Calculation of Effective Lengths of Stepped Columns

KRISHNA M. AGRAWAL AND ANDREW P. STAFIEJ

Travelling cranes are frequently used to move heavy loads in industrial buildings. To accomplish a general movement, the crane traverses a crane bridge, which in turn moves on rails along the length of the building, supported by the main building columns.

Designers frequently choose stepped columns, with the wider lower section serving a dual purpose: (a) to support the crane rail and (b) to provide the necessary strength to support the extra load from the crane. The design of the stepped columns is time-consuming and complicated. Effective lengths, which must be calculated for each segment, depend upon the following: the end fixity types at the two ends, the ratio of segment lengths (l_1/l_2), the ratio of the segment inertias (I_1/I_2), and the ratio of the applied axial loads (P_1/P_2) applied at the top of the column and at the stepped levels.

Various cases of end fixities are encountered in practice (Fig. 1). Anderson and Woodward[1] have presented equations for five end-fixity types to be used in calculating effective lengths. These types are: (1) Pin-Pin, (2) Fix-Free, (3) Fix-Pin, (4) Fix-Slider, and (5) Fix-Fix. Two other cases which have not been dealt with previously are: (6) Pin-Fix and (7) Pin-Slider (Fig. 1). Industrial building frames are often designed as pinned at the bottom, supporting a deep roof truss at the top which provides for a fix or slider end condition.

The characteristic equation in Ref. 1 for case (5) when $P_2 = 0$ [Eq. (A-15) and FUNCTION FC5(x)] appears to be in error. This technical note is intended to correct the equation for the end-fixity case (5) and to extend the directory of characteristic equations for end-fixities to include two additional cases: (6) Pin-Fix and (7) Pin-Slider. The derivation of the equations is omitted in this paper, since the process has been adequately described in Ref. 1. The nomenclature of Ref. 1 is used throughout to maintain continuity.

Krishna M. Agrawal is Senior Structural Analyst, H. A. Simons (International) Ltd., Vancouver, B.C., Canada.

Andrew P. Stafiej is Structural Engineer, H. A. Simons (International) Ltd., Vancouver, B.C., Canada.

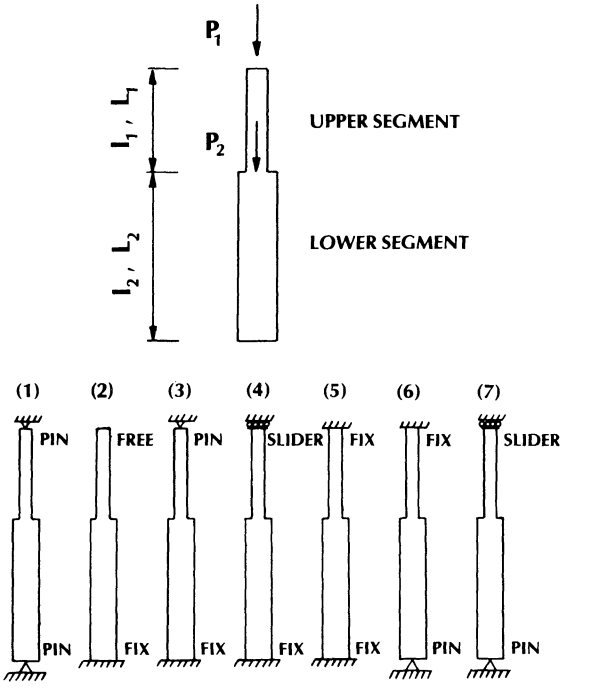

Fig. 1. End condition types

CHARACTERISTIC EQUATIONS

The parameters in the equations have the following definitions:

$$\text{IR} = I_1/I_2; \quad \text{LR} = l_1/l_2; \quad \text{PR} = P_1/P_2$$

$$\text{Z} = \Upsilon_1 l_1; \quad \text{BZ} = \Upsilon_2 l_2$$

$$\beta = \text{B} = (l_2/l_1) \cdot \sqrt{(I_1/I_2)[1 + (P_2/P_1)]}$$

$$\Upsilon_1^2 = P_1/EI_1; \quad \Upsilon_2^2 = (P_1 + P_2)/EI_2$$

Finding the lowest root of the characteristic equation Z = ZRT allows the calculation of buckling load

$$P_{1cr} = \left(\frac{\text{ZRT}}{l_1}\right)^2 \cdot EI_1$$

$$(P_1 + P_2)_{cr} = \left(\frac{\text{ZRT} \cdot \beta}{l_2}\right)^2 \cdot EI_2$$

Equating these critical loads to the Euler buckling formula $P_{cr} = \pi^2 EI/L^2$, one obtains:

$$l_{1(eff)} = \pi l_1/\mathrm{ZRT}$$

$$l_{2(eff)} = \pi l_2/(\beta \mathrm{Z})$$

These effective lengths are used in the AISC interaction formula for designing beam columns.

The variable definitions are:

I_1, I_2	Moments of inertia of upper and lower segments, respectively
P_1, P_2	Applied axial loads at the top and at step level
P_T	Total column axial load = $(P_1 + P_2)$
l_1, l_2	Lengths of upper and lower segments, respectively
L_T	Total column length = $(l_1 + l_2)$
$l_{1(eff)}$, $l_{2(eff)}$	Effective lengths of the upper and lower segments for Euler buckling formula, respectively
K_1, K_2	Effective length factors for upper and lower segments, respectively, with the following definition:

$$K_1 = l_{1(\mathrm{eff})}/(l_1 + l_2)$$

$$K_2 = l_{2(\mathrm{eff})}/(l_1 + l_2)$$

Case 5—Fix-Fix [corrected characteristic equation to replace Eq. (A-15) in Ref. 1]:

c. $P_2 = 0$

$$[\mathrm{COS(Z)} - \mathrm{COS(BZ)}]\{\mathrm{Z}[(1 + \mathrm{LR})/\mathrm{LR}] \cdot \mathrm{SIN(Z)} + \mathrm{COS(Z)} - \mathrm{COS(BZ)}\} + [\mathrm{LR} \cdot \mathrm{SIN(Z)} - \mathrm{Z}(1 + \mathrm{LR})\mathrm{COS(Z)} + \mathrm{SIN(BZ)}/\mathrm{B}][\mathrm{SIN(Z)}/\mathrm{LR} + \mathrm{B} \cdot \mathrm{sin(BZ)}] = 0 \quad \text{(A-8)}$$

Case 6—Pinned-Fixed:

a. General ($P_1 > 0$; $P_2 > 0$):

$$\mathrm{SIN(BZ)}\{2/\mathrm{PR} - \mathrm{Z} \cdot \mathrm{SIN(Z)} \cdot [(1 + \mathrm{LR})/\mathrm{LR} + 1/\mathrm{PR}] - \mathrm{COS(Z)} \cdot [\mathrm{PR}/(1 + \mathrm{PR}) + 2/\mathrm{PR}]\} + \mathrm{COS(BZ)}\{-\mathrm{B} \cdot \mathrm{LR} \cdot \mathrm{SIN(Z)} + \mathrm{BZ} \cdot \mathrm{COS(Z)}\,[1 + \mathrm{LR} - 1/(1 + \mathrm{PR})]\} = 0 \quad \text{(A-16)}$$

b. $P_1 = 0$:

$$\mathrm{SIN(BZ)} \cdot \{(\mathrm{BZ} \cdot \mathrm{LR})^2 - 6\mathrm{IR}/(\mathrm{BZ} \cdot \mathrm{LR})^2 - 6[1 + (1/\mathrm{LR})]\} + 2\mathrm{BZ} \cdot \mathrm{COS(BZ)} \cdot \{[3\mathrm{IR}/(\mathrm{BZ} \cdot \mathrm{LR})^2] - \mathrm{LR}\} = 0 \quad \text{(A-17)}$$

c. $P_2 = 0$:

$$\mathrm{B} \cdot \mathrm{LR} \cdot \mathrm{COS(BZ)} \cdot [\mathrm{SIN(Z)} - \mathrm{Z} \cdot \mathrm{COS(Z)} \cdot (1 + \mathrm{LR})/\mathrm{LR}] + \mathrm{SIN(BZ)} \cdot [\mathrm{COS(Z)} + \mathrm{Z} \cdot \mathrm{SIN(Z)} \cdot (1 + \mathrm{LR})/\mathrm{LR}] = 0 \quad \text{(A-18)}$$

Case 7—Pinned-Slider:

a. General ($P_1 > 0$; $P_2 > 0$):

$$[1/(1 + \mathrm{PR})] \cdot \mathrm{Z} \cdot \mathrm{SIN(Z)} \cdot \mathrm{SIN(BZ)} - \mathrm{LR} \cdot \mathrm{BZ} \cdot \mathrm{COS(Z)} \cdot \mathrm{COS(BZ)} = 0 \quad \text{(A-19)}$$

b. $P_1 = 0$:

$$\mathrm{LR} \cdot \mathrm{BZ} \cdot \mathrm{SIN(BZ)} - \mathrm{IR} \cdot \mathrm{COS(BZ)} = 0 \quad \text{(A-20)}$$

c. $P_2 = 0$:

$$\mathrm{Z} \cdot \mathrm{SIN(Z)} \cdot \mathrm{SIN(BZ)} - \mathrm{BZ} \cdot \mathrm{LR} \cdot \mathrm{COS(Z)} \cdot \mathrm{COS(BZ)} = 0 \quad \text{(A-21)}$$

Reference 4 outlines a computer program similar to the one described in Ref. 1. This program was developed to calculate the roots of the characteristic equations. The solution routine which serves to find the lowest root was modified to improve the speed of convergence to the root. Residual values were calculated by spacing points at equal intervals until a sign change in the residual was observed. At this point, instead of halving the incremental value of Z, a new value for Z was calculated by interpolating the two values of Z, which gave residuals of differing signs. The process was repeated retaining two values for Z, which produced the smallest residuals for further interpolation. The last step was repeated several times, producing a much faster convergence to the characteristic root.

The output from this program (Table 1) lists the slenderness ratios for all seven end-fixity types (Fig. 1) for a wide selection of segment inertia ratios, segment length ratios, and top- and step-level axial-load ratios. Any intermediate value can be easily interpolated from the values presented.

Note that the axial load ratio $P_2/P_T = P_2/(P_1 + P_2)$ varies from 0 to 1. A value of zero corresponds to $P_1 > 0$, $P_2 = 0$ and a value of 1 corresponds to $P_1 = 0$ and $P_2 > 0$. All other values of the ratio correspond to P_1 and P_2 both greater than zero.

ACKNOWLEDGMENT

Checks provided by Tony Katramadakis, Curt Shelton, and Steve Lang are gratefully acknowledged. Financial support for this project was provided by H. A. Simons (International) Ltd.

REFERENCES

1. *Anderson, J. P. and J. H. Woodward* Calculation of Effective Lengths and Effective Slenderness Ratios of Stepped Columns *Engineering Journal/American Institute of Steel Construction, Oct. 1972, pp. 157–166.*
2. *Dalal, S. T.* Some Non-Conventional Cases of Column Design *Engineering Journal/American Institute of Steel Construction, Jan. 1969, pp. 28–39.*
3. *Huang, H. C.* Determination of Slenderness Ratios for Design of Heavy Mill Building Stepped Columns *Iron and Steel Engineering, Nov. 1968, pp. 123–134.*
4. *Agrawal, K. M.* STEPCOL—Computer Program to Calculate Slenderness Ratio of Stepped Columns *H. A. Simons (International) Ltd. Engineering Computing Department, Vancouver, B.C., Canada, March 1980.*

Table 1. Equivalent Length Factors for Various End Conditions
$[K_1 = l_{1(eff)}/L_T; K_2 = l_{2(eff)}/L_T]$

END CONDITION (BOTTOM - TOP)			(1) PIN - PIN		(2) FIX - FREE		(3) FIX - PIN		(4)FIX -SLIDER		(5) FIX - FIX		(6) PIN - FIX		(7) PIN-SLIDER	
I1 / I2	ℓ2 / LT	P2 / PT	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2
0.1	0.1	0.0	0.997	3.153	1.820	5.755	0.637	2.014	0.910	2.878	0.455	1.439	0.695	2.198	1.999	6.320
		0.2	1.020	2.886	1.820	5.148	0.637	1.802	0.910	2.574	0.455	1.287	0.719	2.033	2.047	5.791
		0.4	1.059	2.595	1.820	4.458	0.637	1.560	0.910	2.229	0.455	1.115	0.759	1.858	2.127	5.211
		0.6	1.137	2.275	1.820	3.640	0.637	1.274	0.910	1.820	0.455	0.910	0.836	1.672	2.282	4.565
		0.8	1.360	1.923	1.820	2.574	0.637	0.901	0.910	1.287	0.455	0.644	1.045	1.478	2.708	3.829
		1.0	0.000	1.560	0.000	0.200	0.000	0.197	0.000	0.199	0.000	0.196	0.000	1.283	0.000	2.997
0.1	0.3	0.0	0.941	2.975	1.461	4.621	0.523	1.654	0.732	2.314	0.370	1.170	0.629	1.989	1.964	6.212
		0.2	0.982	2.779	1.461	4.133	0.524	1.483	0.732	2.071	0.372	1.051	0.664	1.879	2.088	5.906
		0.4	1.050	2.572	1.461	3.580	0.526	1.289	0.733	1.795	0.375	0.918	0.720	1.764	2.283	5.593
		0.6	1.176	2.353	1.462	2.924	0.531	1.061	0.735	1.469	0.384	0.769	0.822	1.644	2.633	5.267
		0.8	1.500	2.121	1.464	2.071	0.552	0.780	0.742	1.049	0.439	0.621	1.074	1.519	3.487	4.932
		1.0	0.000	1.880	0.000	0.600	0.000	0.556	0.000	0.590	0.000	0.531	0.000	1.389	0.000	4.631
0.1	0.5	0.0	0.793	2.506	1.107	3.502	0.468	1.480	0.566	1.788	0.342	1.081	0.493	1.561	1.850	5.851
		0.2	0.826	2.336	1.109	3.135	0.479	1.356	0.572	1.616	0.356	1.006	0.518	1.464	2.008	5.679
		0.4	0.880	2.156	1.112	2.724	0.499	1.223	0.583	1.429	0.379	0.928	0.556	1.361	2.255	5.524
		0.6	0.980	1.960	1.120	2.239	0.540	1.080	0.614	1.229	0.422	0.845	0.626	1.252	2.691	5.381
		0.8	1.234	1.746	1.148	1.623	0.659	0.932	0.747	1.056	0.535	0.757	0.801	1.133	3.696	5.227
		1.0	0.000	1.513	0.000	1.000	0.000	0.786	0.000	0.962	0.000	0.665	0.000	1.004	0.000	5.074
0.1	0.7	0.0	0.561	1.775	0.785	2.482	0.436	1.378	0.482	1.524	0.279	0.883	0.331	1.045	1.588	5.022
		0.2	0.579	1.638	0.800	2.263	0.448	1.268	0.518	1.467	0.289	0.816	0.345	0.975	1.755	4.963
		0.4	0.608	1.489	0.828	2.029	0.469	1.148	0.577	1.414	0.303	0.743	0.369	0.903	2.006	4.914
		0.6	0.662	1.325	0.893	1.786	0.508	1.016	0.683	1.367	0.331	0.663	0.417	0.833	2.429	4.858
		0.8	0.807	1.141	1.103	1.560	0.612	0.865	0.937	1.325	0.405	0.573	0.546	0.772	3.395	4.802
		1.0	0.000	0.939	0.000	1.400	0.000	0.689	0.000	1.289	0.000	0.469	0.000	0.729	0.000	4.734
0.1	0.9	0.0	0.329	1.039	0.638	2.016	0.245	0.774	0.461	1.456	0.194	0.614	0.275	0.869	1.094	3.459
		0.2	0.356	1.007	0.697	1.971	0.257	0.727	0.512	1.448	0.214	0.607	0.305	0.863	1.219	3.446
		0.4	0.399	0.977	0.787	1.927	0.279	0.683	0.588	1.441	0.245	0.600	0.350	0.858	1.405	3.441
		0.6	0.476	0.951	0.942	1.884	0.324	0.648	0.716	1.433	0.297	0.594	0.426	0.853	1.718	3.437
		0.8	0.657	0.929	1.302	1.841	0.440	0.623	1.008	1.425	0.417	0.589	0.599	0.848	2.425	3.430
		1.0	0.000	0.909	0.000	1.800	0.000	0.606	0.000	1.418	0.000	0.000	0.000	0.843	0.000	3.417
0.2	0.1	0.0	0.998	2.230	1.840	4.115	0.645	1.441	0.920	2.057	0.460	1.029	0.696	1.555	1.999	4.470
		0.2	1.021	2.042	1.840	3.680	0.645	1.289	0.920	1.840	0.460	0.921	0.720	1.439	2.047	4.095
		0.4	1.060	1.836	1.840	3.187	0.645	1.117	0.920	1.594	0.461	0.798	0.760	1.316	2.128	3.685
		0.6	1.139	1.610	1.840	2.602	0.645	0.912	0.920	1.302	0.461	0.652	0.838	1.185	2.283	3.229
		0.8	1.363	1.363	1.840	1.840	0.646	0.646	0.921	0.921	0.462	0.462	1.049	1.049	2.710	2.710
		1.0	0.000	1.109	0.000	0.200	0.000	0.194	0.000	0.198	0.000	0.192	0.000	0.914	0.000	2.123
0.2	0.3	0.0	0.947	2.117	1.523	3.406	0.557	1.246	0.765	1.711	0.396	0.887	0.635	1.421	1.969	4.404
		0.2	0.990	1.979	1.523	3.047	0.561	1.121	0.767	1.534	0.403	0.805	0.672	1.344	2.093	4.187
		0.4	1.059	1.835	1.525	2.641	0.567	0.983	0.771	1.335	0.415	0.719	0.730	1.265	2.290	3.966
		0.6	1.190	1.683	1.528	2.160	0.584	0.826	0.778	1.100	0.446	0.630	0.836	1.183	2.646	3.743
		0.8	1.523	1.523	1.537	1.537	0.653	0.653	0.812	0.812	0.549	0.549	1.097	1.097	3.508	3.508
		1.0	0.000	1.358	0.000	0.600	0.000	0.522	0.000	0.581	0.000	0.484	0.000	1.010	0.000	3.297

Table 1. Equivalent Length Factors for Various End Conditions
$[K_1 = l_{1(eff)}/L_T; K_2 = l_{2(eff)}/L_T]$

END CONDITION (BOTTOM - TOP)			(1) PIN - PIN		(2) FIX - FREE		(3) FIX - PIN		(4)FIX -SLIDER		(5) FIX - FIX		(6) PIN - FIX		(7) PIN-SLIDER	
I1 / I2	ℓ2 / LT	P2 / PT	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2
0.2	0.5	0.0	0.813	1.818	1.221	2.730	0.539	1.204	0.645	1.443	0.384	0.860	0.512	1.145	1.860	4.159
		0.2	0.848	1.696	1.227	2.453	0.555	1.110	0.664	1.328	0.402	0.804	0.539	1.077	2.030	4.060
		0.4	0.906	1.569	1.239	2.146	0.584	1.012	0.698	1.210	0.430	0.745	0.581	1.007	2.289	3.964
		0.6	1.015	1.435	1.265	1.788	0.641	0.906	0.775	1.096	0.482	0.682	0.659	0.932	2.738	3.872
		0.8	1.287	1.287	1.365	1.365	0.794	0.794	1.000	1.000	0.616	0.616	0.852	0.852	3.762	3.762
		1.0	0.000	1.126	0.000	1.000	0.000	0.678	0.000	0.930	0.000	0.545	0.000	0.767	0.000	3.640
0.2	0.7	0.0	0.609	1.361	0.983	2.197	0.478	1.069	0.618	1.382	0.304	0.680	0.387	0.864	1.640	3.667
		0.2	0.631	1.263	1.017	2.034	0.493	0.986	0.671	1.343	0.316	0.632	0.413	0.826	1.809	3.619
		0.4	0.670	1.160	1.078	1.868	0.517	0.896	0.754	1.306	0.335	0.580	0.456	0.790	2.075	3.594
		0.6	0.742	1.050	1.201	1.699	0.563	0.796	0.899	1.271	0.373	0.527	0.536	0.757	2.507	3.545
		0.8	0.935	0.935	1.538	1.538	0.684	0.684	1.238	1.238	0.481	0.481	0.730	0.730	3.513	3.513
		1.0	0.000	0.827	0.000	1.400	0.000	0.555	0.000	1.208	0.000	0.458	0.000	0.708	0.000	3.473
0.2	0.9	0.0	0.454	1.015	0.899	2.010	0.324	0.724	0.574	1.284	0.262	0.585	0.368	0.823	1.220	2.729
		0.2	0.495	0.991	0.983	1.965	0.347	0.694	0.639	1.278	0.290	0.580	0.409	0.818	1.361	2.723
		0.4	0.559	0.968	1.110	1.923	0.385	0.667	0.735	1.273	0.332	0.575	0.470	0.814	1.571	2.721
		0.6	0.670	0.947	1.332	1.883	0.455	0.643	0.897	1.269	0.403	0.571	0.573	0.811	1.923	2.719
		0.8	0.927	0.927	1.843	1.843	0.622	0.622	1.264	1.264	0.566	0.566	0.807	0.807	2.716	2.716
		1.0	0.000	0.909	0.000	1.800	0.000	0.605	0.000	1.259	0.000	0.562	0.000	0.803	0.000	2.706
0.3	0.1	0.0	0.998	1.822	1.860	3.396	0.652	1.191	0.930	1.698	0.465	0.850	0.696	1.271	1.999	3.650
		0.2	1.021	1.668	1.860	3.038	0.652	1.065	0.930	1.519	0.466	0.761	0.720	1.176	2.048	3.344
		0.4	1.061	1.500	1.860	2.631	0.653	0.923	0.930	1.316	0.466	0.659	0.761	1.076	2.128	3.010
		0.6	1.140	1.316	1.860	2.148	0.653	0.754	0.931	1.075	0.467	0.539	0.840	0.970	2.284	2.637
		0.8	1.367	1.116	1.861	1.519	0.655	0.535	0.932	0.761	0.469	0.383	1.053	0.860	2.711	2.214
		1.0	0.000	0.910	0.000	0.200	0.000	0.191	0.000	0.197	0.000	0.188	0.000	0.751	0.000	1.736
0.3	0.3	0.0	0.953	1.740	1.587	2.897	0.587	1.072	0.800	1.460	0.420	0.768	0.642	1.172	1.974	3.604
		0.2	0.997	1.628	1.586	2.590	0.593	0.969	0.803	1.312	0.431	0.703	0.680	1.111	2.099	3.428
		0.4	1.070	1.513	1.590	2.248	0.605	0.856	0.811	1.146	0.450	0.636	0.741	1.048	2.300	3.252
		0.6	1.204	1.390	1.595	1.842	0.633	0.731	0.827	0.955	0.492	0.568	0.851	0.983	2.658	3.069
		0.8	1.547	1.263	1.616	1.319	0.734	0.600	0.899	0.734	0.619	0.505	1.122	0.916	3.529	2.881
		1.0	0.000	1.133	0.000	0.600	0.000	0.495	0.000	0.572	0.000	0.450	0.000	0.846	0.000	2.710
0.3	0.5	0.0	0.833	1.521	1.335	2.438	0.578	1.055	0.718	1.311	0.407	0.744	0.533	0.973	1.890	3.450
		0.2	0.872	1.424	1.345	2.197	0.598	0.977	0.746	1.219	0.427	0.697	0.563	0.919	2.048	3.344
		0.4	0.934	1.321	1.368	1.935	0.632	0.894	0.797	1.128	0.458	0.647	0.610	0.863	2.318	3.278
		0.6	1.051	1.214	1.416	1.635	0.697	0.805	0.901	1.040	0.516	0.595	0.697	0.805	2.772	3.201
		0.8	1.342	1.096	1.586	1.295	0.870	0.711	1.181	0.964	0.661	0.540	0.911	0.744	3.829	3.126
		1.0	0.000	0.971	0.000	1.000	0.000	0.613	0.000	0.903	0.000	0.481	0.000	0.681	0.000	3.014
0.3	0.7	0.0	0.660	1.205	1.156	2.111	0.509	0.929	0.708	1.292	0.331	0.604	0.444	0.810	1.696	3.097
		0.2	0.690	1.126	1.205	1.968	0.526	0.859	0.771	1.260	0.347	0.567	0.479	0.783	1.862	3.040
		0.4	0.739	1.045	1.289	1.824	0.553	0.783	0.869	1.229	0.375	0.530	0.535	0.757	2.141	3.027
		0.6	0.832	0.961	1.451	1.675	0.606	0.699	1.039	1.199	0.430	0.496	0.636	0.734	2.611	3.015
		0.8	1.074	0.877	1.876	1.532	0.744	0.607	1.436	1.172	0.576	0.470	0.874	0.714	3.658	2.987
		1.0	0.000	0.801	0.000	1.400	0.000	0.509	0.000	1.146	0.000	0.454	0.000	0.695	0.000	2.933

Table 1. Equivalent Length Factors for Various End Conditions
$[K_1 = l_{1(eff)}/L_T; K_2 = l_{2(eff)}/L_T]$

END CONDITION (BOTTOM - TOP)			(1) PIN - PIN		(2) FIX - FREE		(3) FIX - PIN		(4)FIX -SLIDER		(5) FIX - FIX		(6) PIN - FIX		(7) PIN-SLIDER	
I1	ℓ2	P2														
--	--	--	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2
I2	LT	PT														
0.3	0.9	0.0	0.553	1.009	1.101	2.010	0.390	0.712	0.651	1.189	0.309	0.564	0.433	0.790	1.342	2.450
		0.2	0.604	0.986	1.202	1.963	0.421	0.687	0.726	1.186	0.343	0.560	0.482	0.787	1.495	2.441
		0.4	0.683	0.966	1.362	1.926	0.469	0.663	0.836	1.183	0.393	0.556	0.554	0.784	1.729	2.445
		0.6	0.819	0.946	1.632	1.884	0.556	0.641	1.021	1.179	0.479	0.553	0.677	0.781	2.115	2.442
		0.8	1.135	0.927	2.255	1.841	0.762	0.622	1.441	1.176	0.673	0.549	0.953	0.779	2.988	2.440
		1.0	0.000	0.909	0.000	1.800	0.000	0.605	0.000	1.172	0.000	0.546	0.000	0.775	0.000	2.430
0.4	0.1	0.0	0.998	1.578	1.880	2.973	0.660	1.043	0.940	1.487	0.471	0.744	0.696	1.101	1.999	3.161
		0.2	1.022	1.445	1.880	2.659	0.660	0.933	0.941	1.330	0.471	0.666	0.721	1.020	2.048	2.896
		0.4	1.061	1.300	1.880	2.303	0.660	0.809	0.941	1.152	0.472	0.578	0.762	0.933	2.128	2.607
		0.6	1.141	1.141	1.881	1.881	0.661	0.661	0.941	0.941	0.473	0.473	0.842	0.842	2.285	2.285
		0.8	1.370	0.969	1.882	1.330	0.664	0.469	0.943	0.667	0.478	0.338	1.058	0.748	2.713	1.919
		1.0	0.000	0.792	0.000	0.200	0.000	0.189	0.000	0.197	0.000	0.185	0.000	0.656	0.000	1.506
0.4	0.3	0.0	0.959	1.517	1.648	2.606	0.612	0.968	0.833	1.318	0.440	0.695	0.649	1.027	1.980	3.131
		0.2	1.005	1.421	1.649	2.332	0.621	0.878	0.840	1.187	0.453	0.640	0.689	0.975	2.105	2.977
		0.4	1.079	1.322	1.655	2.027	0.637	0.780	0.851	1.042	0.476	0.583	0.752	0.922	2.308	2.826
		0.6	1.218	1.218	1.665	1.665	0.674	0.674	0.878	0.878	0.526	0.526	0.867	0.867	2.666	2.666
		0.8	1.571	1.111	1.698	1.201	0.798	0.564	0.985	0.696	0.668	0.472	1.146	0.810	3.549	2.509
		1.0	0.000	1.002	0.000	0.600	0.000	0.472	0.000	0.563	0.000	0.424	0.000	0.753	0.000	2.362
0.4	0.5	0.0	0.857	1.355	1.451	2.294	0.605	0.957	0.779	1.232	0.424	0.670	0.556	0.879	1.908	3.017
		0.2	0.897	1.268	1.460	2.065	0.628	0.888	0.815	1.153	0.445	0.629	0.589	0.833	2.076	2.936
		0.4	0.964	1.181	1.494	1.830	0.665	0.815	0.878	1.075	0.478	0.586	0.642	0.787	2.352	2.881
		0.6	1.088	1.088	1.560	1.560	0.737	0.737	1.002	1.002	0.541	0.541	0.738	0.738	2.829	2.829
		0.8	1.401	0.991	1.789	1.265	0.925	0.654	1.320	0.933	0.697	0.493	0.974	0.688	3.903	2.760
		1.0	0.000	0.885	0.000	1.000	0.000	0.568	0.000	0.877	0.000	0.442	0.000	0.639	0.000	2.650
0.4	0.7	0.0	0.713	1.127	1.309	2.069	0.537	0.849	0.775	1.225	0.359	0.568	0.494	0.781	1.751	2.768
		0.2	0.749	1.060	1.370	1.938	0.556	0.787	0.845	1.195	0.380	0.538	0.537	0.759	1.919	2.714
		0.4	0.809	0.990	1.475	1.806	0.588	0.720	0.954	1.169	0.416	0.510	0.603	0.738	2.212	2.709
		0.6	0.921	0.921	1.670	1.670	0.648	0.648	1.143	1.143	0.484	0.484	0.719	0.719	2.700	2.700
		0.8	1.206	0.853	2.164	1.530	0.807	0.570	1.584	1.120	0.657	0.464	0.991	0.701	3.785	2.677
		1.0	0.000	0.791	0.000	1.400	0.000	0.494	0.000	1.097	0.000	0.450	0.000	0.685	0.000	2.622
0.4	0.9	0.0	0.637	1.006	1.272	2.011	0.447	0.707	0.715	1.130	0.347	0.548	0.484	0.766	1.460	2.309
		0.2	0.696	0.984	1.389	1.964	0.484	0.684	0.798	1.128	0.385	0.545	0.540	0.764	1.618	2.289
		0.4	0.788	0.965	1.570	1.922	0.540	0.662	0.920	1.127	0.442	0.542	0.622	0.761	1.877	2.298
		0.6	0.946	0.946	1.891	1.891	0.641	0.641	1.124	1.124	0.539	0.539	0.760	0.760	2.305	2.305
		0.8	1.312	0.928	2.613	1.847	0.880	0.622	1.586	1.122	0.758	0.536	1.071	0.758	3.257	2.303
		1.0	0.000	0.909	0.000	1.800	0.000	0.605	0.000	1.118	0.000	0.533	0.000	0.754	0.000	2.282
0.5	0.1	0.0	0.998	1.412	1.901	2.688	0.667	0.943	0.951	1.344	0.476	0.673	0.697	0.985	2.000	2.828
		0.2	1.022	1.293	1.900	2.404	0.667	0.844	0.951	1.202	0.477	0.603	0.722	0.913	2.048	2.591
		0.4	1.062	1.164	1.901	2.082	0.668	0.731	0.951	1.042	0.477	0.523	0.763	0.836	2.129	2.332
		0.6	1.143	1.022	1.901	1.700	0.669	0.598	0.952	0.851	0.479	0.429	0.844	0.755	2.285	2.044
		0.8	1.373	0.868	1.902	1.203	0.673	0.426	0.955	0.604	0.487	0.308	1.062	0.672	2.715	1.717
		1.0	0.000	0.712	0.000	0.200	0.000	0.186	0.000	0.196	0.000	0.182	0.000	0.590	0.000	1.349

Table 1. Equivalent Length Factors for Various End Conditions
$[K_1 = l_{1(eff)}/L_T; K_2 = l_{2(eff)}/L_T]$

END CONDITION (BOTTOM - TOP)			(1) PIN - PIN		(2) FIX - FREE		(3) FIX - PIN		(4)FIX -SLIDER		(5) FIX - FIX		(6) PIN - FIX		(7) PIN-SLIDER	
I1 / I2	ℓ2 / LT	P2 / PT	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2
0.5	0.3	0.0	0.965	1.365	1.712	2.421	0.633	0.895	0.866	1.224	0.455	0.643	0.657	0.929	1.986	2.808
		0.2	1.013	1.281	1.712	2.166	0.644	0.814	0.874	1.106	0.470	0.595	0.698	0.883	2.111	2.670
		0.4	1.090	1.194	1.719	1.883	0.664	0.727	0.890	0.975	0.497	0.544	0.764	0.837	2.315	2.536
		0.6	1.233	1.103	1.733	1.550	0.707	0.632	0.927	0.829	0.553	0.494	0.883	0.790	2.682	2.399
		0.8	1.595	1.009	1.783	1.127	0.847	0.536	1.064	0.673	0.706	0.446	1.172	0.741	3.569	2.257
		1.0	0.000	0.913	0.000	0.600	0.000	0.454	0.000	0.556	0.000	0.403	0.000	0.691	0.000	2.126
0.5	0.5	0.0	0.878	1.242	1.562	2.208	0.626	0.886	0.832	1.177	0.437	0.618	0.580	0.820	1.949	2.757
		0.2	0.923	1.168	1.570	1.985	0.651	0.823	0.873	1.104	0.460	0.582	0.617	0.780	2.099	2.655
		0.4	0.995	1.090	1.612	1.765	0.691	0.757	0.945	1.035	0.496	0.544	0.675	0.740	2.385	2.612
		0.6	1.128	1.009	1.703	1.523	0.768	0.687	1.084	0.969	0.563	0.503	0.781	0.698	2.831	2.532
		0.8	1.460	0.923	1.974	1.248	0.969	0.613	1.434	0.907	0.729	0.461	1.038	0.656	3.908	2.472
		1.0	0.000	0.833	0.000	1.000	0.000	0.536	0.000	0.857	0.000	0.416	0.000	0.615	0.000	2.412
0.5	0.7	0.0	0.766	1.083	1.452	2.053	0.565	0.799	0.829	1.172	0.387	0.547	0.539	0.762	1.775	2.510
		0.2	0.808	1.022	1.520	1.923	0.587	0.742	0.905	1.144	0.413	0.522	0.586	0.742	1.969	2.491
		0.4	0.877	0.961	1.639	1.795	0.623	0.682	1.023	1.120	0.455	0.498	0.660	0.723	2.262	2.478
		0.6	1.006	0.899	1.855	1.659	0.691	0.618	1.229	1.099	0.533	0.477	0.789	0.706	2.754	2.464
		0.8	1.328	0.840	2.425	1.534	0.871	0.551	1.707	1.080	0.727	0.460	1.093	0.691	3.873	2.450
		1.0	0.000	0.785	0.000	1.400	0.000	0.488	0.000	1.057	0.000	0.446	0.000	0.675	0.000	2.422
0.5	0.9	0.0	0.710	1.004	1.416	2.002	0.498	0.704	0.772	1.092	0.379	0.536	0.529	0.748	1.572	2.223
		0.2	0.777	0.983	1.553	1.964	0.539	0.682	0.861	1.089	0.421	0.533	0.590	0.746	1.736	2.196
		0.4	0.879	0.964	1.763	1.931	0.603	0.661	0.993	1.088	0.484	0.530	0.680	0.745	2.019	2.211
		0.6	1.056	0.945	2.117	1.893	0.716	0.641	1.215	1.087	0.590	0.528	0.831	0.743	2.482	2.220
		0.8	1.467	0.928	2.933	1.855	0.984	0.622	1.716	1.085	0.831	0.525	1.173	0.742	3.509	2.219
		1.0	0.000	0.909	0.000	1.800	0.000	0.605	0.000	1.081	0.000	0.523	0.000	0.739	0.000	2.190
0.6	0.1	0.0	0.999	1.289	1.921	2.480	0.674	0.870	0.961	1.240	0.481	0.621	0.697	0.900	2.000	2.581
		0.2	1.023	1.181	1.921	2.218	0.674	0.778	0.961	1.110	0.482	0.556	0.722	0.834	2.049	2.365
		0.4	1.063	1.063	1.921	1.921	0.675	0.675	0.961	0.961	0.483	0.483	0.764	0.764	2.129	2.129
		0.6	1.144	0.934	1.921	1.569	0.677	0.553	0.963	0.786	0.486	0.396	0.846	0.691	2.286	1.867
		0.8	1.376	0.795	1.923	1.110	0.683	0.394	0.966	0.558	0.497	0.287	1.066	0.616	2.717	1.569
		1.0	0.000	0.654	0.000	0.200	0.000	0.184	0.000	0.195	0.000	0.179	0.000	0.543	0.000	1.234
0.6	0.3	0.0	0.973	1.256	1.774	2.290	0.651	0.840	0.897	1.158	0.467	0.603	0.665	0.858	1.992	2.572
		0.2	1.021	1.179	1.774	2.048	0.663	0.766	0.907	1.048	0.484	0.559	0.708	0.817	2.115	2.442
		0.4	1.100	1.100	1.785	1.785	0.686	0.686	0.927	0.927	0.514	0.514	0.776	0.776	2.323	2.323
		0.6	1.248	1.019	1.803	1.472	0.735	0.600	0.973	0.794	0.574	0.468	0.899	0.734	2.691	2.197
		0.8	1.620	0.935	1.867	1.078	0.889	0.513	1.135	0.655	0.737	0.425	1.197	0.691	3.590	2.073
		1.0	0.000	0.850	0.000	0.600	0.000	0.438	0.000	0.549	0.000	0.386	0.000	0.648	0.000	1.953
0.6	0.5	0.0	0.903	1.166	1.654	2.135	0.644	0.831	0.877	1.132	0.450	0.581	0.604	0.780	1.967	2.539
		0.2	0.950	1.097	1.678	1.937	0.670	0.774	0.922	1.064	0.475	0.548	0.645	0.745	2.118	2.445
		0.4	1.027	1.027	1.726	1.726	0.713	0.713	1.001	1.001	0.513	0.513	0.709	0.709	2.411	2.411
		0.6	1.167	0.953	1.837	1.500	0.795	0.649	1.153	0.941	0.584	0.476	0.822	0.671	2.864	2.338
		0.8	1.522	0.879	2.151	1.242	1.007	0.581	1.531	0.884	0.759	0.438	1.100	0.635	3.958	2.285
		1.0	0.000	0.796	0.000	1.000	0.000	0.512	0.000	0.836	0.000	0.398	0.000	0.599	0.000	2.238

Table 1. Equivalent Length Factors for Various End Conditions
$[K_1 = l_{1(eff)}/L_T; K_2 = l_{2(eff)}/L_T]$

END CONDITION (BOTTOM - TOP)			(1) PIN - PIN		(2) FIX - FREE		(3) FIX - PIN		(4)FIX -SLIDER		(5) FIX - FIX		(6) PIN - FIX		(7) PIN-SLIDER	
I1 / I2	ℓ2 / LT	P2 / PT	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2
0.6	0.7	0.0	0.817	1.055	1.578	2.037	0.592	0.765	0.873	1.128	0.413	0.534	0.578	0.746	1.860	2.401
		0.2	0.864	0.998	1.656	1.913	0.617	0.712	0.954	1.101	0.443	0.511	0.630	0.728	2.026	2.339
		0.4	0.942	0.942	1.793	1.793	0.657	0.657	1.083	1.083	0.490	0.490	0.711	0.711	2.350	2.350
		0.6	1.087	0.887	2.027	1.656	0.734	0.599	1.302	1.063	0.577	0.471	0.851	0.695	2.887	2.358
		0.8	1.441	0.832	2.660	1.536	0.935	0.540	1.804	1.041	0.789	0.455	1.180	0.682	4.058	2.343
		1.0	0.000	0.782	0.000	1.400	0.000	0.486	0.000	1.024	0.000	0.442	0.000	0.667	0.000	2.278
0.6	0.9	0.0	0.776	1.002	1.560	2.014	0.544	0.702	0.825	1.065	0.407	0.525	0.568	0.734	1.663	2.147
		0.2	0.851	0.982	1.704	1.968	0.590	0.681	0.919	1.061	0.453	0.523	0.634	0.733	1.850	2.136
		0.4	0.963	0.963	1.934	1.934	0.660	0.660	1.062	1.062	0.521	0.521	0.731	0.731	2.148	2.148
		0.6	1.156	0.944	2.330	1.903	0.784	0.640	1.298	1.060	0.636	0.519	0.895	0.731	2.626	2.144
		0.8	1.605	0.926	3.227	1.863	1.077	0.622	1.833	1.059	0.896	0.517	1.263	0.729	3.711	2.143
		1.0	0.000	0.909	0.000	1.800	0.000	0.605	0.000	1.055	0.000	0.515	0.000	0.726	0.000	2.127
0.7	0.1	0.0	0.999	1.194	1.941	2.320	0.680	0.813	0.971	1.160	0.486	0.581	0.698	0.834	2.000	2.390
		0.2	1.023	1.094	1.941	2.075	0.681	0.728	0.971	1.038	0.487	0.521	0.723	0.773	2.049	2.190
		0.4	1.064	0.985	1.941	1.797	0.682	0.632	0.972	0.900	0.488	0.452	0.765	0.709	2.130	1.972
		0.6	1.145	0.866	1.942	1.468	0.684	0.517	0.973	0.736	0.492	0.372	0.848	0.641	2.287	1.729
		0.8	1.380	0.737	1.944	1.039	0.692	0.370	0.978	0.523	0.507	0.271	1.071	0.572	2.719	1.453
		1.0	0.000	0.608	0.000	0.200	0.000	0.182	0.000	0.194	0.000	0.176	0.000	0.506	0.000	1.145
0.7	0.3	0.0	0.979	1.171	1.835	2.194	0.665	0.795	0.925	1.106	0.477	0.570	0.673	0.804	1.988	2.376
		0.2	1.029	1.100	1.834	1.961	0.679	0.726	0.938	1.003	0.496	0.530	0.718	0.767	2.122	2.268
		0.4	1.112	1.029	1.847	1.709	0.704	0.652	0.962	0.891	0.528	0.488	0.789	0.730	2.333	2.160
		0.6	1.264	0.956	1.872	1.415	0.758	0.573	1.016	0.768	0.592	0.447	0.916	0.692	2.703	2.043
		0.8	1.644	0.879	1.951	1.043	0.924	0.494	1.199	0.641	0.763	0.408	1.224	0.654	3.611	1.930
		1.0	0.000	0.803	0.000	0.600	0.000	0.424	0.000	0.541	0.000	0.371	0.000	0.615	0.000	1.820
0.7	0.5	0.0	0.926	1.107	1.743	2.083	0.659	0.788	0.911	1.089	0.463	0.553	0.629	0.752	1.956	2.338
		0.2	0.977	1.045	1.773	1.895	0.687	0.734	0.964	1.030	0.488	0.522	0.673	0.719	2.142	2.290
		0.4	1.059	0.980	1.836	1.700	0.733	0.678	1.049	0.971	0.529	0.490	0.741	0.686	2.428	2.248
		0.6	1.208	0.913	1.958	1.480	0.819	0.619	1.210	0.915	0.604	0.457	0.864	0.653	2.901	2.193
		0.8	1.584	0.846	2.312	1.236	1.042	0.557	1.615	0.863	0.789	0.422	1.159	0.620	4.018	2.148
		1.0	0.000	0.772	0.000	1.000	0.000	0.493	0.000	0.820	0.000	0.386	0.000	0.587	0.000	2.105
0.7	0.7	0.0	0.866	1.036	1.692	2.023	0.620	0.741	0.913	1.091	0.438	0.523	0.612	0.732	1.890	2.259
		0.2	0.918	0.982	1.785	1.908	0.647	0.692	0.998	1.066	0.470	0.502	0.669	0.715	2.086	2.230
		0.4	1.004	0.929	1.926	1.784	0.692	0.641	1.134	1.050	0.522	0.483	0.756	0.700	2.387	2.210
		0.6	1.161	0.878	2.205	1.667	0.776	0.587	1.363	1.030	0.617	0.466	0.907	0.685	2.949	2.229
		0.8	1.550	0.829	2.880	1.539	0.997	0.533	1.895	1.013	0.845	0.451	1.257	0.672	4.153	2.220
		1.0	0.000	0.780	0.000	1.400	0.000	0.484	0.000	0.996	0.000	0.439	0.000	0.660	0.000	2.169
0.7	0.9	0.0	0.839	1.003	1.677	2.005	0.587	0.701	0.873	1.044	0.433	0.517	0.604	0.723	1.779	2.127
		0.2	0.918	0.982	1.840	1.968	0.636	0.680	0.973	1.040	0.482	0.515	0.675	0.721	1.955	2.090
		0.4	1.040	0.963	2.100	1.944	0.712	0.660	1.125	1.042	0.555	0.513	0.778	0.720	2.267	2.099
		0.6	1.249	0.944	2.523	1.907	0.847	0.640	1.376	1.040	0.677	0.512	0.951	0.719	2.812	2.126
		0.8	1.740	0.930	3.475	1.857	1.164	0.622	1.944	1.039	0.954	0.510	1.343	0.718	3.976	2.125
		1.0	0.000	0.909	0.000	1.800	0.000	0.605	0.000	1.035	0.000	0.509	0.000	0.716	0.000	2.081

Table 1. Equivalent Length Factors for Various End Conditions

$[K_1 = l_{1(eff)}/L_T; K_2 = l_{2(eff)}/L_T]$

END CONDITION (BOTTOM - TOP)			(1) PIN - PIN		(2) FIX - FREE		(3) FIX - PIN		(4)FIX -SLIDER		(5) FIX - FIX		(6) PIN - FIX		(7) PIN-SLIDER	
I1 / I2	ℓ2 / LT	P2 / PT	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2
0.8	0.1	0.0	0.999	1.117	1.961	2.192	0.687	0.768	0.980	1.096	0.491	0.549	0.698	0.781	2.000	2.236
		0.2	1.023	1.023	1.961	1.961	0.688	0.688	0.981	0.981	0.492	0.492	0.724	0.724	2.049	2.049
		0.4	1.064	0.922	1.961	1.699	0.689	0.597	0.982	0.850	0.494	0.428	0.766	0.664	2.130	1.845
		0.6	1.147	0.811	1.962	1.388	0.692	0.489	0.984	0.696	0.498	0.352	0.850	0.601	2.288	1.618
		0.8	1.383	0.692	1.965	0.982	0.702	0.351	0.990	0.495	0.518	0.259	1.075	0.538	2.721	1.360
		1.0	0.000	0.572	0.000	0.200	0.000	0.180	0.000	0.193	0.000	0.173	0.000	0.477	0.000	1.072
0.8	0.3	0.0	0.987	1.103	1.890	2.113	0.678	0.758	0.953	1.065	0.486	0.543	0.681	0.762	2.004	2.240
		0.2	1.038	1.038	1.896	1.896	0.693	0.693	0.967	0.967	0.506	0.506	0.728	0.728	2.128	2.128
		0.4	1.122	0.972	1.911	1.655	0.721	0.624	0.995	0.862	0.540	0.467	0.802	0.694	2.338	2.025
		0.6	1.279	0.904	1.940	1.372	0.779	0.551	1.056	0.747	0.607	0.429	0.933	0.660	2.721	1.924
		0.8	1.671	0.835	2.035	1.018	0.954	0.477	1.258	0.629	0.786	0.393	1.250	0.625	3.619	1.810
		1.0	0.000	0.765	0.000	0.600	0.000	0.412	0.000	0.536	0.000	0.359	0.000	0.589	0.000	1.713
0.8	0.5	0.0	0.950	1.062	1.836	2.053	0.673	0.752	0.947	1.059	0.475	0.531	0.653	0.730	1.966	2.199
		0.2	1.005	1.005	1.868	1.868	0.703	0.703	1.002	1.002	0.502	0.502	0.700	0.700	2.165	2.165
		0.4	1.092	0.945	1.933	1.674	0.751	0.650	1.092	0.946	0.546	0.473	0.773	0.669	2.442	2.115
		0.6	1.250	0.884	2.078	1.469	0.842	0.595	1.262	0.893	0.624	0.442	0.903	0.639	2.930	2.072
		0.8	1.645	0.822	2.457	1.228	1.075	0.538	1.689	0.845	0.819	0.410	1.219	0.609	4.083	2.042
		1.0	0.000	0.753	0.000	1.000	0.000	0.478	0.000	0.803	0.000	0.376	0.000	0.579	0.000	2.000
0.8	0.7	0.0	0.913	1.021	1.811	2.025	0.647	0.723	0.943	1.054	0.460	0.514	0.644	0.720	1.936	2.164
		0.2	0.970	0.970	1.895	1.895	0.677	0.677	1.035	1.035	0.495	0.495	0.705	0.705	2.133	2.133
		0.4	1.063	0.920	2.067	1.790	0.726	0.628	1.181	1.023	0.551	0.477	0.797	0.691	2.494	2.160
		0.6	1.234	0.872	2.364	1.671	0.818	0.578	1.424	1.007	0.652	0.461	0.957	0.677	2.992	2.115
		0.8	1.651	0.826	3.058	1.529	1.056	0.528	1.975	0.988	0.895	0.447	1.329	0.664	4.198	2.099
		1.0	0.000	0.778	0.000	1.400	0.000	0.483	0.000	0.973	0.000	0.436	0.000	0.653	0.000	2.084
0.8	0.9	0.0	0.895	1.001	1.804	2.017	0.626	0.700	0.917	1.026	0.456	0.510	0.638	0.713	1.839	2.056
		0.2	0.981	0.981	1.971	1.971	0.679	0.679	1.024	1.024	0.509	0.509	0.713	0.713	2.053	2.053
		0.4	1.111	0.962	2.220	1.923	0.761	0.659	1.186	1.027	0.586	0.507	0.821	0.711	2.406	2.084
		0.6	1.335	0.944	2.705	1.913	0.905	0.640	1.447	1.023	0.716	0.506	1.004	0.710	2.907	2.056
		0.8	1.862	0.931	3.751	1.876	1.244	0.622	2.045	1.023	1.010	0.505	1.422	0.711	4.110	2.055
		1.0	0.000	0.909	0.000	1.800	0.000	0.605	0.000	1.019	0.000	0.503	0.000	0.708	0.000	2.047
0.9	0.1	0.0	1.000	1.054	1.980	2.087	0.693	0.731	0.990	1.044	0.495	0.522	0.699	0.737	2.001	2.109
		0.2	1.024	0.965	1.981	1.867	0.694	0.654	0.991	0.934	0.497	0.468	0.724	0.683	2.049	1.932
		0.4	1.065	0.870	1.981	1.618	0.696	0.568	0.992	0.810	0.499	0.408	0.767	0.627	2.131	1.740
		0.6	1.148	0.766	1.982	1.322	0.699	0.466	0.994	0.663	0.505	0.336	0.852	0.568	2.289	1.526
		0.8	1.386	0.654	1.986	0.936	0.711	0.335	1.003	0.473	0.528	0.249	1.080	0.509	2.722	1.283
		1.0	0.000	0.542	0.000	0.200	0.000	0.178	0.000	0.193	0.000	0.171	0.000	0.452	0.000	1.013
0.9	0.3	0.0	0.994	1.048	1.954	2.059	0.689	0.727	0.978	1.031	0.493	0.520	0.690	0.728	2.002	2.110
		0.2	1.047	0.987	1.954	1.842	0.706	0.665	0.994	0.937	0.514	0.485	0.739	0.697	2.132	2.010
		0.4	1.134	0.926	1.971	1.610	0.735	0.600	1.025	0.837	0.550	0.449	0.815	0.665	2.348	1.918
		0.6	1.294	0.863	2.007	1.338	0.797	0.531	1.093	0.729	0.621	0.414	0.951	0.634	2.726	1.817
		0.8	1.696	0.799	2.115	0.997	0.981	0.463	1.311	0.618	0.807	0.380	1.276	0.602	3.654	1.722
		1.0	0.000	0.735	0.000	0.600	0.000	0.402	0.000	0.529	0.000	0.349	0.000	0.569	0.000	1.625

Table 1. Equivalent Length Factors for Various End Conditions
$[K_1 = l_{1(eff)}/L_T; K_2 = l_{2(eff)}/L_T]$

END CONDITION (BOTTOM - TOP)			(1) PIN - PIN		(2) FIX - FREE		(3) FIX - PIN		(4)FIX -SLIDER		(5) FIX - FIX		(6) PIN - FIX		(7) PIN-SLIDER	
I1	ℓ2	P2														
--	--	--	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2
I2	LT	PT														
0.9	0.5	0.0	0.976	1.028	1.922	2.026	0.686	0.724	0.974	1.027	0.488	0.514	0.677	0.713	1.991	2.098
		0.2	1.033	0.974	1.959	1.847	0.718	0.677	1.035	0.975	0.516	0.487	0.727	0.685	2.193	2.067
		0.4	1.125	0.918	2.037	1.663	0.769	0.627	1.130	0.922	0.562	0.459	0.804	0.656	2.471	2.018
		0.6	1.290	0.860	2.196	1.464	0.864	0.576	1.310	0.873	0.645	0.430	0.941	0.627	2.965	1.977
		0.8	1.699	0.801	2.618	1.234	1.107	0.522	1.756	0.828	0.849	0.400	1.271	0.599	4.110	1.938
		1.0	0.000	0.738	0.000	1.000	0.000	0.466	0.000	0.788	0.000	0.370	0.000	0.572	0.000	1.913
0.9	0.7	0.0	0.957	1.009	1.906	2.009	0.673	0.710	0.972	1.025	0.481	0.507	0.673	0.709	1.963	2.069
		0.2	1.020	0.961	2.012	1.897	0.706	0.665	1.069	1.008	0.518	0.489	0.737	0.695	2.194	2.069
		0.4	1.119	0.914	2.193	1.790	0.759	0.619	1.221	0.997	0.578	0.472	0.835	0.681	2.549	2.081
		0.6	1.300	0.867	2.514	1.676	0.858	0.572	1.469	0.979	0.686	0.457	1.003	0.669	3.060	2.040
		0.8	1.747	0.823	3.290	1.551	1.113	0.525	2.048	0.966	0.943	0.445	1.394	0.657	4.308	2.031
		1.0	0.000	0.777	0.000	1.400	0.000	0.482	0.000	0.953	0.000	0.433	0.000	0.646	0.000	2.015
0.9	0.9	0.0	0.950	1.001	1.902	2.005	0.664	0.700	0.967	1.019	0.479	0.505	0.669	0.706	1.983	2.091
		0.2	1.041	0.981	2.095	1.975	0.720	0.679	1.071	1.010	0.534	0.503	0.748	0.705	2.162	2.039
		0.4	1.179	0.962	2.401	1.960	0.807	0.659	1.242	1.015	0.615	0.502	0.862	0.704	2.522	2.059
		0.6	1.416	0.944	2.843	1.895	0.960	0.640	1.526	1.017	0.751	0.501	1.056	0.704	3.135	2.090
		0.8	1.965	0.926	3.909	1.843	1.319	0.622	2.157	1.017	1.060	0.500	1.492	0.703	4.433	2.090
		1.0	0.000	0.909	0.000	1.800	0.000	0.605	0.000	1.007	0.000	0.498	0.000	0.701	0.000	2.020
1.0	0.1	0.0	1.000	1.000	2.001	2.001	0.699	0.699	1.000	1.000	0.500	0.500	0.699	0.699	2.001	2.001
		0.2	1.024	0.916	2.001	1.789	0.700	0.626	1.001	0.895	0.502	0.449	0.725	0.649	2.049	1.833
		0.4	1.066	0.826	2.001	1.550	0.702	0.544	1.002	0.776	0.504	0.391	0.769	0.595	2.131	1.651
		0.6	1.150	0.727	2.003	1.267	0.706	0.447	1.005	0.636	0.511	0.323	0.854	0.540	2.289	1.448
		0.8	1.390	0.622	2.007	0.897	0.720	0.322	1.015	0.454	0.538	0.241	1.084	0.485	2.724	1.218
		1.0	0.000	0.517	0.000	0.200	0.000	0.176	0.000	0.192	0.000	0.169	0.000	0.432	0.000	0.962
1.0	0.3	0.0	1.002	1.002	2.010	2.010	0.699	0.699	1.002	1.002	0.500	0.500	0.700	0.700	2.010	2.010
		0.2	1.056	0.944	2.011	1.799	0.717	0.641	1.020	0.912	0.522	0.467	0.750	0.671	2.138	1.913
		0.4	1.145	0.887	2.031	1.573	0.748	0.579	1.054	0.816	0.560	0.434	0.828	0.642	2.354	1.824
		0.6	1.310	0.828	2.069	1.309	0.813	0.514	1.127	0.713	0.634	0.401	0.967	0.612	2.744	1.735
		0.8	1.721	0.770	2.196	0.982	1.005	0.450	1.360	0.608	0.826	0.370	1.301	0.582	3.662	1.638
		1.0	0.000	0.711	0.000	0.600	0.000	0.393	0.000	0.522	0.000	0.340	0.000	0.552	0.000	1.551
1.0	0.5	0.0	1.000	1.000	2.013	2.013	0.699	0.699	1.000	1.000	0.500	0.500	0.699	0.699	2.013	2.013
		0.2	1.061	0.949	2.047	1.831	0.732	0.655	1.065	0.952	0.530	0.475	0.752	0.673	2.202	1.969
		0.4	1.157	0.896	2.125	1.646	0.785	0.608	1.164	0.901	0.578	0.448	0.833	0.645	2.520	1.952
		0.6	1.330	0.841	2.282	1.443	0.885	0.560	1.352	0.855	0.665	0.421	0.978	0.619	3.012	1.905
		0.8	1.765	0.789	2.735	1.223	1.138	0.509	1.816	0.812	0.879	0.393	1.323	0.591	4.180	1.869
		1.0	0.000	0.728	0.000	1.000	0.000	0.457	0.000	0.774	0.000	0.364	0.000	0.565	0.000	1.841
1.0	0.7	0.0	1.000	1.000	2.013	2.013	0.699	0.699	1.000	1.000	0.500	0.500	0.699	0.699	2.013	2.013
		0.2	1.067	0.955	2.110	1.888	0.734	0.657	1.103	0.987	0.540	0.483	0.767	0.686	2.228	1.993
		0.4	1.173	0.909	2.288	1.772	0.791	0.613	1.261	0.977	0.603	0.467	0.870	0.674	2.577	1.996
		0.6	1.364	0.863	2.609	1.650	0.897	0.567	1.517	0.959	0.716	0.453	1.046	0.661	3.146	1.990
		0.8	1.832	0.819	3.455	1.545	1.168	0.522	2.119	0.947	0.987	0.441	1.454	0.650	4.432	1.982
		1.0	0.000	0.776	0.000	1.400	0.000	0.481	0.000	0.936	0.000	0.430	0.000	0.640	0.000	1.960

Table 1. Equivalent Length Factors for Various End Conditions

$[K_1 = l_{1(eff)}/L_T; K_2 = l_{2(eff)}/L_T]$

END CONDITION (BOTTOM - TOP)			(1) PIN - PIN		(2) FIX - FREE		(3) FIX - PIN		(4)FIX -SLIDER		(5) FIX - FIX		(6) PIN - FIX		(7) PIN-SLIDER	
I1	ℓ2	P2														
--	--	--	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2	K1	K2
I2	LT	PT														
1.0	0.9	0.0	1.000	1.000	2.013	2.013	0.699	0.699	1.000	1.000	0.500	0.500	0.699	0.699	2.083	2.083
		0.2	1.097	0.981	2.198	1.966	0.759	0.679	1.117	0.999	0.558	0.499	0.781	0.699	2.236	2.000
		0.4	1.242	0.962	2.508	1.943	0.851	0.659	1.292	1.001	0.642	0.498	0.901	0.698	2.592	2.008
		0.6	1.493	0.944	3.089	1.954	1.012	0.640	1.579	0.998	0.785	0.496	1.104	0.698	3.294	2.083
		0.8	2.086	0.933	4.262	1.906	1.391	0.622	2.231	0.998	1.108	0.495	1.560	0.698	4.659	2.083
		1.0	0.000	0.909	0.000	1.800	0.000	0.605	0.000	0.997	0.000	0.495	0.000	0.695	0.000	1.998