Elastic Buckling of a Column Under Varying Axial Force

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When a truss or open-web steel joist is used as a frame member, parts of the bottom chord near the ends carry compressive forces and adequate bracing must be provided to prevent lateral buckling of the chord. The compression in the chord varies from panel to panel; the problem is to determine the buckling load factor for a given loading and assumed location of the bracing point. Although the problem can be solved exactly, a simple procedure is given herein which may be used in the design office for rapid, hand computation of the buckling load factor in the elastic range.

ANALYSIS

To formulate the problem, let **AB** be a simply supported column of prismatic section with **A** as the support and **B** as the bracing point providing restraint against lateral displacement at **B**. Let **1, 2, . . . N** be the panel points and let P_1, P_2, . . . P_N be the axial loads applied at the respective panel points (Fig. 1). With this scheme, the compressive force in the kth panel is $P_1 + P_2 + \ldots P_k$.

Assuming a mode shape

$$y = C \sin(\pi x/l) + D \sin(2\pi x/l) \qquad (1)$$

where C and D are arbitrary constants, and applying the standard Rayleigh–Ritz procedure gives the following characteristic equation for determining λ, the buckling load factor:

$$\left[\Sigma P_n[\alpha_n + (^1\!/_2\pi)\sin(2\pi\alpha_n)] - \frac{\pi^2 EI}{\lambda l^2}\right] \times \left[\Sigma P_n[\alpha_n + (^1\!/_4\pi)\sin(4\pi\alpha_n)] - \frac{4\pi^2 EI}{\lambda l^2}\right] - \Sigma P_n[(1/\pi)\sin(\pi\alpha_n) + (^1\!/_3\pi)\sin(3\pi\alpha_n)] = 0 \qquad (2)$$

where Σ stands for summation from $n = 0$ to N and $\alpha_n = l_n/l$ are as shown in Fig. 1.

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In order to facilitate computations, Eq. (2) can be written as

$$\left[(\Sigma P_n a_n) - \frac{\pi^2 EI}{\lambda l^2}\right]\left[(\Sigma P_n b_n) - \frac{4\pi^2 EI}{\lambda l^2}\right] - (\Sigma P_n c_n)^2 = 0 \qquad (3)$$

where

$$a_n = \alpha_n + (^1\!/_2\pi)\sin(2\pi\alpha_n)$$
$$b_n = \alpha_n + (^1\!/_4\pi)\sin(4\pi\alpha_n) \qquad (4)$$
$$c_n = (1/\pi)\sin(\pi\alpha_n) + (^1\!/_3\pi)\sin(3\pi\alpha_n)$$

are functions of α_n alone and can be computed independent of loads. These are tabulated in Table 1.

Table 1. Functions a_n, b_n, c_n

α_n	a_n	b_n	c_n	α_n	a_n	b_n	c_n
1.00	1.000	1.000	0.000	0.50	0.500	0.500	0.212
0.98	0.960	0.960	0.040	0.48	0.500	0.460	0.213
0.96	0.920	0.922	0.079	0.46	0.500	0.422	0.217
0.94	0.881	0.886	0.116	0.44	0.499	0.386	0.223
0.92	0.843	0.853	0.152	0.42	0.497	0.353	0.231
0.90	0.806	0.824	0.184	0.40	0.494	0.324	0.240
0.88	0.771	0.801	0.213	0.38	0.489	0.301	0.251
0.86	0.737	0.782	0.238	0.36	0.483	0.282	0.262
0.84	0.706	0.768	0.259	0.34	0.474	0.268	0.272
0.82	0.676	0.759	0.276	0.32	0.464	0.259	0.282
0.80	0.649	0.753	0.288	0.30	0.451	0.253	0.290
0.78	0.624	0.751	0.296	0.28	0.436	0.251	0.296
0.76	0.601	0.750	0.300	0.26	0.419	0.250	0.300
0.74	0.581	0.750	0.300	0.24	0.399	0.250	0.300
0.72	0.564	0.749	0.296	0.22	0.376	0.249	0.296
0.70	0.549	0.747	0.290	0.20	0.351	0.247	0.288
0.68	0.536	0.741	0.282	0.18	0.324	0.241	0.276
0.66	0.526	0.732	0.272	0.16	0.294	0.232	0.259
0.64	0.517	0.718	0.262	0.14	0.263	0.218	0.238
0.62	0.511	0.699	0.251	0.12	0.229	0.199	0.213
0.60	0.506	0.676	0.240	0.10	0.194	0.176	0.184
0.58	0.503	0.647	0.231	0.08	0.157	0.147	0.152
0.56	0.501	0.614	0.223	0.06	0.119	0.114	0.116
0.54	0.500	0.578	0.217	0.04	0.080	0.078	0.079
0.52	0.500	0.540	0.213	0.02	0.040	0.040	0.040

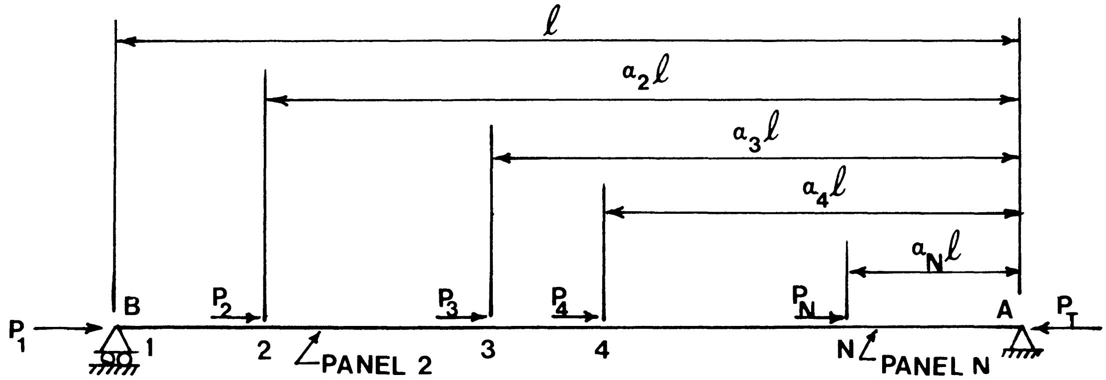

Fig. 1. Notation

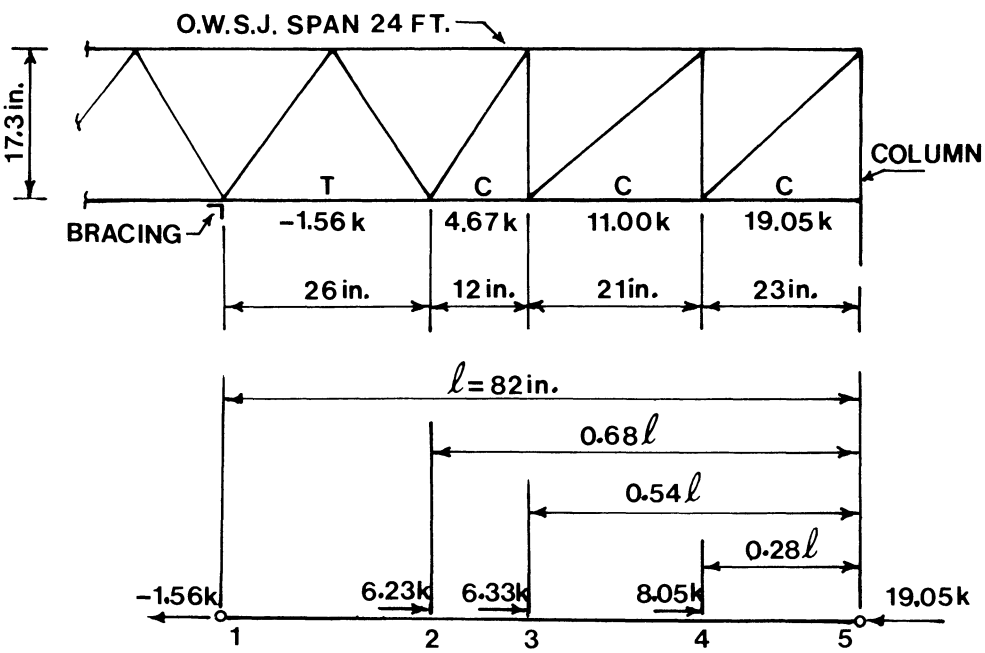

Fig. 2. Example

Introducing

$$A = \Sigma P_n a_n$$
$$B = \Sigma P_n b_n \qquad (5)$$
$$C = \Sigma P_n c_n$$

the buckling load factor, λ, is given by

$$\frac{EI\pi^2}{\lambda l^2} = \frac{(4A + B) \pm \sqrt{(4A + B)^2 - 16(AB - C^2)}}{8} \qquad (6)$$

where the sign is chosen to yield the smaller, positive value of λ.

NUMERICAL EXAMPLE

To demonstrate the procedure, consider a chord with unsupported length l = 82 in. and loaded as shown in Fig. 2. Using Table 1, the computations may be carried out in the following tabular form.

n	α_n	a_n	b_n	c_n	P_n kips	a_nP_n kips	b_nP_n kips	c_nP_n kips
1	1.0	1.0	0.0	0.0	−1.56	−1.560	0.0	0.0
2	0.68	0.536	0.741	0.282	6.23	3.339	4.616	1.757
3	0.54	0.500	0.578	0.217	6.33	3.165	3.658	1.373
4	0.28	0.436	0.251	0.296	8.05	3.509	2.020	2.383
					19.05	8.453	10.294	5.513
					P_T	A	B	C

Using Eq. (6), one obtains

$$\frac{\pi^2 EI}{\lambda l^2} = 9.543 \qquad (7)$$

For a chord comprised of two $1\frac{1}{4} \times 1\frac{1}{4} \times \frac{1}{8}$-in. angles back-to-back, the applicable moment of inertia, I, is 0.37 in.^{4} Taking E = 29,000 ksi for steel, the buckling load factor for lateral buckling of such a chord is $\lambda = 1.65$, a value which is higher than the exact value, obtained by computer analysis, by less than 4%. Various other cases have been verified with similar results.

It may be noted that the above analysis disregards the out-of-plane stiffnesses of web members at the panel points. However, in the case of open web joists, these are small and their neglect is justified.