Eccentrically Loaded Weld Groups—AISC Design Tables

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Historically the design tables for eccentric loads on weld groups presented by the American Institute of Steel Construction (AISC) in the *Manual of Steel Construction* have been based on the assumption that the bolt or weld element furthest from the group centroid controlled the design load of the total group. The remaining bolts or weld elements were assumed to carry a proportional amount of load depending on the relative distance from the centroid. The overall stiffness of the configuration was considered to be a function of the polar moment of inertia. This procedure was known to give a generally conservative solution to a complex problem. It was known that, with the exception of nearly square configurations and relatively low loads, the polar moment of inertia does not adequately represent the stiffness of a joint, especially if inelastic deformations are considered.

The AISC in the 7th Edition Manual[1] partially rectified the situation for bolts by using a reduced effective eccentric distance to adjust for the difference between the purely elastic concept and the actual inelastic behavior.

The 8th Edition of the Manual[2] departs from the previous approach by adopting the instantaneous center concept along with an inelastic load deformation relationship for each element. When an eccentric load is applied to a weld group, each element will be subjected to strains due to reaction shear as well as strains due to moment. The resultant of these strains will be rotation of the group about some point in space. The instantaneous center is defined as that point in space about which all the elements are instantaneously rotating. The instantaneous center concept and inelastic load deformation relationships are well documented by Crawford[3] for bolts and by Butler[4,5] for welds. The intent of this paper is to discuss some aspects of eccentrically loaded weld groups and the reader is referred to several other papers[6,7] for additional discussion of eccentrically loaded bolted connections.

RESEARCH REVIEW

The description of some aspects of the experimental program from Refs. 4 and 5 will be more suitably presented near the end of the paper, after a proper theoretical, computational and interpretative foundation has been prepared.

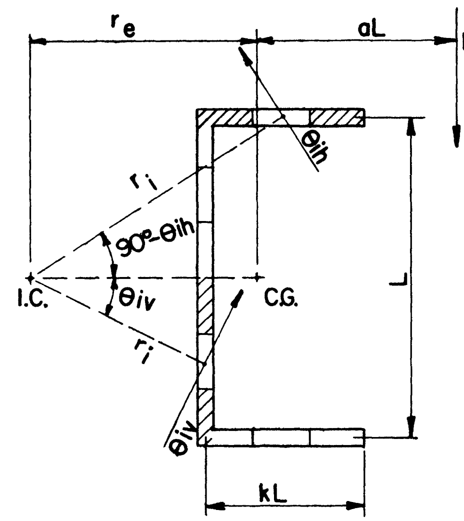

Fig. 1. Basic weld element geometry

The strength, deformation, etc. of a ¼-in. fillet weld made with an E60 electrode is given by Butler[4,5] as the following relationships, where r_i (in.) is the distance from the instantaneous center to any weld element and θ_i (degrees) is the angle between the axis of the weld element and the resultant force on the element (see Fig. 1).

$$\Delta_i = 0.225\,(\theta_i + 5)^{-0.47} \tag{1}$$

$$(R_{ult})_i = \frac{10 + \theta_i}{0.92 + 0.0603\,\theta_i} \tag{2}$$

where

Δ_i = elemental ultimate deformation

$(R_{ult})_i$ = elemental ultimate strength

After all the Δ_i's are computed, the ratio Δ_i/r_i is minimized to obtain the controlling element. The Δ_i and r_i values for the controlling element become:

$$\Delta_{max} = \Delta_i \tag{3}$$

$$r_{max} = r_i \tag{4}$$

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Generally, Δ_{max} and r_{max} are obtained from the element furthest from the instantaneous center, except when the instantaneous center is inside the boundary of the weld pattern, which occurs with large eccentricities, and then it becomes dependent on both θ and r, and as a result cannot be explicitly located.

Using Eqs. (3) and (4) gives:

$$\Delta_{ir} = \frac{r_i \Delta_{max}}{r_{max}} \tag{5}$$

$$\mu_i = 75e^{0.0114\theta_i} \tag{6}$$

$$\lambda_i = 0.4e^{0.0146\theta_i} \tag{7}$$

$$R_i = (R_{ult})_i(1 - e^{-\mu_i \Delta_{ir}})^{\lambda_i} \tag{8}$$

where

λ_i , μ_i = regression coefficients from experimental data

Δ_{ir} = revised proportional elemental deformation

R_i = elemental resisting strength

In spite of the fact that r_{max} is not always associated with the maximum value of r_i , the maximum value or R_i generally (but not always for a small number of elements) is associated with the weld element having r_i equal to r_{max}. When the instantaneous center is found, the three equations of equilibrium ($\Sigma M = 0$, $\Sigma F_x = 0$, $\Sigma F_y = 0$) are satisfied. Since the moment equilibrium equation is less sensitive to the exact location of the instantaneous center, it provides a more accurate solution to the weld group capacity.

The preceding discussion has been a brief review to acquaint the reader with the equations that were used to develop the tables for the eccentrically loaded welds. An examination of Eq. (2) indicates that $(R_{ult})_i$ varies from 10.87 kip/in. to 15.76 kip/in. for a ¼-in. fillet weld for values of θ equal to 0 and 90 degrees respectively, θ depending on the location of the instantaneous center relative to the location of the critical weld element. The most critical condition is obviously when θ is near 90 degrees for the controlling element and, from Eq. (8), $R_i = 0.994\ (R_{ult})_i$, or 15.67 kip/in. for a ¼-in. fillet weld. Traditionally, weld computations are based on the strength of a $1/16$-in. weld and an E70 electrode. Converting to this basis is accomplished by multiplying 15.67 by $70/(60 \times 4)$, resulting in an ultimate value of 4.57 kip/$1/16$-in. of fillet weld/lin. in.

The least critical condition occurs when θ is 0 degrees, which is the case of weld element minimum resisting strength and maximum ductility. This minimum ultimate strength for a $1/16$-in. weld element with an E70 electrode and a 0 degree load angle is 3.17 kip/lin. in. obtained from Eq. (8). The relationship between the strength of an element of weld and the angle of load application can be easily seen by reviewing Fig. 2, which is a plot of Eq. (8). The angle increments (0, 10, 30, and 90 degrees) were chosen only to reflect equally all portions of the effect of the angle.

The design of welds, however, is controlled by Specification[8] provisions covering both the weld metal and the adjacent base metal. The specified design shear strength of a $1/16$-in. fillet weld is given in Sect. 1.5.3 as

$$F_v = 0.3\ F_u A_w \tag{9}$$

where

F_v = design shear strength of the weld metal
= 0.928 kip/in. for E70 weld metal

F_u = specified minimum tensile strength of the weld metal
= 70 ksi for E70 weld metal

A_w = area of effective weld metal throat
= 0.707/16 in.²/lin. in. for $1/16$-in. equal leg fillet weld

The ultimate values for a $1/16$-in. weld loaded at angles of 0 and 90 degrees, when converted to an equivalent working load by multiplying by 0.3, become 0.951 and 1.37 kip/in., respectively. The relationship between the ultimate values and the equivalent working values is demonstrated in Fig. 2.

Similarly, the base metal shear strength is restricted by Sect. 1.5.3 to a maximum design value of:

$$F_{vb} = 0.4\ F_y A_{bm} \tag{10}$$

where

F_{vb} = design shear strength of the base metal
= 0.900 kip for A36 steel

F_y = specified minimum yield stress of the base metal
= 36 ksi for A36 steel

A_{bm} = area of the nominal fusion zone of the base metal
= $1/16$ in.²/lin. in. for a nominal $1/16$-in. fillet weld

In the case of equal leg fillet welds with A36 steel and E70 weld metal, the allowable load per lineal inch on the throat of the weld, F_v , and the allowable load per lineal inch along the leg on the base metal, F_{vb} , have a near match (approximately 3% difference), which is not significant, and because the exact fusion surface is not precisely known, the larger value has been used in the Manual.[2]

At this point it is appropriate to discuss the experimental programs[4,5] which form the basis of the design philosophy.

These reports indicate that the weld tests were made using base metal conforming to CSA G40.12,[9] $F_y = 44$ ksi, and AWS E60 electrodes. The full scale test beams were S24 x 100's conforming to ASTM A36. The objective of the test program was to evaluate the strength of eccentrically loaded weld groups *per se*. Accordingly, the specimens were intentionally designed to maximize the strains within the weld to assure that failure would not result from buckling or rupture of the connection material or the beam web. The following steps were taken:

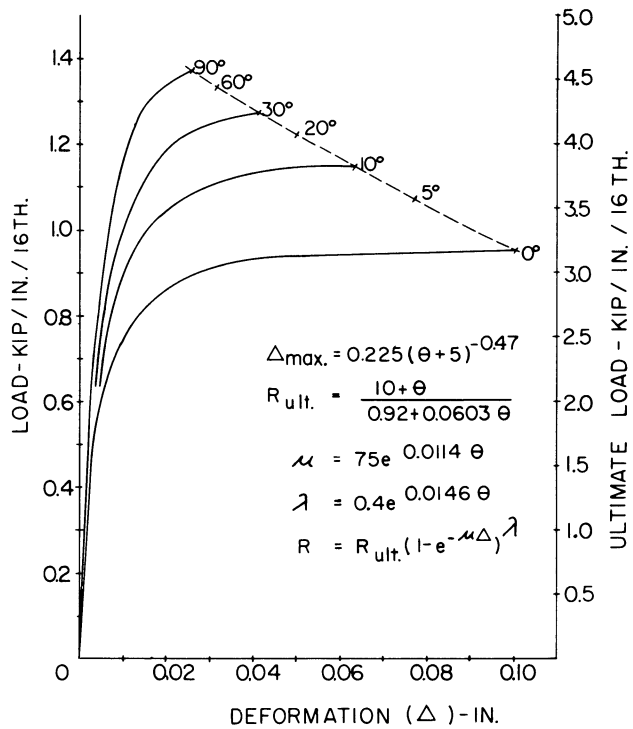

Fig. 2. Effect of load angle on weld load-deformation relationship

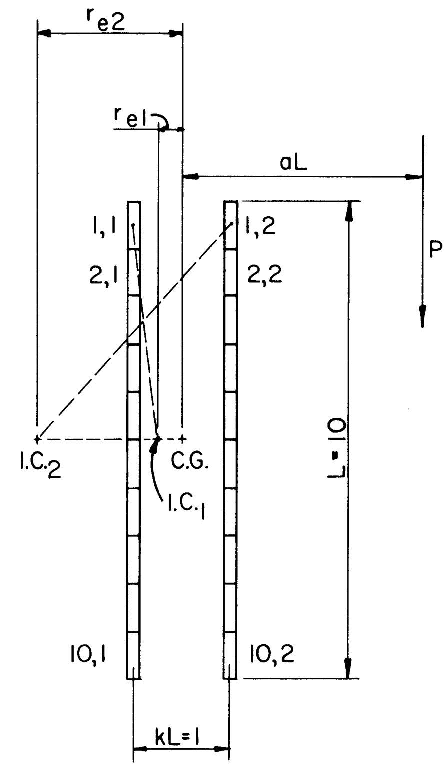

Fig. 3. Case study weld element identification

1. In some cases ½-in. doubler plates were attached to the connecting brackets to prevent them from buckling, even though the brackets already were ½-in. thick.
2. The test beam web was 0.747 in. thick, with ¼-in. fillet welds on either side.

Thirteen full scale tests were conducted on two different weld configurations. One configuration was the same as Fig. 1, while the other was equivalent to Fig. 3, with k equal to zero (a single fillet weld on each side of the test beam web). The eccentricity and k-value (Fig. 1 only) was varied for the test series.

The test results indicate that, under ideal conditions, the weld metal strength is considerably greater than the base metal design strength when θ is near 90 degrees and reasonably matched when θ is near 0 degrees. Also, when θ is near 90 degrees, ductility is less than when θ is near zero degrees.

DESIGN RECOMMENDATIONS

To recognize the significance of the two conditions discussed above and especially that of paragraph 1, and to satisfy the limitations of the Specification[8] the values in the 8th Edition Manual[2] were reduced below those of the factored test results. As a result, the tabular values in the Manual do not exceed the Specification limits as given by Eq. (9).

The solutions were obtained from a computer program using the Fortran language. Once the instantaneous center was located to the required accuracy, the weld group ultimate coefficient was computed from the moment equilibrium equation ($\Sigma M = 0$). The tabulated coefficient (C) was then computed using the following equation:

$$C = C_{ult} F_e F_c F_{max} / N \tag{11}$$

where

C = tabular value, kip/lin. in./$^1/_{16}$-in. of weld (a non-dimensionalized coefficient)

C_{ult} = ultimate capacity of eccentrically loaded weld group, kip/¼-in. of weld (computed from curves for ¼-in. weld made with E60 electrode)

N = number of elements in weld group base segment

F_e = factor for converting ¼-in. weld with E60 electrode to $^1/_{16}$-in. weld with E70 electrode

= $70/(60 \times 4) = 0.2917$

F_c = factor for reducing ultimate strength of weld group to allowable design strength basis

= 0.3 for AISC Specification (Sect. 1.5.3)

F_{max} = ratio of specification allowable stress on throat of fillet weld to calculated stress at ultimate on throat of critical element reduced by factor

F_c of the weld group, equal to or less than 1.0

Three specific cases will be reviewed in detail to demonstrate the significance of several pertinent factors encountered in the development of the tabular coefficients. In all cases, values of R_i are kips/lin. in. for 1/4-in. welds and E60 electrodes.

Case 1

k = 0.1, a = 3.0 (see Fig. 3)
C_{ult} = 24.74, N = 10
r_e = 0.214 in.
θ = 86.36 degrees, element (1,1)
R_i = 15.62
F_e = 70/240, F_c = 0.30
F_{max} = 10.61/15.62 = 0.679
C = 0.1469

See Fig. 4a for a partial plot of the weld group elemental stresses.

Case 2

k = 0.1, a = 0.2 (see Fig. 3)
C_{ult} = 227.2, N = 10
r_e = 3.936 in.
θ = 45.41 degrees, element (1,2)
R_i = 15.01
F_e = 70/240, F_c = 0.30
F_{max} = 10.61/15.01 = 0.707
C = 1.404

See Fig. 4b for a partial plot of the weld group elemental stresses.

Case 3

k = 0.5, a = 0.06 (see Fig. 1)
C_{ult} = 11342, N = 10
r_e = 42.09 in.
θ = 83.71 degrees, element (3,1)
R_i = 15.60
F_e = 70/240, F_c = 0.30
F_{max} = 10.61/15.60 = 0.680
C = 1.580

The case study data cited above was obtained from a computer listing using 10 weld elements as a basic number in one of the weld strips and was found to provide adequate accuracy for review purposes. Comparing data using 5, 10, 20, and 40 elements indicated that in general only the third significant figure would change in going from 5 to 40 elements. Values of 0.145, 1.39, and 1.58 were obtained using 40 elements for the three cases studied. Greater variation was found to occur when less than optimum convergence criterion was employed.

These three cases represent increases of 38.1%, 20.9%, and 3.59%, respectively, over the values published in the 7th Edition Manual.[1] The 7th Edition value for Case 3 is based on a slightly larger k-value (0.1).

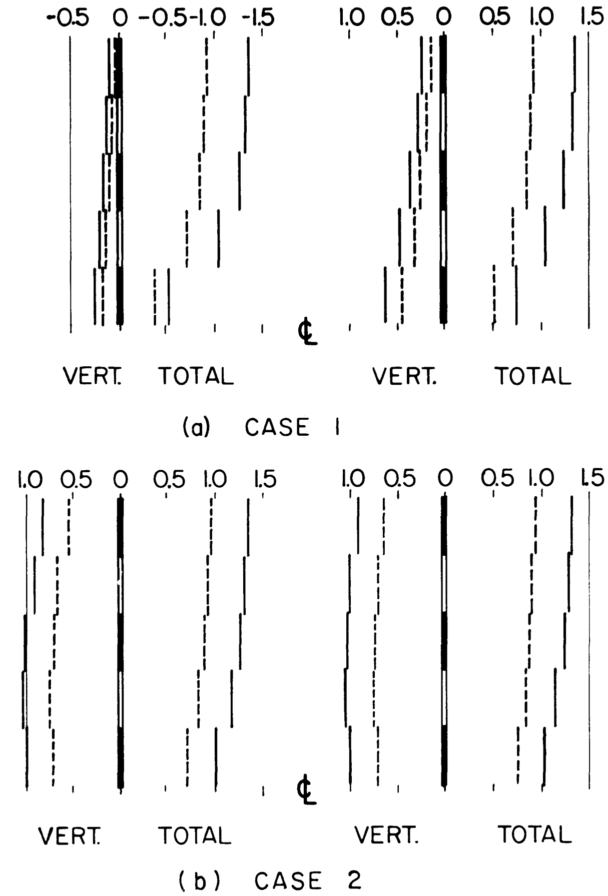

Fig. 4. Typical beam connections

The effect of the instantaneous center location on the elemental stress distribution is shown in Fig. 4 for Case 1 and Case 2 which represents the upper half of a weld group configuration as shown in Fig. 3. The lower half distribution would be identical because of symmetry. The solid bars to the right of each base line represent the "*total*" force (kip/in./1/16-in.) inclined at the appropriate angle as defined in Fig. 1. The elemental ultimate force (kip/in./1/4-in.) is modified by F_e = 70/240 and F_c = 0.3. The dashed bars to the right of the base line indicate a further modification by $F_{max} = 10.61/(R_i)_{max}$ to insure that the maximum elemental force (stress) is approximately equal to the specification limiting value of 0.928 kip/in./1/16-in. The solid bars and the dashed bars to the left of the base line represent the vertical component of the forces shown to the right of the base line. In addition, the negative sign of the force magnitude numbers for the left segment of Case 1 indicates that the forces for these elements are opposite in sense to the others even though, for convenience, they are plotted on the same side.

Case 1 represents a large eccentricity with the resultant instantanous center being located within the group geometry. As a result there is a significant relative difference between the minimum and maximum instantaneous center-elemental distance (0.576 and 4.55 in.) and, coupled with the effect of the angle of the applied load (35.0 degrees to 86.4 degrees), the elemental forces range from 0.560 to 1.37 kip/in./$^1/_{16}$-in. (solid bars on the right in Fig. 4a). The distribution is not linear, but a complex non-linear function obtained from Eq. (8). The three uppermost elements on both the right and left segments are nearly equal and considerably above the code maximum value of 0.928 kip/in./$^1/_{16}$-in. To satisfy the Specification provisions, each elemental value is adjusted downward by the factor F_{max}, so that no elemental maximum force is greatly over 0.928. These reduced values are shown as dashed bars on the right of the base lines. On the left of the base line the vertical component of the total elemental force is plotted for both the unfactored (solid bars) and for the factored (dashed) conditions. The large angle between the weld element and the axis of the applied elemental force results in vertical components that are relatively small compared to the total elemental forces plotted on the right. The sum of the vertical components equals the external applied load (factored) but only a small portion of the total inclined force.

Case 2 represents the other extreme of the elemental force distribution. The small eccentricity requires a relatively large instantaneous center distance. The relative difference between the minimum and maximum instantaneous center-elemental distance (3.47 in. and 6.32 in.) is reduced compared to the difference in Case 1. The angles of the applied forces for the elements vary from 6.42 to 52.6 degrees; however, because of the combination of elemental distances and the respective angles, the range of the total elemental forces is from 1.02 to 1.32 kip/in./$^1/_{16}$-in. (solid bars on right) and is not as great as in Case 1. When the total elemental forces are factored down by F_{max} (dashed bars) there again is a more uniform distribution close in to the maximum Specification value of 0.928 kip/in./$^1/_{16}$-in. The vertical components are reduced, but not as dramatically as in Case 1, because the angles are smaller and the cosine function is used. As a result, the sum of the vertical components is again equal to the external applied load and of the same magnitude as the total inclined force which is considerably different than for Case 1 previously reviewed.

Case 3 represents another condition that initially seems to indicate inconsistency in the results. An eccentricity factor of 0.06 for all practical purposes implies a concentric load. A smaller eccentricity factor is not necessary as adequate convergence had been accomplished. The seeming inconsistancy stems from the fact that, if the allowable load as per the Specification (0.928 kip/in./$^1/_{16}$-in.) is applied to this weld configuration (Fig. 1) with $k = 0.5$, the capacity would normally be assumed to equal to 0.928 (1 + 0.5 + 0.5) = 1.86 versus 1.58 published. A small applied load eccentricity means a large instantaneous center distance and, therefore, all the elements have practically identical (between 40 and 46 in.) radius vectors. The horizontal weld elements will have a load angle near 90 degrees (all above 83) while the vertical elements will have angles near 0 degrees (all below 7). An examination of Fig. 2 will indicate that, when the weld elements (horizontal) having a force applied at near 90 degrees reach their ultimate deformation, the weld elements (vertical) having a force applied at near 0 degrees, and restricted to the same deformation by compatibility, will not have reached their ultimate and obviously lower capacity. Then, when the critical horizontal element is factored down by F_{max} to the Specification limit of 0.928 kip/in./$^1/_{16}$-in., the vertical elements are similarily reduced to some lesser value (0.65), resulting in a coefficient (C) value (1.58) less than what normally would be assumed (1.86 kip/in./$^1/_{16}$-in.). Obviously, for the cases with small eccentricities this is a conservative procedure.

The three case studies have explained how the experimental results have been converted into design tables via the equations and computer program. Because the factored capacity, as determined by the summation of strength of weld elements based on actual test results, would exceed the Specification provisions governing the allowable stresses in both the weld metal and the base metal by considerable margins, they were further reduced to allowable levels. Depending upon which configuration and eccentricity of load, the results in some cases are significantly increased over the values listed in the 7th Edition Manual.[1] However, the user can still have confidence in knowing that the stresses in the weld metal are matched to the allowable stress in the base metal. As in the past, for example, a $^1/_4$-in. fillet weld on either side of a $^1/_2$-in. thick A36 plate would result in allowable stress limits being reached simultaneously. The difference now being a more uniform factor of safety throughout all the tables than was provided in the past.

As pointed out in Ref. 5, "The performance of the connected parts (e.g., shear capacity of web, buckling of plates, etc.) must also be checked." Therefore, the following must be considered in the design of real connections:

1. Common configurations such as those shown in Fig. 5 need evaluation. Recent research[6,7] has shown that for similar high-strength-bolted connections (coped top flange and the connection concentrated in the upper portion of the beam web), a failure mode known as "block shear" is possible. Tests are currently being run to evaluate the need for similar concern with welded connections.

2. The load deformation relationships for weld elements loaded at different angles were obtained from linear welds tested independently of each other. Since, in actual connections, some weld groups are made up of weld elements intersecting at various angles with each other, small strain incompatibilities do occur mathematically, but obviously can not occur physically.

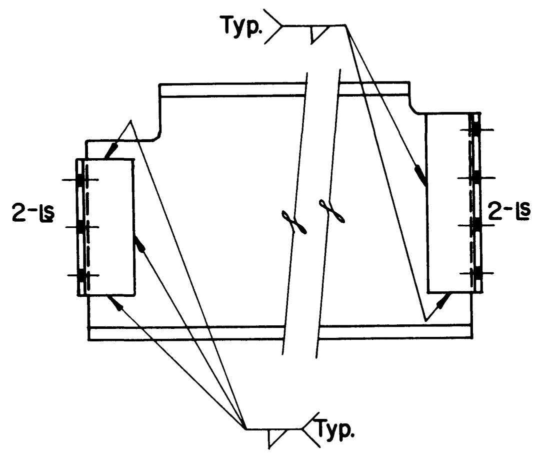

Fig. 5. Comparison of weld element load distribution (kip/$^1/_{16}$-in./lin. in.)

SUMMARY AND CONCLUSIONS

The preceding discussion has explained how the design tables for eccentrically loaded weld groups have been developed for use in the new 8th Edition *Manual.*[2] The tables result in substantially increased values, in some cases depending upon the group geometry and load eccentricity. However, because of the limited available research, which involved only A36 steel and E60 electrodes, a certain amount of conservatism had to be introduced to keep stresses within the Specification limits.

It is probable that in the future, when selective research is completed, the Specification limits can be revised to allow higher shear stress values. The key points to be resolved are as follows:

1. Are the test results applicable to geometry configurations that have not been tested?
2. What limits need be applied to the through thickness strength of the base metal with welds on either side? Should the allowable shear stress in the base metal in the highly localized area close to the weld fusion boundary be limited to the traditionally recognized allowable shear stress away from the weld?
3. Are the independent load-deformation relationships completely applicable when the axes of weld segments intersect at various angles?

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