Plastic Design Aids for Pinned-Base Gabled Frames

GEORGE D. MANOLIS AND DIMITRIOS E. BESKOS

Single-span rigid steel gabled frames are widely used in the construction of structures covering large areas with no obstructions, such as industrial buildings, auditoriums, warehouses, etc. It is the intent of this paper to facilitate the design of such structures by offering charts for the ultimate strength requirements of the frameworks that comprise them.

In the case of statically indeterminate structures, such as most rigid frames, the methods of plastic design are more rational procedures than their elastic counterparts and yield lighter, yet perfectly adequate, designs. Furthermore, it is possible to realize additional weight savings by using haunches at the eaves of the frame, which strengthen the structure, and by using different column and rafter sections that result in a more efficient distribution of material.

Only pinned-base frames are investigated, since complete end fixity is not only difficult to achieve in practice, but is also expensive. Besides, in the case of a differential settlement, the results are less detrimental to the frame if the bases of the columns are pinned rather than fixed.

The subject of plastic analysis and design is well established. References 1 through 5 comprise a representative list of the earlier publications. Specifications that govern plastic design in this country, the United Kingdom, and Western Europe are given in Refs. 6, 7, and 8, respectively. Design aids for multistory, multispan steel frames have also appeared in the literature. Ketter[9] was the first to produce charts pertaining to the rapid design of one-story, pitched roof, multispan frames composed of uniform section members. Single-span gabled frames with pinned or fixed bases and uniform member sections under vertical (gravity) or horizontal (wind) loads were treated as special cases. At a later time the American Institute of Steel Construction[10] presented the frame charts of Ref. 9 in an improved and more refined way. Driscoll *et al*[11] published a two-volume comprehensive work on the plastic analysis and design of rectangular steel multistory frames. In the United Kingdom, three British Constructional Steelwork Association publications[12–14] pertain to the design of pitched roof portal frames. References 12 and 13 treat the case of frames made of uniform section members under a vertical load. Two formulas are given, depending on whether the frame in question is pinned or fixed at the ends, and wind action must be checked when the design is completed. Reference 14 contains a brief description of a computer program for the minimum weight design of any type of single-span steel gabled frame under any type of loading. Design tables for some pinned-base frames in terms of British sections and grade 43A British steel (yield stress = 43 ksi) were produced by this program in conjunction with the results of a cost investigation study involving British steel fabricators. Obviously, these tables are not complete and too specialized to be used outside the United Kingdom.

Some of the most frequently used methods of plastic analysis are the equilibrium or statical method of solution, the mechanism method, the method of inequalities, and the moment balancing technique. Application of the equilibrium method to the plastic design of a particular gabled frame with haunches is described in Refs. 2 and 10. In this paper, the mechanism method is used to obtain the required ultimate moment capacities M_p and KM_p for the columns and rafters of a pinned-base gabled frame with haunches, respectively. The parameter K is defined as the ratio of the strength of the rafters to the strength of the columns. Actually, six ultimate moment capacities M_p are obtained in closed form as functions of geometric and load parameters, one for each possible failure mechanism, and the largest positive M_p is the correct one. A computer program was written to handle all of the numerical computations involved and results for cases more frequently encountered in practice were plotted in the form of design charts. Ketter's charts[9,10] for single-bay frames can be obtained from this computer program as a special case.

In subsequent examples, two frames are designed with the aid of the charts, and a brief treatment on the subject of stability is also included. Additional design considerations, such as bracing, deflections, connections, the design of the haunches, the effects of axial and shear forces on the bending rigidity of the sections, etc., are not considered in the present work but can be found, for example, in Refs. 6 and 15.

George D. Manolis is Graduate Student, Dept. of Civil and Mineral Engineering, University of Minnesota, Minneapolis, Minn.

Dimitrios E. Beskos is Asst. Professor, Dept. of Civil and Mineral Engineering, University of Minnesota, Minneapolis, Minn.

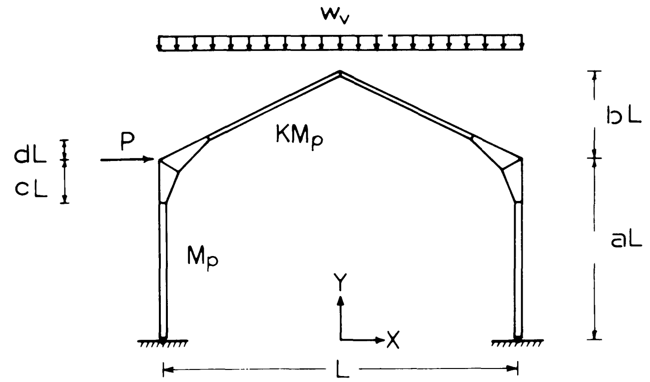

Fig. 1. Single-span, pinned-base gabled frame with haunches and different section members

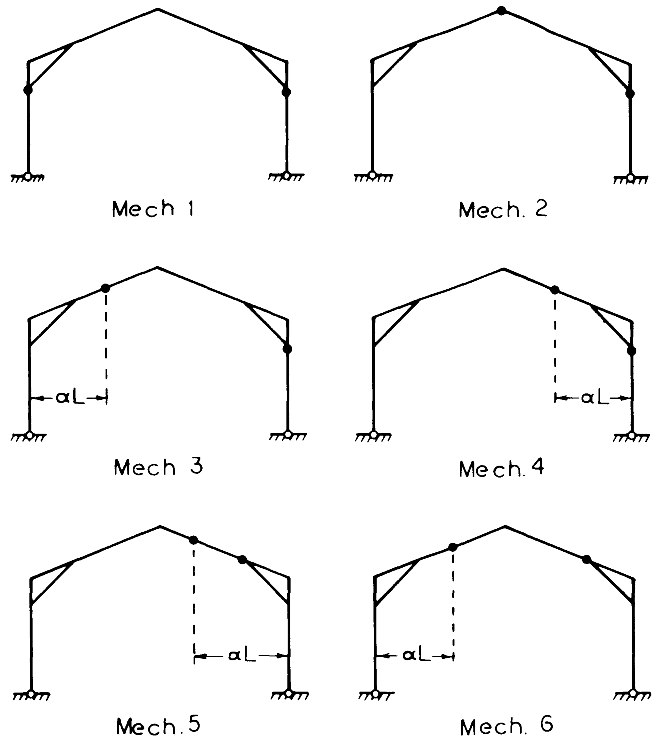

Fig. 2. The six independent failure mechanisms

FORMULATION AND SOLUTION

Consider the gabled frame shown in Fig. 1. The geometrical parameters involved are the span L, the column height aL, the rafter height bL, the length of the column portion of the haunch cL, and the height of the rafter portion of the haunch dL. Moment capacities M_p and KM_p are prescribed for the columns and rafters, respectively. The loading consists of a uniformly distributed vertical service load w_v and a horizontally distributed service wind load w_h , which can be replaced by an equivalent concentrated load P acting at the eaves. Load P produces the same overturning moment about the base of the structure as w_h does and is given by the formula

$$P = w_h(aL + bL)^2/2aL \tag{1}$$

A single loading parameter A is introduced by combining the two loads P and w_v as follows:

$$A = 2aP/w_vL \tag{2}$$

In order to determine the required dimensionless ultimate strength M_p/w_uL^2 of the frame in question, where w_u is the ultimate vertically distributed load, the mechanism method of plastic analysis is employed. The following usual assumptions concerning first-order rigid plastic theory are made:

1. The structural material is ductile with a rigid-plastic stress-strain curve.
2. Normal and shear forces do not influence the ultimate bending resistance of a member.
3. Instability does not occur prior to the development of a mechanism.
4. Loads are increased proportionally.
5. Deformations are small, so that the equilibrium equations can be formulated for the undeformed structure.
6. Connections are continuous, so that they can transmit plastic moments. An additional assumption in connection with the design of haunched frames is that the haunches are stronger than either the columns or the rafters, so that plastic hinges in haunches cannot be formed.

The first step in the mechanism method procedure is to ascertain the location of all possible plastic hinges that form as the loads on the structure approach their ultimate values. Then, all possible combinations of these hinges that result in failure configurations, i.e., collapse mechanisms, must be identified. For the frame in question, the number of possible plastic hinges is 7 and the number of redundancies is 1, so there are $7 - 1 = 6$ possible independent mechanisms, as shown in Fig. 2. It should be noted that in the last four mechanisms there are plastic hinges whose location αL must be determined. For each failure configuration, the external loads are related to the internal stiffnesses of the members by the principle of virtual work. This way an M_p value is obtained which must not be exceeded by any other moment in the frame. Since the structure is to sway, no combination of these independent mechanisms is geometrically possible, and the highest positive M_p value obtained from investigating all six mechanisms is the correct required ultimate strength. Thus, the upper bound collapse load P_p corresponding to that value of M_p will coincide with the true ultimate load of the frame.

In the following, a representative mechanism will be treated in detail for illustrative purposes.

Mechanism 3: Beam mechanism—By giving the structure a virtual displacement compatible with its geometry, the location of the instantaneous center of rotation I.C. is specified from purely geometrical considerations (similar triangles) and is shown in Fig. 3.

The angles of rotation of all pertinent hinges are:

$$
\begin{aligned}
\theta_B &= \theta \\
\theta_{IC} &= \frac{a - c}{\dfrac{a}{\alpha} - a + 2b + c}\,\theta \\
\theta_A &= \frac{\left(\dfrac{1}{\alpha} - 1\right)(a - c)}{\dfrac{a}{\alpha} - a + 2b + c}\,\theta \\
\theta_3 &= \frac{\dfrac{1}{\alpha}(a - c)}{\dfrac{a}{\alpha} - a + 2b + c}\,\theta \\
\theta_7 &= \frac{\dfrac{a}{\alpha} + 2b}{\dfrac{a}{\alpha} - a + 2b + c}\,\theta
\end{aligned}
\tag{3}
$$

Upon equating the work done by the external loads to the energy stored inside the members of the structure due to the rotations at the plastic hinges, the following equation is obtained:

$$
KM_p\theta_3 + M_p\theta_7 = P_u aL\theta_A + w_u \frac{\alpha^2 L^2}{2}\theta_A + w_u \frac{(1-\alpha)^2 L^2}{2}\theta_{IC} \tag{4}
$$

where P_u is the service wind load P multiplied by the overload or safety factor $F.S.$ Combination of Eqs. (3) and (4) and use of Eq. (2) finally lead to

$$
\frac{M_p}{w_u L^2} = \frac{a - c}{2} \times \frac{(1-\alpha)(A+\alpha)}{K(a-c) + a + 2b\alpha} \tag{5}
$$

Next, the distance αL that locates the plastic hinge 3 must be found. Since α is an independent variable and since the structure will fail at the first opportunity, the correct α will be determined from the expression

$$
\frac{\partial M_p}{\partial \alpha} = 0 \tag{6}
$$

In view of Eq. (6), Eq. (5) yields:

$$
\alpha = \frac{-[a + K(a-c)] \pm \sqrt{[a + K(a-c)]^2 - 2b[(A-1)((a-c)K + a) + 2bA]}}{2b} \tag{7}
$$

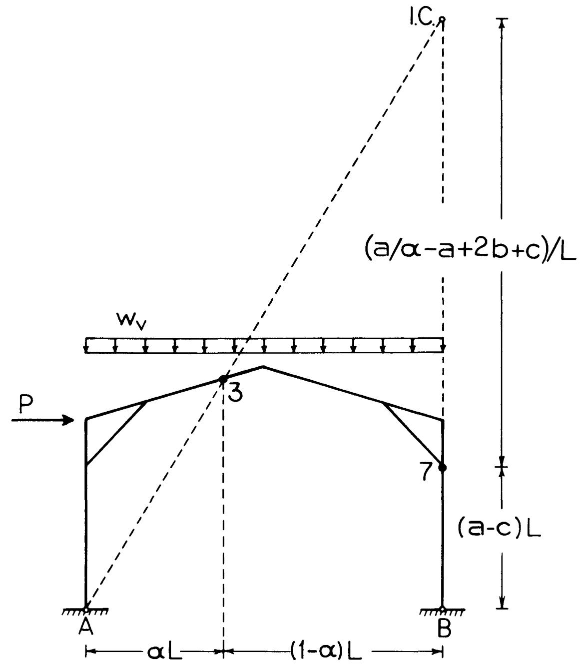

Fig. 3. Location of the instantaneous center of rotation of mechanism 3

The factor α ranges between $0.5(d/b)$ and 0.5; thus, the negative sign in front of the square root in Eq. (7) is rejected. When $b = 0$, then $\alpha = (1 - A)/2$.

Following the approach delineated above, the expressions for the non-dimensional moment capacity $M_p/w_u L^2$ and the plastic hinge location ratio α, where applicable, are derived and presented below for the remaining five mechanisms.

Mechanism 1: Sway mechanism

$$
\frac{M_p}{w_u L^2} = \frac{A}{4}\left(1 - \frac{c}{a}\right) \tag{8}
$$

Mechanism 2: Gable mechanism

$$
\frac{M_p}{w_u L^2} = \frac{1}{4} \cdot \frac{A + 0.5}{K + (a+b)/(a-c)} \tag{9}
$$

Mechanism 4: Beam mechanism

$$
\frac{M_p}{w_u L^2} = \frac{1}{2} \cdot \frac{(a-c)\alpha(A + 1 - \alpha)}{K(a-c) + a + 2b\alpha} \tag{10}
$$

$$
\alpha = \frac{kc - 2(k+1)a + \sqrt{(2(K+1)a - Kc)^2 + 8b(1+A)((K+1)a - Kc)}}{4b} \tag{11}
$$

Mechanism 5: Beam mechanism

$$\frac{M_p}{w_u L^2} = \left[\frac{1}{2K} \cdot \frac{(A + (1 - \alpha)^2)(a + d)(X - \alpha - 0.5) + (a - d/2b)^2(1 - \alpha)(a + d)}{(a + d)(X + 0.5) + Y(1 - \alpha)}\right]$$

$$+ \left[-\frac{1}{2K} \cdot \frac{(d/2b)^2(1 - \alpha)(Y - a - d)}{(a + d)(X + 0.5) + Y(1 - \alpha)}\right] \quad (12)$$

where X and Y are the coordinates of the instantaneous center of rotation, which are given by the following expressions:

$$X = \frac{1}{2} \cdot \frac{(a + d)(\alpha - 1) + (2b\alpha + a)[0.5 - (b - d)/2b]}{(a + d)(\alpha - 1) - (2b\alpha + a)[0.5 - (b - d)/2b]}$$

$$Y = \frac{-(a + d)(2b\alpha + a)}{(a + d)(\alpha - 1) - (2b\alpha + a)[0.5 - (b - d)/2b]} \quad (13)$$

It is not practically feasible to employ Eq. (6) to solve for α. Instead, α will be assigned a sequence of values ranging from $0.5(d/b)$ to 0.5 and the correct one will maximize Eq. (12).

Mechanism 6: Beam mechanism

$$\frac{M_p}{w_u L^2} = \left[\frac{1}{2K} \cdot \frac{(A + \alpha)^2(0.5 - \alpha + X)(a + d) + (1 - \alpha - d/2b)^2\alpha(a + d)}{(a + d)(0.5 + X) + \alpha Y}\right]$$

$$+ \left[-\frac{1}{2K} \cdot \frac{(d/2b)^2\alpha(Y - a - d)}{(a + d)(0.5 + X) + \alpha Y}\right] \quad (14)$$

where X and Y are given by:

$$X = \frac{1}{2} \cdot \frac{\alpha(a + d) + (a + 2b\alpha)((b - d)/2b - 0.5)}{\alpha(a + d) - (a + 2b\alpha)((b - d)/2b - 0.5)}$$

$$Y = \frac{(a + 2b\alpha)(a + d)}{\alpha(a + d) - (a + 2b\alpha)((b - d)/2b - 0.5)} \quad (15)$$

and α is found by the same procedure as described in mechanism 5.

All of the above equations reduce to the ones given by Ketter[9,10] if c and d are allowed to go to zero and K is allowed to go to unity.

THE COMPUTER PROGRAM

A computer program written in Fortran was prepared to compute the dimensionless ultimate strength ratio $M_p/w_u L^2$ of a pinned-base gabled frame for any combination of the geometric and loading parameters involved. In order to construct the design charts given in this paper, it was necessary to restrict these parameters to the following selected values, which represent cases most frequently encountered in practice:

1. Factor K assumes the values 0.75, 1.00, and 1.25. Values of K less than unity imply that the rafters have a smaller section modulus than the columns.
2. The loading parameter A assumes the values 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, and 0.6.
3. Parameter a ranges from 0.10 to 0.40 in steps of 0.02.
4. Parameter b is either 0.13 or 0.20, corresponding to a rafter slope of 15° or 22.5°, respectively.
5. Finally, the parameters c and d are either both 0.00 (no haunches at all) or 0.03 and 0.04, respectively, when $b = 0.13$, and are 0.03 and 0.06, respectively, when $b = 0.20$.

For any combination of the above six parameters, the computer program determines six values for $M_p/w_u L^2$, one for each mechanism. These six values are compared and the largest positive number is output, along with the number of the controlling mechanism and the plastic hinge location α, where applicable. Negative values for $M_p/w_u L^2$ are rejected, since they correspond to work done (and not absorbed) by the mechanism. Values of $M_p/w_u L^2$ which correspond to a computed α lying outside the interval $0.5d/b$ to 0.5 are also rejected for geometrical reasons.

DESIGN PROCEDURE

The ultimate strength requirements of a pinned-base gabled frame can be rapidly found by using the charts of Figs. 4 through 13. The first five charts, Figs. 4–8, are set up so as to give the ratio $M_p/w_u L^2$ as a function of the ratio a, with A as the additional parameter. Each chart has a heading which states the pertinent values of K, b, c, and d. Usually the choices of K, c, and d are at the discretion of the designer. If further information about the collapse mechanism is needed, as in the case of deflection computations, the remaining five charts, Figs. 9–13, can be used. Those charts plot the plastic hinge location ratio α as a function of the ratio a, with A as the additional parameter. Once more, each chart is identified by the values of K, b, c, and d printed at the top. When $\alpha = 0$, mechanism 1 controls. When α is different from zero, the solid lines pertain to mechanism 3 and the dashed lines pertain either to mechanism 6, in the case of haunched frames, or to mechanism 4, in the case of frames without haunches ($c = d = 0$). It should be noted that when a frame does not have haunches, the right-hand side plastic hinge in mechanisms 1 through 6 and the left-hand side plastic hinge in mechanism 1 will all move to the rafter-column intersection. Finally, for cases not covered in the charts, interpolation or use of the computer program are two alternatives.

In principle, unbraced frames must be designed according to second-order elasto-plastic methods, to account for the additional moments induced by the axial loads displacing through an amount Δ (P-Δ effects). It is well known[11,16,17] that the reduction of the failure load P_f due

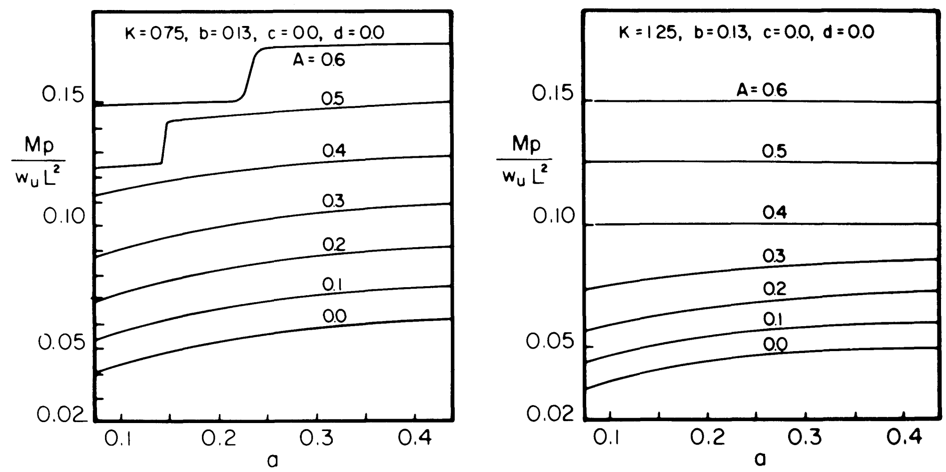

Fig. 4. M_p/w_uL^2 *versus column height ratio a for pinned-base gabled frames*

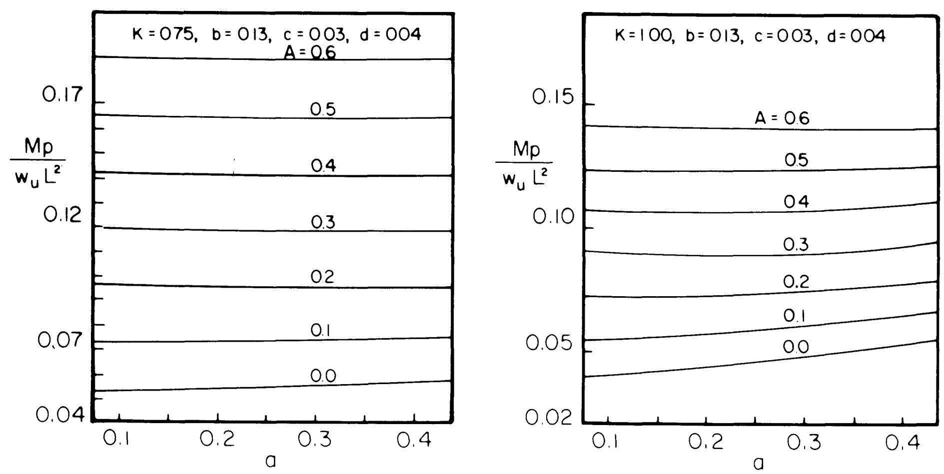

Fig. 5. M_p/w_uL^2 *versus column height ratio a for pinned-base gabled frames*

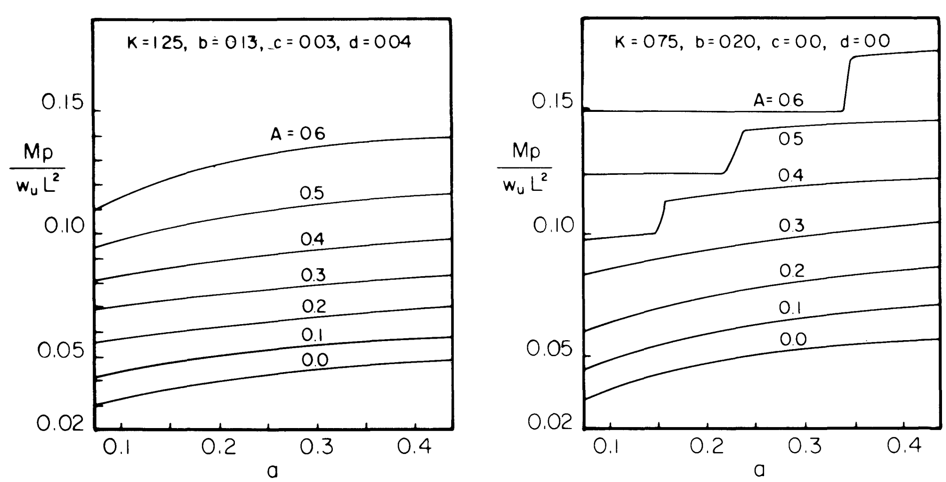

Fig. 6. M_p/w_uL^2 *versus column height ratio a for pinned-base gabled frames*

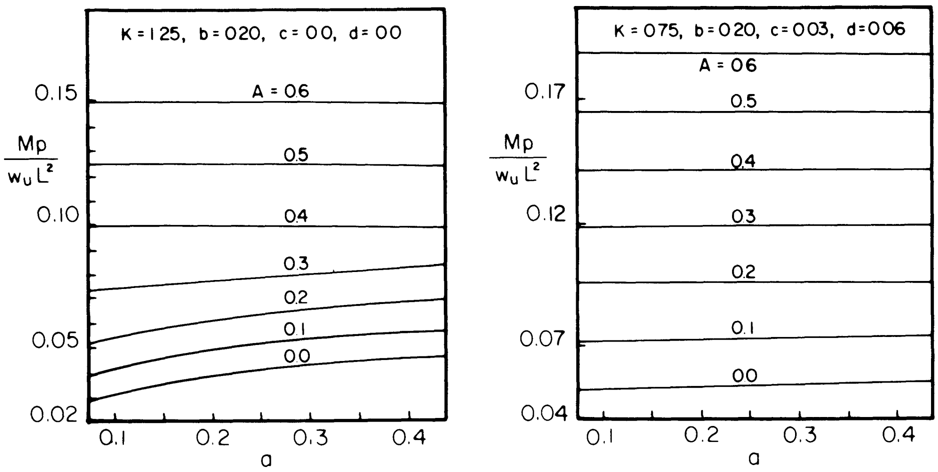

Fig. 7. M_p/w_uL^2 versus column height ratio a for pinned-base gabled frames

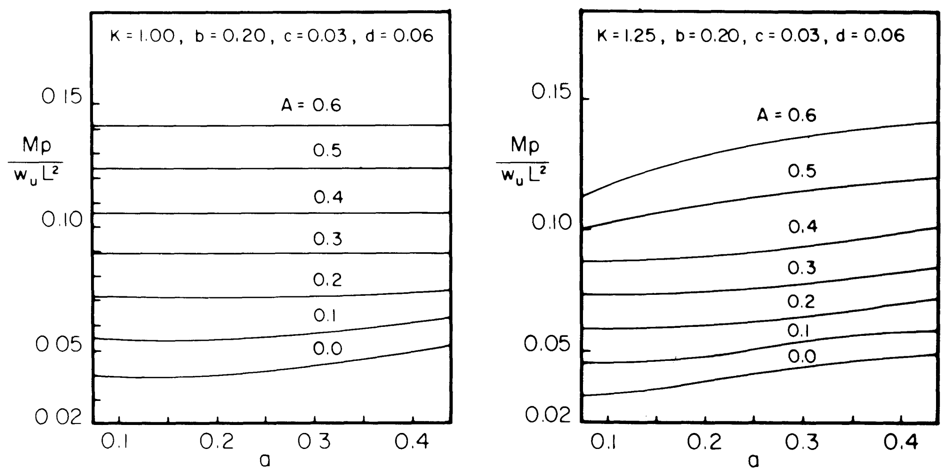

Fig. 8. M_p/w_uL^2 versus column height ratio a for pinned-base gabled frames

to these P-Δ effects can be taken into account in a reliable manner by the Rankine-Merchant formula

$$\frac{1}{P_f} = \frac{1}{P_p} + \frac{1}{P_{cr}} \tag{16}$$

where P_p is the collapse load obtained by rigid-plastic theory and P_{cr} is the elastic critical load obtained from a first-order analysis. Since actual frames benefit from the effect of partitions and from the strain hardening of the steel, Ref. 8 has adopted the following modified version of Eq. (16):

$$P_f = \frac{P_p}{0.9 + P_p/P_{cr}} \tag{17}$$

Equation (17) can be used if $4 \leq P_{cr}/P_p \leq 10$, provided plastic hinges are only allowed in the girders and the frame is braced in the perpendicular direction. If $P_{cr}/P_p > 10$, P_f is limited to P_p , and if $P_{cr}/P_p < 4$, an elastoplastic second-order method must be used. Fortunately, for most practical cases of single-story gabled frames, $P_{cr}/P_p > 10$ and the failure load P_f coincides with the plastic load P_p.

For completeness, P_{cr} is be computed for the structures designed in the Structural Examples section by using the computer program of Ref. 18. The basic procedure for evaluating P_{cr} is to increment the service loads until the determinant of the stiffness matrix of the frame in question becomes zero. The determination of the internal axial force distribution in the frame for every value of the applied load is accomplished in an iterative manner. Since the program can accept only nodal data, the distributed load w_v is substituted by four point loads equal to $w_vL/4$ acting at the apex and the eaves. When haunched frames are examined, the average haunch area and flexural rigidity EI are as-

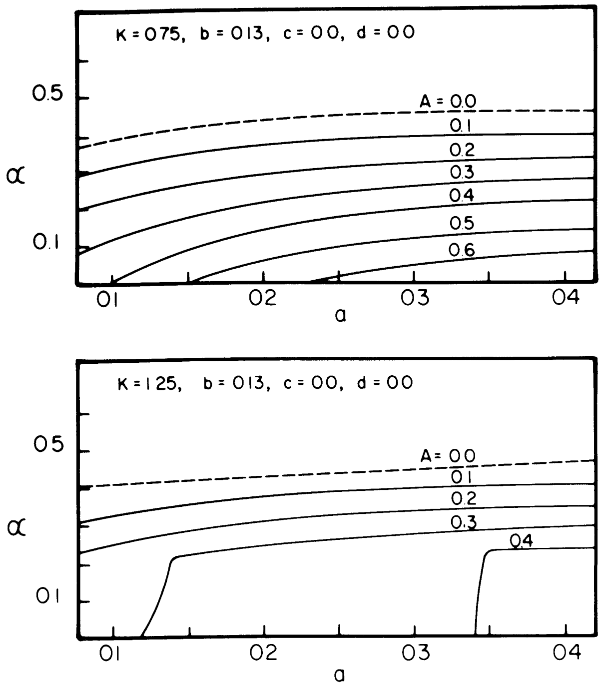

Fig. 9. Rafter plastic hinge location ratio α versus column height ratio a

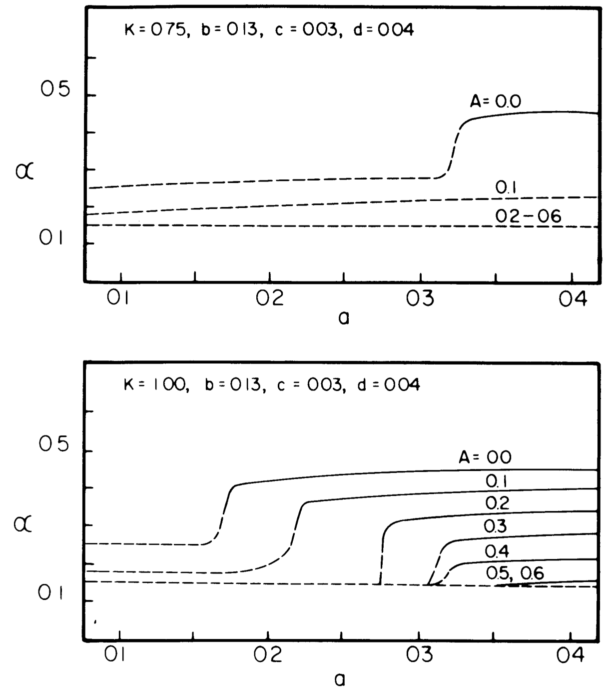

Fig. 10. Rafter plastic hinge location ratio α versus column height ratio a

sumed to be 150% and 225% greater than those of the framing members, respectively. In order to compute the ratio P_{cr}/P_p, the load P_p is taken as $w_uL/4$, where w_u pertains to the governing of the two loading cases considered in the examples.

STRUCTURAL EXAMPLES

Example 1—A pinned-base steel gabled frame with a span of 100 ft, a column height of 20 ft, and a slope of rafters of 15° will be designed. This sets the geometrical parameters a and b equal to 0.20 and 0.13, respectively. The service loads consist of a live load w_v = 1.00 kips/ft and a wind load w_h = 0.60 kips/ft. The equivalent concentrated load P which replaces w_h through the use of Eq. (1) becomes 16.33 kips. By assuming that the sections will weigh an average of 90 lbs/ft, w_v is modified to 1.13 kips/ft.

The first option is to design a haunched frame. Parameters c and d are selected to be 0.03 and 0.04, respectively, corresponding to a column haunch length of 3 ft and a rafter haunch length of 15.5 ft. Two loading cases are considered. The case that requires the larger M_p capacity for the members of the frame in question is the governing one.

1. In loading case I, the dead and live loads are taken with a factor of safety $F.S.$ = 1.70 and no wind loads are considered. This makes w_u = 1.70 (1.13) = 1.92 kips/ft and P_u = 0.0. Using Eq. (2), A = 0. Part of Table 1 can be constructed from the data obtained by entering the appropriate graphs on Figs. 5 and 6.
2. In loading case II, the combined dead, live, and wind loads are taken with an $F.S.$ = 1.30. This makes w_u = 1.30 (1.13) = 1.47 kips/ft, P_u = 1.30 (16.33) = 21.23 kips, and from Eq. (2), A = 0.06. From an inspection of Figs. 5 and 6, it is apparent that loading case II does not govern the design.

Table 1 suggests that for the haunched option, the lightest structure will result when K = 1.00. Then, M_p (columns) = M_p (rafters) = 0.043 (1.92) $(100)^2$ = 825.6 ft-kips. From Ref. 6, pp. 2-16, 2-17, for A36 steel, a W27x94 section (M_p = 834 ft-kips) is chosen for both the columns and the rafters. Including the small additional weight of the haunches, the weight of the structure is 14.08 kips.

Table 1. Selections for Example 1, Loading Case I

	Haunches		No Haunches	
K factor (1)	Mp/w_uL^2 columns (2)	KMp/w_uL^2 rafters (3)	M_p/w_uL^2 columns (4)	KMp/w_uL^2 rafters (5)
0.75	0.054	0.040	0.053	0.040
1.00	0.043	0.043	0.048	0.048
1.25	0.040	0.050	0.044	0.055

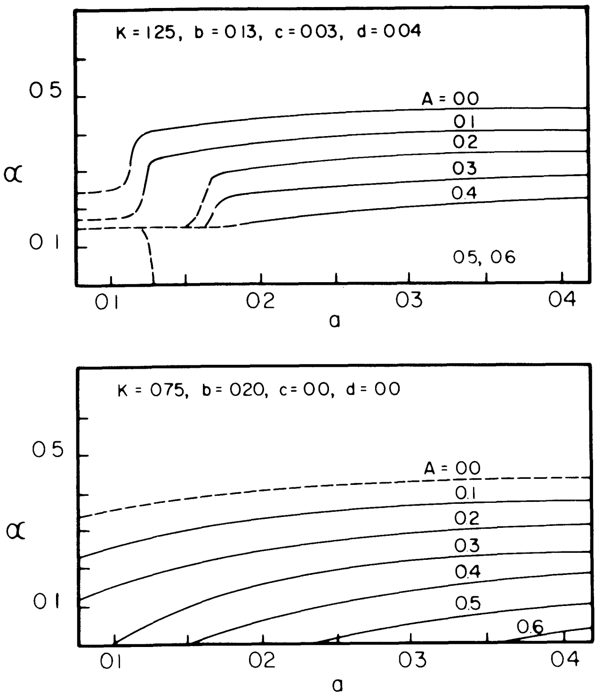

Fig. 11. Rafter plastic hinge location ratio α versus column height ratio a

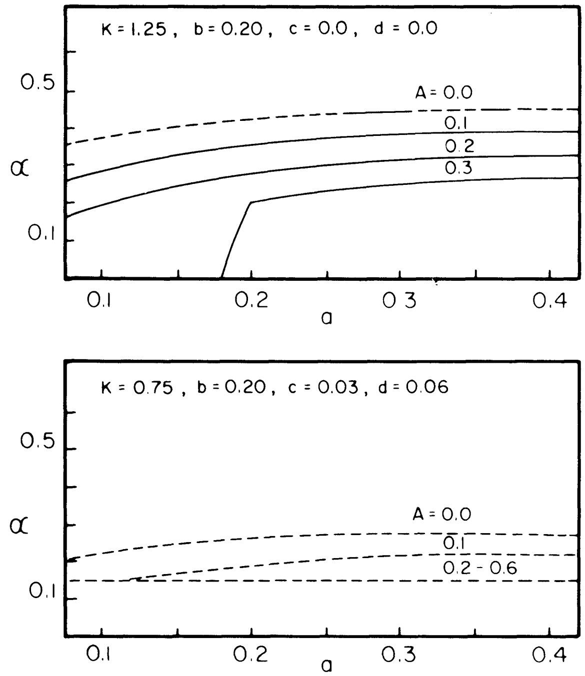

Fig. 12. Rafter plastic hinge location ratio α versus column height ratio a

The second option is to design a frame without haunches. In this case, $c = 0$ and $d = 0$. The same two loading cases are considered, and since the loading and parameter A are common to both design options, loading case I will govern. The remaining part of Table 1 can now be constructed by using Fig. 4 and Ref. 9, p. 43, or Ref. 10, p. A-12 for the case of uniform section members ($K = 1.00$). The lightest structure will result when $K = 0.75$. Therefore, M_p (columns) = 0.053 (1.92) $(100)^2$ = 1017.6 ft-kips and a W30x108 section is selected; M_p (rafters) = 0.040 (1.92) $(100)^2$ = 768 ft-kips and a W27x94 section is selected. The total weight of this structure is 14.04 kips. If $K = 1.00$ is selected, then for both columns and rafters M_p = 921.6 ft-kips and a W30x99 section is required, bringing the total weight to 14.20 kips.

Concluding, the design with haunches results in steel savings of 0% and 0.9% as compared with a different section member design without haunches and with a conventional uniform section member design with no haunches, respectively.

Furthermore, by employing the computer program of Ref. 18, one can evaluate for the haunched frame a P_{cr} equal to 760 kips and, for the frame without haunches and different section members, a P_{cr} equal to 604 kips. For either frame, since loading case I governs, P_p = 48 kips and the ratio P_{cr}/P_p is greater than 10 for both structures. Therefore the common $P_f = P_p$ = 48 kips.

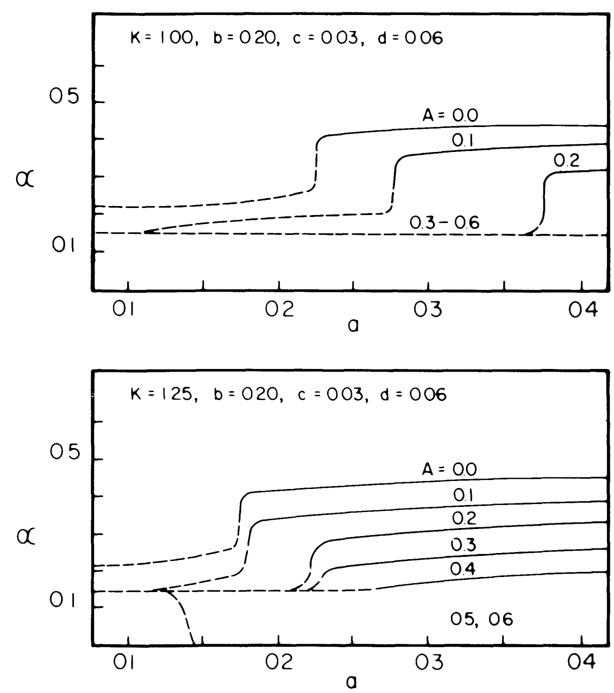

Fig. 13. Rafter plastic hinge location ratio α versus column height ratio a

Table 2. Selections for Example 2, Loading Case I

	Haunches		No Haunches	
K factor (1)	M_p/w_uL^2 columns (2)	KMp/w_uL^2 rafters (3)	M_p/w_uL^2 columns (4)	KMp/w_uL^2 rafters (5)
0.75	0.055	0.041	0.056	0.042
1.00	0.048	0.048	0.051	0.051
1.25	0.044	0.055	0.046	0.057

Table 3. Selections for Example 2, Loading Case II

	Haunches		No Haunches	
K factor (1)	Mp/w_uL^2 columns (2)	KMp/w_uL^2 rafters (3)	Mp/w_uL^2 columns (4)	KMp/w_uL^2 rafters (5)
0.75	0.142	0.107	0.123	0.092
1.00	0.106	0.106	0.109	0.109
1.25	0.094	0.118	0.100	0.125

Example 2—In this example, a frame with a 40 ft span, a 16 ft column height, and a 22.5° rafter slope will be designed. The loads consist of $w_v = 1.00$ kips/ft and $w_h = 1.20$ kips/ft, which can be replaced by a $P = 21.60$ kips acting at the eaves according to Eq. (1). The weight of the structure is estimated at 40 lbs/ft, which modifies w_v to 1.08 kips/ft. The geometrical parameters a and b become 0.40 and 0.20, respectively.

If a haunched frame is the first option, then for $c = 0.03$ and $d = 0.04$, the haunch lengths are 1.2 ft and 6.3 ft, respectively. For loading case I, $w_u = 1.84$ kips/ft and A from Eq. (4) is 0.0. With the aid of the charts in Figs. 7 and 8, part of Table 2 is constructed. For loading case II, $w_u = 1.40$ kips/ft and $A = 0.40$. Thus, consulting the same figures, part of Table 3 is constructed. The factor K is selected as equal to 1.00 for the design. Then, for loading case I, M_p for both columns and rafters is 141.3 ft-kips and, for loading case II, $M_p = 235.9$. Therefore, case II controls the design and the selection is a W18x40 section. The weight of the resulting structure is 3.08 kips.

A frame without haunches will be the next option. For this case, $c = 0$ and $d = 0$. The remaining parts of Tables 2 and 3 can be completed from Figs. 6 and 7 and Ref. 9 or 10, after noting that the loading and loading parameter A are common to both design options. For a $K = 1.00$, loading case I requires an $M_p = 150.1$ ft-kips and loading case II requires an $M_p = 244.2$ ft-kips. Case II governs the design and a W16x45 section is chosen, bringing the total weight of the structure to 3.38 kips. The steel savings thus realized from the use of haunches are 9.7%.

As far as stability is concerned, use of the computer program of Ref. 18 leads to a $P_{cr} = 343$ kips for the haunched frame and to a $P_{cr} = 239$ kips for the frame without the haunches. Since loading case II governed both design options, $P_p = 14$ kips and the ratio P_{cr}/P_p is greater than 10 for either design. Therefore, P_f coincides with P_p and is equal to 14 kips.

CONCLUSIONS

The design of pinned-base gabled frames with different column and rafter sections along with haunches at the eaves is readily accomplished with the aid of charts which provide the required plastic moment as a function of a few geometrical and loading parameters for cases most frequently encountered in practice. This approach is much easier than the alternative procedure for designing haunched gabled frames, namely the statical or equilibrium method of plastic analysis. In case of frames not covered in the charts, either interpolation or the computer program may be used.

The advantage of using different column and rafter sections along with haunches at the eaves is that, in general, savings in steel can be realized as compared with the case of no haunches and uniform sections. This, however, may not necessarily imply an overall more economical structure, since the fabrication of haunches is an additional expense which can be eliminated only if specialized production techniques are adopted. A careful observation of Figs. 4–8 indicates that the use of haunches will result in a lighter structure when the rafters are stronger than the columns and that, in most cases, it will not result in a lighter structure when the columns are stronger than the rafters. Finally, it should be noted that, in general, steel savings will be more pronounced as the wind load increases with respect to the vertical load and as the span length of the frame increases.

ACKNOWLEDGEMENTS

The senior author (D. E. B.) is grateful to Messrs. D. S. Kiriazis, President, and E. Kardaras, Chief Engineer of Dexion Hellas S.A., Athens, Greece, for their encouragement and support during the early stages of this investigation. The University of Minnesota Computer Center is also acknowledged for making its facilities available to the authors.

NOMENCLATURE

A = $2aP/w_vL$
a = ratio of column height to span length
b = ratio of rafter height to span length
c = ratio of column haunch length to span length
d = ratio of rafter haunch height to span length
E = modulus of elasticity
$F.S.$ = factor of safety
I = moment of inertia
K = ratio of rafter stiffness to column stiffness

L = span length
M_p = plastic moment capacity
P = horizontal service load
P_{cr} = elastic critical load
P_f = failure load
P_p = collapse load by simple plastic theory
P_u = horizontal ultimate load
w_h = horizontal distributed service load
w_u = ultimate distributed load
w_v = vertical distributed service load
α = ratio of hinge location distance to span length
θ = angle of hinge rotation

REFERENCES

1. *Baker, J. F., M. R. Horne, and J. Heyman* The Steel Skeleton *Vol. 2, Plastic Behaviour and Design, Cambridge University Press, Cambridge, England, 1956.*
2. *Beedle, L. S.* Plastic Design of Steel Frames *John Wiley and Sons, New York, N.Y., 1958.*
3. *Hodge, P. G., Jr.* Plastic Analysis of Structures *McGraw-Hill, New York, N.Y., 1959.*
4. *Horne, M. R.* Plastic Theory of Structures *M.I.T. Press, Cambridge, Mass., 1971.*
5. *Neal, B. G.* The Plastic Methods of Structural Analysis *3rd ed., Halstead Press, New York, N.Y., 1977.*
6. Manual of Steel Construction *7th ed. American Institute of Steel Construction, New York, N.Y., 1970.*
7. *Horne, M. R.* The Plastic Design of Columns *BCSA Publication No. 23, British Constructional Steelwork Association, London, England, 1964.*
8. *Massonnet, C.* European Recommendations (E.C.C.S.) for the Plastic Design of Steel Frames *Acier-Stahl-Steel, Vol. 41, No. 4, April, 1976, pp. 146–153.*
9. *Ketter, R. L.* Plastic Design of Multi-span Rigid Frames *Thesis presented to Lehigh University, at Bethlehem, Penn., in 1956, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.*
10. Plastic Design in Steel *American Institute of Steel Construction, New York, N.Y., 1959.*
11. *Driscoll, G. C., et al* Plastic Design of Multistory Frames *Vol. 1, Lecture Notes; Vol. 2, Design Aids, Lehigh University, Bethlehem, Penn., 1965.*
12. *Baker, J. F.* Plastic Design in Steel to B.S. 968 *BCSA Publication No. 21, British Constructional Steelwork Association, London, England, 1963.*
13. *Horne, M. R. and M. W. Chin* Plastic Design of Portal Frames in Steel to B.S. 968 *BCSA Publication No. 29, British Constructional Steelwork Association, London, England, 1966.*
14. *Blockley, D. I.* The Design of Single Storey Pitched Roof Portal Frames *British Constructional Steelwork Association, London, England, 1970.*
15. Plastic Design in Steel, a Guide and Commentary *2nd ed., American Society of Civil Engineers and Welding Research Council, New York, N.Y., 1971.*
16. *Horne, M. R. and W. Merchant* The Stability of Frames *Pergamon Press, Oxford, England, 1965.*
17. *Majid, K. I.* Non-Linear Structures *Wiley-Interscience, New York, N.Y., 1972.*
18. *Putman, S.* On the Elastic Stability of Frameworks *Thesis presented to the University of Minnesota, at Minneapolis, Minn., in 1977, in partial fulfillment of the requirements for the degree of Master of Science.*