A Method for the Plastic Design of Unbraced Multistory Frames

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In a building without vertical bracing or shear walls, the frames must be able to resist gravity loads and the combination of vertical and horizontal loads, plus second order effects due to vertical load-lateral displacement interaction. Besides, frame stiffness must be sufficient to keep lateral displacements under working loads below maximum allowable values.

Two different load factors are used in plastic design—one for vertical loads only and a smaller one for the combination of vertical permanent and horizontal accidental loads. Design of two or three stories at the top of unbraced buildings is generally governed by gravity loads, because the beams and columns necessary to support vertical loads are also able to resist gravity plus horizontal loads under a reduced load factor. The importance of horizontal forces increases in lower stories, and their design is governed by the combination of both types of loads.

Design of upper stories is usually made with no consideration of lateral displacements. A revision is carried out later in order to verify that the overall critical load is not smaller than the collapse mechanism load. If necessary, the structure is modified or the critical load is taken as the limit of structural usefulness.

When design is governed by combined gravity and lateral loads, collapse takes place by instability, characterized by increasing lateral displacements under horizontal loads that grow to a maximum and decrease afterwards. Behavior of the structure can be ascertained studying the formation of successive plastic hinges due to increasing horizontal forces which act upon the structure, loaded from the beginning with complete factored vertical loads. Beams and columns are assumed to remain in the elastic range between plastic hinges. Influence of axial loads on column bending strength, second order moments, and compatibility conditions must be taken into account.

A method for the design of beams and columns in stories of regular frames governed by the combination of gravity and lateral loads is presented in this paper. To this end, horizontal load-lateral displacement curves (Q-Δ curves) corresponding to preliminary sizes of beams and columns are plotted for all or some of the stories. If the story behavior, as depicted by the Q-Δ curve, is not satisfactory from the point of view of strength or lateral stiffness, preliminary sizes are modified and a new curve is plotted.

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The method can be easily programmed for use in computerized plastic design. Nevertheless, one of the main advantages of the method is its simplicity, which makes it suitable for simple manual computations. It is, therefore, a powerful tool for the design of medium size, regular buildings that do not justify the use of computers, and for the approximate revision of buildings designed by computer, employing elastic or plastic methods.

The method described in this paper is related to a method originally developed at Lehigh University,[1-5] which was later simplified by the writer and others.[6-9] The amount of numerical work is drastically reduced, although keeping enough accuracy for practical purposes, and computations are systematized by arranging them in tabular form. Besides, the method is based on the condition that plastic hinges shall appear only in beams (with the exception of column bases). This condition is in accordance with modern design philosophy, especially in seismic areas.

SECOND-ORDER ANALYSIS

Analysis of multistory rigid frames has traditionally been made using first-order elastic theory, but second-order effects can be significant, especially in unbraced frames.

In current design practice, second-order effects are usually considered, in an indirect and approximate way, by using interaction equations for column design. Moments computed by a first-order elastic or plastic analysis are more or less arbitrarily amplified, and effective lengths longer than actual lengths are used. Beams are designed using the original first-order moments.[10] Incorrect results are obtained when each column is treated individually, especially if the frames are geometrically irregular or column and beam stiffnesses change considerably in each story or in adjacent stories. Also, design of beams to support first-order moments is irrational, as they have to equilibrate the amplified moments that columns apply to the joints.

The number of factors that has to be taken into account in an exact elastoplastic second-order analysis is high, but most of them are usually neglected in ordinary design problems.[11,12] The two most important factors in multistory frame behavior are formation of an increasing number of plastic hinges and interaction of vertical loads and story lateral displacements ($P\Delta$ effect). Only these two factors will be considered in this paper.

EVALUATION OF $P\Delta$ EFFECTS

$P\Delta$ effects can be evaluated making a first-order analysis of the structure under actual vertical loads and horizontal loads, increased in the amount necessary to reproduce, approximately, second-order effects.

The fictitious additional shear force, V_i, that has to be applied to story **i** of a multistory frame is given by

$$V_i = \frac{P_i}{h_i}\,\Delta_{i,i-1} \tag{1}$$

where

P_i = weight of the level under consideration plus every level above it
$\Delta_{i,i-1}$ = relative horizontal displacement between the upper and lower levels of the story
h_i = story height (Fig. 1)

BASIC EQUATIONS

Columns in any story of a building subjected to the combined action of gravity loads and horizontal wind or earthquake forces must resist bending moments produced by the horizontal shear force Q, plus those due to the total vertical load P acting upon the laterally deformed structure (Fig. 2). $P\Delta$ moments are similar to those produced by a fictitious shear force $P\Delta/h$.

P and Δ are equal to P_i and $\Delta_{i,i-1}$ in Eq. (1).

Equilibrium of horizontal loads gives:

$$\Sigma M_c = Qh + P\Delta \tag{2}$$

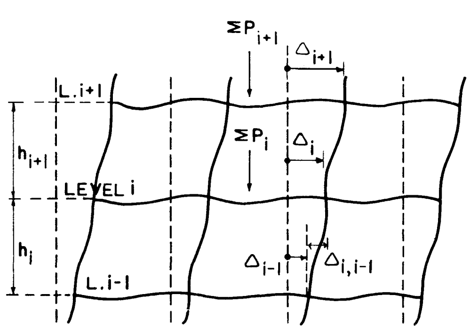


Fig. 1. Lateral displacement of a multistory rigid frame

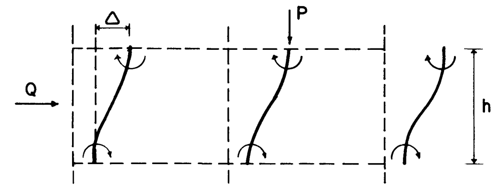


Fig. 2. Forces which produce bending in columns

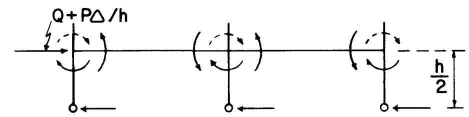


Fig. 3. Substructure corresponding to an intermediate story (vertical loads not shown)

$$Q = \frac{\Sigma M_c}{h} - P\frac{\Delta}{h} \tag{3}$$

ΣM_c is the sum of the moments in both ends of all columns in the story.

Equation (3) shows clearly that the $P\Delta$ effect reduces the structure's capability to resist lateral load.

The substructure in Fig. 3 is obtained assuming that the point of inflection in each column is at mid-height of the column[2,8] and isolating the upper part of the story. The $P\Delta$ effect is included by increasing the horizontal load. Vertical loads are not shown.

From the equilibrium of horizontal forces:

$$\Sigma M_c = \left(Q + P\frac{\Delta}{h}\right)\frac{h}{2} = \frac{Qh}{2} + P\frac{\Delta}{2} \tag{4}$$

In this equation, and in the rest of the paper, ΣM_c refers only to the moments acting in the upper end of the story columns.

Joint moments are also in equilibrium; then,

$$\Sigma M_v = (\Sigma M_c)_L + (\Sigma M_c)_U$$

where ΣM_v is the sum of moments at both ends of every beam in the level under study, due to horizontal forces, including the fictitious one, $P\Delta/h$, and $(\Sigma M_c)_L$ and $(\Sigma M_c)_U$ are the sums of moments at the ends of the columns connecting to the joints of that level, below and above it, also due to horizontal loads.

Assuming that $(\Sigma M_c)_U = (\Sigma M_c)_L = \Sigma M_c$,

$$\Sigma M_v = 2\Sigma M_c \tag{5}$$

The assumption that leads to Eq. (5) is conservative, but sufficiently accurate for design purposes.[2]

From Eqs. (4) and (5), $Q = (\Sigma M_c - P\Delta/2)/(h/2)$ and $\Sigma M_c = \Sigma M_v/2$; then,

$$Q = \frac{\Sigma M_v - P\Delta}{h} = \frac{\Sigma M_v}{h} - \frac{P\Delta}{h} \tag{6}$$

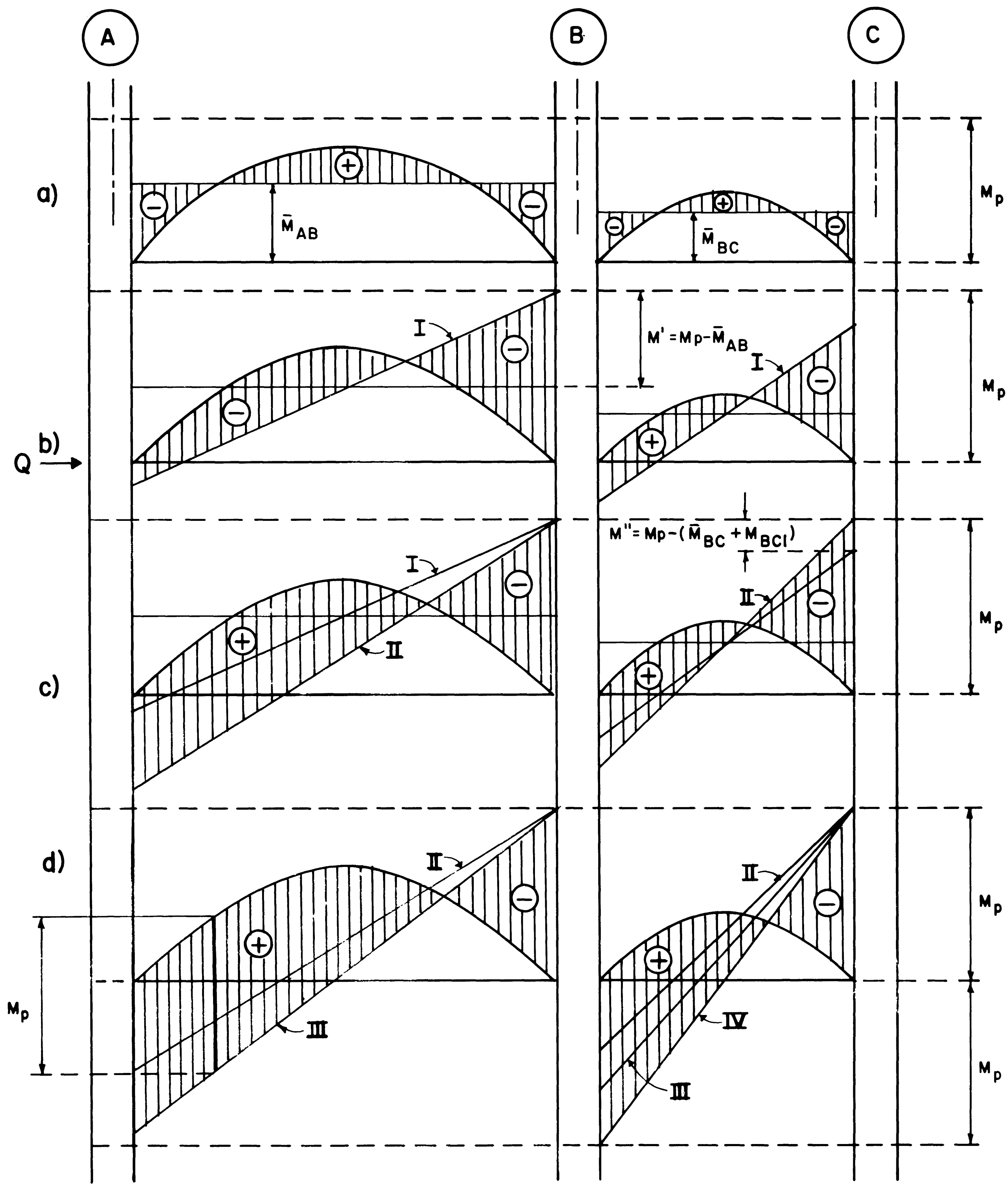


Fig. 4. Bending moment diagrams for the different loading stages

Application of slope-deflection equations to an isolated column leads to[6,8]

$$\frac{\Delta}{h} = \frac{\Sigma_c M_v h}{12EI_c} + \theta \tag{7}$$

where $\Sigma_c M_v$ is the end moment of the beam that connects to an exterior column, or the sum of the end moments of two beams if it is an interior one, θ is the angle of rotation at the column's upper end, and I_c its moment of inertia.

To obtain Eq. (7) it has been assumed that the column behaves elastically and that its stiffness is independent of the axial load. Neither assumption is strictly true, but they do not introduce significant errors in columns with slenderness ratios and axial loads in the range which is usual in buildings.[8]

Equation (7) can be generalized to cover the complete story:

$$\frac{\Delta}{h} = \frac{h}{12E} \Sigma \frac{M_v}{I_c} + \theta \tag{8}$$

ΣM_v is now the sum of moments at both ends of all beams in the level, and ΣI_c is the sum of moments of inertia of all the columns that contribute to the story's lateral stiffness. It has been assumed that θ is the same for every joint in the level.[8]

Equation (8) can finally be written as:

$$\frac{\Delta}{h} = \frac{h}{12E\Sigma I_c} \Sigma M_v + \theta \tag{9}$$

The two basic equations to obtain the Q-Δ curve are Eqs. (6) and (9), developed for a complete building story. They are applied to an isolated frame in the numerical examples at the end of this paper.

Q-Δ CURVE OF A STORY

Loads initially applied to the story are the working gravity loads multiplied by the load factor corresponding to the combination of permanent and accidental loads. Bending moment diagrams are determined using clear beam spans and assuming no rotation at the joints. Unbalanced moments are resisted by the columns meeting at each joint (Fig. 4a).

Upon application of the horizontal load the story deflects laterally and additional moments have to be computed and added to those due to vertical load (Fig. 4b).

The first stage in the loading process ends with the formation of the first plastic hinge. It develops at the leeward end of one of the girders, where vertical and horizontal load moments are additive. The horizontal load additional moment necessary to develop a plastic hinge at the leeward end of each girder is $M' = M_p - \overline{M}$, where M_p and $\overline{M}$ are the girder plastic moment and fixed end moment.

The joint rotation corresponding to each M' moment is now computed:

$$M' = S\theta' = C_1 EK\theta' \tag{10}$$

$$\theta' = M'/C_1 EK$$

where S is the girder stiffness and C_1 a numerical factor.

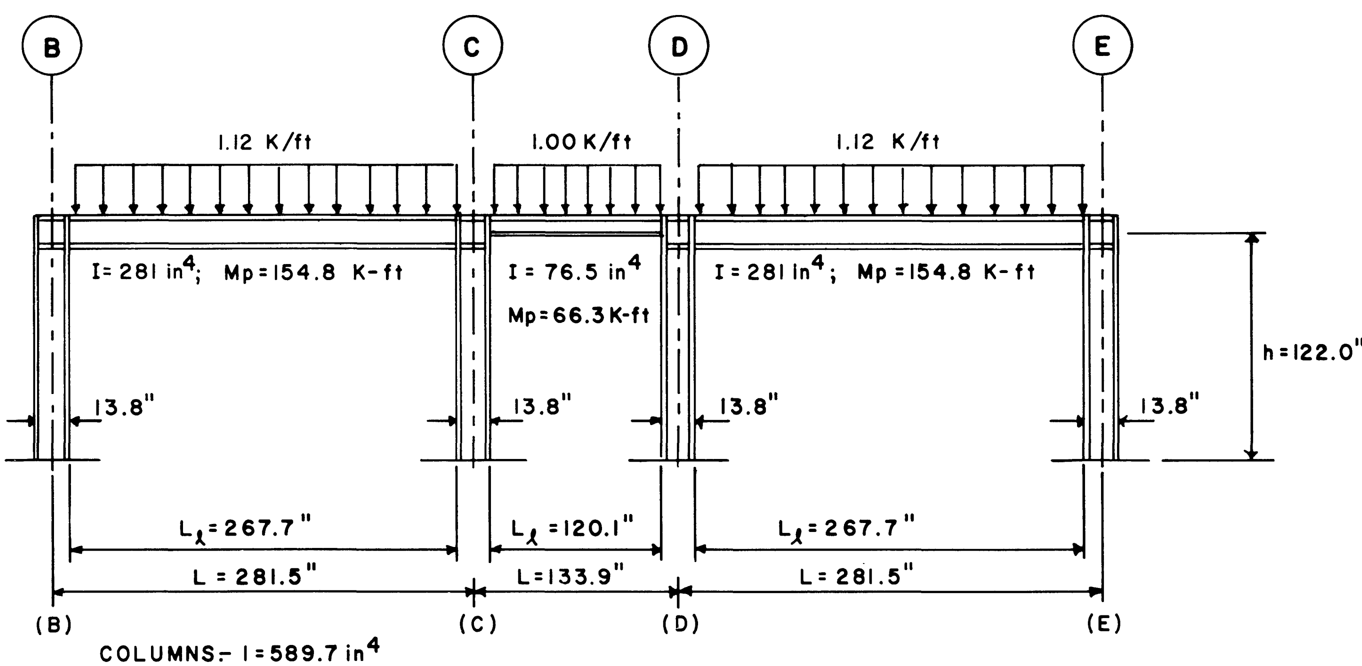


Fig. 5. Frame analyzed in illustrative example No. 1

Table 1

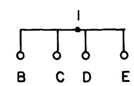
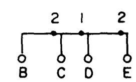
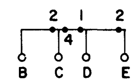
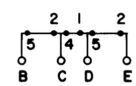
ROW	CONCEPT	UNITS		(B)			(C)			(D)			(E) SEQUENCE OF PLASTIC HINGE FORMATION
1	h	in	122.0										
2	P(TOTAL LOAD)	Kips	502.2										
3	$K_v = I_v/L_\ell$	in^3			1.05			0.64			1.05		
4	M_p	K-ft			154.8			66.3			154.8		
5	$M_i = WL_\ell^2/8$	K-ft			69.7			12.5			69.7		
6	$\bar{M} = WL_\ell^2/12$	K-ft		46.5		46.5	8.3		8.3	46.5		46.5	
7	C_1	—		6.0		6.0	6.0		6.0	6.0		6.0	
8	$S_1 = C_1 E K_v$	K-ft		15225		15225	9280		9280	15225		15225	
9	$M' = M_p - \bar{M}$	K-ft		108.3		108.3	58.0		58.0	108.3		108.3	
10	$\theta' = M'/S_1$	—		0.00711		0.00711	0.00625		0.00625	0.00711		0.00711	
11	θ_1	—	0.00625										
12	$M_1 = S_1\theta_1$	K-ft		95.2		95.2	58.0		58.0	95.2		95.2	
13	ΣM_1	K-ft	496.8										
14	ΣI_c	in^4	2358.8										
15	$(\Delta/h)_1$	—	0.00714										
16	Q_1	Kips	45.28										
17	$M_{R1} = \bar{M} + M_1$	K-ft				141.7			$\boxed{66.3}$			141.7	
18	$M_{L1} = \bar{M} - M_1$	K-ft		-48.7			-49.7			-48.7			
19	C_2	—		6.0		6.0	3.0		—	6.0		6.0	
20	$S_2 = C_2 E K_v$	K-ft		15225		15225	4640		—	15225		15225	
21	$M_R'' = M_p - M_{R1}$	K-ft				13.1			0			13.1	
22	$M_L'' = M_p + M_{L1}$	K-ft		106.1			16.6			106.1			
23	$\theta'' = M''/S_2$	—		0.00697		0.00086	0.00358		—	0.00697		0.00086	
24	θ_2	—	0.00086										
25	$M_2 = S_2\theta_2$	K-ft		13.1		13.1	4.0		—	13.1		13.1	
26	ΣM_2	K-ft	56.4										
27	ΣI_c	in^4	2358.8										
28	$(\Delta/h)_2$	—	0.00096										
29	Q_2	Kips	5.1										
30	$(\Delta/h)_T$	—	0.00810										
31	$Q_T = Q_1 + Q_2$	Kips	50.38										
32	$M_{R2} = M_{R1} + M_2$	K-ft				$\boxed{154.8}$			$\boxed{66.3}$			$\boxed{154.8}$	
33	$M_{L2} = M_{L1} - M_2$	K-ft		-61.8			-53.7			-61.8			

Table 1 (*cont'd*)

ROW	CONCEPT	UNITS		(B)		(C)		(D)		(E) SEQUENCE OF PLASTIC HINGE FORMATION
34	C_3	—		3.0	—	3.0	—	3.0	—	
35	$S_3=C_3EKv$	K-ft		7612.5	—	4640	—	7612.5	—	
36	$M_R'''=Mp-M_{R2}$	K-ft			0		0		0	
37	$M_L'''=Mp+M_{L2}$	K-ft		93.0		12.6		93.0		
38	$\theta'''=M'''/S_3$	—		0.01222	—	0.00272	—	0.01222	—	
39	θ_3	—	0.00272							
40	$M_3=S_3\theta_3$	K-ft		20.7	—	12.6	—	20.7	—	
41	ΣM_3	K-ft	54.0							
42	ΣIc	in^4	1769.1							
43	$(\Delta/h)_3$	—	0.00285							
44	Q_3	Kips	3.9							
45	$(\Delta/h)_T$	—	0.01095							
46	Q_T	Kips	54.28							
47	$M_{R3}=M_{R2}+M_3$	K-ft			$\boxed{154.8}$		$\boxed{66.3}$		$\boxed{154.8}$	
48	$M_{L3}=M_{L2}-M_3$	K-ft		-82.5		$\boxed{-66.3}$		-82.5		
49	C_4	—		3.0	—	0	—	3.0	—	
50	$S_4=C_4EKv$	K-ft		7612.5	—	—	—	7612.5	—	
51	$M_R^{IV}=Mp-M_{R3}$	K-ft			0		0		0	
52	$M_L^{IV}=Mp+M_{L3}$	K-ft		72.3				72.3		
53	$\theta^{IV}=M^{IV}/S_4$	—		0.00950	—	—	—	0.00950	—	
54	θ_4	—	0.00950							
55	$M_4=S_4\theta_4$	K-ft		72.3	—	—	—	72.3	—	
56	ΣM_4	K-ft	144.6							
57	ΣIc	in^4	1179.4							
58	$(\Delta/h)_4$	—	0.01002							
59	Q_4	Kips	9.2							
60	$(\Delta/h)_T$	—	0.02097							
61	Q_T	Kips	63.48							
62	$M_{R4}=M_{R3}+M_4$	K-ft			$\boxed{154.8}$		$\boxed{66.3}$		$\boxed{154.8}$	
63	$M_{L4}=M_{L3}-M_4$	K-ft		$\boxed{-154.8}$		$\boxed{-66.3}$		$\boxed{-154.8}$		
64	$\Sigma Mv/L_\ell$	Kips		13.88 $\uparrow$	13.88 $\downarrow$	13.25 $\uparrow$	13.25 $\downarrow$	13.88 $\uparrow$	13.88 $\downarrow$	
65	$WL_\ell/2$	Kips		12.49 $\downarrow$	12.49 $\downarrow$	5.00 $\downarrow$	5.00 $\downarrow$	12.49 $\downarrow$	12.49 $\downarrow$	
66	V	Kips		1.39 $\uparrow$	26.37 $\downarrow$	8.25 $\uparrow$	18.25 $\downarrow$	1.39 $\uparrow$	26.37 $\downarrow$	$\Sigma V=59.96\downarrow$
67	$dc/2$	ft		0.58	0.58	0.58	0.58	0.58	0.58	
68	$M=M_4 \pm Vdc/2$	K-ft		155.6	170.1	71.1	76.9	155.6	170.1	
69	M_{COL}	K-ft	155.6 /2 = 77.8		241.2/2=120.6		232.5/2=116.3		170.1/2 = 85.1	

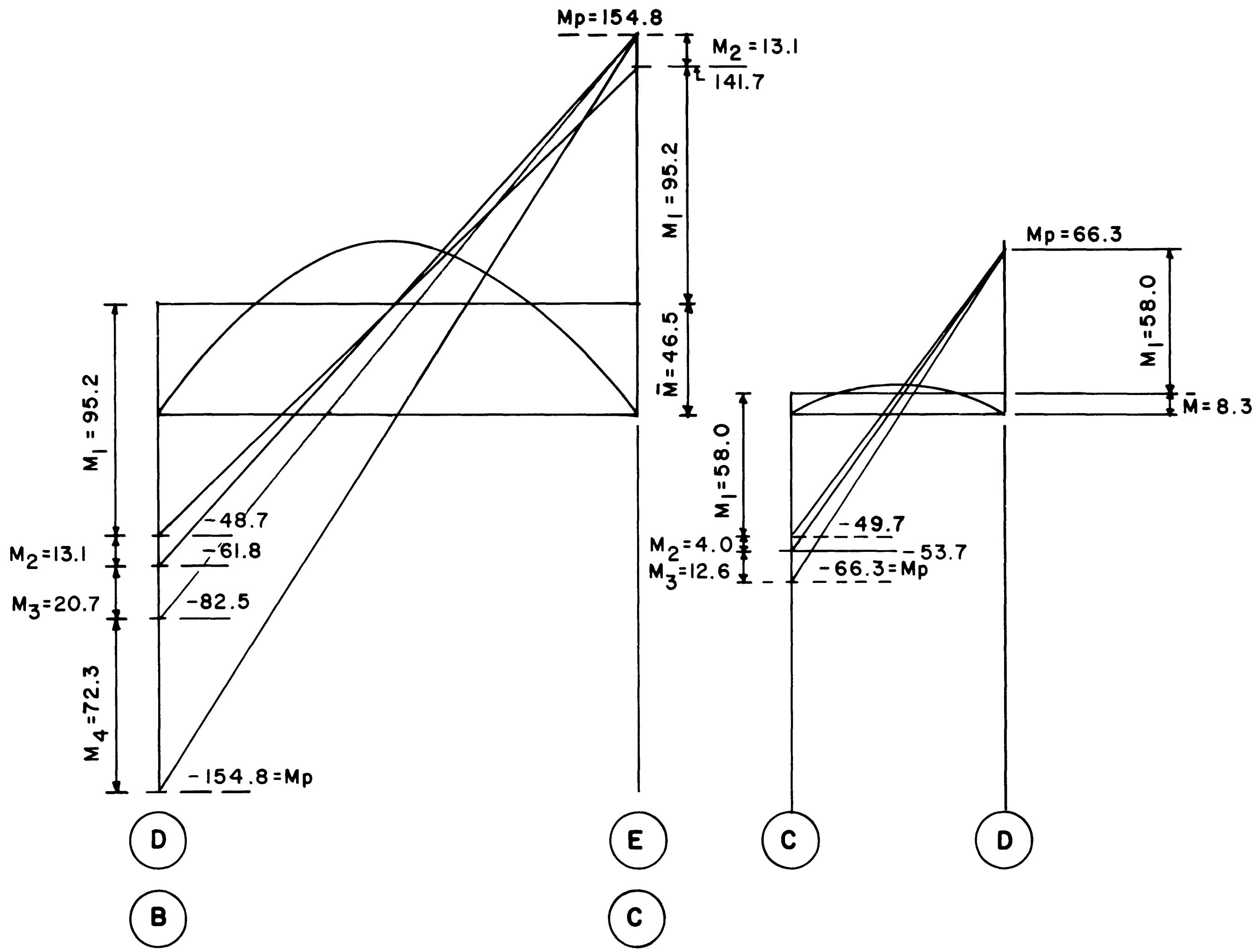


Fig. 6. Bending moment diagrams for illustrative example No. 1

If the girder cross section is constant, $C_1 = 6$, $K = I/L$.

The smallest θ' angle is the one corresponding to the first plastic hinge (in Fig. 4 it has been assumed that the first hinge develops at the leeward end of beam AB). When that angle is known, Eq. (10) is used to compute the moments at the ends of each beam corresponding to it. The sum of the beams end moments, ΣM_v, is taken into Eq. (9), and the lateral displacement determined as Δ/h. Finally, Eq. (6) gives the horizontal load Q that produces that displacement. Coordinates of a point in the Q-Δ/h curve are now known: the straight line from the origin to that point is a good representation of the first part of the Q-Δ/h curve.

The bending moment diagram corresponding to the formation of the first plastic hinge is diagram I, Fig. 4b.

The second stage is similar to the first, but the stiffness of beam AB is reduced because of the plastic hinge developed at the leeward end (if the moment of inertia of the beam is constant, the stiffness is $3EI/L = 3EK$); also, the moment at the plastic hinge location does not change. The second stage ends when a new plastic hinge develops, at the leeward end of beam BC, for instance (Fig. 4c).

In the third stage of the loading process, both beams have plastic hinges at the leeward end, and column C does not contribute any longer to the story's lateral rigidity. Its moment of inertia is not included in ΣI_c, Eq. (9).

A number of plastic hinges sufficient to transform the story into a mechanism eventually develops (Fig. 4d). Until then, the Q-Δ/h curve consists of several straight lines connecting the points which represent the termination of each stage. Upon formation of the mechanism, the relationship between horizontal load and lateral displacement is given by a descending straight line that passes through the point corresponding to the last plastic hinge. The equation of this line is:

$$Q = -\frac{P\Delta}{h} + \frac{M_r}{h}$$

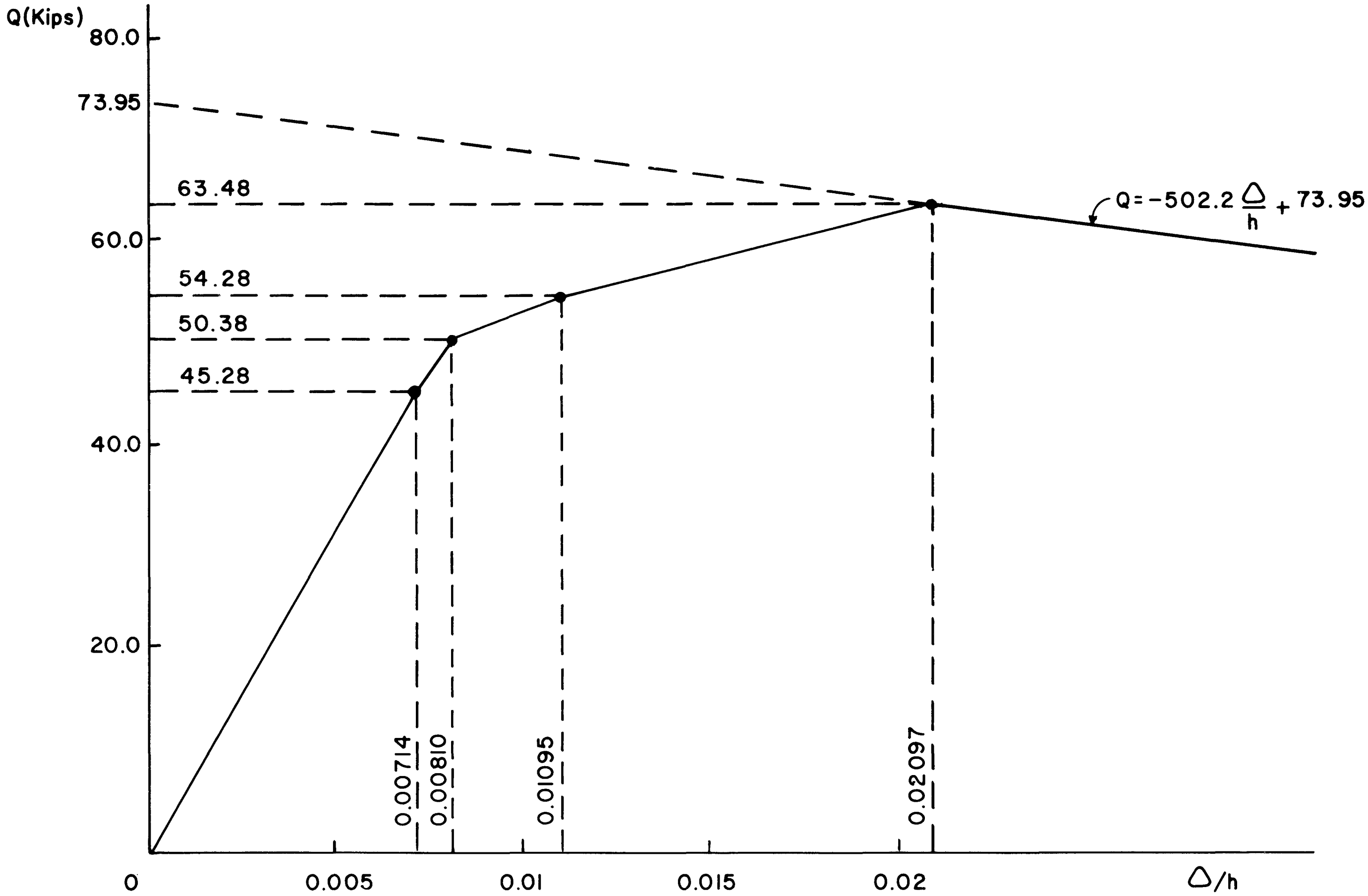


Fig. 7. Horizontal load—lateral deflection curve for illustrative example No. 1

where M_r is the total restraining moment provided by all of the beams in the story when the last plastic hinge develops.

The complete horizontal load-lateral displacement curve can now be plotted.

Numerical solution of a given problem is considerably facilitated by tabulating the computations, as shown in the illustrative example. It is generally convenient to plot simultaneously the bending moment diagrams, as in Fig. 4, to check the numerical results. The diagram is necessary if the second plastic hinge in one or more beams develops in an intermediate section, instead of the windward end, because the hinge position and windward moment are then graphically determined (Fig. 4d). Also, bending moment diagrams are necessary when the beams and the floor slab work as composite members, in order to find the zones of positive and negative bending moment.*

* *A paper on this topic will be submitted for publication in the near future.*

COLUMN DESIGN

Columns must be able to resist axial loads and bending moments applied to them by the beams until formation of the story collapse mechanism. The moments at the column faces must be increased by $Vd_c/2$ to obtain the design moments at the column center line, where V is the shear force at the girder end and d_c is the column depth.

As the $P\Delta$ effect has already been considered, column sizes are checked using a formula for beam-columns whose ends can not displace laterally.

If the designer wants to make sure that no plastic hinges will develop at the column ends, he can use a load factor bigger than that employed in beam design (if this precaution is not taken, some plastic hinges can possibly develop in the columns because of differences between the assumed and actual response of the structure and factors not considered in analysis and design, such as differences between real and specified yield points or handbook and actual geometric properties of rolled shapes).

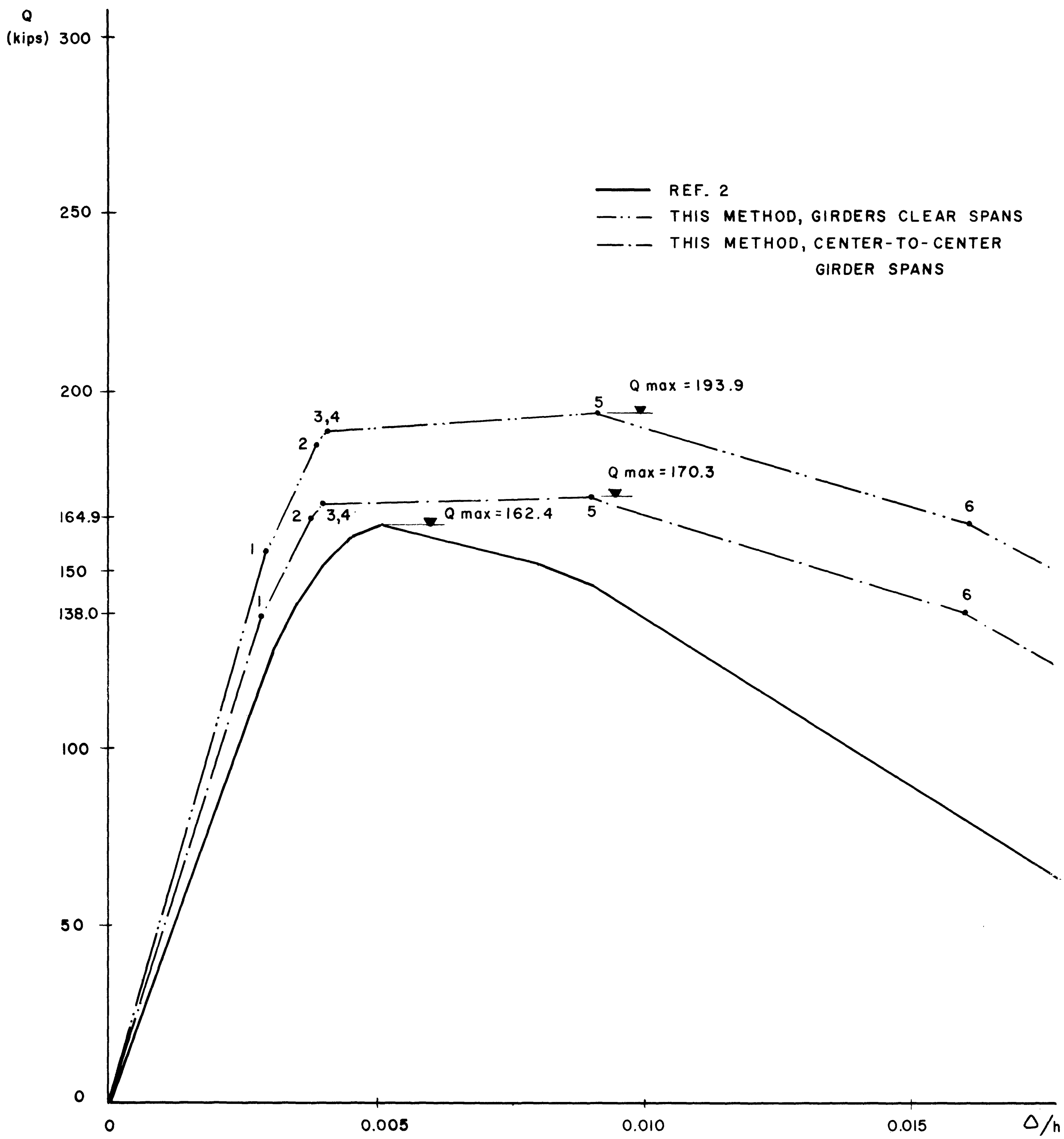


Fig. 8. Horizontal load—lateral deflection curves for illustrative example No. 2

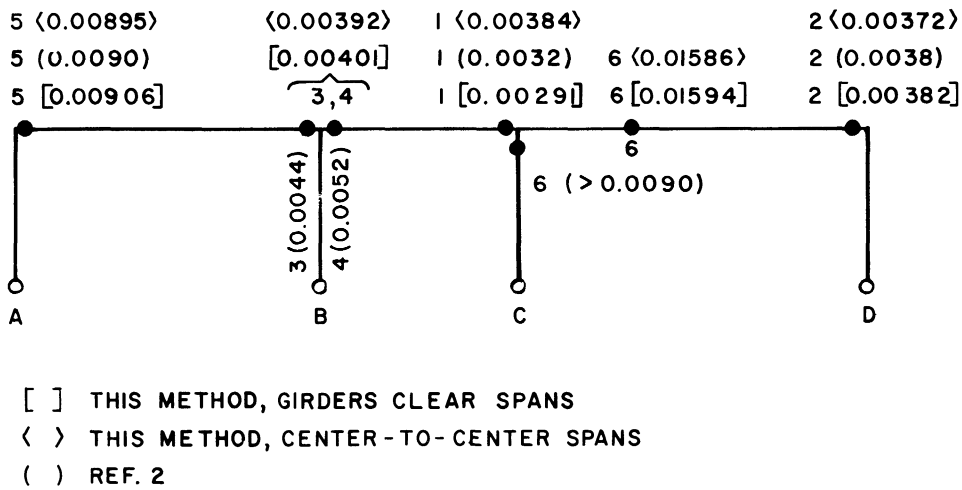


Fig. 9. Sequences of plastic hinge formation for illustrative example No. 2

ILLUSTRATIVE EXAMPLES

Example 1—Figure 5 shows a story of a multistory frame belonging to a building which was designed by the allowable stress method in the writer's office. Vertical loads are multiplied by the load factor corresponding to the combination of vertical permanent and horizontal accidental loads.

Computations are shown in Table 1, bending moment diagrams for every loading stage are depicted in Fig. 6, and the horizontal load-lateral deflection curve is shown in Fig. 7.

Example 2—The structure used as an illustrative example in Ref. 2 was analyzed in the following two ways, using the method described in this paper:

(a) Employing the clear spans of girders, as suggested in this paper.
(b) Computing restraining moments using center-to-center girder spans, as in Ref. 2.

Results of both analyses are shown in Figs. 8 and 9, which also contain the Q-Δ curve and the sequence of plastic hinge formation found in Ref. 2. Agreement is fairly good. It is the writer's belief that results based on clear spans are closer to the structure's true behavior.

REFERENCES

1. *Driscoll, G. J., Jr. et al.* Plastic Design of Multi-Story Frames *Lecture Notes and Design Aids, Report Nos. 273.20 and 273.24, Fritz Engineering Laboratory, Lehigh University, 1965.*
2. *Daniels, J. H., and L. W. Lu* The Subassemblage Method of Designing Unbraced Multi-Story Frames *Report No. 273.37, Fritz Engineering Laboratory, Lehigh University, March 1966.*
3. *Daniels, J. H. and L. W. Lu* Design Charts for the Subassemblage Method of Designing Multi-Story Frames *Report No. 273.54, Fritz Engineering Laboratory, Lehigh University, Dec. 1966.*
4. *Daniels, J. H.* A Plastic Method for Unbraced Frame Design *Engineering Journal, AISC, Vol. 3, No. 4, Oct. 1966.*
5. *Driscoll, G. C., J. O. Armacost and W. C. Hansell* Plastic Design of Multi-Story Frames by Computer *Journal of the Structural Division, ASCE, Vol. 96, No. ST1, Jan. 1970.*
6. *De Buen, O.* Diseño plástico de marcos rígidos no contraventeados *II Congreso Nacional de Ingeniería Sísmica, Veracruz, May 1968.*
7. *De Buen, O.* Antiseismic Design of Multi-Story Steel Frames by Plastic Methods *IV World Conference on Earthquake Engineering, Santiago, Chile, Jan. 1969.*
8. *De Buen, O.* A Modification to the Subassemblage Method of Designing Unbraced Multi-Story Frames *Engineering Journal, AISC, Vol. 6, No. 4, Oct. 1969.*
9. *Disque, R. O.* Applied Plastic Design of Unbraced Multistory Frames *Engineering Journal, AISC, Vol. 8, No. 4, Oct. 1971.*
10. Specification for the Design, Fabrication and Erection of Structural Steel for Buildings *AISC, Feb. 1969 (including Supplements 1, 2 and 3, adopted Nov. 1970, Dec. 1971, and June 1974).*
11. *Mac Gregor, J. G.* Stability of Reinforced Concrete Building Frames *State of Art Report No. 1, Technical Committee No. 23, International Conference on Planning and Design of Tall Buildings, Lehigh University, Aug. 1972.*
12. *Adams, P. F.* Stability of Three-Dimensional Building Frames *State of Art Report No. 4, Technical Committee No. 16, International Conference on Planning and Design of Tall Buildings, Lehigh University, Aug. 1972.*