Plastic Design of Steel Floor Beams

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The potential for savings in material and improvement in performance which can be realized by use of plastic design for continuous floor beams, warrants increased attention from fabricators and designers. Various facets of application of this concept will be discussed.

The top flanges of floor beams are generally the same nominal elevation as those of the girders. Consequently, whether framing to girders or to columns, the beams could be spliced at a plastic hinge point. Such an arrangement, utilizing A36 steel beams, is assumed in this paper.

Items such as reduced restraint for end spans, deductions for bolt holes, and problems in developing full plastic moment capacity at connections are details which require attention if the full efficiency of plastic design is to be achieved. Some compromises, as discussed below, are desirable to avoid overcomplicating the fabrication and erection, in the attempt to save a last increment of beam weight.

When framing to girders, the use of a simple connection at the exterior of the end span is a practical necessity. A simple connection is also desirable for framing to exterior columns. Though this will increase the end span positive moment, no moment will be induced in exterior columns which are generally low in flexural efficiency.

Plastic design allows the allocation of the total moment, $WL/8$, between positive bending and negative bending, on the basis of flexural capacity provided in each of these regions of maximum moment. Therefore, the designer may simplify the connections at points of support by developing the capacity of only the flanges, not providing for the flexural strength of the web. Designing the connections in this manner will force the plastic hinge at the support to form in the connections, permitting the beam to remain essentially elastic. Thus, the requirement for drilling or reaming of bolt holes in the tension area of the beam would not apply; the area of these holes would not be deductible from a critical section, and a beam web stiffener at the plastic hinge at points of support would not be mandatory.

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However, sufficient ductility must be provided *within* the connection to allow the necessary plastic deformation.

The plastic modulus of the flanges is readily calculated by multiplying the area of one flange by the distance between their centers, $Z_f = A_f(d - t_f)$. Values of Z_f are tabulated in Table 1 for some of the more commonly used sections. The ratio of the plastic modulus of the flanges to the plastic modulus of the overall section varies from about 0.6 to 0.8. The plastic moment diagram in Fig. 1 is based on an average ratio of 0.7, and shows increases in positive moments of only 5% on the end span and 18% on the interior span over that of members fully developed at the interior supports. The small significance of the 18% increase is demonstrated by the fact that simple connections would result in a 100% increase.

In this paper, all discussion and examples assume the use of A36 steel. Other grades of steel are, of course, permitted under the AISC Specification; the procedures described are equally applicable to those steels.

Table 1. Section Properties

Size	d	t_f	A_f	Z	Z_f	$Z + Z_f$
W12×16	11.99	0.265	1.057	20.1	12.4	32.5
W12×19	12.16	0.350	1.402	24.7	16.6	41.3
W12×22	12.31	0.425	1.713	29.3	20.4	49.7
W12×26	12.22	0.380	2.466	37.2	29.2	66.4
W14×22	13.74	0.335	1.675	33.2	22.5	55.7
W14×26	13.91	0.420	2.111	40.2	28.5	68.7
W14×30	13.84	0.385	2.591	47.3	34.9	82.2
W14×34	13.98	0.455	3.069	54.6	41.5	96.1
W16×26	15.69	0.345	1.898	44.2	29.1	73.3
W16×31	15.88	0.440	2.431	54.0	37.5	91.5
W16×36	15.86	0.430	3.004	64.0	46.3	110
W16×40	16.01	0.505	3.532	72.9	54.8	128
W16×45	16.13	0.565	3.975	82.3	61.9	144
W18×35	17.70	0.425	2.550	66.5	44.1	111
W18×40	17.90	0.525	3.158	78.4	54.9	133
W18×46	18.06	0.605	3.666	90.7	64.0	155
W18×50	17.99	0.570	4.272	101	74.4	175
W21×44	20.66	0.450	2.925	95.4	59.1	155
W21×50	20.83	0.535	3.494	110	70.9	181
W21×57	21.06	0.650	4.261	129	87.0	216

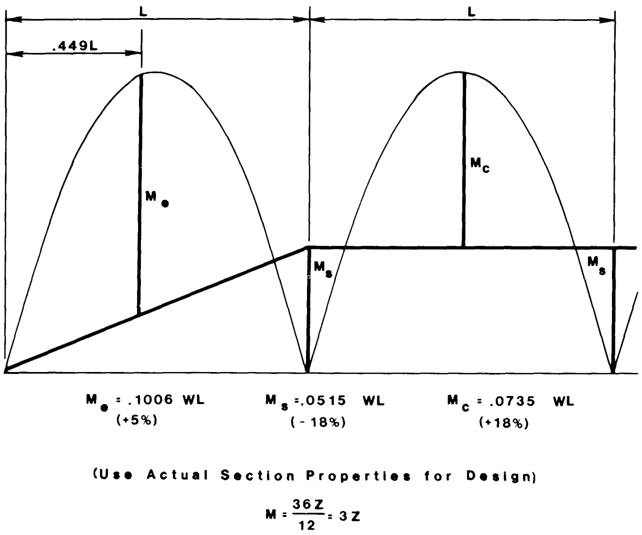

Fig. 1. Plastic moment diagram for $M_s = 0.7M_c$

DESIGN

The design procedure for this concept is simple and logical. Calculate the factored simple span moment required for an interior span ($M = W_uL/8$). Using Table 1, choose a beam for which the sum of the positive and negative bending capacities ($ZF_y + Z_fF_y$) = $12M$, where M is in kip-ft. For A36 steel, $Z + Z_f = 12M/36$. The moment capacity at the first interior support is determined by the value Z_f of the interior beam. A slightly conservative value for the plastic modulus required for the end beams can be calculated by subtracting $0.45Z_f$ (interior) from the Z required for a simple span. An exact solution may be obtained by area under the shear diagram. See Design Examples.

ELASTIC PERFORMANCE

A common misconception about plastically designed members is that plastic hinges form under service loading. To the contrary, the action under design loading is predominantly elastic. The purpose of plastic analysis and design is to assure that a mechanism will not form at less than 1.7 times design load. Even after the formation of a so-called mechanism, collapse does not occur; rather, some reserve capacity still exists because of strain hardening.

A generalized analysis of members designed in accordance with the proposed criteria shows that plastic hinges will not form upon application of uniform design load. Checkerboard loading, with the design live load applied simultaneously on only the end and the first interior spans, could result in plastic rotation at the first interior support upon imposition of the last 15% increment of load. However, the resulting deflection still would be substantially less than that of a simple beam of equal strength. Discussion of deflection appears in a subsequent section.

In summary, for all practical purposes, plastically designed beams are elastic elements as far as performance under design loading is concerned. The concepts of plastic action and the load factor are only tools for providing a true factor of safety against ultimate strength, while assuring that unacceptably large deflections will not occur at design loads.

PLASTIC-COMPOSITE DESIGN

An additive reduction in beam sizes may be achieved by use of composite design.

As recommended in the foregoing non-composite design procedure, the suggested procedure outlined below divides the factored simple span moment between positive and negative moment values. The negative moment is governed by the moment capacity of the beam flanges. The positive moment is used for design of the composite section.

The AISC Specification recommends an allowable calculated bending stress of 24 ksi for composite beams. Use of the section modulus of the transformed section to design a member for the factored moment would be equivalent to using an allowable calculated stress of 36/1.7 or 21.2 ksi. The design can be modified to incorporate an allowable calculated stress of 23.3 ksi by using a shape factor of 1.1. (The actual shape factor of the composite section for the range of shapes under consideration is in the order of 1.4.) Since the assumed relationship $Z_{tr} = 1.1S_{tr}$ yields results in essential accord with the current Specification, its use is advocated in the suggested design procedure which follows.

Refer to Fig. 2 and to the Design Examples for illustration of the procedure as it applies to a specific design problem.

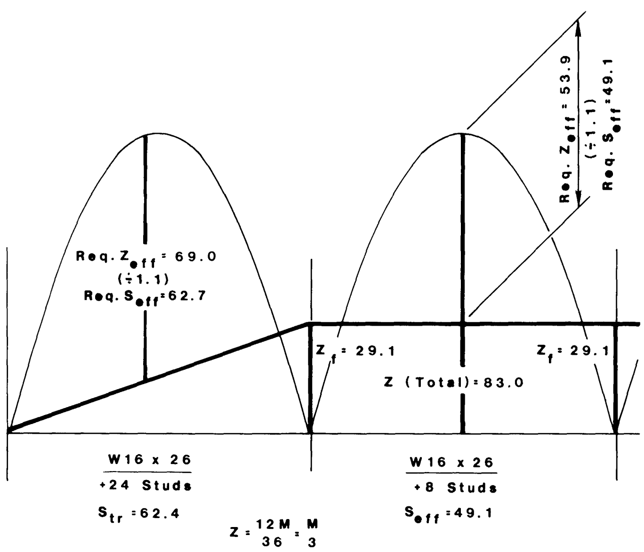

Fig. 2. Plastic moment diagram for plastic-composite design example

1. Calculate the factored simple span moment.

$$M = (1.7)(WL/8) \text{ kip-ft}$$

2. Choose a trial section for the interior spans, based upon a plastic modulus of the bare beam, adequate for 0.40 to 0.50 of the moment from (1) above.

$$Z < 0.5 \times 12M/36$$

3. Subtract the value Z_f (plastic modulus of flanges only) of trial section, from the required total as calculated in (1). The value of Z_f is recommended not to exceed $1.1 \times S_{tr}$ of the trial section.
4. Calculate the shear connector requirement for full or partial action, using as the required S_{eff} the value obtained from (3) divided by 1.1.
5. For the end span, subtract from the Z calculated in (1), 0.45 times the Z_f-value used in (3). (The exact value can be determined from the area under the shear diagram.)
6. Choose a beam having Z_{tr} or $(1.1S_{tr})$ not less than value obtained from (5). Design the connectors for full or partial composite action.
7. Start shear connector spacing a distance *not less than* $L/8$ from *continuous* connections at supports. (Analyses show that, even for extreme conditions, this distance from support will consistently assure that all connectors are in the region of positive bending.) Where support is provided by a simple connection, the spacing should start at the support.

The question logically arises as to whether or not it is necessary to shore continuous composite construction to avoid plastic hinge formation in the negative moment region at service loading. A disproportionate share of the moment unquestionably is initially distributed to the negative region, prior to hardening of the concrete. However, because of the large increase of stiffness in the region of positive moment, this initial condition tends to be offset by distribution of a smaller share of moment from loads applied after hardening of concrete. To quantify this problem, an analysis was made assuming 30% of the total load imposed on the non-composite beam, and the moment of inertia of the transformed section as 3 times that of the bare beam. This analysis shows that calculated stresses in the negative region are comparable to those in an equivalent plastic non-composite design. Therefore, the criteria for shoring generally need be no more stringent than for a simple composite beam of equal capacity.

DEFLECTIONS

In calculating the relative deflections cited below, the simple beams were assumed to be 20% deeper than the continuous plastically designed beams, and live load assumed as 60% of total load.

Compared to the deeper simple beam, deflections of interior spans are dramatically reduced (±60%) with all spans loaded, and substantially reduced (±20%) with alternate spans loaded. The reduction for end spans is ±15% with all spans loaded, and ±5% with alternate spans loaded.

The subject of deflections can best be summarized by the following from Ref. 2:

> "One of the most important questions about deflections in plastic design is whether or not they will constitute an undue design limitation. The answer to this question on the basis of available test results and calculations is: No."

OCCUPANT INDUCED VIBRATIONS

Dr. Thomas M. Murray has published two papers on occupant induced vibrations which have appeared in the AISC *Engineering Journal*.[3,4] Engineers who have used his recommendations for evaluation and design of floor systems have found them to be very useful. Another paper by Wang and Carrle[5] indicates that end restraints on steel joists could be useful in reducing the perceptibility of vibrations. This conclusion should apply equally to steel beams.

Dr. Murray has further refined acceptability criteria in a recent paper,[6] wherein he concludes that a floor system will be satisfactory if $D > 35A_of + 2.5$, where D is the summation of damping provided by the various components of the system (%), A_o is initial amplitude from a heel drop, and f is the first natural frequency in hz. The coefficient of 1.57 in the formula for frequency is the same for a series of *equal* length continuous spans as it is for simple spans, and the amplitude, $A_o = \alpha(600L^3)/EI \div (N_{eff} \times DLF)$. Values of α are 0.015 for end spans and 0.0115 for interior spans N_{eff} is the number of effective beams, and DLF is the dynamic load factor. See Refs. 3 and 4 for evaluation of f, N_{eff} , and DLF, and for estimating a value for D.

As in the case of cantilevered and overhanging beams,[4] the transformed section is used in both the positive and negative moment regions, whether or not either is designed as composite.

Damping, D, for an open office floor area, as assumed in the Design Examples, might be:

Slab and beam	2.0%
Hung ceiling	1.0
Ducts and mechanical	1.5
$D =$	4.5%

Other research in which the damping of office floors was measured indicates that the actual damping is probably slightly higher than this conservative assumed value of 4.5%.

Using the sizes obtained in the Design Examples yields results as given below. Values of the parameter $(35A_of + 2.5)$ should be equal to or less than 4.5% for a satisfactory system.

Table 2. Cost Comparison for 4 spans

Framing System	Bm. Depth in.	Steel Wt. lb	Steel Cost $	Stud Qty.	Stud Cost $	Steel + Studs $
Simple	18	5520	1104	—	—	1104
Continuous plastic	16	4260	852	—	—	852
Simple composite	16	3720	744	104	104	848
Plastic composite	16	3120	624	64	64	688

Plain material 20¢/lb. Studs $1.00 ea. installed. No fabrication or erection.

Framing Method	$(35A_0f + 2.5)$
Simple span (W18×46)	5.0% (high)
Simple span composite (W16×31)	4.8% (high)
Continuous span composite, end span (W16×26)	4.1% (acceptable)

Investigation for a wide range of spans and loadings shows that this reduction in damping requirements for continuous beams is a trend, and vibration perceptibility is consistently reduced with continuous construction.

ECONOMY

Referring to the Design Example calculations, assuming four spans of equal length (three interior joints), and using approximate cost figures for *unfabricated* structural steel and for field installed shear connectors, the data shown in Table 2 are obtained. Note that the *installed* cost of the shear connectors is included for composite designs, leaving connections as the only variable. Quantities are given to allow modification to local markets.

In order to arrive at a cost differential for the erected framing, estimates were obtained from several fabricators and erectors on the cost of connections. Even though there was an anticipated diversity of opinion, a consensus indicated that the *additional erected* cost of continuous connections for each of the three interior joints of the four span comparisons is, for this case, about $20 to $30 over the cost for simple beam connections. Using $25 as an average, the net overall savings, including material, fabrication and erection, for the plastic design over the simple beam design is $177 [1104 − (852 + 75)] for non-composite construction, and $85 [848 − (688 + 75)] for composite construction. These differentials are conservative, as the fabricator's cost for overhead and for handling the additional tonnage required by the simple beam designs will tend to increase the spread.

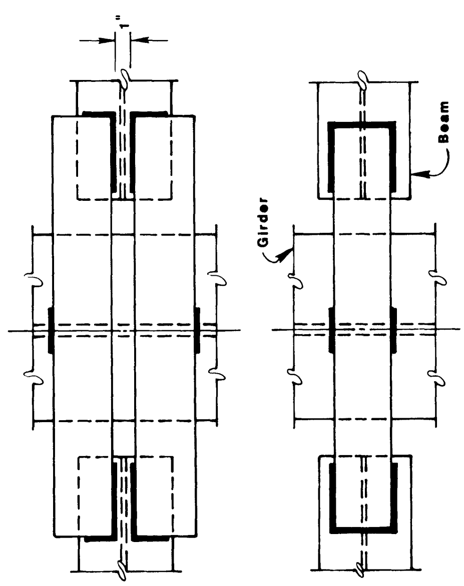

Fig. 3. Splice plates for top flange

CONNECTIONS

Criteria for Connections—For the beams to perform in accordance with the foregoing concepts, the connections should have the capacity to:

1. Support and transmit calculated loads.
2. Prohibit significant non-elastic movement prior to calculated plastic hinge formation.
3. Allow deformation for plastic hinge formation.

The AISC Specification provides for multiplying allowable stresses for bolts and welds by 1.7 for use with the factored loading. To be in conformity with criteria (2) above, bolts, if used in the moment splice should have a factor of safety against slippage of about 1.7. Since the working value for ¾ *A*325 bolts on clean mill scale surfaces provides a factor of safety against slippage of 1.17,* a bolted moment splice was not considered. Alternative moment connections in the discussion which follows were limited to welded and direct bearing connections.

Top Connection—The welded connections in Fig. 3 are recommended for splicing the top flange, as the plate length between the ends of the spliced beams provides the ductility necessary to allow plastic hinge formation *within* the connection. An easy, though slightly conservative, method

* *The slip coefficient for clean mill scale surfaces is 0.322.[7] The load at slip is obtained by multiplying this coefficient by the specified mininum tension (28 kips). The factor of safety against slippage is obtained by dividing this load at slip by the allowable shear for ¾-in. diam. A325 bolts in friction-type connections with standard holes.*

for sizing the splice plates when the plates are of the same grade material as the beams, is to match the flange area of the *smaller* of the two beams being spliced.

Personal experience has shown that plates of approximately $\frac{5}{16}$-in. thickness will not interfere with the application of metal deck. It probably would be necessary to cut the deck around plates of appreciably greater thickness. The purpose of the two-plate connection is to limit the thickness and to avoid overhead field welding, which would be necessary with a single plate of greater width than the beam flange. In some cases, use of a plate of higher strength material will provide sufficient reduction in required area to meet the thickness limitation, and allow the use of the preferred single splice plate.

The welds shown in Fig. 3 for the double-plate alternate are eccentrically loaded. The specimen for the load test (which will be discussed later) was designed with this eccentricity neglected, with no adverse consequences. Tests[8] have shown the use of full eccentricity on welds produces non-uniform factors of safety ranging from 3.6 to 7.6, while a uniform and adequate factor of safety will be provided if 20% of the bending component is considered. Whether all or 20% of the bending is considered, tables for eccentrically loaded welds[1] may be used for design.

Bottom Connection—Conventional details for connecting the bottom flange by welding or by end-plate bearing are shown in Figs. 4 and 5. Both methods require saw-cutting or accurately trimming the end of the beams and coping the top flange. Other considerations for the seat angle connection are: preparing the bottom flange for a butt weld, providing sufficient girder depth for the vertical leg of the seat angle,* and performing a separate field welding operation each side of the beam web. The end-plate connection provides no erection seat, has very tight erection clearances even with shims, and requires use of both field welding for the top flange splice plate, and high-strength bolting for the end plate. (High-strength bolts are needed to draw the parts together and prevent slippage. Slippage could result in weld cracks in the top flange splice, so the designer may choose to use a factor of safety against slippage larger than that provided by code for friction-type connections.) This end-plate connection does, however, accommodate a minimal depth differential between the beams and the girder.

While the connections discussed above individually have desirable features, such as providing an erection seat or the ability to accommodate small beam-girder depth differentials, neither combines all features desirable for general application. Such is the case with most, if not all conventional connections for continuous beams.

Lack of a practical and versatile standard connection is the probable reason for the lack of general usage of continuous steel beam deisgns for buildings. Studying this need for a better connection led to the development of the M-Seat.

The M-Seat Connector shown in Fig. 6 is a proprietary special purpose angle designed specifically for this application. It has few limitations and offers the better features of conventional details, plus the following unique advantages:

1. In the vast majority of cases, the 2-in. clearance plus the 3:12 slope eliminates the need for coping the beams.

** Large thickness requirements for the seat angle will also require large leg sizes. For example, a 1-in. thickness will require 6 × 6 or 8 × 4 size angles.*

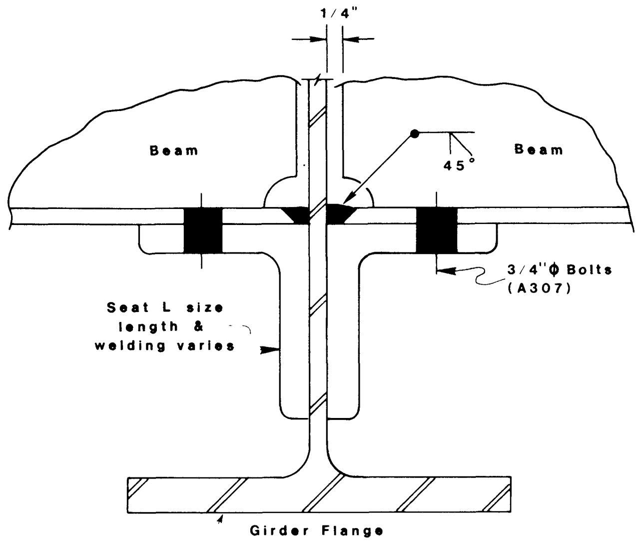

Fig. 4. Seat angle connection

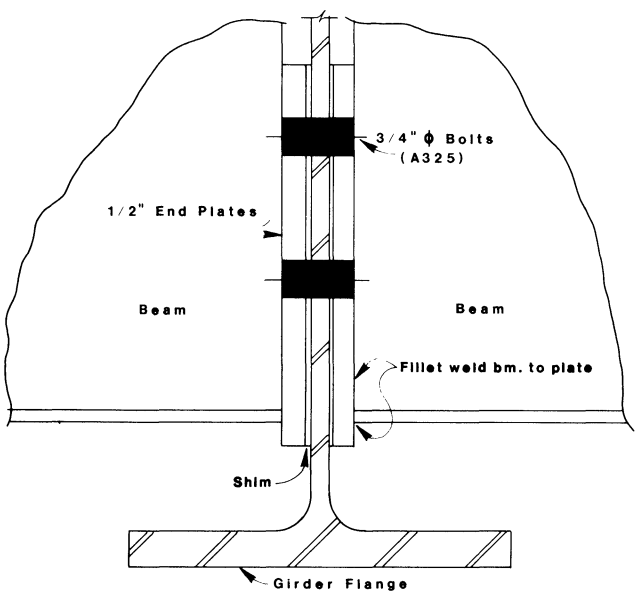

Fig. 5. End plate connection

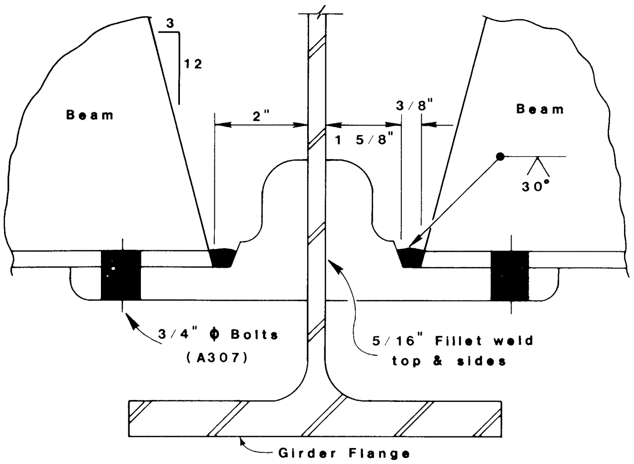

Fig. 6. M-Seat connection

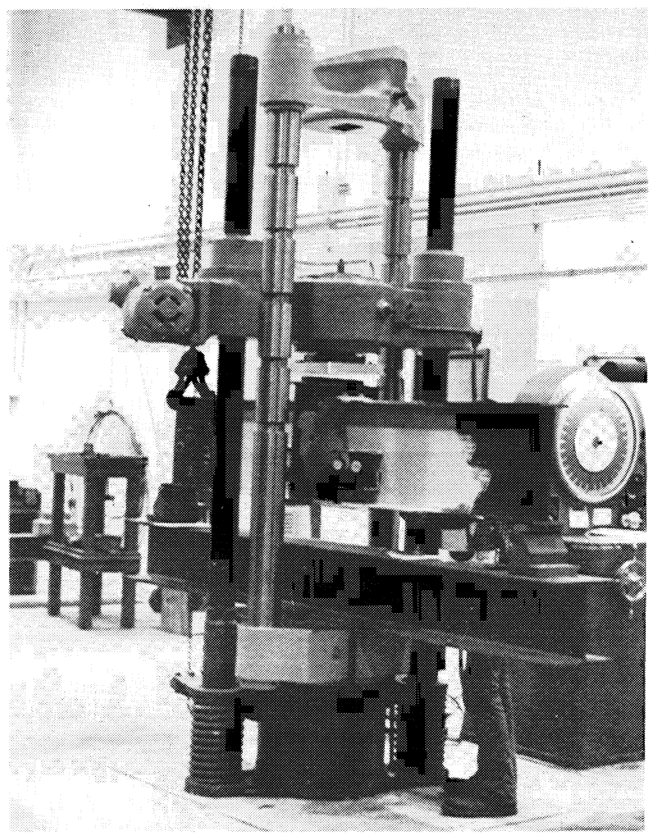

Fig. 7. Overall view of test specimen

2. The shoulder on the seat complements the 3:12 slope, providing a 30° groove angle for a prequalified butt-weld detail.
3. The butt-weld is made from directly above, and there is sufficient clearance and visibility for welding the full width of the flange in each pass.
4. The beam fabrication is limited to trimming the ends and punching of bolt holes.
5. The erection clearances are generous.
6. It produces a standard of quality and performance by which alternate connections, if considered, may be judged.
7. The estimated erected cost is equal to or less than other methods.

Load Test Of Connection—The specimen shown in Fig. 7 was fabricated by Metal Fabricators, Inc., Englewood, Col., and was tested in the Structural Testing Laboratory of the University of Colorado at Boulder. Short lengths of W18×40's were framed to both sides of a 1 ft-6 in. length of W21×44. Each compression flange was attached to an M-Seat connector with two $\frac{3}{4}$-in. A307 fit-up bolts and manually welded with a groove weld. The two M-Seat connectors were identical except for outstanding leg thicknesses, which were 0.45 in. and 0.65 in. The connectors were 7 in. long, machined from a steel billet. They were attached to the W21×44 by $\frac{5}{16}$-in. semiautomatic submerged arc fillet welds across top and down sides.

The tension flanges of the W18×40's were spliced by two 5 × $\frac{5}{16}$-in. bars manually welded with $\frac{1}{4}$-in. fillet welds. On one end, the bars were cut square, while on the opposite end, they were cut at an angle. The total length of weld applied to each end was the same, calculated for the direct tension strength of the plates. The bars also were welded in the middle to the W21×44 with 2-in. long welds. See Fig. 8.

No special care was taken in selecting steel components of the test specimen from stock, nor in fabrication. Consequently, it is felt that the specimen was typical of a production piece. The imperfections in the steel sections, while within usual tolerances, did cause some difficulties during testing, attributable to the test set-up, and cannot be related to performance in an actual structure.

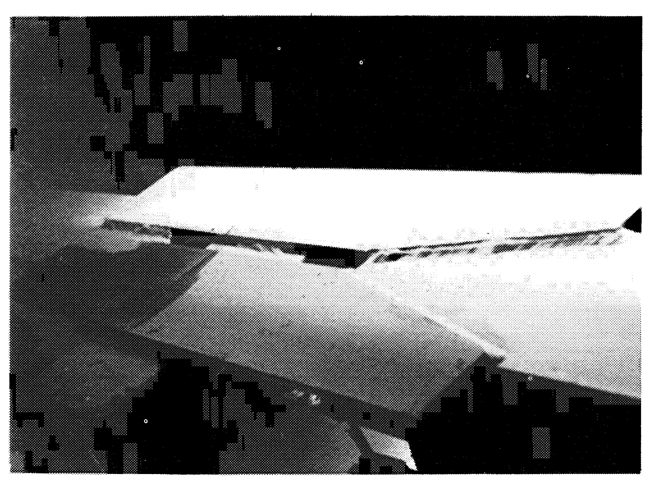

Fig. 8. Splice plates—no load

Three sequences of load were applied to the test specimen. The first was to observe the behavior of the entire specimen up to a load that would produce a stress of approximately 24 ksi in the tension flange connection. The next was to observe behavior of the M-Seat connection angles under severe shear loading, and the last was to observe behavior of the entire specimen as it was loaded well into the plastic range of the tension flange connection to form a plastic hinge. Before the specimen was installed in the testing machine, its middle portion was painted with a thin plaster of paris solution.

Photographs of the seats after loading are shown in Figs. 9 and 10. Figure 9 shows considerable deformation of the 0.45-in. leg, and only slight deformation of the 0.65-in. leg. Figure 10 shows yielding of the beam web and flange which developed over the 0.65-in. thick M-Seat angle leg, at loads well in excess of design loads. There was no evidence of web crippling or buckling.

The test gave insight into the manner in which the loads are transferred through the connection and provided information upon which the following conclusions are based:

1. The difference in performance of the diagonally cut tension flange splice bars and the square cut is of no significance.
2. Yielding across the entire width of splice bars, as is necessary for plastic hinge formation, can be achieved.
3. A 7-in. long M-Seat with an outstanding leg of 0.65-in. thickness welded with $^5/_{16}$-in. fillet welds across the top and down the sides will support an ultimate end reaction of no less than 50 kips. (It has been decided to make the outstanding leg 0.75 in., which should further increase capacity.)
4. This M-Seat Connector will support an ultimate load of 50 kips without buckling or crippling an unstiffened beam web of 0.315-in. thickness.
5. The vertical offset of 0.20 in. (such as might be encountered with beams of different weights) between the M-Seats on opposite sides of the web had no apparent effect.

Detailing and Specifying—A method for detailing and specifying continuous connections is shown in Figs. 11 and 12. The limitation noted in Fig. 12 applies only to the building frame and is not applicable to intermediate floor beams. Therefore, plastic design may be used for the intermediate beams without restriction, regardless of the design method used for the frame. In the event the fabricator wishes to submit an alternate detail, criteria can be included in the specifications to cover alternate, non-proprietary connections.

It is suggested that details provide for a $^1/_8$-in. differential in elevation of the top flange of the beams above the top flange of the girders to allow for beam depth tolerance and out-of-square tolerance of the girder flange. Seats should be at least $^1/_2$-in. wider than the beam flange to provide a run-off extension for the butt weld.

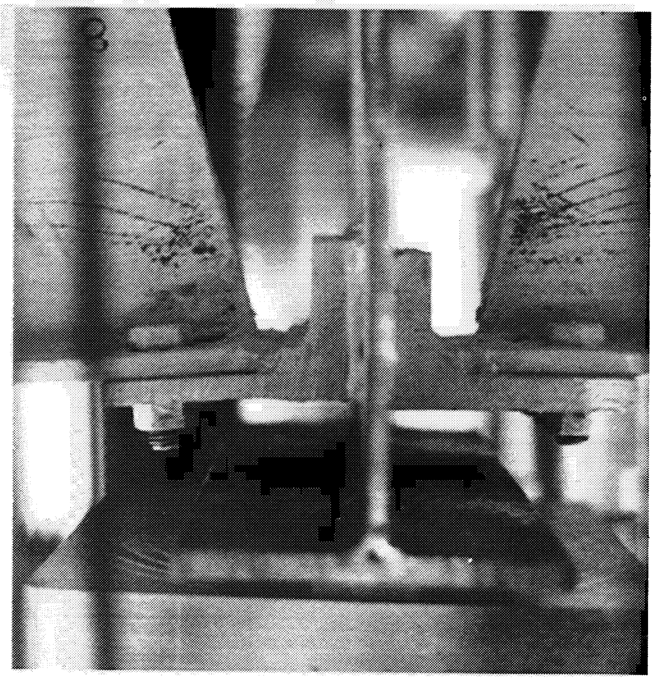

Fig. 9. M-seats at 90,000 lbs (45,000 lbs each beam)

The M-Seats are made from carbon steel forgings conforming to ASTM A668, meeting supplementary requirement S4 for weldability. They are furnished in lengths of 5, 7, and 9 in., which will accommodate a range of flange widths from 4 to $8^1/_2$ in. The shoulder is 0.60 in. in height, which equals or exceeds the flange thickness of the more common sections.

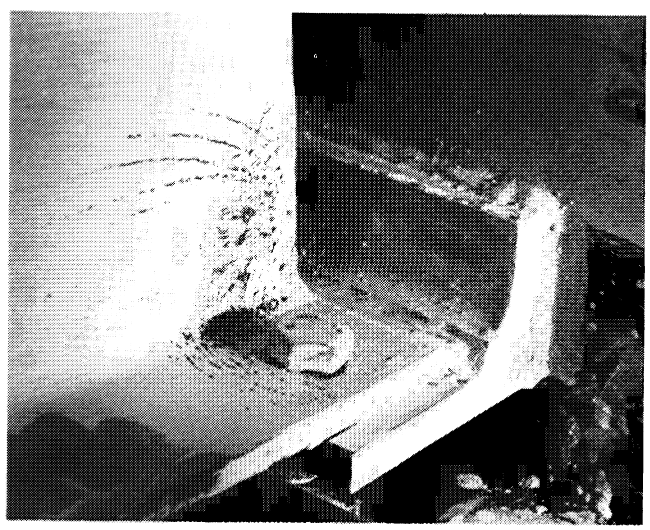

Fig. 10. M-seat (0.65-in. leg) at 100,000 lbs (50,000 lbs each beam)

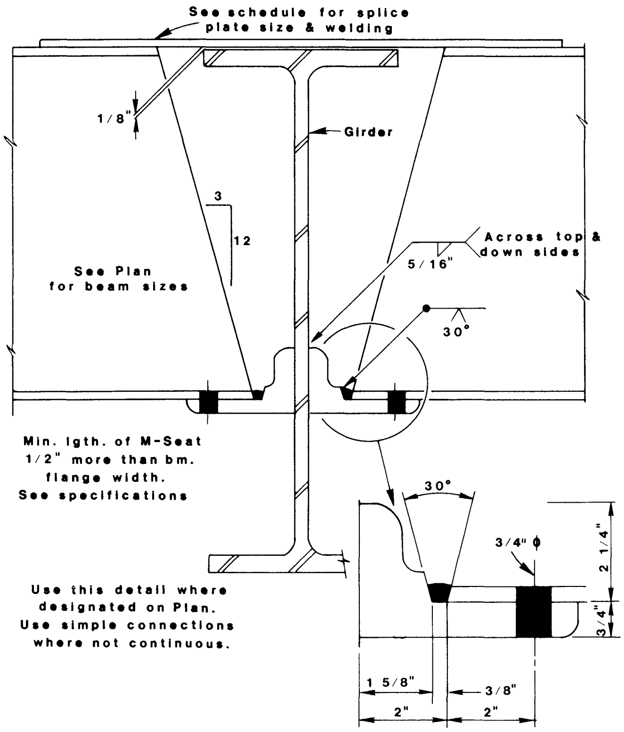

Fig. 11. Detail for beam-to-girder connection

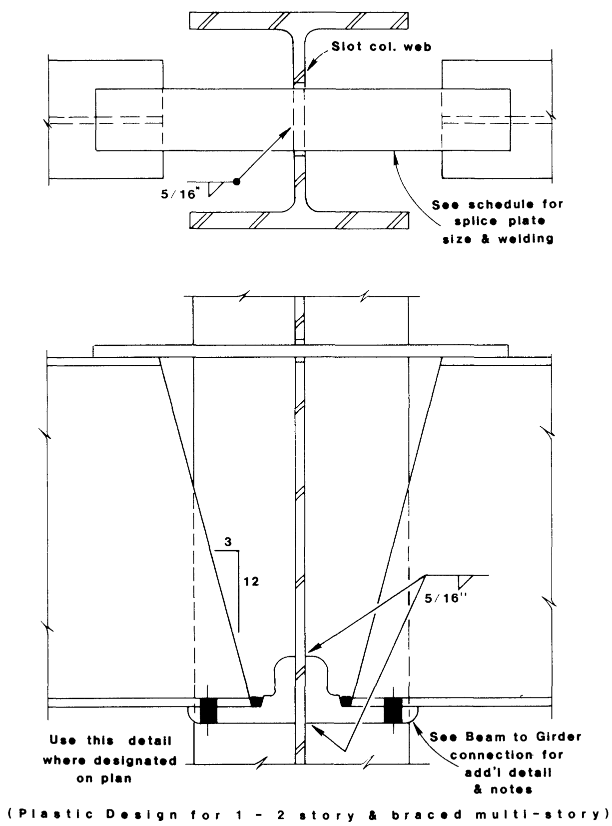

Fig. 12. Detail for beam-to-column connection

CONCLUSIONS

From the foregoing, it is evident that plastic design in general, and plastic-composite design in particular, does reduce the cost and improve the performance of steel beam floor systems.

The continuous connections need not be complicated. Using the M-Seat as a base detail, a means for specifying and detailing connections has been presented, which allows the fabricator and erector to use a method best suiting their fabrication and erection practices and capabilities, while adhering to sound standards for performance and integrity.

Continuous plastically designed floor beams have been used in numerous structures with favorable results in economy and performance. The first application of the M-Seat Connector was on an office and computer building for McDonnell-Douglas Corporation in St. Louis. The Architect was Hellmuth, Obata, & Kassabaum; Engineer, Jack D. Gillum and Associates; and Fabricator, Mississippi Valley Structural Steel Co.

DESIGN EXAMPLES

Given:

Spans: 30′-0″
Beam spacing: 10′-0″
Slab: 2½″ stone concrete on 2″ metal deck ($f'_c = 3{,}000$ psi)
Steel: $A36$ ($F_y = 36$ ksi)
Shear Connectors: ¾″ diam. stud
Dead load:

Slab and deck:	44 psf
Ceiling:	5
Mech. and elec.:	2
Beams:	3
Total D.L.	54 psf

Live load: 100 psf
ANSI reduced live load: 76 psf
Total load = 54 + 76 = 130 psf
$w = 130 \times 10 = 1300$ lbs/ft = 1.3 kips/ft

Comparison of Designs by Conventional Procedures:

Simple Spans:

$M = [(1.3)(30)^2]/8 = 146$ kip-ft **Use W18×46**

Plastic Design:

M_p (total) $= (1.7)\,[(1.3)(30)^2]/8 = 249$ kip-ft

Interior Spans:

Req. $Z + Z_f = (249)(12/36) = 83.0$ in.^{3}

From Table 1: **Use W16×31**

End Span:

Req. $Z = 83.0 - (0.45)(37.5)$
$= 66.1$ in.^{3} **Use W16×40**

Simple Span Composite:

$M = 146$ kip-ft

Req. $S_{tr} = (146)(12/24) = 73.0$ in.^{3}

From AISC table: **Use W16×31**

Using partial composite action **Use 26 studs**

Suggested Plastic-Composite Design Procedure:

(For explanation of procedure, see discussion entitled "Plastic-Composite Design".)

M_P (total) = 249 kip-ft

Interior spans:

Trial section:

Req. Z (total) = 83.0 in.^{3}

Z (interior) $\approx$ (0.45 to 0.50) $\times$ Z (total) $\approx$ 41 in.^{3}

Try W16×26: $Z = 44.2$ in.^{3}; $Z_f = 29.1$ in.^{3}

Z (req) $- Z_f = 83.0 - 29.1 = 53.9$ in.^{3}

Req. $S_{eff} = 53.9/1.1 = 49.1$ in.^{3}

For W16×26 w/4½″ slab:

$S_{tr} = 62.4$ in.^{3}; $S_s = 38.3$ in.^{3}; $A = 7.68$ in.^{2}

$V_h = [(7.68)(36)]/2 = 138$ kips

$V_h' = [(S_{eff} - S_s)/(S_{tr} - S_s)]^2 \, V_h = 27.4$ kips

Min. $V_h' = 138/4 = 34.5$ kips

$N_1 = 34.5/11.5 = 3$ studs; 6 studs/bm

Max. spacing = 8(4.5) = 36 in.

$\therefore$ Use 8 studs minimum for 22.5 ft

End Spans:

$$Z \text{ (req.)} - (0.45 Z_f) = 83.0 - [0.45 \times 29.1] = 69.9 \text{ in.}^3$$

S_{eff} (req.) = 69.9/1.1 = 63.5 in.^{3}

Using area under shear diagram:

$+ M_P = 207$ kip-ft

Req. $+ Z = 69.0$ in.^{3}

Req. $S_{eff} = 69.0/1.1 = 62.7$ in.^{3}

For W16×26: $S_{tr} = 62.4$ in.^{3}; $A = 7.68$ in.^{2}

$V_h = [(7.68)(36)]/Z = 138$ kips

$N_1 = 138/11.5 = 12$ studs; 24 studs/bm

Design Summary:

Interior: W16×26 + 8 studs

End: W16×26 + 24 studs

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