Torsional Strength and Stiffness of Steel Structures

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The torsional strength and stiffness of steel structures might seem to many to be a rather forbidding topic, but it can be quite simple for a wide variety of common steel structures such as bridges, towers, laced structural members, hydraulic gates and others. In undergraduate courses in Strength of Materials, all engineering students learn the derivation and use of the torsion formulas as applied to circular shafts. Some may learn that in a thin-walled tube of any cross section the shearing force per unit of length around the periphery of a cross section is a constant and is equal to the torsional moment divided by twice the area enclosed within the cross section. That is about as far as most undergraduate students go with the torsion problem. However, that same simple thin-walled tube formula, $v = M/2A_o$, is applied to box girders and it also can be applied to many trussed structures, or to structures containing both plates and truss forms.

Consider a trussed box as shown in Fig. 1. With the front face open, that is, without being braced, the box offers no effective resistance to the twisting shown. Let the amount of twist be measured by the angle, ϕ, as seen in the top view. Other displacements can then be determined in terms of ϕ and the overall dimensions of the box by going through steps 1 through 10, as indicated on the figure. The angle ϕ is measured in radians and is small enough so that, without significant error, the sin ϕ and the tan ϕ can be set equal to ϕ and the cos ϕ can be taken as unity.

Let the twisting forces, P, which form torsional moments $M_y = Pa$ with respect to the Y-axis and $M_x = Pb$ with respect to the X-axis, be resisted by the shearing forces V_x and V_y acting on the edges of the front face, as shown. These shearing forces may be supplied by a diagonal member, by some other truss form in the front face, or by a plate.

For moment equilibrium with respect to the Z-axis,

$$V_y a - V_x b = 0 \tag{1}$$

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Also, since the total force system is in equilibrium, the work done during the displacement shown must be equal to zero.

$$P(\phi a) - V_x(\phi c) - V_y(\phi a/b)c = 0 \tag{2}$$

Dividing the equation by ϕ and substituting the value of V_y from Eq. (1),

$$M_y - 2V_x c = 0 \tag{3}$$

If the shearing force per unit of length along the top and bottom edges is designated v_x, so that $v_x a = V_x$, and this

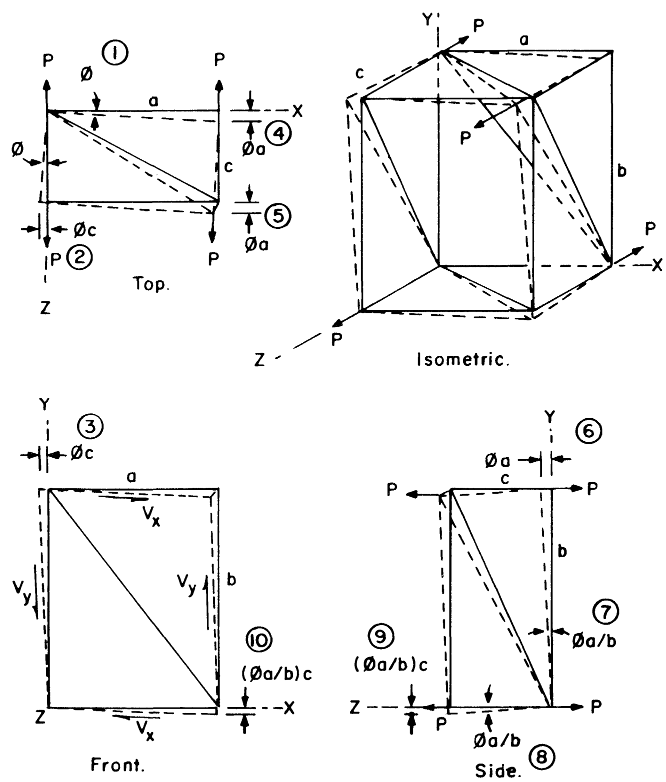


Fig. 1. Twisted rectangular box

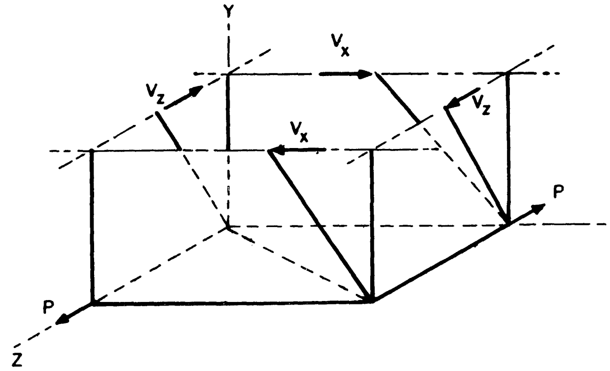


Fig. 2. Shearing forces on any cross section perpendicular to the Y-axis. (A diagonal member is added to the front face to indicate how shearing resistance in that face might be provided)

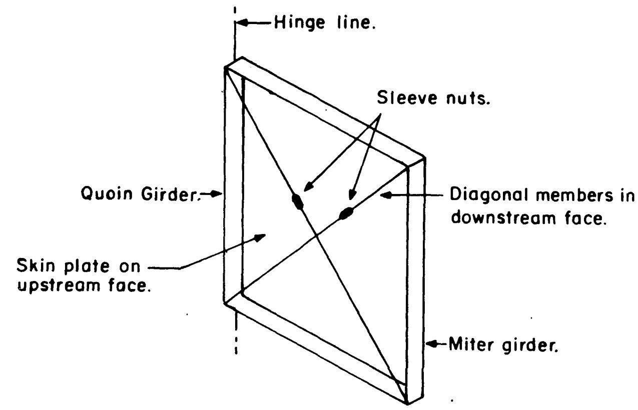


Fig. 3. Schematic of lock-gate leaf, showing sleeve nuts used in prestressing

value of V_x is substituted into Eq. (3) and the equation is then solved for v_x,

$$v_x = M_y/2ac = M_y/2A_o \tag{4}$$

From Eq. (1), $V_y/b = V_x/a$, so that $v_y = v_x$.

If a plane perpendicular to the Y-axis is passed through the box and the portion of the box below the plane is considered as a free body, as in Fig. 2, equilibrium in the X-direction requires a unit shear in the back face equal to v_x. Moment equilibrium about the Y-axis and equilibrium in the Z-direction require a unit shear, v_z, in the side panels, which is equal to v_x. Thus, around the periphery of the cross section there is a unit shearing force of

$$v_x = v_z = M_y/2ac = Pa/2ac = P/2c$$

from which the total shearing forces shown are determined. A diagonal member has been added to the front face to resist the shear. The vertical components of the forces in the diagonal members and in the vertical members can be computed from simple statics.

There is no external moment applied with respect to the Z-axis. On a cross section perpendicular to the Z-axis, the shearing forces produce equal and opposite couples whose sum is zero, as in the front view of Fig. 1.

Any of the faces of the box could be some other truss form or a plate and the formula would still apply. If two diagonals are used in a panel and the diagonals are slender, so that they cannot resist compression, they may be prestressed so they will resist compressive forces up to the amount of the prestress. This is done routinely on lock gates. Stress under maximum loads is the same as it would be without prestress, but rigidity is greatly increased.

In the discussion so far, no account has been taken of the torsional or warping resistance of the various components of the structure. However, unless these components have a thickness which is unusually large, their contribution to overall torsional strength and stiffness is not significant.

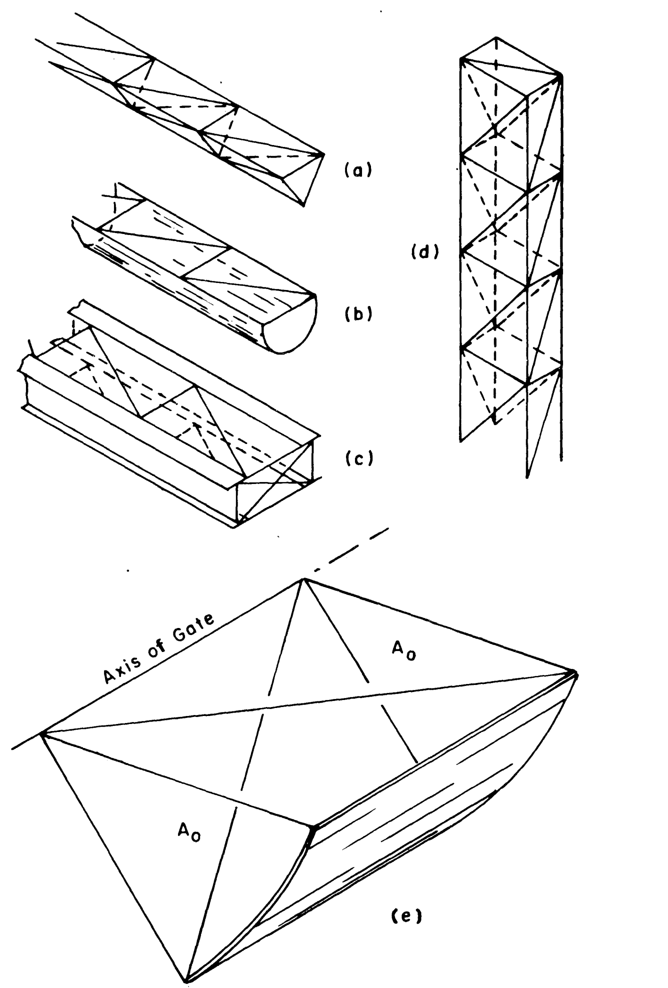


Fig. 4. Some structural types to which $v = M/2A_o$ is applicable

Even in the case of lock gates, which are massive structures with heavy members, the leaves twist noticeably out of plumb under just their own weight without the diagonal members in place and adjusted. This same analysis has been applied successfully to lock gates for the past forty years, but the formula used looks different. The thin-walled tube formula can, however, be extracted from the lock-gate formula. A schematic drawing of a lock-gate leaf is shown in Fig. 3.

It can be shown that the formula, $v = M/2A_o$, can be applied to any closed box-like structure regardless of the shape of the cross section. Thus, it can be applied to the torsional analysis and design of triangular-section trusses (Fig. 4a), to trussed troughs (Fig. 4b), to trussed girders (Fig. 4c), to towers (Fig. 4d), or even to tainter gates (Fig. 4e).

It is important to remember that all faces or surfaces of any box or tube, including the ends, must be closed; that is, the faces must consist of plates, truss forms, or rigid frames. This is equally true of tubes usually thought to be torsionally strong and rigid, such as those of Fig. 5. Either the tube must be thick enough (compared with other dimensions of the cross section) so that the shape of the cross section is maintained, or the cross sections at all load points must be braced. Except at load points the cross bracing is unstressed, but at occasional intermediate points cross bracing would seem desirable for stability.

It used to be thought that W sections or I sections could be made torsionally stiff by forming tubes out of them with side plates, particularly at the ends, as illustrated in Fig. 6. The side plates do add some torsional resistance to the members, but only the amount of the warping or torsional resistance of the side plates themselves, and this is not very much, as Morris Ojalvo points out. As he shows, however, torsional stiffness will be increased significantly by closing both ends of each tube thus formed with end plates welded in place.

Another example is provided by the small box truss illustrated in Fig. 7, which was tested in the laboratory at Ohio University. Two ordinary open-web steel joists placed three feet apart formed the sides. Cross bracing and diagonal bracing between the top chords divided the truss into five panels, each 4 ft long. For the first test, there was no horizontal diagonal bracing between the bottom flanges. Two 2-kip loads were applied above one joist, as shown. The loaded joist deflected a predictable amount with the middle cross sections rotating, as shown in Fig. 8a. The unloaded joist did not deflect vertically at all, and there was no significant resistance to twisting of the individual joists and resulting lateral displacement of their lower chords, Fig. 8b.

For the second test, $^5/_8$-in. rods were added as diagonal cross bracing between the bottom chords (just as the horizontal bracing at the top-chord level) and the pair of 2-kip loads was applied again. The formerly-open bottom face of the box now resisted the horizontal displacements of Fig. 8b and the whole structure became torsionally stiff. The

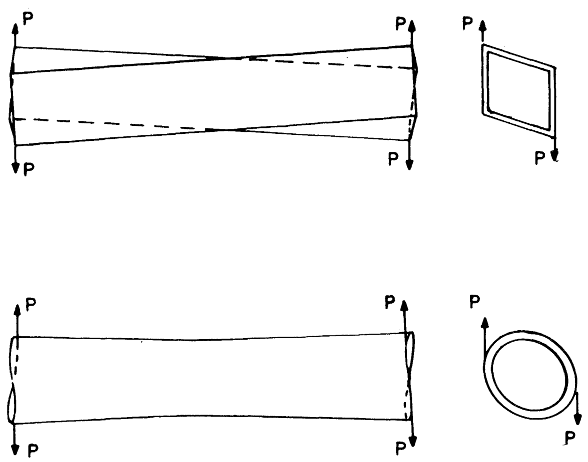


Fig. 5. Schematics of open-end rectangular and circular tubes, indicating flexibility under applied torsional couples

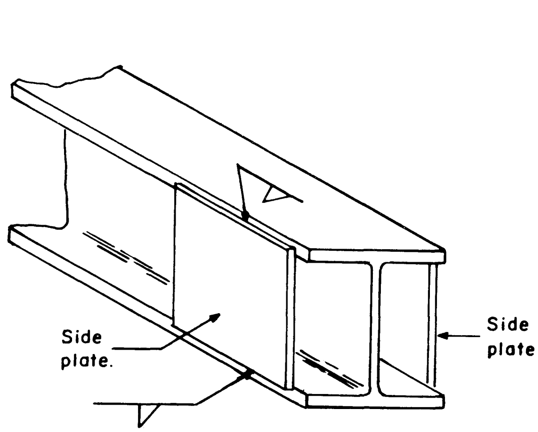


Fig. 6. Torsional stiffening of beam by addition of side plates

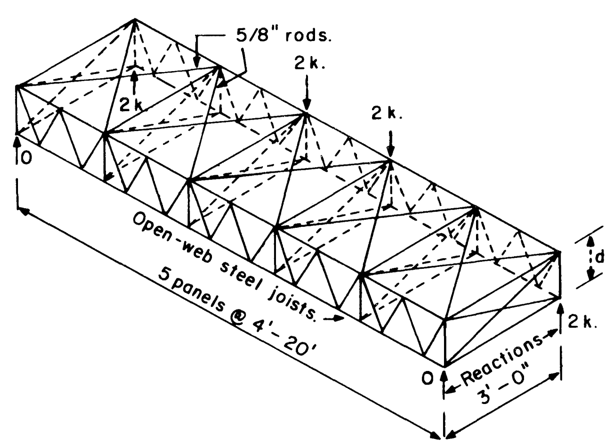


Fig. 7. Laboratory model of box truss

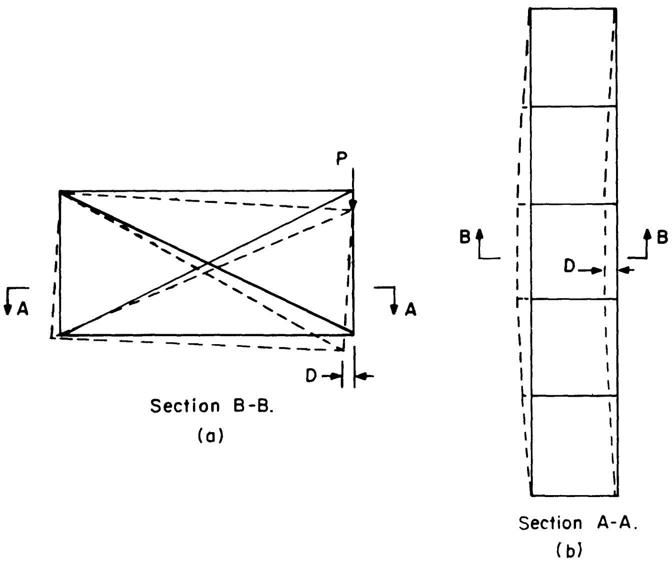


Fig. 8. Horizontal displacement of lower chords (no diagonal bracing in plane of lower chords)

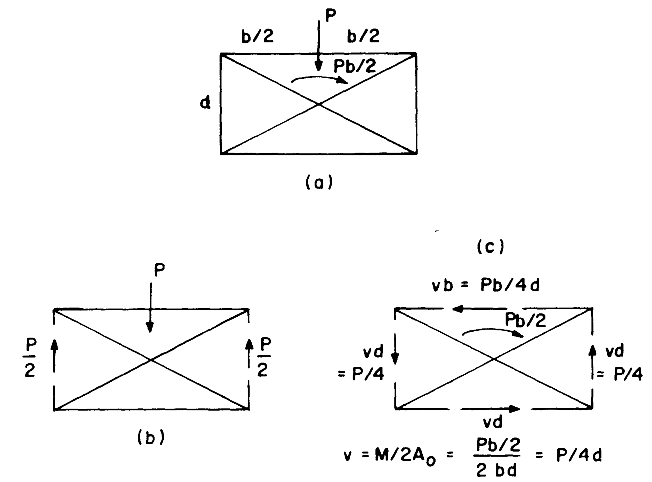


Fig. 9. Shear distribution (bottom chords braced)

loaded joist deflected only ¾ as much as it did in the first test, and the unloaded joist carried ¼ of the load.

To explain this different behavior, consider Fig. 7. Between the load points and the supports, the vertical shear in the structure is 2 kips or P. With no lower-chord diagonal bracing, the loaded joist carries all of this load with little or no stress in other parts of the structure.

With the bottom of the box closed by diagonal bracing, the load can be resolved into a vertical component, P, acting through the shear center, and a moment, $Pb/2$, Fig. 9a. The centric component, P, is supported equally by both joists, Fig. 9b. The moment component is resisted by a unit

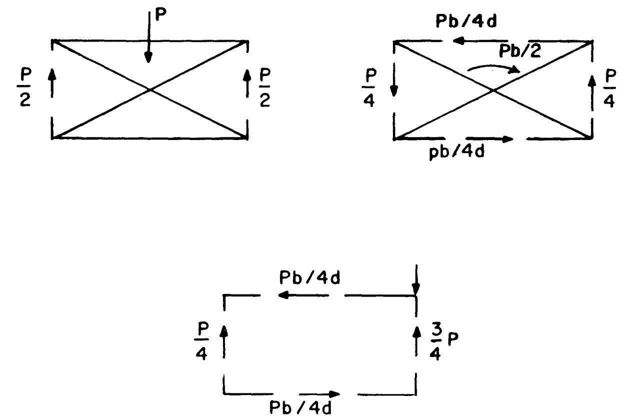
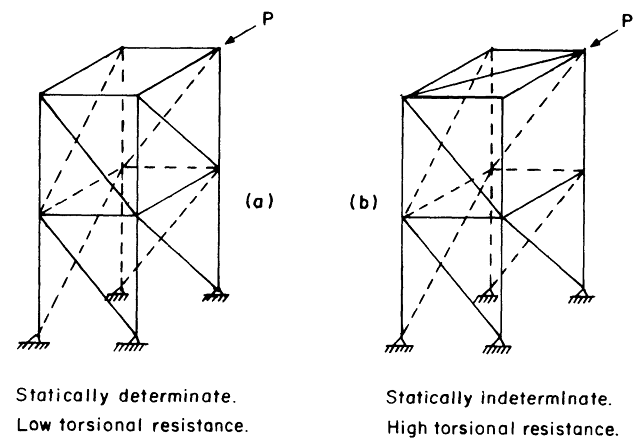


Fig. 10. Resultant shear distribution (bottom chords braced)



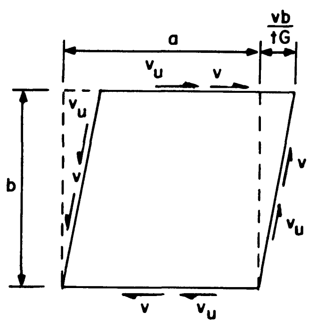
$$I \times D = \Sigma\,(u) \times (SL/AE)$$

Fig. 11. Two-story tower with eccentric load

shearing force of $(Pb/2)/(2A_0) = P/4d$ around the periphery of the cross section, Fig. 9c. This unit shearing force multiplied by the length of the sides of the cross section gives shearing forces which, when added to the shears produced by the vertical load component, produce the results shown in Fig. 10, indicating that the unloaded joist has in it a vertical shear of $0.25P$ with the loaded-joist shear being $0.75P$.

One may be puzzled by the fact that the unloaded joist carries one-fourth of the load, although the reaction under it at the end is zero. However, it is not difficult to trace the forces through the members of the structure to see that such is indeed the case, as equilibrium of the whole structure requires.

Thus far it has been assumed that the various surfaces or truss panels which form a box structure offer only negligible resistance to warping. The single box unit of Fig. 1 resists torsional loads only because all six faces resist



$$1 \times D = \Sigma (u) \times (SL/AE) + \Sigma (v_u a) \times (vb/tG)$$

Figure 12

shear. The structure is statically determinate, and it would be unstable if even one face were open.

Now consider the tower of Fig. 11a. The bottom face of the tower, defined by the four points of support, not only resists a change of shape in its own plane, but it resists warping as well. Hence, an extra component of resistance to the load has been added, and the structure is stable even though the top face of the tower is open. Also, the structure is statically determinate, but any load is resisted solely by that side of the tower in the plane of which it lies. Thus the torsion formula, $v = M/2A_o$, does not apply in this case.

If the tower is made torsionally strong by adding a diagonal in the top, Fig. 11b, the structure is also made statically indeterminate and the torsion formula, derived from Fig. 1 in which all faces warp freely, does not apply strictly to this case either. However, for a structure of two stories or more in height, the error in computed member forces is not more than a few percent. There would be nothing gained by applying the torsion formula to a one-story structure.

Actually, the thin-walled-tube formula applies strictly only to structures such as that of Fig. 1, in which all faces are closed (braced) and are free to warp. Such is the case for only a very few structures. Therefore, the formula does not apply strictly to box trusses, box girders, or any other structures having four points of support, or having a member running through the interior space of the box unit from one corner to the diagonally-opposite corner. Unless cross sections are free to warp, the thin-walled-tube formula does not even apply strictly to all thin-walled tubes. However, the error in using the formula to compute the load-carrying capacity of the usual structure is not more than a few percent.

Probably the easiest way in which to compute the displacement produced by a set of loads on a structure is to determine the work done by a unit virtual force as the set of loads is applied. The virtual force should be applied at the point and in the direction of the desired displacement. The work done by the unit virtual force as the set of real loads is applied is $1 \times D$, in which D is the desired displacement. The work done by the unit force is also equal to $u \times SL/AE$ for all of the members in which u is the member force produced by the virtual load and S is the member force produced by the set of real loads. From the equation $1 \times D = \Sigma u \times (SL/AE)$, the value of D can be found.

If one or more of the surfaces of the box or tube is a plate, either flat or curved, and the shear per unit of length is v_u for the unit load and v for the set of real loads, the work done on the plate by the virtual force as the set of real loads is applied (see Fig. 12), is $v_u a \times vb/tG$, and this must be added to the work done on the members. Thus,

$$1 \times D = \Sigma u \times SL/AE + \Sigma v_u a \times vb/tG \qquad (5)$$

in which a and b are the width and length of the developed plate, t is its thickness, and G is the shearing modulus.

If the angle of twist, ϕ, of a section of the structure is desired, a unit virtual moment is applied at the section. The work done by this moment as the real loads are applied is a measure of the rotation caused by the real loads.

$$1 \times \phi = \Sigma u \times SL/AE + \Sigma v_u a \times vb/tG \qquad (6)$$

In this equation, u is the member force produced by the unit virtual moment and $v_u = \frac{1}{2}A_o$ is the unit shear force caused by the virtual moment. In both Eqs. (5) and (6) the products, $u \times S$ and $v_u \times v$, are positive if the forces are in the same direction and negative if they are in opposite directions. If all faces are plates, the first term on the right-hand side of Eq. (6) is zero and the second term becomes the one used in determining the angle of twist of box girders.