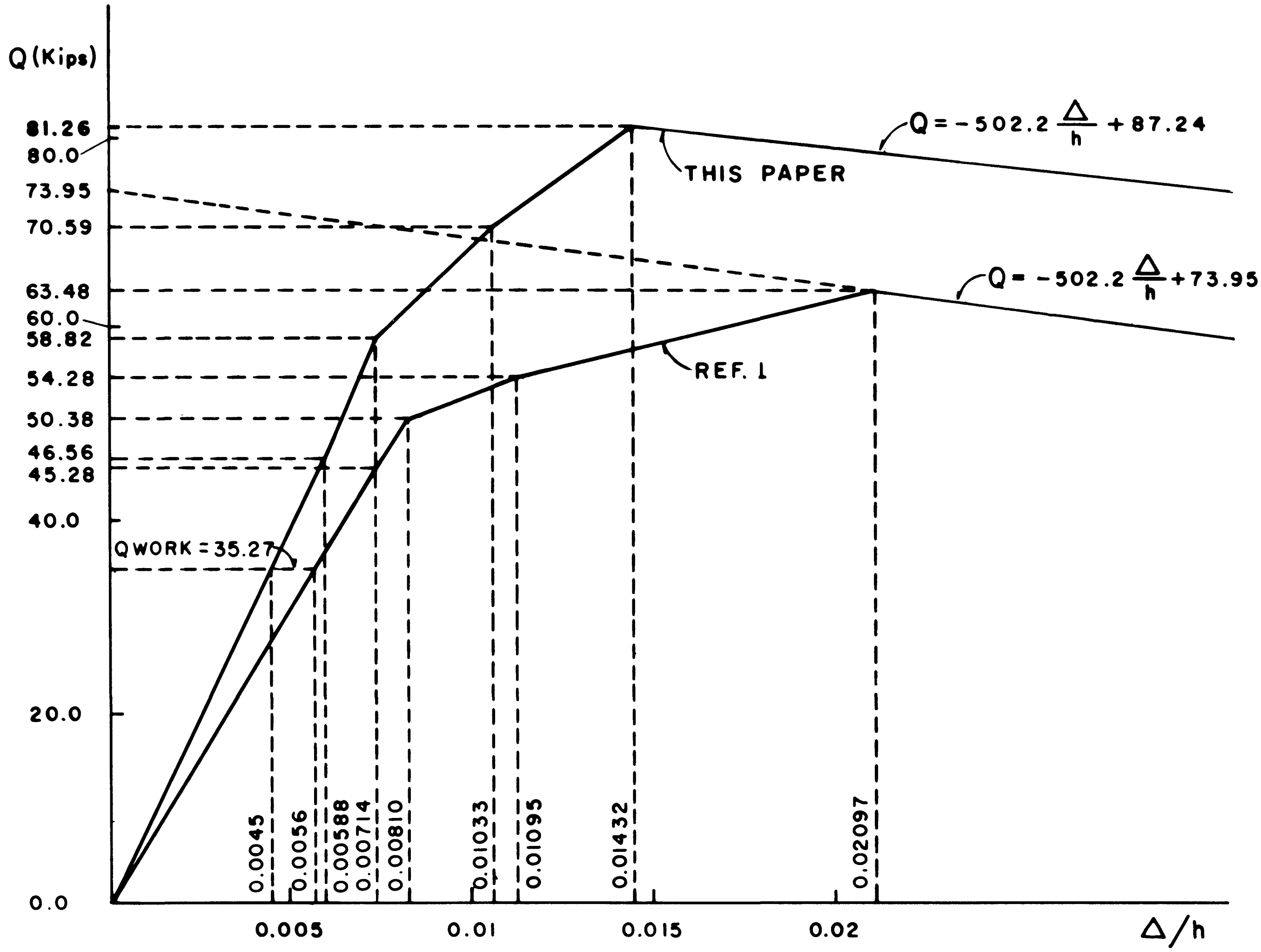

Fig. 9. Q-Δ curve and comparison with Ref. 1, illustrative example

REFERENCES

1. *De Buen, O.* A Method for the Plastic Design of Unbraced Multistory Frames *Engineering Journal, AISC, Vol. 15, No. 3, Third Quarter, 1978.*
2. *Flores, Alberto J.* Column Analogy *Class notes, School of Engineering, National University of Mexico, Mexico City, 1946.*
3. *Kinney, J. S.* Indeterminate Structural Analysis *Addison-Wesley Publishing Co., Inc., 1957.*
4. *Schaffhausen, R. J. and A. W. Wegmüller* Multistory Rigid Frames with Composite Girders under Gravity and Lateral Forces *AISC Engineering Journal, Vol. 14, No. 2, Second Quarter, 1977.*
5. *Du Plessis, D. P. and J. Hartley Daniels* Experiments on Composite Beams under Positive End Moment *Lehigh University, Fritz Eng. Laboratory Report No. 374.4, Aug. 1974.*
6. Specification for the Design, Fabrication and Erection of Structural Steel for Buildings *AISC, New York, Nov. 1978.*
7. *Slutter, R. G.* Composite Steel-Concrete Members *Chapter 13 of the book "Structural Steel Design", L. Tall, Ed., 2nd. ed., The Ronald Press Co., New York, 1974.*