

# Discussion

## A Fresh Look at Bolted End-Plate Behavior and Design

Paper presented by N. KRISHNAMURTHY  
(2nd Quarter, 1978 issue)

Discussion by **Henning Agerskov**

The author must be congratulated for his investigations on the behavior of bolted tee-hanger and end-plate connections, which have resulted in interesting findings.

During recent years the writer has carried out an investigation on the strength and stiffness characteristics of these types of connections at the Technical University of Denmark. The main results obtained in this investigation have recently been reported in two ASCE Journal of the Structural Division papers (Refs. 4 and 5), and the writer wishes to comment briefly on some of the author's findings, in the light of the results of his own research.

The author has done valuable service in emphasizing that, when considering the tee-hanger model, the bending moment,  $M_1$ , at the load line is always larger than the moment,  $M_2$ , at the bolt line. Many previous approaches presuming values of  $M_2 > M_1$  have led to quite unrealistic results.

The writer does not agree with the author when stating that "except for very thick plates and at or near failure loads, the reactive pressures are distributed over extensive areas between the plate edges and the bolts." The writer finds the assumption that the resultant of the prying forces is at the yield load of the connection acting at the edge of the plate to be justified by his own and other investigations. However, the question has been discussed in detail in Ref. 11, and this discussion shall not be repeated here.

A number of details in the results presented might be discussed. However, the writer would like to concentrate on two items: the question of the magnitude of the resulting bolt forces in the connection and the deformation characteristics of the connection.

In the proposed design method, the theoretical bolt area  $a_t$  per row required is determined from Eq. (14):  $a_t =$

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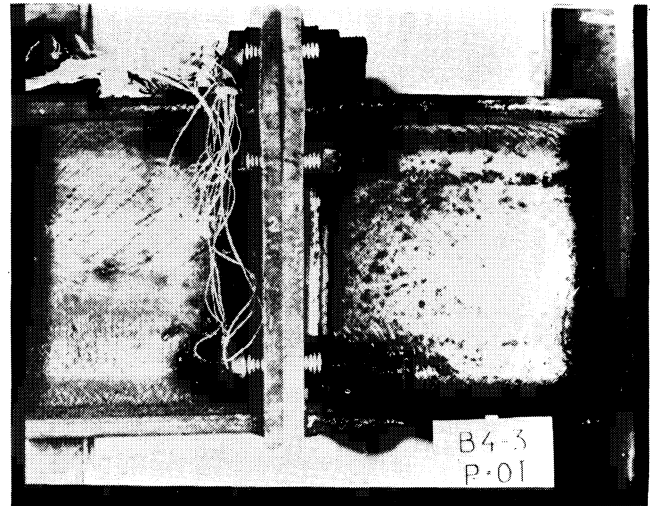


Fig. 7. End-plate connection from test series B4, after unloading (Ref. 4)

$0.5F_f/F_{bt}$ , i.e., the bolt area is calculated as half the nominal force in the beam flange divided by the allowable tensile stress in the bolt. In Eq. (14) the increase in bolt forces due to the prying action in the connection is quite neglected. This will only be acceptable in case of a connection with a heavy end-plate (e.g., see Figs. 7-9 of Ref. 5). The analytical and experimental investigations carried out at the Technical University of Denmark showed that the increase in bolt forces due to prying action may amount to about 50 percent, depending on the material properties and the geometry of the connection, and this increase may not in general be neglected. Further, the writer finds the statements concerning the distribution of the tensile force in the beam flange on the inner and outer rows of bolts questionable. It should not be accepted that, in some cases, "the inner row will be somewhat overstressed and the outer row understressed by the same amount." The author found the ratio  $C_1$  to vary from 0.3 to 0.5. If a value of 0.3 is supposed, the outer row of bolts carries 30 percent and the inner row 70 percent of the tensile force, while both are designed to take 50 percent of the force. Thus, according to the author's findings, the overstressing of the inner row of bolts due to uneven distribution of the flange force may be up to 40 percent of the bolt design force.

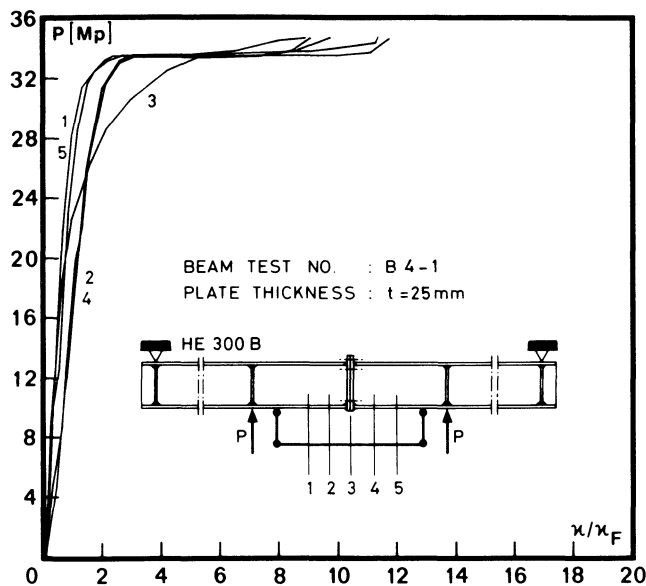


Fig. 8. Applied load vs. curvature in test B4-1 (Ref. 4)

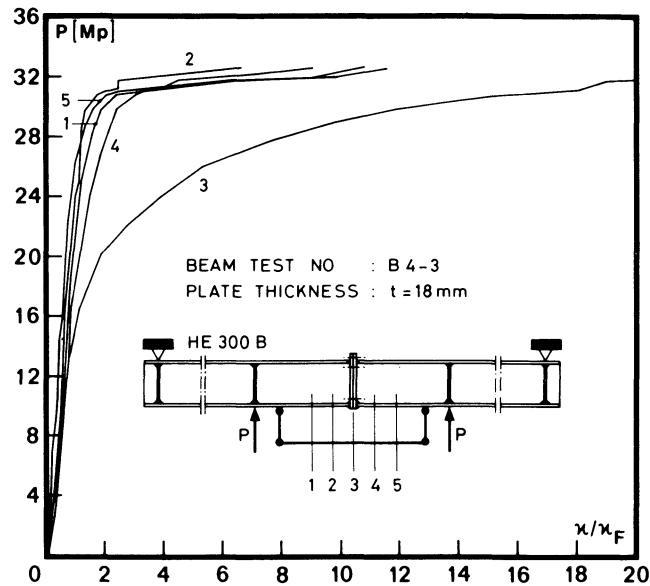


Fig. 9. Applied load vs. curvature in test B4-3 (Ref. 4)

The design method proposed by the author results in a considerable reduction in end-plate thickness. The author states that “the decrease in end-plate thickness by the proposed method will frequently be accompanied by some reduction in connection rigidity”. The writer finds that this will in reality always be the case and that this reduction in many cases may be considerable, and thus critical to the structure. In the experimental investigation carried out at the Technical University of Denmark, the influence of the end-plate thickness on the deformation characteristics of the connection was studied. It was found in the tests with relatively thin end-plates that the load could be increased beyond the yield load as defined in the theory proposed by the writer,<sup>4,5,12</sup> but this increase was accompanied by heavy plastic deformations arising after the yield load of the connection was reached and while strain hardening was taking place in the end-plate. Figure 7 shows the end-plate connection from one of the tests, after unloading. Considerable plastic deformations may be observed. It must be emphasized that this connection did not have an extremely thin end-plate. In the connection shown in Fig. 7, the end-plate thickness was equal to the thickness of the beam flange, approx. 18 mm (0.71 in.). More details on the test specimens may be found in Ref. 4.

The reduction in the connection rigidity due to the thinner end-plates further appears from Figs. 8 and 9. The diagrams show the relationship between the applied load,  $P$ , i.e., the moment in the central part of the test beam and the curvature,  $\kappa$  (a dimensionless ratio  $\kappa/\kappa_F$  is used). Curves 1 and 5 are by and large unaffected by the introduction of the connection in the beam, while the deviation between these curves and curve 3 shows the influence of the end-plate connection on the deformations. The test specimens

of tests B4-1 and B4-3 were practically identical, with the exception of a thinner end-plate used in test B4-3.

On the basis of the results of the experimental investigation at the Technical University of Denmark, the writer wishes to emphasize that a significant reduction in connection rigidity may be the result of a considerable reduction in end-plate thickness. This reduction in rigidity may be unacceptable in many structures, e.g., high-rise buildings.

#### REFERENCES

11. Agerskov, H. Closure of “High-Strength Bolted Connections Subject to Prying” (Ref. 4), *Journal of the Structural Division, ASCE, Vol. 103, No. ST 10, pp. 2066–2068, Oct. 1977.*
12. Agerskov, H. Analysis of High-Strength Bolted End Plate Connections *Bygningsstatistiske Meddelelser, Vol. 48, No. 3, pp. 63–93, Oct. 1977.*

#### Discussion by W. McGuire

The end-plate connection procedure proposed by Professor Krishnamurthy could be of major significance in design. Other investigators who have worked on this problem will probably agree that existing methods may be conservative. Any proposal that can justify more liberal procedures deserves careful attention. The purpose of this discussion is to make some observations and raise some questions that

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require satisfactory answers before the proposed procedure can gain unqualified acceptance.

Dr. Krishnamurthy notes that, "in reality, except possibly at very low load levels, the applied force overcomes the bolt clamping effects, and the bolts stretch and bend, permitting some rotation of the plates". Since, after initial tightening, the bolts may be at or above the proof load, any significant stretching or bending would appear to result in permanent fastener deformation and the possibility of loosening of the nuts under repeated loads. The writer wonders whether this could happen and, if so, how it is prevented in the proposed method. Some of the existing methods contain implicit provisions intended to retain the clamping effect up to working loads. The author's views on the importance of this aim would be helpful.

Essentially the same questions are raised by the author's statement that use of his method may result in some reduction in connection rigidity. Is there any possibility that this could be the result of bolt yielding and loosening? If so, what would be the consequences of such action under the number of repetitions of live and wind loads that might be expected in a building? Can the designer view connections designed by the author's method as AISC Type 1 connections, or should he be prepared to treat them as Type 3 or Type 2 and satisfy the AISC requirements for such types?

Dr. Krishnamurthy concludes that prying action is of limited importance. In the simplified split-tee model, however, any reduction of his term  $s/p_e$  below the value of unity results in the equilibrium requirement of either a prying effect or bending of the bolts. It is recognized that he views the split-tee model as mainly a convenience for routine design. Nevertheless, if one is to deny a requirement of statics, one should have a good reason for doing so. There does not appear to be one in the paper.

The author's coefficients  $C_1$ ,  $C_2$ , and  $C_3$ , along with possible beneficial strain hardening, seem to categorize the major behavioral phenomena in end-plate connections. Unfortunately, no attempt is made to explain behavior in terms of these coefficients. Rather, after introducing them and combining them as  $\alpha_m$  in Eq. (11a), they immediately disappear from the paper as  $\alpha_m$  is redefined in terms of physically less meaningful coefficients in Eq. (13). If, from Dr. Krishnamurthy's extensive analyses and tests, he can explain how each of the factors  $C_1$ ,  $C_2$ , and  $C_3$  enters into the reduction of the connection's actual complex behavior to the simplified split-tee model, and if he can do this quantitatively, he will have made an outstanding contribution to the understanding of an important and difficult problem.

Lastly, the work of Grundy, Thomas, and Bennetts on the same problem deserves attention (Ref. 13). Based on their studies, they suggest assuming full double-curvature plastic moment capacity in the design of end plates. For bolt design they suggest a flat 20% allowance for prying. Dr. Krishnamurthy's comments on Ref. 13 would be useful.

## REFERENCE

13. Grundy, Paul, Ian R. Thomas, and Ian D. Bennetts The Design of Beam-to-Column Moment Connections Using End Plates and High Strength Bolts *Second Conference on Steel Developments, Aust. Inst. of Steel Construction and Monash University, Melbourne, May 1977.*

### Discussion by John D. Griffiths and J. M. Wooten

Dr. Krishnamurthy's paper provides structural engineers with valuable new information on the behavior of end-plate connections. He is to be congratulated for his tireless energy in the completion of a most arduous task.

His work is comforting to those engineers of several decades past who have designed split-tee connections (Fig. 10) for use in tier buildings, powerhouses, and other large structures. This is so even though the author's study is not directly applicable to split-tee connections *per se*. His work is also comforting to those associated more recently with the design of end-plate moment connections for similar large structures and for most of the single-span rigid frames produced by the Metal Building manufacturers. Many thousands of these assemblies designed by the simple split-tee analogy have given a good account of themselves, even though theoretical questions have been raised as to their design. Thanks to Dr. Krishnamurthy's work, we now realize more clearly why all these connections have performed so well for so many years.

During several years of research, Dr. Krishnamurthy has progressively rationalized and simplified explanations of behavior and recommendations for design. His paper presents a design procedure that is convenient, in its present form, for use with a programmable computer. His simplified method can also be applied, albeit laboriously, with most pocket calculators.

The purpose of this discussion is to present a procedure based on the author's paper which requires little computation and a knowledge of only simple statics, so that the designer can retain a "feel" for the problem.

Dr. Krishnamurthy's work covers a wide range of variables associated with different beam and end-plate materials, bolt properties, and geometry. For the typical case, designs can be simplified if specific values are assigned for end-plate material and bolt pitch (see Fig. 11).

Unlike main material, plate material is usually taken from stock and custom fabricators ordinarily stock only A36 with thickness increments of  $1/8$ -in., except for thin plates.

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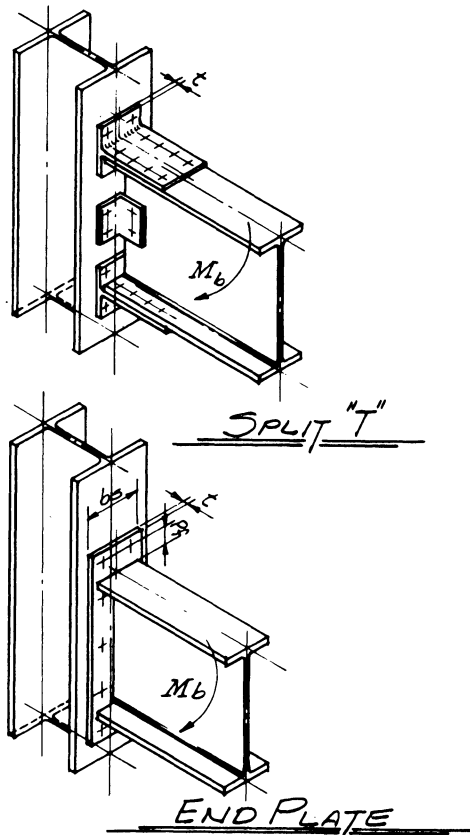


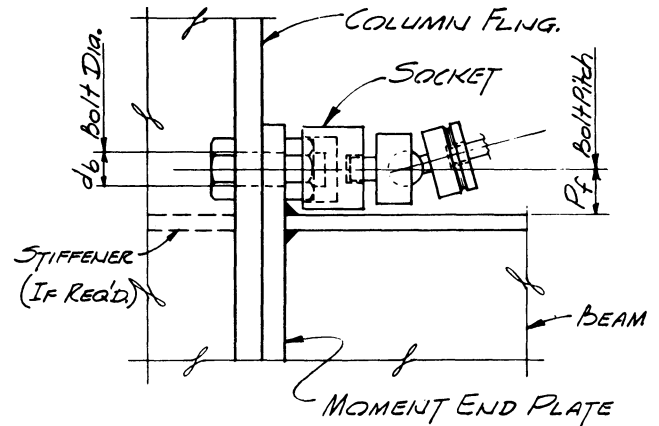
Figure 10

End-plate design is very sensitive to bolt pitch, with the minimum pitch providing greatest economy. As shown in Fig. 11, the minimum pitch (with a reasonable allowance for standardization and shop-field tolerance) can be taken as the bolt diameter plus 1/2-in. Therefore, for this discussion, it will be assumed that:

1. End-plate material is A36.
2. Beam material is A36 or 50 ksi yield.
3. Bolts are 3/4-in. or larger.
4. Bolt pitch  $p_f = d_b + 1/2$ -in.
5. Minimum actual plate width and maximum effective plate width  $b_e = b_f \times 1.15$ .

One word of warning! As stated before, the end-plate design is most sensitive to the bolt pitch  $p_f$ . A change from the stated assumption that  $p_f = d_b + 1/2$  cannot be tolerated by the design recommendations of this discussion. Any increase in  $p_f$  might result in an unconservative plate thickness; a decrease in  $p_f$  would likely result in unacceptable difficulties in field assembly. In fact, no change in the above stated assumptions should be made if the recommendations of this discussion are to remain valid.

In this paper, the author refers to a "modified split-tee analogy" and provides a formula for a preliminary plate moment  $M_t$ :



$d_b$	PF BASED ON 1/4" DRIVE CL. $d_b + 1/2$ "		MAXIMUM FILLET WELD
3/4" $\phi$	1 9/16"	1 1/4"	1/2"
7/8" $\phi$	1 5/16"	1 3/8"	1/2"
1" $\phi$	1 7/16"	1 1/2"	1/2"
1 1/8" $\phi$	1 9/16"	1 5/8"	1/2"
1 1/4" $\phi$	1 5/4"	1 3/4"	1/2"
1 3/8" $\phi$	1 7/8"	1 7/8"	1/2"
1 1/2" $\phi$	2	2	1/2"

Figure 11

$$M_t = \frac{F_f}{2} \times \frac{P_e}{2}$$

where  $F_f$  = total flange force  
 $p_e$  = effective bolt pitch ( $p_f - d_b/4$ )

He then multiplies the value of  $M_t$  by a coefficient  $\alpha_m$  to obtain a final plate design moment  $M_d$ :

$$M_d = \alpha_m M_t$$

where:

$$\alpha_m = 1.29 \left[ \frac{F_y}{F_{bu}} \right]^{0.4} \left[ \frac{F_{bt}}{F_p} \right]^{0.5} \left[ \frac{b_f}{b_s} \right]^{0.5} \left[ \frac{A_f}{A_w} \right]^{0.32} \left[ \frac{p_e}{d_b} \right]^{0.25}$$

It was previously noted that calculations for the above could be made on a non-programmable calculator. The writers have done so many times, but have not been able to establish confidence in their results because of errors they have often made, due to the number of variables and the complexity of the procedure.

The "non-modified split-tee analogy" of past years directly computed a plate design moment  $M$  based on a lever arm  $p_f$  instead of ( $p_f - d_b/4$ ):

$$M = \frac{F_f}{2} \times \frac{p_f}{2}$$

Table 1

Section	$A_f/A_w$	$R$	Section	$A_f/A_w$	$R$	Section	$A_f/A_w$	$R$
W36 × 300	0.887	0.98	× 101	0.995	1.00	W12 × 87	1.748	1.09
× 280	0.882	0.98	× 93	0.683	0.94	× 79	1.732	1.09
× 260	0.850	0.97	× 83	0.686	0.94	× 72	1.720	1.09
× 245	0.835	0.97	× 73	0.683	0.94	× 65	1.706	1.09
× 230	0.818	0.96	× 68	0.667	0.93	× 58	1.631	1.08
× 210	0.588	0.92	× 62	0.641	0.93	× 53	1.527	1.07
× 194	0.587	0.92	× 57	0.532	0.90	× 50	1.281	1.04
× 182	0.579	0.91	× 50	0.465	0.88	× 45	1.266	1.03
× 170	0.573	0.91	× 44	0.423	0.87	× 40	1.281	1.04
× 160	0.554	0.91				× 35	0.992	1.00
× 150	0.530	0.90	W18 × 119	1.082	1.01	× 30	0.963	0.99
× 135	0.463	0.88	× 106	1.059	1.01	× 26	0.936	0.99
			× 97	1.076	1.01	× 22	0.575	0.91
W33 × 241	0.853	0.97	× 86	1.056	1.01	× 19	0.520	0.90
× 221	0.829	0.97	× 76	1.048	1.00	× 16	0.419	0.87
× 201	0.807	0.96	× 71	0.741	0.95	× 14	0.390	0.86
× 152	0.612	0.92	× 65	0.751	0.95			
× 141	0.583	0.91	× 60	0.751	0.95	W10 × 60	1.842	1.10
× 130	0.541	0.90	× 55	0.722	0.95	× 54	1.882	1.10
× 118	0.492	0.89	× 50	0.714	0.94	× 49	1.859	1.10
			× 46	0.604	0.92	× 45	1.603	1.07
W30 × 211	0.905	0.98	× 40	0.595	0.92	× 39	1.516	1.07
× 191	0.887	0.98	× 35	0.504	0.89	× 33	1.348	1.05
× 173	0.861	0.97				× 30	1.045	1.00
× 132	0.606	0.92	W16 × 100	1.170	1.02	× 26	1.033	1.00
× 124	0.590	0.92	× 89	1.152	1.02	× 22	0.913	0.98
× 116	0.558	0.91	× 77	1.146	1.02	× 19	0.672	0.94
× 108	0.516	0.90	× 67	1.149	1.02	× 17	0.583	0.91
× 99	0.476	0.88	× 57	0.789	0.96	× 15	0.497	0.89
			× 50	0.781	0.96	× 12	0.463	0.88
W27 × 178	0.909	0.98	× 45	0.768	0.96			
× 161	0.902	0.98	× 40	0.772	0.96	W8 × 35	1.796	1.09
× 146	0.885	0.98	× 36	0.679	0.94	× 31	1.711	1.09
× 114	0.646	0.93	× 31	0.589	0.92	× 28	1.495	1.06
× 102	0.635	0.93	× 26	0.506	0.89	× 24	1.487	1.06
× 94	0.597	0.92				× 21	1.127	1.02
× 84	0.545	0.90	W14 × 120	1.855	1.10	× 18	1.007	1.00
			× 109	1.899	1.10	× 15	0.690	0.94
W24 × 162	0.994	1.00	× 99	1.859	1.10	× 13	0.593	0.92
× 146	0.959	0.99	× 90	1.860	1.10	× 10	0.635	0.93
× 131	0.904	0.98	× 82	1.348	1.05			
× 117	0.877	0.98	× 74	1.394	1.05	W6 × 25	1.580	1.07
× 104	0.848	0.97	× 68	1.382	1.05	× 20	1.545	1.07
× 94	0.683	0.94	× 61	1.364	1.05	× 15	1.238	1.03
× 84	0.655	0.93	× 53	1.141	1.02	× 16	1.148	1.02
× 76	0.616	0.92	× 48	1.115	1.01	× 12	0.890	0.98
× 68	0.560	0.91	× 43	1.103	1.01	× 9	0.911	0.98
× 62	0.428	0.87	× 38	0.861	0.97			
× 55	0.397	0.86	× 34	0.824	0.97	W5 × 19	1.867	1.10
			× 30	0.734	0.95	× 16	1.748	1.09
W21 × 147	1.011	1.00	× 26	0.633	0.93			
× 132	1.002	1.00	× 22	0.557	0.91	W4 × 13	1.442	1.06
× 122	1.003	1.00						
× 111	0.994	1.00						

It can be shown that this moment  $M$  closely approximates  $M_d$  in the case of ( $A_f/A_w$ ) values for most wide-flange sections used as beams.

In the case of fillet welds, the author further modifies the lever arm  $p_f$  by subtracting  $w_t$  from the value ( $p_f - d_b/4$ ). A similar procedure can be used for the value of  $M$ , except that the weld leg size may be deducted instead of the weld

throat size, i.e.,  $M = [F_f(p_f - w_s)]/4$ , with the added limitation that the weld size not exceed  $1/2$ -in. When fillet welds would exceed  $1/2$ -in., many fabricators consider it economical to change to groove welds. In the case where fillet weld reinforcing is used for such groove welds, the vertical leg dimensions of the fillet weld may be deducted from  $p_f$ .

Table 1 lists the W sections to be included in Part 2 (Beam and Girder Design) of the 8th Edition of the AISC Manual. The values of  $A_f/A_w$  and  $R$  are shown for each section.

$$R = t_s/t \text{ or } t_s = Rt$$

where  $t_s$  = plate thickness based on  $M_d$  (the basic paper).  
 $t$  = plate thickness based on  $M$  (the discussion).

The values of  $R$  are maximum values based on the least favorable combinations of variables.

Thus, where the value of  $R$  is 1 or less, the plate thickness based on  $M$  is conservative. These  $R$ -values are not presented with the thought that they will be used as factors to achieve a more accurate result. Rather, the listing was developed to show that the "split-tee analogy" (not "modified" except for welds) can be applied with safety over a wide range of sections. Note that for all beams with depths from 16 through 36-in., the maximum value of  $R$  exceeds unity by only 2 percent. Also, for at least the lightest series of sections in the 6 through 14-in. depths,  $R$ -values are within this 2 percent range. These lighter sections are the ones most often used for end-plate designs.

So much for the conservative aspect of this procedure. How about economy? Although variables have been reduced to be in line with the practice of custom fabricators, some variables, such as bolt materials and weld and bolt sizes, remain. It was mentioned previously that  $R$  was chosen as a maximum. If the true value of  $R$  is less than the maximum value recorded in Table 1, the plate thickness will be greater than required. The greatest difference between maximum and minimum values of  $R$  for any individual wide-flange section shown in Table 1 is 7%.

Design examples for end plates, developed by the authors, by AISC, and by others, are consistently based on the case where full design moment of the beam must be accommodated. In practice, design for full beam moment capacity is often not required. It should be realized that Dr. Krishnamurthy's procedure and the procedure herein discussed are applicable with complete safety for any moment equal to or less than the maximum beam capacity.

This suggested design method will be of slight interest to engineers with Metal Building manufacturers. These firms are computer-equipped to a high degree. Also, for good reasons, they are extremely weight conscious, whereas custom fabricators often experience increased costs if too much emphasis is placed on saving the last pound of steel. Likewise, engineers with large design firms will have little interest, provided their firms use enough end-plates to warrant computer availability on this one item to all hands at all times.

We believe that many professionals would prefer a less complex method for design and would also like to have a procedure for a quick check of proposed details prepared by others. The simple, easily remembered method pre-

sented above retains most of the benefits of Dr. Krishnamurthy's investigation, but eliminates the necessity for charts and tables. Of greatest importance, it reduces the possibility of errors inherent in the calculation of lengthy formulas.

#### DESIGN EXAMPLE\*

Given:

W16X45 section  
 Applied moment: 1723 kip-in.  
 1/2-in. welds

Solution:

$$F_f = \frac{1723}{(16.12 - 0.563)} = 111 \text{ kips}$$

$$111/4 = 27.8 \text{ kips}$$

Use 4 - 1-in. A325 bolts

$$p_f = d_b + 0.5 - w_s = 1.0 + 0.5 - 0.5 = 1 \text{ in.}$$

$$M = 0.25 \times 111 \times 1 = 27.8 \text{ kip-in.}$$

$$\text{Maximum } b_e = 7.039 \times 1.15 = 8.09 \text{ in.}$$

Use 8-in. plate.

$$t = \sqrt{\frac{6 \times 27.8}{8 \times 27}} = 0.879 \text{ in.}$$

Use 8 x 7/8-in. plate.

The original design used an 8 1/2 x 13/16-in. plate. On the basis that plates are stocked in 1/8-in. thickness increments, most fabricators would have furnished an 8 1/2 x 7/8-in. plate.

#### Discussion by Norman W. Rimmer

Dr. Krishnamurthy is to be congratulated for the in-depth study on the bolted end-plate moment connection and for the relatively simple design formula that he has developed for a very complex design problem.

I have reviewed the final report written by Dr. Krishnamurthy, which reports the findings of the testing of 9 test specimens that were fabricated with end-plates based on the newly developed end-plate design procedure. It is quite clear from this report that, when this method of design is used, the connection has a capacity to develop the full moment strength of the beams they are connecting.

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\* Same example as Dr. Krishnamurthy's Appendix E.

I thought it would be of interest to compare the end-plate thickness required for these 9 specimens to that which would be required utilizing the design method developed by Butler Manufacturing Company in 1954. This comparison indicates that the two methods give results within 10 percent of each other. The two largest differences between the two procedures had a magnitude of 7.47 percent and 10.11 percent, and in both cases the Butler procedure indicated that a thinner plate could be used. Where the new method developed by Dr. Krishnamurthy indicated the thinner of the two procedures, the maximum difference was 6.6 percent.

The method developed by Butler Manufacturing Company was first used in 1954 for the development of the new line of buildings that went into production in 1955. All Butler metal buildings have been designed since that time utilizing this procedure. We have load tested to failure many rigid frames at our research center. In all of the tests that have been conducted, we have never had a failure of a bolted end-plate moment connection. The failures have always occurred in the beam section.

Since the introduction of this type of bolted end-plate connection beginning with our production in 1955, we have produced literally thousands of buildings each year during the last 24 years. In all these years, with these many thousands of buildings, we have never to our knowledge experienced a bolt or an end-plate failure in a bolted end-plate splice connection. We believe this speaks very well for the use of this type of connection in our structures.

#### Discussion by N. Krishnamurthy

The author thanks the discussers for their many useful comments, and will attempt to clarify and amplify his statements, supplying supportive information wherever possible. He will first respond to the discussions by Professors Agerskov and McGuire, because they have overlapping concerns and they raise questions requiring answers.

To Professor Agerskov's comment on the reactive pressure distribution, the author concedes that his statement regarding its nature was oversimplified. Not only the plate thickness and load level, but the bolt distance, pretension force, and many other parameters also affect the reactive pressures in a highly nonlinear fashion. The point that the author was making was that except under certain unusual combinations of these various parameters, and only at the *yield load* (emphasis Agerskov's), the prying force was a distributed pressure bulb, and not critical as far as the plate moment was concerned.

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The author agrees with Professor McGuire that any plate moment at the bolt line (corresponding to an  $s/p_e$  value less than unity) must imply the presence of a reaction to satisfy static equilibrium. But he also wishes to point out that this moment of resistance can be offered by a combination, in varying degrees, of: (a) unsymmetric clamping force around the bolt head, (b) bending of the bolt shank, and (c) eccentric reactive pressure or force at the back of the plate.

In a photoelastic study<sup>14</sup> that the author directed — which to his knowledge was the first time the pressure distributions at the back of the plate under various applied loads were directly observed and recorded — the reactive pressures in the plate projection and the bending stresses at the bolt line were found to be so small that they could not be easily quantified. The author's numerous finite element analyses and many strain measurements on the plates of his steel test specimens also overwhelmingly support this conclusion.

In any case, the author believes he has demonstrated conclusively his contention that, where strength is concerned, his proposed plate thickness, completely devoid of any prying force consideration in its design, is quite adequate. In particular, as was mentioned in Appendix F of the paper and described in greater detail in the author's discussion<sup>15</sup> of Professor Agerskov's paper,<sup>5</sup> the 7/8-in. plate designed by the author's procedure (and predicted by Agerskov's method to develop only 62% of the capacity of the attached W16 X45 beam), developed the full capacity of the beam with no overall distress to the plate.

On this basis, the author would prefer to drop the concept of "prying force" as the prime consideration covering both the plate thickness and bolt size as assumed at present, and instead discuss the bolt force increase as a separate phenomenon not directly or necessarily related to prying.

That the bolt force increases under applied load is a fact, but it in itself should not be cause for concern. The magnitude of the increase of course merits further discussion.

As the author has pointed out in his discussion,<sup>15</sup> the bolt force increases reported by Professor Agerskov himself in his paper<sup>5</sup> are of a low order, mostly less than 10%, at the maximum applied loads reported. Without more detailed information to the contrary, it can only be surmised that the 50% bolt force increases that he refers to in his present discussion of the author's paper could happen only under extreme (and unlikely?) circumstances. (Incidentally, it may be noted that a 50% increase over the recommended pretension of  $0.7F_u$  would put the final bolt force at 5% over its nominal ultimate capacity, implying that the actual bolt strength was at least that much higher than nominal.)

The author's analytical and experimental findings also confirm that at beam service load levels, except in rare instances, the force in the bolts designed by his procedure increases by about 5% at service load levels. Beyond service loads, the bolt force increases at a faster rate. Close to the

beam yield, the force increases sharply, reaching or occasionally passing the *nominal* yield of the bolt. But even at beam failure, there is generally a 5 to 10% reserve bolt capacity below ultimate strength. In the W16 X45 beam test referred to earlier, the bolt force increased only by about 15% at beam failure. In all the tests that the author has conducted on connections designed by his procedure, no bolt has completely failed.

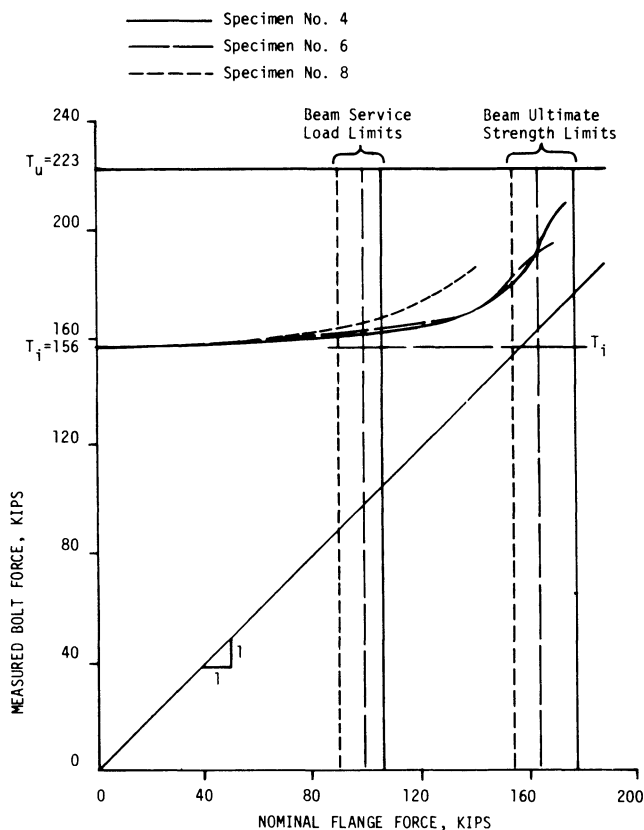


Fig. 12. Variations of measured bolt force with increasing beam loading, for author's specimens, with 7/8-in. bolts.

Figure 12 depicts the bolt force variations from three of the author's tests, illustrating the general behavior described in the preceding paragraph. The nominal flange force is calculated from Eq. (4). The bolt force is calculated as the nominal pretension plus the average increase determined from the strain gages attached to all four bolts around the tension flange. As all plotted quantities for the bolt are based on nominal values, and actual strengths are invariably higher than nominal, additional reserves may be anticipated in a practical situation.

The test connections were designed on the basis of the mill-sheet values of the yield stress  $F_y$  for the beam and plate materials. The service load and ultimate strength limits for beam capacity marked in Fig. 12 were also computed for the appropriate mill-sheet values. In the cases plotted, the actual  $F_y$  values from coupon pull tests were not significantly different from the mill-sheet values.

The author is familiar with the work of Grundy, Thomas and Bennets, and is pleased to note their independent confirmation of many of his findings. In the paper mentioned by Professor McGuire, they state, referring to British practice, "The bolt sizes appear to be less than those required by statics. Nevertheless, tests have shown the connections to be satisfactory . . ." They also (again qualitatively) hypothesize reasons for the prying force "being much less in practice than predicted by simplistic theories," endorsing indirectly the author's general lack of concern about prying force in end-plate connections. They quantify the prying force at 10 to 15% of the greatest bolt force, placing the total force well below the bolt capacity.

In regard to Professor Agerskov's comment on the distribution of the tension flange force to the inner and outer rows, two factors must be borne in mind: (1) The author's lower limit of 0.3 for  $C_1$  corresponding to the outer row force was reached only at low loads in a few cases among the hundreds of connections analyzed. (2) Based on the recommended bolt pretension of 0.7 times the ultimate strength, even the 40% increase that Professor Agerskov computes would not be catastrophic in theory, as the maximum force would be still below the ultimate strength.

In reality however, the concern about reaching so close to the ultimate strength due to unbalanced distribution of the bolt forces is unfounded. For, by the time the beam loading exceeds service load levels, the beam web generally yields at the corner between the beam flange and end plate (as the author has observed in this computer analyses and some tests), redistributing the subsequent beam flange forces more evenly to the outer and inner rows.

On the basis of the foregoing, the author reiterates his conviction that his proposed bolts, sized only for the nominal flange force, would be safe against failure prior to beam failure.

The author can respond to Professor McGuire's comments about the coefficients  $C_1$ ,  $C_2$ , and  $C_3$  more in philosophical than in mathematical terms. The coefficients were intended for convenience of conceptualization, rather than as steps in the development of governing equations. It was hoped that the reader would thus identify the many complex phenomena that would modify the idealized value of the split-tee moment, as described in the paper. Once these concepts were lumped into the single modifier  $\alpha_m$ , the original coefficients had served their purpose and were allowed to "immediately disappear."

In fact, the situation is similar to Professor McGuire's development of the prying force expression in his book,<sup>16</sup> wherein he introduces the two stiffness coefficients  $k_b$  and

$k_p$  symbolically for purposes of discussion, and subsequently replaces them with empirically developed expressions.

The author, too, went through numerous theoretical derivations to develop an expression for  $C_1$  on the basis of a “stepped-beam analogy” for the end plate, with a larger stiffness on the beam web side compared to the plate projection beyond the beam, and another similar expression for  $C_2$  on the basis of the smaller stiffness along the bolt line as compared to the load line. He could also have followed through with a numerical example, very much like the one McGuire used in the prying force section mentioned earlier, (in his words) “to illustrate a principle rather than as a quantitative calculation of representative relative stiffness.” Because such derivations and examples cannot be used for practical applications, and are generally supplanted, or at least considerably modified, by other considerations, the author prefers not to have to go through them at this time.

The massive computer output from the finite element analysis would, of course, have been a better source of valuable and precise information, and the author considered quantifying  $C_1$  and  $C_2$  from that data. At best the author could have determined two more expressions of the same general form as Eq. (13), and possibly even more complicated. This would only have resulted in the proliferation and inbreeding of regression equations, without the redeeming suitability for plate design that  $\alpha_m$  had.

These difficulties, accentuated by the limitations of time and scope, compelled the author to stop with stating the ranges of variation of  $C_1$  and  $C_2$ . Even had they been determined more precisely, there would have been no reasonable way to define  $C_3$ , the catch-all coefficient to explain the force dispersions, deep-beam action, and many other effects.

Further, as the author has emphasized in the paper, the redefinition of  $\alpha_m$  by Eq. (13) was a gratuitous fallout from a purely mathematical exercise. Since no physical basis was imposed on (or could be expected from) the statistical regression analysis, it was, at least to the author, a pleasant surprise that the computed data revealed groupings of dominant parameters with such recognizable physical significance. Far from being “physically less meaningful” than  $C_1$ ,  $C_2$ , and  $C_3$  as Professor McGuire states, the parameters in  $\alpha_m$ , determined to a high degree of statistical correlation, are more meaningful than the originally assumed coefficients. The entire development leading to Eq. (11a) could have been omitted, and the validity of the remaining expressions and interpretations would not have been altered.

With reference to Professor McGuire’s interest in the double curvature assumption and plastic moment approach of Grundy *et al.*,<sup>13</sup> the author can point out that his procedure also involves both concepts in some form.

The split-tee moment  $M_t$  by the author’s Eq. (6) is the double-curvature moment based on an effective bolt dis-

tance  $p_e$ , instead of the actual bolt distance  $p_f$ . Further, the author’s prediction equation (A.2) for  $M_d$  covers the entire range of elastic-plastic behavior. In any case, if Eq. (A.2) is rewritten with  $F_p$  of  $(0.75F_y)$ , we get

$$M_d = F_y(b_s t_s^2 / 4.5) \quad (18)$$

which would be very close to the plastic design capacity  $(F_y Z)$ ,  $Z$  being the plastic section modulus  $(b_s t_s^2 / 4)$ . The author and his assistants have also examined applications of yield line theories to end-plate connections, and have resolved the complex mechanisms that Grundy *et al.* disregarded in their paper. Some of these developments will be submitted for publication shortly.

There is no question that thinner end plates *generally* result in decreased connection rigidity. In his paper, the author has not only pointed this out, but warned designers against using thinner plates where deformations could be critical. Professor Agerskov’s comments reinforce this point.

It must be pointed out, however, that Professor Agerskov’s Figs. 8 and 9 must not be interpreted as an illustration of the larger flexibility of connections designed by the author’s procedure, compared to Agerskov’s or other methods. In fact, to develop the service capacity of the test beam HE300B, the plate thickness by the author’s procedure would be 35 mm, considerably thicker than the two plate thicknesses compared, namely 24 mm and 18 mm. With such thicker plates, the loss of rigidity would be of less significance than in the cases represented by Figs. 8 and 9. Situations also occur when neither of two plates of different thicknesses cause any rotation up to working load levels — this being the author’s reason for admitting to reduction of connection rigidity with thinner plates “generally” rather than “always.”

Incidentally, the test specimen of Figs. 8 and 9 had four bolts per row, and the author took the following approach in applying his procedure to this case:

With  $n$  bolts per row, Eq. (A.7) becomes

$$a_b = (n/4)\pi d_b^2 \quad (A.7a)$$

and it may be shown that consequently  $\alpha_m$  would be *reduced*, by the factor  $(2/n)^{0.1235}$ . The important point to note is that more than two bolts per row would improve the connection, which may also be intuitively appreciated. However, without further validation tests, the author would not recommend the designer taking advantage of this reduction, which would in any event be only 4% in plate thickness with four bolts per row.

Thus, without arguing against the general validity of statements concerning the increased flexibility of thinner plates, the author would still maintain that, in many cases, such increased flexibility may be insignificant and/or not intolerable. Further, as the author commented on “excessive” deformation in his discussion<sup>15</sup> of Agerskov’s paper, the limiting values of deformations are a very subjective

topic. As long as the designer is assured of equal strength of two connections with different stiffnesses, he should have the option to choose the one that would suit his circumstances, very much as he has alternatives in other structural details such as with friction-type versus bearing-type connections.

Professor McGuire raises questions regarding the performance of the proposed connections under repeated loads. Because all of the author's tests were run under monotonically increasing loads (except for rare unloading and reloading for some adjustment), his procedure is not applicable to dynamic loading.

On the other hand, the repeated loads that Professor McGuire mentions as examples, namely live and wind loads, are usually considered as static equivalents. For these and all other loads which would be treated as static loads in conventional analysis and design, the author's procedure is equally applicable, in his opinion.

Certainly the proposed connections would be much more rigid than a Type 2 simple connection. But whether a connection with the proposed plate is Type 1 or Type 3 may be a question of terminology. Many of the proposed connections would hold the original angles virtually unchanged, within the working load levels; many would not. But as more and more data are being gathered, the moment-rotation characteristics of various connections can be documented and in due course predicted, and the computational skills and tools are now sufficiently sophisticated for the actual connection behavior to be incorporated into the structural design. Reference may be made to the author's recent paper<sup>17</sup> on the prediction of moment-rotation characteristics of end-plate connections, in which prediction equations and general methods of designing connections with desired rigidity ratios are outlined, some permitting frames to be analyzed as conventional rigid frames.

While hoping that by the preceding explanations the author has allayed the concerns occasioned by his proposed liberalization of the current end-plate design procedure, he would also offer the following as heuristic arguments:

The proposed procedure is based on the findings from investigations as logical and precise as those underlying many other semi-empirical formulas by which engineers design complicated components today. The common denominators to this evolutionary process are: (a) formulation of an idealized model (rigid frame, finite element, etc.), (b) development of governing equations for general behavior (by structural mechanics, regression analysis of parametric data etc.), (c) validation by (and/or adjustment of the prediction equations to reflect) test results, and (d) simplification to suit the user's convenience and accepted practice.

If the proof of the rightness of the conclusions and the efficiency of the recommendations from such a process lies in their practical demonstration, the author's proposals have already been vindicated. Apart from the many controlled tests conducted by the author to validate his procedure,

millions of end-plate connections with plates and bolts smaller than by the AISC recommended procedure, have performed satisfactorily in service (mainly in low-rise structures) for decades now, with no loosening of bolts or other deterioration.

The author appreciates the complimentary remarks by Messrs. Griffiths and Wooten. In turn he is happy to acknowledge the constant guidance and support he has received from them, as well as from other members of the AISC/MBMA Task Committee on End-Plate Connections, in the long and complex investigation culminating in the proposed design procedure.

Griffiths and Wooten have presented a further simplification of the author's procedure, applicable to the specific set of material and dimensional parameters detailed in their discussion.

Stated briefly, for the particular assumptions they have listed, and for sections with  $(A_f/A_w)$  ratio smaller than about 1.15, the split-tee method (modified only for fillet weld size), would give from 14 percent overdesign to 2 percent underdesign of the end plate, in comparison with the author's procedure. Under these conditions, when simplicity and speed are more important than maximum economy, their approach would indeed be more convenient.

As they have carefully pointed out, their suggestion is *not* a substitute for the author's proposed method in the general case. Any and every deviation from their assumed limits would affect the outcome. Moreover, the influence of  $(A_f/A_w)$  ratio has been omitted, conservatively, for the limits they have defined; to appreciate the significance of this omission, it may be noted that a plot of the variation of the discussers'  $R$  values with  $(A_f/A_w)$  ratio would be very similar to the author's Fig. 5 or 6.

To extend the benefit of the "feel" Messrs. Griffiths and Wooten cite for the split-tee method to his procedure also, the author would point out that the proposal is still the split-tee, with two simple modifications:

- (a) Use of an effective bolt distance, increasing the economy, but not essential to the method
- (b) Use of a moment modification factor to account for the numerous complex interactions of material properties, beam proportions, etc.

While the discussers' simplification (or similar upper-bound techniques) are valuable short cuts for specific situations, other design aids may also be developed, retaining the generality and economy of the basic procedure. The author's Appendices C and D were intended to be such aids. Appendix C may also be presented as a series of charts of  $(A_f/A_w)$  versus  $\alpha_m$  curves for various values of  $(p_e/d_b)$ , one chart for each value of  $C_a C_b$ . It is feasible to prepare tables of minimum bolt sizes and plate thicknesses to develop the full capacity of standard rolled sections for general use, and of typical built-up sections for in-house use by pre-engineered metal-building companies.

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