Allowable Axial Stresses in Segmented Columns

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An axially loaded column which is stepped, or one for which axial loads and/or reactions are applied at points other than its ends, may be referred to as a segmented column. Each segment of such a column has constant properties and constant load along its length. Crane columns and pipe supports are typical examples. Frequently such columns have eccentric or lateral loads which also produce bending, but for determining allowable axial stresses the columns may be reduced to forms similar to Fig. 1.

For elastic behavior, the critical condition in a segmented column with a given combination of proportional loads usually depends upon a complex interaction between the segments such that the whole column may be regarded as buckled. The contribution of each segment is a function of its properties, its load, and the position of the segment along the length of the column. The critical condition for an individual segment is generally unknown, because each segment relies on the other segments for its end conditions.

Many methods for determining the critical (Euler) loading conditions in elastic segmented columns are available (see Refs. 1 through 9).

When inelastic behavior is involved in one or more of the segments, the determination of the ultimate loading condition is complicated by the fact that the extent of inelastic behavior is dependent upon the unknown load level. This problem can be solved by a trial-and-error process, but the calculation effort virtually precludes its use as an everyday design tool.

Unfortunately, inelastic behavior in columns is the rule rather than the exception. Several semiemperical methods for establishing allowable stresses in segmented columns have been proposed,[1,10,11] but they are restricted to two segments and only one set of boundary conditions.

The purpose of this article is to suggest a somewhat more rational approach for finding allowable stresses in inelastic segmented columns, an approach which has as its basis similar principles to those employed in developing the AISC allowable stresses in compression for axially loaded, prismatic, non-segmented columns. The method proposed here requires a knowledge of the elastic critical conditions for the column and is also approximate. The basis for the approximation and the limits of the error are included. Although cantilever columns are used for illustrative purposes, the method is independent of the end conditions.

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DEFINITIONS AND CONCEPTS OF FAILURE

In order to understand the ultimate condition in a segmented column, we must take a fresh look at some familiar concepts. We begin by defining a characteristic load P, various multiples of which may be acting on the various segments, such as in Fig. 1(c). In this example, the top segment experiences a load of $2P$, the middle segment $3P$, and the bottom segment carries a total of $6P$. In general, a segment load will be nP, where n is characteristic of the segment, while P is characteristic of the whole column. It is assumed that all segment loads will be proportional to P and, therefore, in fixed proportion to one another. For purposes of discussion, P may be any value between zero and the failure condition, however defined.

It can be shown, though it is reasonable to expect, that the elastic or Euler value P_e for a particular segmented column, with (1) specified end conditions, (2) given values of the moment of inertia for the various segments, and (3) *fixed* proportions between the segment lengths, will vary

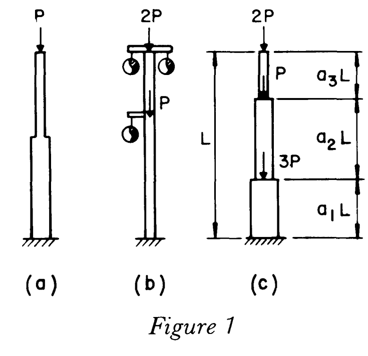


Figure 1

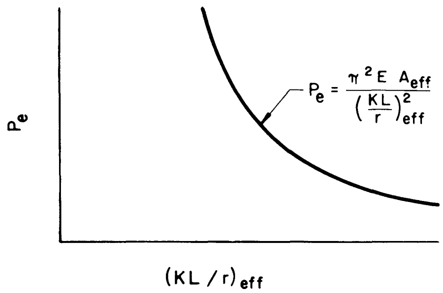


Figure 2

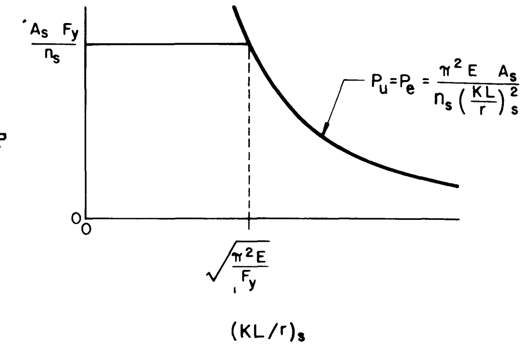


Figure 3

inversely with the square of the length of the overall column. In fact, one could say that

$$P_e = \frac{\pi^2 E I_{eff}}{(KL)^2} \tag{1}$$

in which K is the effective length factor and I_{eff} is some effective value of moment of inertia for the whole column. The difficulty with such a statement is that both K and I_{eff} are unknown properties, peculiar to the particular geometry of the column under consideration. The determination of either one would require knowing explicitly how each of the segments, with its own particular properties, loading, and end conditions, contributed to the critical conditions of the whole column. Such knowledge is not available at present.

An equally vague but slightly more useful statement would be that

$$P_e = \frac{\pi^2 E A_{eff}}{(KL/r)^2_{eff}} \tag{2}$$

in which A_{eff} is some effective cross-sectional area for the whole column and r is the related effective radius of gyration. If P_e is determined using one of the techniques cited above, and A_{eff} is *assumed,* it becomes possible to calculate the quantity $(KL/r)_{eff}$. The dependence of P_e upon this fictional Kl/r ratio could be depicted as in Fig. 2. Obviously, the horizontal coordinates depend upon the particular value of A_{eff} assumed, as well as upon the actual length of the column.

Elastic buckling, however, is the ultimate condition only for columns which are long enough to fail that way. For shorter columns, inelastic behavior takes over and the maximum ultimate load, P_u, which a segmented column can sustain, no matter how short it is, occurs when the most highly stressed (MHS) segment reaches the yield point. The MHS segment is that segment for which nP/A is a maximum. We designate its area as A_s and its characteristic load multiple as n_s. Therefore,

$$\text{Max } P_u = \frac{A_s F_y}{n_s} \tag{3}$$

If we now assume in Eq. (2), or in Fig. 2, that $A_{eff} = A_s/n_s$, we can show the upper bound on P_u as a function of an effective KL/r, for a given column. This bound consists of two parts, which are indicated in Fig. 3 and are defined by Eqs. (2) and (3). The ultimate value of the characteristic load for a specific column might fall below one of these two limits, but it would never be larger.

It should be noted here that the quantity $(KL/r)_s$ is actually the dependent variable in Fig. 3, and is the repository of all of our ignorance about the interdependence of the various segments. It is completely without physical significance and might better be represented by a different symbol, except for the convenience it will provide later. $(KL/r)_s$ is defined by the expression

$$(KL/r)_s = \sqrt{\frac{\pi^2 E A_s}{n_s P_e}} \tag{4}$$

To determine the actual value of P_u for any length of column in the inelastic range, it is necessary to specify the criteria for inelastic behavior. We adopt the Column Research Council's recommendation[12,13] that such behavior be reflected in a reduced effective modulus of elasticity, called the tangent modulus, according to the expression

$$E_t = \frac{4Ef_a}{F_y}\left(1 - \frac{f_a}{F_y}\right) \quad \text{for} \quad \frac{F_y}{2} \leq f_a \leq F_y \tag{5}$$

(When $f_a \leq F_y/2, E_t = E$; when $f_a = F_y$, $E_t = 0$.) Using tangent moduli for segments stressed above $F_y/2$, it is possible to find P_u from elastic analysis.*

If we consider a special case of the segmented column, one which consists of a single segment, or one in which nP/A is the same for all segments, then the inelastic ulti-

* *The authors have done this by programming Newmark's numerical integration technique[6] for a cantilever column with a potential of up to 100 segments. After P_e has been found, each segment stressed at $f_a > F_y/2$ has been assigned an estimated value of E_t and an approximate P_u is determined. From the approximate P_u, new stresses are calculated, new E_t's assigned, and the process repeated until P_u converges. Since Newmark's method for elastic columns is itself an iterative procedure, this approach is not recommended for hand calculations.*

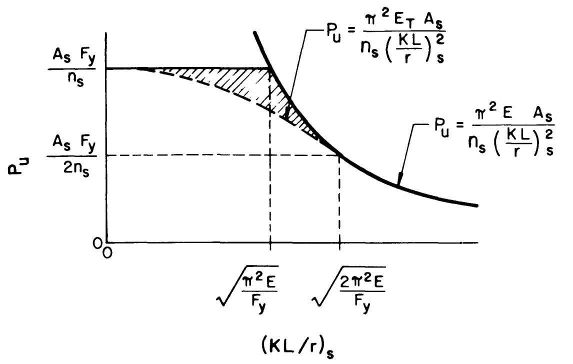


Figure 4

mate load can be found from the equation

$$P_u = \frac{\pi^2 E_t A_s}{n_s (KL/r)_s{}^2} \tag{6}$$

in which $(KL/r)_s$ can be determined from Eq. (4). Equation (6) appears in our plot of P_u versus $(KL/r)_s$ as the dashed line in Fig. 4. It is the familiar parabolic transition between simple yielding in very short columns $[(KL/r)_s = 0]$ and Euler buckling in long columns $[KL/r)_s \leq \sqrt{2\pi^2 E/F_y}]$. The latter limit is better known as C_c.

While Eq. (6) is strictly valid only for the special cases in which the stresses in all segments are equal, it represents a lower bound for the P_u of any segmented column in the inelastic range. Any column with at least one segment not stressed as much as the MHS segment must necessarily have an ultimate load equal to or greater than Eq. (6). This is true because any not-so-highly-stressed segment, if it makes any contribution to the ultimate condition at all, does so with a modulus of elasticity which is greater than if it were at the same stress as the MHS segment. Thus, in general, the true ultimate load for any segmented column in the inelastic range will lie within the cross-hatched area of Fig. 4, an area whose limits are defined in terms of P_e for the whole column and the properties of the MHS segment.

ALLOWABLE STRESS CONCEPTS

In the absence of a readily available technique for determining P_u exactly, it is conservative to assume that P_u for any particular column is defined by Eq. (6). If we substitute Eq. (5) into Eq. (6), making use of the relation $f_a = n_s P_u / A_s$, the result is

$$P_u = \frac{A_s F_y}{n_s}\left[1 - \frac{F_y (KL/r)_s{}^2}{4\pi^2 E}\right] \tag{7}$$

Except for columns in which all segments have the same axial stress, Eq. (7) will be approximate. In principle, the maximum possible error one could introduce by using Eq. (7) is 33 percent (of the lower bound value), when $(KL/r)_s$ for the column is $\sqrt{\pi^2 E/F_y}$ (see Fig. 4). In practice the error tends to be much smaller, as will be shown later.

To find an allowable load for the column, we divide Eq. (7) by the familiar factor of safety which appears in Sect. 1.5.1.3.1 of the AISC Specification,[14] with the result that

$$P_a = \frac{\frac{A_s F_y}{n_s}\left[1 - \frac{F_y (KL/r)_s{}^2}{4\pi^2 E}\right]}{\frac{5}{3} + \frac{3(KL/r)_s}{8C_c} - \frac{(KL/r)_s{}^3}{8C_c{}^3}} \tag{8}$$

Although this allowable load appears to be expressed in terms of the MHS segment, it is actually the allowable characteristic load for the whole column. The allowable stress for the MHS segment is found by dividing P_a by A_s/n_s for that segment. Using the relation $C_c = \sqrt{2\pi^2 E/F_y}$ we have

$$F_{as} = \frac{F_y\left[1 - \frac{(KL/r)_s{}^2}{2C_c{}^2}\right]}{\frac{5}{3} + \frac{3(KL/r)_s}{8C_c} - \frac{(KL/r)_s{}^3}{8C_c{}^3}} \tag{9}$$

The similarity between Eq. (9) and Formula (1.5–1) of the AISC Specification is intentional, for the purpose of emphasizing their common bases and limits. From a physical point of view, the similarity is superficial. As we have noted above, the quantity $(KL/r)_s$ is not found from the length, radius of gyration, or end conditions of either the whole column, or of the MHS segment. It is found from substituting the critical or Euler characteristic load for the entire column and the A/n for the MHS segment into Eq. (4).

When F_y of the column corresponds to one of the values for Table 1 in Appendix A of the Specification, F_{as} can be determined from the appropriate table using $(KL/r)_s$. If F_y is not standard, it is more convenient to recognize that

$$\frac{(KL/r)_s}{C_c} = \sqrt{\frac{A_s F_y}{2n_s P_e}} \tag{10}$$

so that Eq. (9) becomes

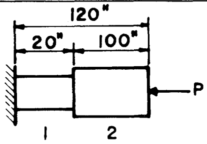
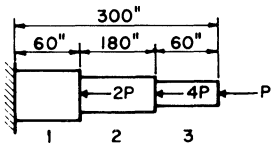
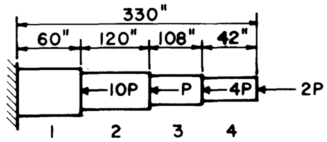
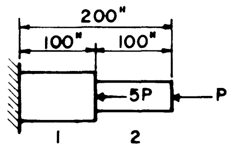
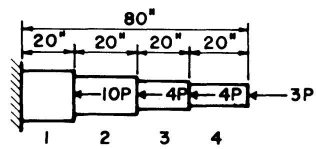
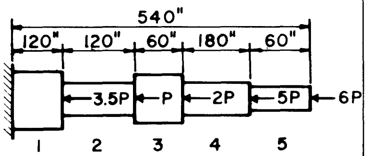
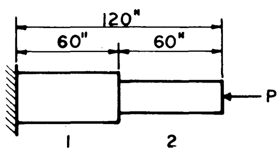
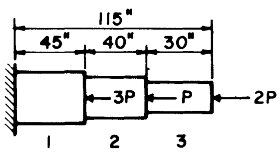
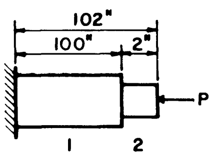
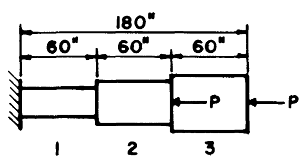
$$F_{as} = \frac{F_y\left[1 - \frac{A_s F_y}{4n_s P_e}\right]}{\frac{5}{3} + \frac{3}{8}\left(\frac{A_s F_y}{2n_s P_e}\right)^{1/2} - \frac{1}{8}\left(\frac{A_s F_y}{2n_s P_e}\right)^{3/2}} \tag{11}$$

Equations (9) and (11) are valid between the limits $P_e = \infty$ [for $(KL/r)_s = 0$] and $P_e = A_s F_y / 2n_s$ [for $(KL/r)_s = C_c$].

When $P_e \leq A_s F_y / 2n_s$, the column is taken as elastic and a factor of safety of 23/12 should be used. Thus,

$$P_a = \frac{12P_e}{23} \tag{12}$$

Table 1. Summary Of Results For Arbitrary Column Configurations

	Example Problem $F_y = 36$ kips/in.^{2}	Area Segment (in.^{2})	Mom. of Inertia Segment (in.^{4})	P_e (kips)	n	MHS Segment	$(KL/r)_s$	F_a (kips/in.^{2})
I		$A_1 = 5.$ $A_2 = 50.$	$I_1 = 11.2$ $I_2 = 312.5$	142.8	$n_1 = 1$ $n_2 = 1$	1	100.1	$F_{a1} = 12.84$ $F_{a2} = 1.28$
II		$A_1 = 18.6$ $A_2 = 11.9$ $A_3 = 5.6$	$I_1 = 429.$ $I_2 = 161.$ $I_3 = 28.$	49.6	$n_1 = 7$ $n_2 = 5$ $n_3 = 1$	2	117.2	$F_{a1} = 9.56$ $F_{a2} = 10.67$ $F_{a3} = 4.54$
III		$A_1 = 24.4$ $A_2 = 18.6$ $A_3 = 15.7$ $A_4 = 11.9$	$I_1 = 732.$ $I_2 = 429.$ $I_3 = 300.$ $I_4 = 161.$	61.3	$n_1 = 17$ $n_2 = 7$ $n_3 = 6$ $n_4 = 2$	1	81.9	$F_{a1} = 15.12$ $F_{a2} = 8.17$ $F_{a3} = 8.29$ $F_{a4} = 3.65$
IV		$A_1 = 5.$ $A_2 = 3.$	$I_1 = 20.$ $I_2 = 6.8$	15.9	$n_1 = 6$ $n_2 = 1$	1	122.5	$F_{a1} = 9.96$ $F_{a2} = 2.77$
V		$A_1 = 3.1$ $A_2 = 2.8$ $A_3 = 2.6$ $A_4 = 2.0$	$I_1 = 90.$ $I_2 = 52.5$ $I_3 = 37.5$ $I_4 = 30.$	120.3	$n_1 = 21$ $n_2 = 11$ $n_3 = 7$ $n_4 = 3$	1	18.7	$F_{a1} = 21.0$ $F_{a2} = 12.18$ $F_{a3} = 8.35$ $F_{a4} = 4.65$
VI		$A_1 = 50.4$ $A_2 = 15.7$ $A_3 = 30.8$ $A_4 = 18.6$ $A_5 = 15.7$	$I_1 = 10266.$ $I_2 = 900.$ $I_3 = 3519.$ $I_4 = 1287.$ $I_5 = 900.$	44.1	$n_1 = 17.5$ $n_2 = 14$ $n_3 = 13$ $n_4 = 11$ $n_5 = 6$	2	85.3	$F_{a1} = 5.73$ $F_{a2} = 14.71$ $F_{a3} = 6.96$ $F_{a4} = 9.76$ $F_{a5} = 6.30$
VII		$A_1 = 25.$ $A_2 = 10.$	$I_1 = 100.$ $I_2 = 50.$	416.4	$n_1 = 1$ $n_2 = 1$	2	82.9	$F_{a1} = 6.01$ $F_{a2} = 15.03$
VIII		$A_1 = 25.$ $A_2 = 8.$ $A_3 = 4.$	$I_1 = 100.$ $I_2 = 35.$ $I_3 = 20.$	130.9	$n_1 = 6$ $n_2 = 3$ $n_3 = 2$	3	66.1	$F_{a1} = 8.08$ $F_{a2} = 12.62$ $F_{a3} = 16.83$
IX		$A_1 = 50.$ $A_2 = 19.4$	$I_1 = 100.$ $I_2 = 50.$	701.7	$n_1 = 1$ $n_2 = 1$	2	89.1	$F_{a1} = 5.54$ $F_{a2} = 14.28$
X		$A_1 = 2.77$ $A_2 = 9.0$ $A_3 = 15.$	$I_1 = 25.$ $I_2 = 75.$ $I_3 = 150.$	49.9	$n_1 = 2$ $n_2 = 2$ $n_3 = 1$	1	89.1	$F_{a1} = 14.30$ $F_{a2} = 4.40$ $F_{a3} = 1.32$

and the allowable stress for the MHS segment becomes

$$F_{as} = \frac{12 n_s P_e}{23 A_s} \quad (13)$$

Once the allowable stress in the MHS segment is determined, the allowable stress in any other segment is found by multiplying the appropriate equation [(9), (11), or (13)] by the quantity nA_s/n_sA. This method is valid for all grades of steel covered by Sect. 1.5.1.3 (allowable stresses in compression) of the Specification. If F_y is different for the various segments, the procedure will consistently yield reasonable or conservative results only if the MHS segment is also the one with the lowest yield stress. For other cases of mixed F_y's, the more exact method of determining P_u as a basis for F_a is recommended.

A method similar to the MHS segment procedure is given in Ref. 1. It requires, however, that the allowable stress in each segment be found from an effective (KL/r) for that segment. As a consequence, the allowable stresses for segments other than the MHS segment, as determined by the method of Ref. 1 may be unconservative to a considerable extent.

SAMPLE CALCULATIONS

The following two problems are presented to demonstrate the simplicity of the proposed procedure.

Problem 1

Given: The dimensions and properties of the column shown in Table 1, Example VII.

Find: F_a for both segments.

Solution:

1. Determine P_e by any method available. For this column, $P_e = 416.4$ kips.
2. The most highly stressed (MHS) segment is segment 2, by inspection.
3. Calculate $(KL/r)_s$ from Eq. (4):

$$(KL/r)_s = \sqrt{\frac{\pi^2(29{,}000)(10)}{(1)(416.4)}} = 82.9$$

From Appendix A of the AISC Specification:

$$F_{as} = F_{a2} = 15.03 \text{ ksi*}$$

4. The allowable stress in the other segment is

$$F_{a1} = 15.03 \left[\frac{(1)(10)}{(1)(25)}\right] = 6.01 \text{ ksi}$$

The apparent contradiction posed by the larger segment having a lesser allowable stress can be resolved when we recognize that there is only one characteristic allowable load P_a and it applies to the entire column. Any more than 6.01 ksi in Segment 1 will be accompanied by overstressing of Sgment 2. Unless bending is also present in the column, requiring an interaction formula to determine the allowable combined stress state in each segment, there is no need to calculate F_a for the other segments.

* *As a check, Eq. (11) can be used.*

Problem 2

Given: The dimensions and properties of the column shown in Table 1, Example VIII.

Find: F_a for all segments.

Solution:

1. $P_e = 130.9$
2. For Segment 1: $6P/25 = 0.24P$
   Segment 2: $3P/8 = 0.375P$
   Segment 3: $2P/4 = 0.5P$
   Segment 3 is the MHS.
3. $(KL/r)_s = \sqrt{\dfrac{\pi^2(29{,}000)(4)}{(2)(130.9)}} = 66.1$

   $F_{as} = F_{a3} = 16.83$ ksi (AISC Specification, Appendix A)

$$F_{a1} = 16.83 \left[\frac{(4)(6)}{(2)(25)}\right] = 8.08 \text{ ksi}$$

$$F_{a2} = 16.83 \left[\frac{(4)(3)}{(2)(8)}\right] = 12.62 \text{ ksi}$$

Table 1 shows a variety of columns and loadings that have been investigated. The procedure and F_y in all cases have been the same. Except as noted above, changing F_y does not affect the procedure, only the results.

DISCUSSION OF ERRORS

As was indicated earlier, basing all calculations on P_e may lead to erroneous, albeit conservative, values of F_a. In Table 2 the ultimate loads for the examples in Table 1, as determined by both Eq. (7) and by the "exact" procedure, are given, with the error shown as a percentage of the former.*

As demonstrated by Examples III and V, there is no obvious relation between the error and how divergent a column is from an apparently "rational" (i.e., larger section properties for larger segment loads) design. If there were such a relation, there would be no need to resort to the approximations inherent in Eq. (7).

* *The lower bound value is used as the basis for comparison even though it is the inexact one, because it may be the only one available to the designer.*

Table 2. Compare Theoretical P_u With P_u (MHS Method)

Example (see Table 1 for Sketch)	(1) E (at failure) (kips/in.²)	(2) P_u Ultimate Strength Method (kips)	(3) P_u (MHS Method) (kips)	(4) % Error $\left[\frac{(2)-(3)}{(3)}\right] \times 100$
I	E_1 = 24,939 E_2 = 29,000	123.7	122.0	1.4
II	E_1 = 28,990 E_2 = 28,457 E_3 = 29,000	48.6	48.6	—
III	E_1 = 12,623 E_2 = 29,000 E_3 = 29,000 E_4 = 29,000	45.1	40.6	11.1
IV	E_1 = 28,918 E_2 = 29,000	15.9	15.9	—
V	E_1 = 744 E_2 = 28,327 E_3 = 29,000 E_4 = 29,000	5.3	5.3	—
VI	E_1 = 29,000 E_2 = 19,202 E_3 = 29,000 E_4 = 28,931 E_5 = 29,000	32.0	31.0	3.2
VII	E_1 = 29,000 E_2 = 14,632	306.3	282.3	8.5
VIII	E_1 = 29,000 E_2 = 23,073 E_3 = 3,587	69.7	62.2	12.0
IX	E_1 = 29,000 E_2 = 0.0	697.6	523.2	33.3
X	E_1 = 21,450 E_2 = 29,000 E_3 = 29,000	37.6	37.4	0.5

To depict how the "exact" ultimate loads for the various columns in Tables 1 and 2 compare with one another, and with their approximate values given by Eq. (7), Fig. 5 has been constructed similar to Fig. 4. The ordinate of Fig. 5 is now P_u/P_y , where $P_y = A_sF_y/n_s$. Example IX was selected to demonstrate the maximum possible error when $(KL/r)_s$ is equal to $C_c/\sqrt{2}$. In this case the MHS segment clearly makes a negligible contribution to the actual ultimate strength of the column, but because it is used as the basis for calculating the approximate ultimate strength, the large error results. It is interesting to note in comparison with Example IX that the "irrational" column of Example X has very little error at the same $(KL/r)_s$, because the MHS segment in Example X makes a very significant contribution to the actual ultimate strength of the column.

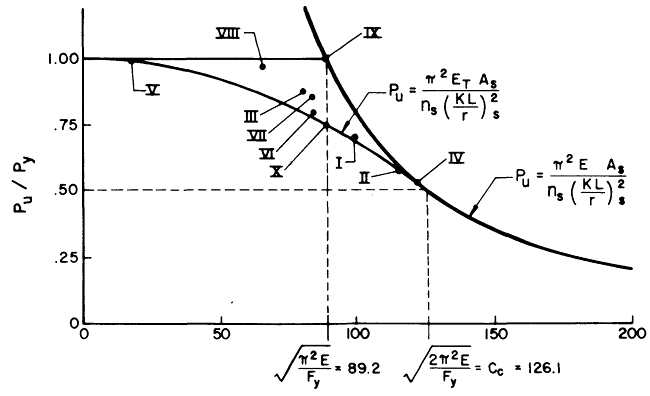


Figure 5

SUMMARY AND CONCLUSION

What has been presented are the background and description of a simple process for determining allowable axial stresses in inelastic segmented columns. The method is independent of end conditions and can be used for columns with any number of segments, if the corresponding Euler buckling load is known or can be determined. As a basis for the approach, similar principles to those employed in the development of the AISC allowable compressive stresses for axially loaded prismatic columns are used. The procedure is outlined as follows:

1. Calculate the Euler buckling load
2. Locate the most highly stressed segment (MHS) by dividing the total load on each segment by the area of that segment ($n \times p/A$)
3. (a) If F_y is any of those available in Table 1 of Appendix A of the AISC Specification.
   i. Calculate $(KL/r)_s$ for the MHS using Eq. (4).
   ii. Knowing $(KL/r)_s$, read a value of F_{as} from Appendix A of the AISC Specification.

   (b) If F_y is not typical, calculate F_{as} using Eq. (11) or (13), as appropriate.
4. If required, calculate a value of F_a for each of the remaining segments by using the expression

$$F_a = \frac{F_{as}A_sn}{n_sA}$$

Except when all segments are at the same stress, this method results in approximate allowable stresses which may be conservative, but are never unconservative. When the various segments of a column have different yield stresses, the procedure may not be valid unless the MHS segment is the one with the lowest yield point.

As with most allowable stress calculations, this procedure does not provide a direct guide to efficient or optimum design, but only permits the designer to evaluate the safety of proportions selected by some other process. If, in addition to axial loads, bending moments are present, the designer will also need to know allowable stresses in bending, which may also be influenced by an absence of lateral support. For a design procedure of one type of segmented column, the crane column, the reader is referred to Ref. 10 or 11. The method of determining allowable axial stresses in these references can be replaced by the method of this article.

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