Computer-Aided Design of Stub-Girder System

LEON RU-LIANG WANG AND JOHN A. GOTSCHALL

The stub-girder system for high-rise buildings is a relatively new and innovative concept in structural floor framing. The system, developed by Dr. Joseph Colaco[2] in 1972, has gained considerable attention in recent years as a viable alternative to the traditional method of structural floor framing.

The stub-girder system, in an effort to minimize overall space requirements while maintaining an independence between the various building systems, combines space for the mechanical ductwork in the design of the structural system. The result is an increase in the efficiency of both the structural frame and the mechanical layout, combined with a reduction in both space and material required.

To date, several major buildings have been designed using the stub-girder system. Yet, the stub-girder system has been unable to gain wide-spread application in the construction industry due to the inability of average design firms to carry out the necessary computer analyses. Based on the report by the junior author,[5] the purpose of this paper is to provide smaller design firms with the opportunity to design and analyze the stub-girder system without

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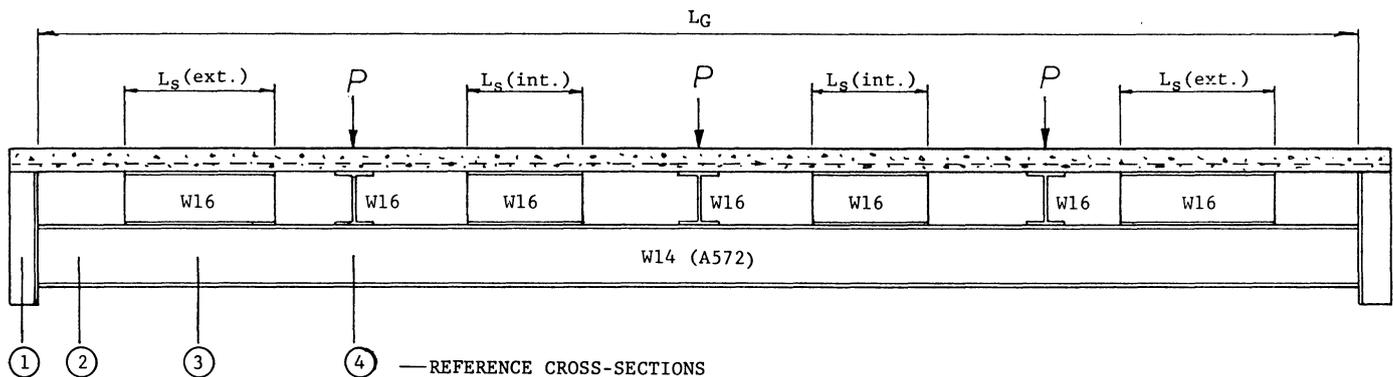
the need for large computer facilities. Computer programs are developed which can be executed in mini-computers without a loss in accuracy.

In addition, influential parameters are investigated to best determine the governing factors in the behavior of the stub-girder system. With this knowledge, a designer will be capable of discerning the major design criteria. The results will provide guidelines toward an initial design.

ANALYSIS MODELS

Following Colaco's paper,[2] three structural analysis models have been used to analyze the stub-girder system. This section presents a general description of each.

Non-Prismatic Beam Model (Fig. 1)—The first and simplest model to analyze the stub-girder system is the non-prismatic beam, or variable beam model. Simple beam theory is applied in the analysis so that only pure flexural stresses are assumed.

The method of numerical integration[10] is used for the analysis of the stub-girder system as a variable cross section beam. In the numerical procedure, distributed and concentrated external loads are replaced by equivalent concentrated loads spaced equally along the beam at specified pivot points. Through several iterations of numerical integration, the resulting state of stresses in the stub-girder can be determined.
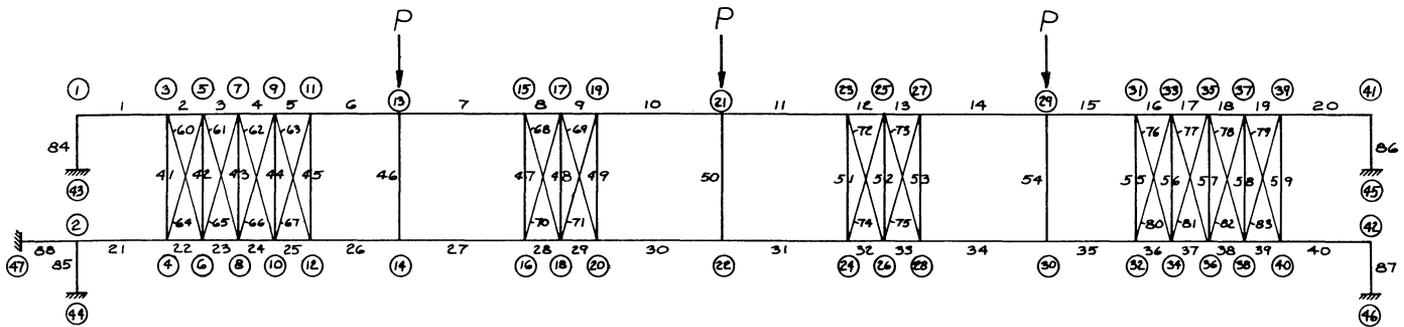

Fig. 1. Variable beam model

Fig. 2. Vierendeel truss model

Vierendeel Truss Model (Fig. 2)—The second model used to analyze the stub-girder system is the Vierendeel truss model. The stub-girder system is especially adaptable to such a model due to its structural nature. In the stub-girder system, the two major structural elements are the concrete slab and the high-strength girder. The concrete slab acts as the compression element and the steel girder acts as the tension element. The shop-welded stub pieces act as connection elements between the top and bottom chords transmitting moments, thrusts, and shears between the two.

In the Vierendeel model, a series of horizontal top and bottom chord elements represent the concrete slab and high-strength girder, respectively. Vertical elements model the stub pieces and spandrel beams with diagonal members to account for shear resistance in the stub pieces.

In the Vierendeel truss model, the stub-girder system is analyzed by the stiffness method for plane frames.[8] The system generates a "system" matrix which relates the external loads to the joint displacements directly.

Once displacements have been defined, it is a simple matter to solve for the member displacements and forces.

Finite Element Model—The third model used to represent the stub-girder system is a finite element model. A general finite element program called SAP IV[12] is used for the analysis. Two-dimensional quadrilateral elements comprise the major portion of the structural model. Each plane stress element has two degrees of freedom (translation in the X and Y directions) per node. All moments, shears, thrusts, and displacements are calculated in terms of the resultant translations of the nodes. Two-dimensional truss bar elements model the flanges of the stub-girder elements. They account for the pure tension-compression function of the flanges.

Comparison of Models—In the comparison of the three analysis models, the non-prismatic beam model provides a rather rough estimate of the stress state in the stub-girder system. In relation to the Vierendeel truss model and the finite element model, the non-prismatic beam model is unconservative. The program consistently predicts smaller

stresses and deflections along the stub-girder length. The discrepancy in the results is due to the simple beam theory approach to the analysis. The variable beam results are highly dependent upon only the primary moment, while neglecting axial effects and secondary moments due to member distortions. The resulting stress analysis for a typical stub-girder member closely follows the shape of the moment diagram for the variable beam model. The only deviations in the distribution occur along the beam where the cross-sectional properties change dramatically. Even this deviation has been found to be minimal, because the major changes in cross section occur near the transformed neutral axis (where the stub-pieces occur), and thus, have a minimal effect upon the stress resultants. For a comparison between the non-prismatic beam results and the Vierendeel truss results, see Fig. 3. Note the minimal effect of section changes on the results of the variable beam analysis.

The Vierendeel truss model provides a step up in accuracy over the non-prismatic beam model. The stiffness method adopted by the Vierendeel truss program provides the user with a great deal of flexibility in modeling the stub-girder system. Local effects can be accurately recorded by choosing adequately small members. The accuracy of the Vierendeel truss program is due, in part, to the inclusion of axial forces as well as moments which occur along the stub-girder. In the variable beam program, the stub-girder is considered as a monolithic member with tensile stresses in the bottom fiber and equivalent compressive stresses in the top fiber. The effects of axial deformation are ignored in the variable beam theory. In reality, the axial forces vary dramatically both along the beam and at every fiber in a cross section. Due to local conditions caused by the occurrence of stub pieces, the axial forces become an important factor in the analysis of the stub-girder. The very nature of the stub-girder system as a Vierendeel dictates that axial forces play an important role in the stress analysis. The Vierendeel truss program accounts for the axial forces which occur in each member. The result is an analysis of higher accuracy and dependability than the non-prismatic beam program.
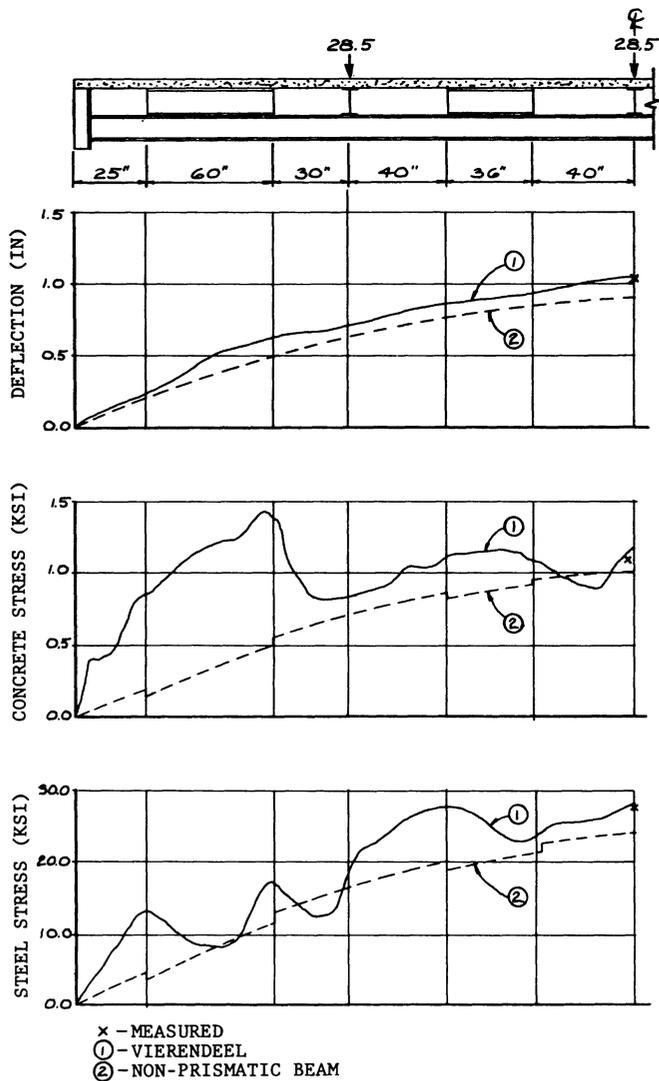
26

Fig. 3. Comparison of Vierendeel truss and variable beam
results

Fig. 4. Comparison of Vierendeel truss and other models

The Vierendeel truss program also has the capability of determining stresses within the stub-girder system. Stresses in the stub pieces or in the spandrel beams can be determined in addition to concrete stresses and girder stresses. In comparison, the variable beam model is only capable of determining the stresses in the extreme top and bottom fibers of the stub-girder system. Determining the stub stresses is important in designing for required stiffeners. This capability provides another advantage of the Vierendeel program over the variable beam program.

Theoretically, the finite element model should provide the most accurate results. In the limiting case of finite element analysis, using minute elements, the exact stress state of a system can be determined. In more practical terms, it is impossible to obtain sufficiently accurate results without the need for exhorbitant computer time. When used as a
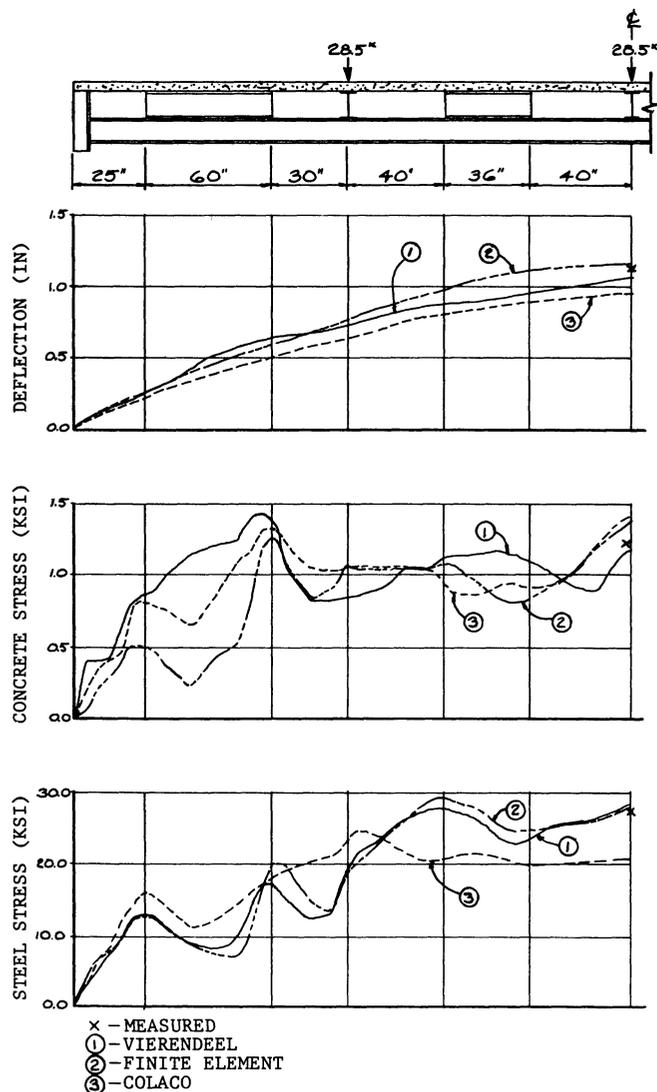
check upon the accuracy of the Vierendeel truss results, the finite element analysis displays the same general trends as the Vierendeel truss analysis. Note, in Fig. 4, the accuracy of the Vierendeel results when compared to both the SAP IV finite element model and Colaco's[2] finite element results. The discrepancies which exist between Colaco's results and the other two analyses is due, in part, to the possible differences in the methods chosen to model the stub pieces. The composite behavior between the concrete slab and the steel frame may also have been modelled differently, accounting for the discrepancies. In this respect, it can be concluded that though the specific method of modeling the stub-girder system directly affects the resulting accuracy, both the Vierendeel truss program and the finite element program are capable of providing accurate results, dependent upon the model chosen.

27

In summary, the non-prismatic beam program provides consistently unconservative results when compared to the more accurate Vierendeel truss program and finite element program. But the simplicity of the program, and its capabilities for use as a design tool, make the non-prismatic beam program a desirable resource. It is only necessary to apply a "correction" factor to the variable beam program to ensure conservative results. A set of correction factors which relates the variable beam results to the more accurate Vierendeel truss results is given in Table 1.

Note that computer program listings and extensive results for all models have been given in Ref. 5. Readers are referred to the original reference for details which will not be repeated in this paper due to limitation of space.

PARAMETRIC STUDIES

Using a set of practical girder sizes as prescribed by Colaco[3] for the stub-girder system, a number of parameters have been developed for study. In all practical applications of the stub-girder system, the optimum beam and girder sizes have remained constant. In each case, a W14 section beam and a W16 section girder have been used. Within this scope, several other parameters have been varied in an effort to observe their effect upon the stub-girder system. Table 2 provides results of the maximum steel and concrete stresses, as well as the maximum deflection for variously defined parameters using the Vierendeel truss program. The values represent maximum values along the girder and thus do not necessarily represent midspan values.

Stub-Girder Stiffness Ratio—Several studies have been made varying the relative stiffnesses between the stub pieces and the main girder. By increasing the size of the stub piece from a W16x26 to a W16x78 (the practical range of stub piece sizes for a typical floor bay), while maintaining a constant girder size of W14x48, a fairly wide range of stiffness ratios has been obtained. Results have consistently shown that the effect of varying stiffness is negligible. In comparing the maximum steel and concrete stresses over
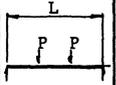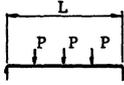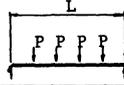
the range of stiffnesses chosen, differences of less than a percentage point have occurred. From these studies, it can be concluded that the size of the stub piece in relation to the girder size is not a governing factor in the behavior of the stub-girder system overall. It should be noted, however, that local stress concentrations which occur in the concrete or girder (especially at stub ends) are affected by the stiffness of the stub piece. This is especially true in the case where vertical web stiffeners are welded to the stub pieces. A high stub piece stiffness tends to transmit a higher percentage of the total load to the stub, reducing the concrete and girder stresses in the local region.

Stub-Girder Length Ratio—The next set of studies concern the relative lengths between the stub pieces and the girder span. The studies have been developed over the full range of practical girder sizes. The stub lengths have been increased in an orderly pattern from $L_S/L_G = 0.3$ to $L_S/L_G = 0.8$. The results consistently show that the increase in total stub length relative to the girder span acts to decrease the stresses and deflections throughout the stub-girder system. This result is consistent with common sense observations that by increasing the total stub length, the system stiffness is also increased.

It can be concluded that the total length of the stub pieces relative to the girder length is a major parameter affecting the behavior of the stub-girder system. The designer's problem is to choose a total stub length which provides the system with sufficient stiffness while maintaining adequate space for mechanical ducts. Above all, an economical solution must be determined in which a minimum amount of steel is required.
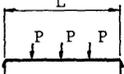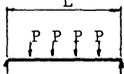
Stub Length Ratio—In addition to noting the effects of changes in the total stub length, comparative studies have been made by varying the relative length between the exterior stub pieces and the interior stub pieces. In each case, the lengths of the stub pieces have been varied in 1-ft increments between 3 ft and 6 ft. A constraint has been placed upon the system which dictates that the exterior stub pieces

Table 1. Correction Factors Relating Variable Beam Results to Vierendeel Results (Slab Thickness = 4 in.)

	L / P P	L / P P P					L / P P P P			
P (kips)	28.8	23.0	24.4	25.9	27.4	28.8	24.2	25.4	26.5	27.6
L (in.)	346	370	394	418	442	466	490	514	538	562
Girder Size	W14x30	W14x30	W14x34	W14x38	W14x43	W14x48	W14x53	W14x61	W14x68	W14x74
Steel Stress	1.28	1.19	1.22	1.23	1.21	1.17	1.14	1.14	1.13	1.12
Concrete Stress	1.55	1.30	1.33	1.35	1.35	1.36	1.49	1.47	1.46	1.44
Deflection	1.44	1.40	1.42	1.40	1.41	1.39	1.17	1.21	1.23	1.30

Vierendeel Result = (coeff.) × Variable Beam Results

28

Table 2. Maximum Stresses and Deflections of Stub-Girders under Various Parametric Conditions (Slab Thickness = 4 in.)

	$\overbrace{\underset{P\ P}{}}^{L}$	$\overbrace{\underset{P\ P\ P}{}}^{L}$					$\overbrace{\underset{P\ P\ P\ P}{}}^{L}$			
P (kips)	28.8	23.0	24.4	25.9	27.4	28.8	24.2	25.4	26.5	27.6
L (in.)	346	370	394	418	442	466	490	514	538	562
Girder Size	W14x30	W14x30	W14x34	W14x38	W14x43	W14x48	W14x53	W14x61	W14x68	W14x74
$L_{(ext.)}/L_{(int.)}$	Steel Stress (Max.), **ksi**									
3/3	28.38	30.85	30.73	31.84	31.41	31.31	29.17	28.32	28.21	30.60
4/3	24.11	28.78	28.65	29.52	29.06	29.05	27.67	26.87	26.88	28.94
5/3	24.10	28.78	28.65	28.50	29.08	28.99	27.67	26.86	26.83	28.94
6/3	22.18	28.77	28.62	29.48	29.06	28.69	27.61	26.81	26.77	28.87
4/4	24.00	28.14	28.21	29.40	29.12	29.17	26.99	26.28	26.36	28.67
5/4	24.00	28.14	28.19	29.37	29.08	29.10	26.92	26.18	26.23	28.53
6/4	22.13	27.60	27.52	28.41	28.04	27.70	26.45	25.82	25.72	27.78
5/5	24.77	27.60	27.51	28.40	28.02	27.93	27.98	27.46	27.81	30.42
6/5	22.65	27.60	27.52	28.40	28.02	27.66	26.74	26.28	26.63	29.19
6/6	21.89	26.45	26.46	27.41	27.11	26.79	26.89	26.44	26.82	29.39
	CONCRETE STRESS (MAX.), KSI									
3/3	1.10	1.13	1.19	1.28	1.37	1.47	1.47	1.61	1.75	1.90
4/3	0.99	1.04	1.10	1.19	1.27	1.40	1.41	1.56	1.70	1.84
5/3	0.99	1.04	1.10	1.19	1.27	1.37	1.41	1.55	1.69	1.84
6/3	0.91	0.99	1.07	1.16	1.25	1.37	1.38	1.51	1.65	1.79
4/4	0.95	1.03	1.10	1.19	1.27	1.40	1.45	1.58	1.71	1.85
5/4	0.96	1.04	1.10	1.19	1.26	1.37	1.45	1.58	1.71	1.85
6/4	0.88	0.99	1.07	1.16	1.25	1.36	1.41	1.54	1.67	1.80
5/5	0.96	1.04	1.08	1.17	1.23	1.29	2.08	2.17	2.24	2.39
6/5	0.88	0.91	0.97	1.07	1.16	1.28	2.02	2.12	2.19	2.34
6/6	0.86	0.91	0.97	1.07	1.16	1.28	2.02	2.11	2.18	2.33
	DEFLECTION (MAX.), IN.									
3/3	0.63	0.69	0.79	0.96	1.09	1.27	1.34	1.51	1.71	1.98
4/3	0.55	0.63	0.72	0.88	1.01	1.17	1.27	1.43	1.62	1.88
5/3	0.50	0.59	0.68	0.83	0.95	1.10	1.23	1.38	1.56	1.82
6/3	0.45	0.56	0.64	0.79	0.90	1.04	1.18	1.34	1.51	1.76
4/4	0.55	0.62	0.71	0.87	1.00	1.16	1.23	1.39	1.57	1.83
5/4	0.49	0.59	0.67	0.82	0.94	1.09	1.19	1.34	1.52	1.77
6/4	0.44	0.55	0.63	0.78	0.89	1.03	1.15	1.30	1.47	1.71
5/5	0.49	0.56	0.65	0.79	0.90	1.05	0.96	1.09	1.25	1.45
6/5	0.44	0.53	0.62	0.76	0.86	1.01	0.93	1.06	1.21	1.40
6/6	0.43	0.53	0.61	0.75	0.86	1.00	0.91	1.03	1.18	1.37

29

must be equal to or larger than the interior stub pieces at all times. Symmetry has also been assumed.

The results of the relative stub length studies show that minor changes do occur in the stub-girder system based upon the arrangement of the stubs. The basis for comparison involves using a roughly constant total stub length and noting the changes due to the different possible combinations of stub lengths. For instance, comparisons can be made between $L_{S(exterior)}/L_{S(interior)} = 5/3$ and $4/4$, $6/3$ and $5/4$, and $6/4$ and $5/5$. Though the results are not as consistent as the previous parametric results, several trends do seem to appear. For the smaller total stub lengths, a balanced stub ratio ($4/4$ and $5/4$) seems to provide lower stresses in the girder. In comparison, an unbalanced stub ratio favoring exterior stub lengths ($5/3$ and $6/4$) seems to reduce the concrete stresses and deflections. Noting that the maximum steel stress occurs at the interior stub piece, a longer interior stub piece will tend to transmit stresses over a larger distance, and thus, reduce stress concentrations. In comparison, the maximum concrete stress occurs at the exterior stub piece. A longer exterior stub piece will tend to reduce this stress concentration in the same manner as the steel stress was reduced. In the case of deflection, the moment diagram becomes the governing factor influencing the system's behavior. A stub length ratio which favors long exterior stubs will tend to reduce the moment diagram by shifting the weight away from the center of the span.

For higher ratios of total stub lengths to girder span, the trends become unpredictable. This may show that as the spaces between stubs become smaller, the effects caused by the ratio of stub length to girder span no longer dictate the resultant stress distribution.

DESIGN PROCEDURE

Description—The greatest application of the stub-girder system exists in structures with shear cores and bay dimensions of approximately 30 ft by 40 ft (Ref. 2). It is in such conditions that the stub-girder system provides maximum structural efficiency. In all cases the major components of the system are assumed to be: (1) a high strength steel (A572) W14 section girder with (2) A36 steel W16 section stub pieces and (3) A36 steel W16 section spandrel beams, followed by (4) a metal deck and (5) a lightweight concrete slab. These components have been found to provide optimum efficiency for the limited restrictions of the stub-girder system.

For a high-rise office building, standard design live loads exist which govern the initial design. Typically, live loads vary between 80 lbs/ft² and 100 lbs/ft² in an office building.[1] Once the loading has been determined, and a desired set of bay dimensions has been defined, the design of the spandrel beams can be accomplished in a straightforward manner. Usually the beam spacing is a designer's choice, though it must meet the requirements of the metal deck specifications. The beams are to be designed by taking into account continuity considerations. Full composite action

is assumed. Since the stub-girder system requires temporary shoring during construction, the full dead and live load is taken by the composite section. Both the non-prismatic beam program and the Vierendeel truss program are based upon this assumption. From the given loading conditions and an analysis based upon both continuity considerations and composite action considerations, a W14 section beam can be designed.

The next step in the design sequence is to design the stub pieces. It has been found that the size (weight) of the stub piece section has little effect upon the stress envelope in the stub-girder system. Any section which is able to resist the existing shears and thrusts is an adequate section. For simplicity in design and placement, the section chosen for the stub should be the same as the spandrel beam section. In comparison, the relative length of the stub pieces is a factor that affects the stresses in the system. The stub length is a function of the rigidity required and the space required for mechanical ducts. In an initial design, a designer must choose a length arbitrarily to best suit the building requirements. Changes can later be made in the stub lengths as part of the final design.

Next, given a span length, loading conditions, and the desired slab thickness (again a typical range exists which limits the practical depth of the concrete slab), an initial choice for the high strength girder can be made. For an initial design, any of the W14 sections given in Table 2 may be used. The true optimum section will be determined in the final design.

The complete system can then be analyzed using the variable beam program. The results will produce a maximum stress and maximum deflection. Using the steel correction factors for the given girder as previously discussed, an adjusted set of stresses can be developed which more closely resemble the true state of the system. The manner in which the stresses are calculated in the variable beam model is:

$$f_1 = \frac{M}{S_1} c_1 \qquad (1)$$

where c_1 is the steel correction factor from Table 1, f_1 is the maximum steel stress in trial 1, and S_1 is the transformed section modulus. Note that at the midspan, for the defined loading conditions and span length, moment M is a constant unaffected by changes in the section properties and can be expressed without correction as:

$$M = \frac{f_1 S_1}{c_1} = \frac{f_2 S_2}{c_2} = \text{constant} \qquad (2)$$

Thus, using the results of trial 1 and using the maximum allowable stress (f_{all}) for f_2, a new section modulus S_2 can be determined and analyzed. Then the procedure can be illustrated as:

$$S_2 = \frac{M}{f_{all}} c_2 = \frac{c_2}{c_1} \frac{f_1}{f_{all}} S_1 \qquad (3)$$

30

Without knowing S_2, c_2 cannot be determined exactly. But, by choosing an estimate of c_2 from the correction factor tables, and by using an iterative process, the true correction factor and the desired section modulus can be determined. Working backwards, the size of the girder which most closely produces the composite section modulus just obtained can be determined.

With this initial design, a more accurate analysis can then be made using the Vierendeel truss program. Any adjustments required in the overall stub-girder system can then be made. The Vierendeel truss program will locate where the stress concentrations occur. From these results, any requirements for web stiffeners or stub length changes can be determined.

Design Example—The design procedure is illustrated in the following example:

Given: Design load = 100 lbs/ft^2
Bay dimensions = 30 ft $\times$ 40 ft
Concrete slab = 4 in.
Beam spacing = 10 ft

Design a W14 section stub girder

Solution:

Assuming composite action and shored construction, the beam can first be designed using composite design procedures. For this example, it is determined that a W16x26 satisfies all the design criteria. Therefore, W16x26 is used for both the stub pieces and the beams.

A beam spacing of 10 ft over the 40 ft girder span determines the concentrated loading points on the stub-girder system. A uniform load of 100 lbs/ft^2 results in concentrated loads of $P = 28.8$ kips at each beam-girder intersection.

For the initial trial, assume a high-strength W14x30 girder with a dead load weight of 0.030 kips/ft. Using the variable beam program, stress and deflection results can be obtained roughly. According to the variable beam results, a maximum steel stress of 40.2 kips occurs at midspan. The next step is to apply a correction factor to this value. From Table 1, a steel correction factor can be found for a W14x30 girder with a three-point load. The steel factor is 1.19. Thus, the corrected steel stress is now 47.8 ksi. Midspan moment is:

$$M = \frac{f_1 S_1}{c_1} = \frac{47.8 \text{ ksi } (210 \text{ in.}^3)}{1.19} = 8435 \text{ kip-in.}$$

The maximum steel stress of 47.8 ksi is much higher than the allowable stress for high strength steel, $f_{all} = 33$ ksi. Thus, a new section must be chosen based upon the results of trial 1. From the constant moment previously determined, the new section can be determined as:

$$S_2 = \frac{M}{f_{all}} c_2 = \frac{8435 \text{ kip-in. } (1.2)}{33 \text{ ksi}} = 306 \text{ in.}^3$$

where c_2 has been estimated conservatively as 1.2. A W14x48 girder with a W16x26 beam and a 4-in. lightweight concrete slab and metal deck provide a midspan section modulus of 310 in.^{3} Using the actual correction factor for a W14x48 with a three-point load, the maximum moment resistance of the midspan section is:

$$M = \frac{f_{all} S_2}{c_{2act}} = \frac{33 \text{ ksi } (310 \text{ in.}^3)}{1.17} = 8743 \text{ ksi} > 8435 \text{ ksi}$$

A more detailed analysis may now be performed using the determined sections and an appropriate ratio of stub lengths as dictated by the mechanical duct requirements. The Vierendeel truss program is used for this purpose. Using, for example, a stub length ratio $(L_{S(exterior)}/L_{S(interior)}) = 5/3$ with a W14x48 high-strength girder and 4-in. concrete slab, the resulting maximum steel stress is 29.0 ksi, which is satisfactory. The deflection must then be checked such that it does not exceed $L/360$.

$$\text{Deflection (all.)} = \frac{(40 \text{ ft}) (12 \text{ ft/in.})}{360} = 1.33 \text{ in.}$$

$$\text{Deflection (act.)} = 1.1 \text{ in.} < 1.33 \text{ in.}$$

Finally, the concrete stresses must be checked and appropriate reinforcing must be designed.

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