A Fresh Look at Bolted End-Plate Behavior and Design

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End-plate connections of the typical configuration shown in Fig. 1 are increasingly used as moment-resistant connections in framed structures. However, end plates designed by the prying force formulas in the AISC *Manual of Steel Construction*[1] may be unrealistically thick. The prying force formulas were proposed by Nair *et al.*,[2] based on their work on tee hangers. Previously, Douty and McGuire[3] and later Agerskov[4,5] have presented other versions of the same basic model, and/or suggested adjusted coefficients to reflect test results. The research in the U.S.A. and abroad on this topic has been summarized by Fisher and Struik.[6]

In the prying force method, the end-plate region around the beam tension flange is considered analogous to a tee hanger, as in Fig. 2. Hence, the terms "tee flange" and "plate" or "end plate" will be used interchangeably in this paper; "tee stem" will likewise correspond to the "beam flange". Figure 3 illustrates the dimensions and forces involved in the application of the prying force method. The section at or near the face of the tee stem at which the applied force is transferred to the tee flange will be designated the "load line", L. (All the notation used in this paper is listed in Appendix A.)

The major assumption of all the analytical models proposed thus far is that a concentrated prying force Q is developed at or near the edge of the tee flange in response to the load on the tee stem. The moment diagram resulting from the action of Q and the bolt force T along the bolt center line (also assumed concentrated) is therefore linear. The critical values of the plate moment at the bolt line and at the load line are given by

$$M_2 = Qa \tag{1a}$$

and

$$M_1 = Qa - Fb \tag{1b}$$

The prying force is computed from the formula

$$Q = F\left(\frac{c_1 b d_b^2 - c_2 w t_p^2}{c_3 a d_b^2 + c_4 w t_p^2}\right) \tag{2}$$

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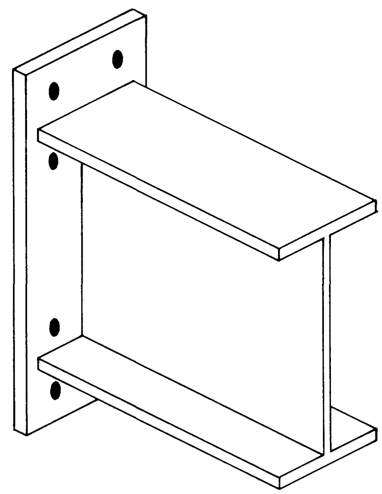

Fig. 1. Typical configuration of end-plate connection

in which the coefficients c_1, c_2, c_3, and c_4 are specified separately for A325 and A490 bolts. If the calculated Q is negative, it is to be taken as zero. The larger of the two moments M_1 and M_2 from Eqs. (1a) and (1b) is the design moment for the end plate.

In recent years, a considerable amount of continuum mechanics and yield line theory has been brought to bear on the problem. Efforts to adjust the plate design procedure to reflect test data have been focused on the modifications of the prying force formulas and of the dimensions a and b. These efforts have indeed provided analysis and design tools where none existed. The theories and formulas are being continually modified to reduce their observed conservatism. But each time a theory is modified, a new set of assumptions must be invoked; every time a coefficient is adjusted, one more theoretical refinement is effectively nullified.

The variations in the bolt forces have been accurately measured in many tests, as reported in the published literature,[3,4] but the magnitude and distribution of the prying force itself are only qualitatively known, either from visual observation or by pressure marks on a paper sheet with

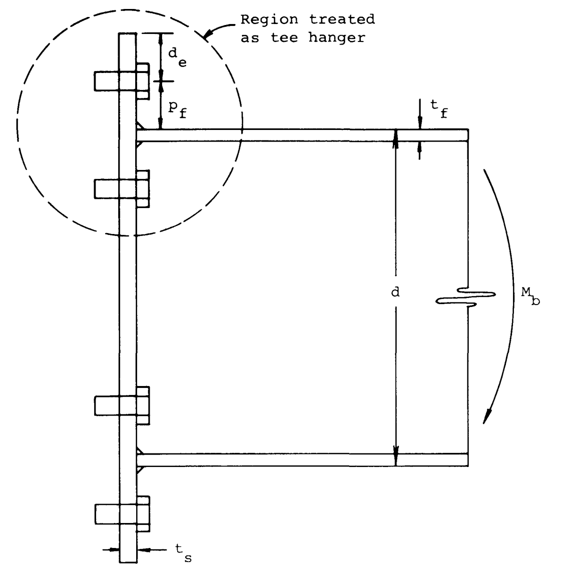

Fig. 2. End-plate connection and region treated as tee hanger

carbon backing. In extenuation of this limited experimental evidence, it must be admitted that the location of the prying forces (at the back of the plate), and of the maximum bending stresses (around the bolt hole and at the junction of the tee flange and stem), are inaccessible to routine instrumentation.

AUTHOR'S RESEARCH

To generate more information on this complicated problem, the author and his research assistants have investigated extensively various aspects of bolted tee-hanger and end-plate connections for the last six years. The research, sponsored by AISC and the Metal Building Manufacturers Association (MBMA), covers the following:

1. Feasibility, sensitivity, convergence, and parameter studies of entire connections and specific components, by the finite element method, including:
   (a) Two-dimensional (2D) analysis of nearly two hundred end-plate connections and more than one hundred tee hangers.
   (b) Three-dimensional (3D) analysis of many bench mark cases and test specimens of end-plate connections and tee hangers.
2. Tests on specimens, namely:
   (a) Twenty-four steel end-plate connections.
   (b) Fourteen steel tee-hangers.
   (c) Eighteen photoelastic models of tee hangers.

The author has published material on the correlation between the 2D and 3D analyses,[7] and on the general methodology of his solution,[8] in addition to discussions.[9,10]

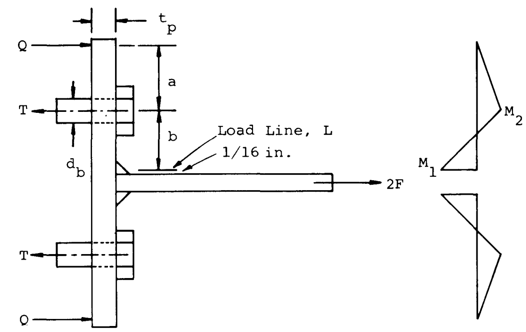

Fig. 3. Assumed forces and bending moment diagram for tee hanger

All the research tasks and the findings therefrom have been reported to the sponsors; most of the information has also been documented as master's degree theses at Auburn University and Vanderbilt University. Papers are under preparation on various details of the project.

The finite element analyses model the geometry, material properties, bolt pretensioning, and the subsequent loadings. The unknown support conditions corresponding to the deformed back of the plate are determined by an iterative process of analysis and support checks. In each cycle, nodes that tend to separate from the support are released, and previously released nodes that tend to migrate into the support are resupported. The 2D programs also incorporate the idealized elastic-perfectly-plastic behavior of the steel.

The tests were aimed at exploring and documenting the behavior of end plates and tee-hanger flanges under the interaction of bolt pretension and externally applied loadings. Deflections (and hence rotations for moment connections), as well as strains (and thus stresses) and bolt forces, were measured; in many of the steel tests, brittle coatings were used to reveal surface yielding. Test findings were used to check the computed results, and improve the analytical models as necessary and to the extent possible.

With so much new information available, it is now possible for a fresh look to be taken at end-plate behavior, and a new procedure to be proposed for its design.

END-PLATE BEHAVIOR

Let us reexamine the basic problem: The end-plate connection represents a highly nonlinear, indeterminate, and complex situation of great practical significance. By the vary nature of the problem, attempts to resolve it by classical theories or rationally reduce it to simple yet familiar formulas would be frustrating and approximate at best. In particular, in the situation where the bending spans are of the same order of magnitude as (and often less than) the plate thickness itself, the simple theory of bending will not apply; it may still be used for convenience and psychological advantage.

The applied force F is delivered as a line load across the width of the tee stem and is dispersed through the plate thickness. With a fillet (weld) at the junction of the beam flange and end plate, the dispersion would start at or close to the toe of the fillet.

The bolt force T is transferred to the plate over the annular area of the bolt head projection, and then begins to disperse through the plate thickness.

The "prying force" Q is actually the action of the pressure bulb developed by the bolt pretension, shifting away from the bolt line in response to the external load. Except for very thick plates and at or near failure loads, the reactive pressures are distributed over extensive areas between the plate edges and the bolts. Moreover, the pretensioning of the bolts tends to "quilt" the material, that is, to lift the plate away from the support at the outer edges; consequently, the lever action on which the prying force concept is based is considerably different from what is assumed.

Thus, all three forces acting on the plate are far from being concentrated. Because of this distributed nature of the forces, the bending moment diagram for the plate is curved, rather than linear as assumed. Hence, for the same force resultants, the actual peak moments at the bolt line and the load line are smaller than their theoretical values computed on the basis of assumed concentrated forces.

Further, the relative magnitudes of the two peak moments are also fairly predictable. Because of the statically redundant state of the plate, stiffnesses are critical in the distribution of bending moments. The plate cross section is reduced at the bolt holes; the bolts restrain the plate only at isolated points along the bolt line; the bending of the plate is biaxial around the bolts. All these factors combine to reduce the plate stiffness at the bolt line considerably below its stiffness at the load line. Thus, the bending moment at the load line is always larger than the moment at the bolt line. In other words, the bolt line moment M_2 (Fig. 2) does not govern the plate design in practice.

BASIC FORMULATION

Based on the postulates presented—every one of which has been amply confirmed by the computer analyses and laboratory tests—the following ultimate simplification can be proposed:

In the final analysis, the end plate must be sized for some shear F_1 and moment M_1 at its junction with the beam flange, as indicated in Fig. 4. If the point of contraflexure is located at a distance s from the load line, then,

$$M_1 = F_1 s \tag{3}$$

Conventionally, the nominal force F_f in the beam flange is taken as

$$F_f = M_b/(d - t_f) \tag{4}$$

In the familiar procedure known as the "split-tee" method, the plate projection is assumed to receive half of this flange force by implied symmetry; further, complete fixity is assumed at the bolt line. Under these circumstances, F_1 is $(F_f/2)$ and s is one-half the bolt distance.

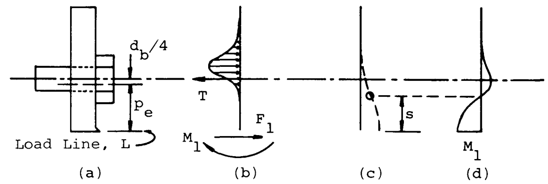

Fig. 4. Modified split-tee method: (a) Geometry, (b) Forces and pressure bulb, (c) Deflected shape, (d) Actual bending moment diagram

The bolt distance itself bears some scrutiny. Again traditionally (in the split-tee method), the bending span is taken as the bolt distance (p_f in Fig. 2) from the outer face of the beam flange to the center of the bolt line. However, as other investigators[2,6] have also noticed, the actual bending span p_e will be less. Correlations of the author's finite element analyses and test results have indicated that the effective span may be safely taken as

$$p_e = p_f - 0.25 d_b - w_t \tag{5}$$

Apart from an intuitive appreciation of this reduced value, a similar value for the bolt reduction has also been recommended by Fisher and Struik.[6] In Eq. (5), w_t, the throat size of the fillet weld between the beam flange and end plate, is zero for an unreinforced groove weld.

The benefit of this effective bolt distance p_e will be incorporated in all further discussions in this paper. Thus the moment at the bolt line by the split-tee method is

$$\begin{aligned} M_t &= (F_f/2)(p_e/2) \\ &= 0.25 F_f p_e \end{aligned} \tag{6}$$

This value will be used as the base or reference value for the proposed design procedure.

MODIFIED SPLIT-TEE METHOD

Equation (6) represents a very reasonable idealized situation. But the real behavior is considerably different, both in terms of the shear force F_1 and the arm s.

In a tee hanger, F_1 is indeed one-half the applied force on the tee stem. But in end-plate connections where the beam web is welded to the plate, firstly the longitudinal stresses due to the beam moment are delivered to the plate partly down the web; secondly, the plate region between the beam flanges, with part of the beam web acting as a rib on the plate, is much stiffer than the plate projection beyond the beam flanges. Both these effects result in a transfer of less force to the plate projection and more to the plate region between the beam flanges.

The author's finite element analyses have shown that the ratio (F_1/F_f) varies from 0.3 to 0.5. Thus we may write,

$$F_1 = C_1 F_f \qquad C_1 \leqslant 0.5 \tag{7}$$

Again, the point of contraflexure would be at mid-height of the bolt distance p_e under ideal fixity conditions at the bolt. But in reality, except possibly at very low load levels, the applied force overcomes the bolt clamping effects, and the bolts stretch and bend, permitting some rotation of the plate. This, combined with the reduced stiffness at the bolt line, as explained earlier, results in a shift of the inflection point towards the bolt, thus increasing the value of s. The computer analyses confirm that the ratio (s/p_e) can vary from 0.5 to 1.0. Hence,

$$s = C_2 p_e \qquad 0.5 \leqslant C_2 \leqslant 1.0 \tag{8}$$

Thus, the theoretical design moment by the simple bending theory would be

$$M_s = F_1 s = C_1 C_2 F_f p_e \tag{9}$$

Because of the force dispersions and deep beam effects referred to earlier, the actual critical moment is likely to be less, say,

$$M_d = C_3 M_s = C_1 C_2 C_3 F_f p_e \tag{10}$$

Combining Eqs. (6) and (10) gives

$$M_d = \alpha_m M_t \tag{11}$$

where

$$\alpha_m = 4 C_1 C_2 C_3 \tag{11a}$$

The coefficient α_m may thus be considered a moment modification factor to compensate for the many assumptions made in the development of M_t by Eq. (6).

MODIFICATION FACTOR

From the regression analyses of the results of numerous finite element studies (whose formulations have been verified or adjusted by tests), the prediction equation for the plate moment M_d was developed as follows:

$$M_d = 1.29 \left(\frac{F_y}{F_{bu}}\right)^{0.4} \left(\frac{F_{bt}}{F_p}\right)^{0.5} \left(\frac{b_f}{b_s}\right)^{0.5} \left(\frac{A_f}{A_w}\right)^{0.32} \times \left(\frac{p_e}{d_b}\right)^{0.25} M_t \tag{12}$$

From Eqs. (11) and (12),

$$\alpha_m = C_a C_b (A_f/A_w)^{0.32} (p_e/d_b)^{0.25} \tag{13}$$

where

$$C_a = 1.29 (F_y/F_{bu})^{0.4} (F_{bt}/F_p)^{0.5} \tag{13a}$$

and

$$C_b = (b_f/b_s)^{0.5} \tag{13b}$$

Appendix B presents details of the development of Eq. (12). Appendices C and D give values of α_m and C_a.

BOLT SELECTION

The bolt diameter d_b in the preceding development is selected as follows:

The theoretical bolt area a_t per row required is determined from

$$a_t = 0.5 F_f / F_{bt} \tag{14}$$

Two bolts (or more, if necessary) are chosen to provide an actual area a_b not less than a_t.

At this point, one question must be examined. If less than half the longitudinal force in the beam is transferred to the plate projection, as suggested earlier, more than half must remain in the plate region between the beam flanges. Theoretically, then, there must be more bolt area between the beam flanges than in the plate projections. (In practice, both the bolt rows at the tension flange would have to be increased to meet the inner row requirement.) On the other hand, the actual bolt area provided will generally be more than the theoretical area required. Under service loads, at worst, the inner row will be somewhat overstressed and the outer row understressed by the same amount. It is also possible that the excess area in the outer row will help resist any increase in the bolt force there, due to possible development of prying action under high loads. Based on these considerations, additional bolt area or unequal bolt area distribution is not recommended.

PHYSICAL INTERPRETATION

The modification factor α_m, as proposed in Eq. (13), is basically the result of regression analysis of computer results. The statistical analysis could only identify the dominant trends, with no concern for any physical basis. However, the parameters in the factor can be physically interpreted to a certain extent, in the light of the hypotheses presented.

The coefficient C_a lumps all material interactions together. For A36 steel and A325 bolts, F_y is 36.0 ksi, F_p (at $0.75F_y$) is 27.0 ksi, F_{bu} is 93.0 ksi, and F_{bt} is 44.0 ksi, leading to a C_a-value of 1.13. For 90.0 ksi steel and A490 bolts, C_a becomes 1.04. This material coefficient can be tabulated as in Appendix D, for various common combinations of materials and bolts. As all the analyses were based on the assumption that the plate and beam materials were the same for any one connection, Eq. (13a) is not directly applicable to cases where the beam and plate are from different grades of steel. It would be consistent with the original formulation to use the average yield stress for F_y and the plate yield stress in setting F_p, with the restriction that C_a must not be smaller than the single material value based on average F_y. However, on the basis of results of tests which covered variations in the plate, beam flange, and beam web yield stresses, it is considered adequate to use the smallest of the yield stresses (and the *corresponding* value of F_p) in the determination of C_a. In any case, the actual value of F_p for the plate must be used in the subsequent computation of the end-plate thickness.

The coefficient C_b is a plate width correction, amounting to 0.95 for b_f/b_s ratio of 0.9. To avoid any adverse consequence of the end plate being too much wider than the beam flange, an effective maximum plate width is recommended as follows:

$$b_e = b_f + 2w_s + t_s \qquad (15)$$

thus allowing for a 45 degree dispersion from the weld toe at the edge of the beam flange. For unreinforced groove welds, w_s is taken as zero.

The (A_f/A_w) ratio quantifies the distribution of the beam material, and determines the fraction of the applied longitudinal force that is transferred to the plate projection. For the majority of rolled and built-up wide-flange or I-shaped sections, A_f/A_w is between 0.25 and 2.50, leading to a variation of 0.63 to 1.36 in the α_m result.

Finally, the p_e/d_b term appears to represent the complex influence of the bolt size and clamping force, in relation to the bolt distance. The ratio generally lies between 0.75 and 2.50, for which the influence on α_m varies from 0.93 to 1.26. *The term p_e/d_b is a very dominant parameter*, and clearly the bolt distance must be kept as small as feasible, for maximum economy.

DESIGN PROCEDURE

The procedure for end-plate design is formulated as follows:

1. Find the nominal flange force F_f from Eq. (4).
2. Find the required bolt area a_t per row from Eq. (14), and hence determine the bolt size d_b.
3. Find the effective bolt distance p_e from Eq. (5).
4. Find the split-tee moment M_t from Eq. (6).
5. Find the moment modification factor α_m from Eqs. (13), (13a), and (13b).
6. Find the design moment M_d for the end plate from Eq. (11).
7. Find the end-plate thickness t_s by the simple theory of bending from the expression

$$t_s = \sqrt{\frac{6M_d}{b_sF_p}} \qquad (16)$$

8. Check the effective plate width b_e by Eq. (15). If b_e is less than the actual plate width b_s, repeat steps 5 through 8 with b_e in place of b_s.
9. Check the maximum shear stress f_s in the plate from

$$f_s = F_f/(2b_st_s) \qquad (17)$$

If f_s exceeds the allowable value $0.4F_y$ for the plate material, increase t_s to satisfy Eq. (17).

Implied in the application of the proposed procedure are the other conditions assumed in the entire analysis: The bolts must be pretensioned to the AISC recommended value of 0.7 times their ultimate strength. The vertical edge distance, although apparently of secondary significance in a certain range, must be kept at about 1.75 times (not less than 1.5 times) the bolt diameter. The beam end must be welded to the end plate, not only at the flanges (all around if by fillet welds), but also down the web.

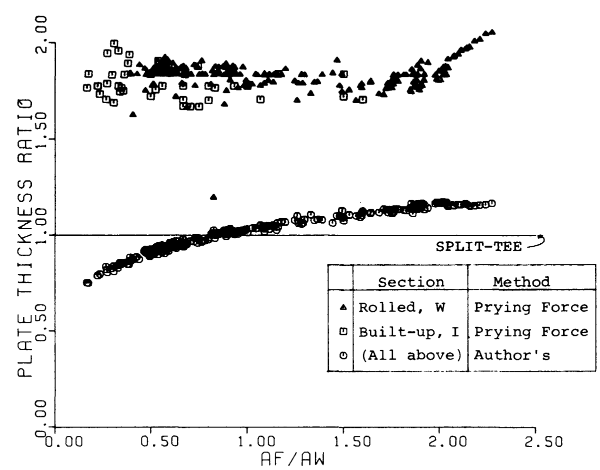

Fig. 5. Comparison of end-plate designs by prying force method and by modified split-tee method, for 36.0 ksi steel, A325 bolts, p_f/d_b = 1.5

Routine design may be simplified in a number of ways. The A_f/A_w ratio, or even its value raised to the 0.32 power as needed in Eq. (13), may be tabulated for standard rolled and built-up sections. Charts may be prepared for α_m as a function of the A_f/A_w ratio, for standard combinations of materials, bolt sizes and bolt distances. Appendix D for the material coefficient is a design aid. Appendix C represents another design aid for the quick determination of α_m for known values of C_aC_b, A_f/A_w, and p_e/d_b.

Appendix E demonstrates the application of the design procedure to the same beam example as in Ref. 1, 1st Printing. The resulting plate thickness is $^{13}/_{16}$-in., as against the value of $1^7/_{16}$ in. by the prying force method.

The proposed procedure results in savings of 25 to 50 percent below the prying force method in most practical situations. Figures 5 and 6 depict comparisons for two specific combinations where beams are attached to the end plates by fillet welds, applied to all the 192 standard rolled wide-flange sections listed in the AISC Manual, and 44 typical built-up sections used by MBMA member companies. In Figs. 5 and 6 the reference split-tee thickness is the value required for the split-tee moment, from Eq. (16), with M_t by Eq. (6) used in place of M_d.

TEST FINDINGS

Notwithstanding the very high correlation obtained in the regression analyses, the real test of the proposed design procedure was whether the thinner plates were strong

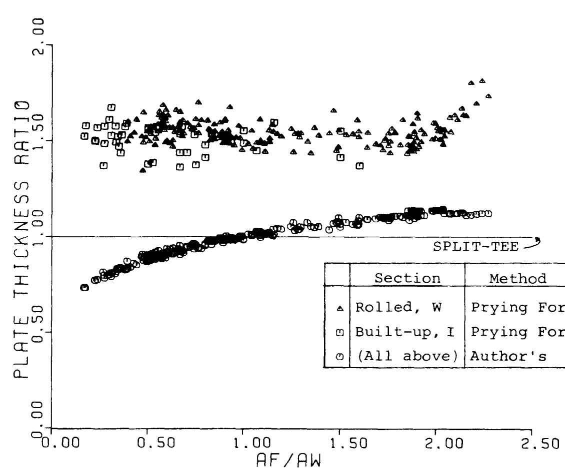

Fig. 6. Comparison of end-plate designs by prying force method and by modified split-tee method, for 50.0 ksi steel, A325 bolts, p_f/d_b = 1.5

enough to develop the ultimate capacities of the beams connected.

To demonstrate this, a series of nine connections, whose end plates were designed according to the proposed procedure, were tested to failure. The beams covered A_f/A_w ratios from 0.5 to 2.0, and the bolt sizes and locations spanned p_e/d_b ratios from 0.8 to 1.4. Table 1 presents significant details of the specimens and the test results.

The first specimen failed by torsional twisting of the beam prematurely; extensive lateral support was provided subsequently to prevent such failure. In all other cases, failure was by local buckling of the beam compression flange and web. In no case was the end plate itself under visible or measured overall distress. In all but three cases (including the aborted first test), the beams developed or exceeded their ultimate strength. The last two, which reached only 80% and 83% of their ultimate strength, were unusually deep beams with very thin webs that buckled between the stiffeners.

CONCLUSION

Admittedly, the proposed design procedure breaks with tradition by deriving empirical relationships based upon statistical analysis of the results from parameter studies on the computer. But the entire process essentially reduces to two concepts: (1) application of simple classical theory to a basic model, and (2) modification by a factor which incorporates dominant influences in relatively compact, experimentally validated proportions. Thus, it accomplishes the same ends as most of the more complicated theoretical methods attempt, namely the development of simple design expressions for complicated phenomena, and their adjustment to reflect observed behavior.

In fact, the design formulas are the direct outcome of analysis; test findings have been incorporated in them only to the extent of refining the computer models to include details such as the bolt heads, and for simplifying the C_a computation.

The proposed procedure is recommended as economical and safe for end-plate connections used as longitudinal splices between beams or frame members, and for beam-to-column connections where the column flanges either are stiffened at the levels of the beam flanges or are otherwise adequate.

The decrease in end-plate thickness by the proposed method will frequently be accompanied by some reduction in connection rigidity. Under circumstances where deformations can be critical, the influence of such increased connection flexibility on the overall behavior of the structure may need review. (The same analysis that led to the proposed procedure for design based on strength have also generated data that quantify the connection stiffness as moment-rotation relationships, which may in turn be used in this suggested reanalysis process.)

It is highly reassuring that in the validation tests none of the designed plates failed or showed any signs of distress, nor did any bolt fracture, even when the attached beams had failed. But could the plates be even thinner, in the typical configuration studied or in modified configurations

Table 1. Details of End-Plate Connection Tests

No.	d	b_f	t_f	t_w	d_b	p_f	b_s	t_s	d_s	w_s	F_y [b]	$\frac{M_m}{M_p}$	Failure Mode [c]
1 [a]	15.65	5.5	0.345	0.250	0.750	1.313	6.0	0.625	21.00	0.438	44.2	0.58	T
2 [a]	11.96	6.5	0.400	0.237	0.875	1.438	7.0	0.750	18.25	0.375	37.6	0.99	T
3 [a]	16.00	7.0	0.503	0.307	1.000	1.500	8.0	0.760	24.00	0.438	39.7	1.01	F
4	13.00	6.0	0.500	0.250	0.875	1.344	7.0	0.622	18.75	0.438	47.4	0.99	F
5 [a]	15.65	5.5	0.345	0.250	0.750	1.063	6.0	0.500	20.50	0.313	44.2	1.07	F
6	9.00	6.0	0.500	0.250	0.875	1.313	7.0	0.688	14.75	0.500	47.4	1.03	F,T
7 [a]	16.00	7.0	0.503	0.307	1.000	1.563	8.0	0.750	22.50	0.500	39.7	1.01	F
8	24.75	6.0	0.375	0.188	0.875	1.250	7.0	0.500	30.50	0.375	52.1	0.80	F,W
9	24.75	6.0	0.375	0.188	0.875	1.750	7.0	0.750	31.34	0.375	52.1	0.83	F,W

[a] Rolled sections: Nos. 1 and 5, W16×26; No. 2, W12×27; Nos. 3 and 7, W16×40. All dimensions actual and inch units.
[b] Yield stress of beam flange, used in computation of M_p.
[c] Failure modes: F — Flange buckling; T — Torsional twisting; W — Web buckling.

such as with stiffened plates or multiple bolt rows between the beam flanges? To explore this question, additional research sponsored by MBMA is under way.

ACKNOWLEDGMENTS

The research alluded to in the paper was financially supported by the American Institute of Steel Construction and the Metal Building Manufacturers Association since 1971. The valuable counsel of the associated task committees is gratefully acknowledged. The test specimens were donated by American Buildings Company of Eufaula, Alabama, and McMurray Steel Company and Englert Engineering Company, both of Nashville, Tennessee. The massive computational services needed were provided by the computer centers of Auburn University, University of Alabama in Birmingham, and Vanderbilt University. Many of the research tasks referred to were carried out by the author's graduate students Graddy, Huang, Jeffrey, Daniel, Avery, Saxena, Oswalt, and Byers.

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APPENDIX A — NOTATION

a = Lever arm for prying force; the smaller of d_e and $2t_p$
a_b = Actual bolt area per row
a_t = Theoretical bolt area per row
A = Cross-sectional area of beam
A_f = Area of tension flange of beam
A_w = Area of beam web
b = Bolt distance from section of maximum moment at face of tee stem, in prying force formula
b_e = Effective width of end plate
b_f = Width of beam flange
b_s = Width of end plate
c_i = Coefficients in prying force formula (i = 1, 2, 3, 4)
C_a = Material coefficient
C_b = Plate width correction factor
C_i = Coefficients in author's formulas (i = 1, 2, 3)
d = Beam depth
d_b = Nominal bolt diameter
d_e = Vertical edge distance of plate beyond outer bolt row
d_s = Depth of end plate
f_b = Extreme fiber bending stress in beam
f_s = Maximum shear stress in end plate
F = Half the force applied in the tee stem, assumed transferred to tee flange
F_b = Allowable extreme fiber bending stress in beam
F_{bt} = Allowable tensile stress in bolt
F_{bu} = Ultimate tensile stress of bolt
F_f = Nominal force in beam flange
F_p = Allowable bending stress in end plate
F_y = Yield stress of beam and plate material; average value if different
F_1 = Shear in plate projection; total force beyond beam tension flange
M_b = Beam bending moment
M_d = Design moment for end plate
M_m = Maximum moment developed by beam in test
M_p = Ultimate moment capacity of beam
M_s = Theoretical design moment for plate
M_t = Plate moment by split-tee method
M_1 = Plate moment at load line
M_2 = Plate moment at bolt line
p_f = Bolt distance from face of beam flange
p_e = Effective bolt distance
s = Distance from load line to point of contraflexure in end plate
S = Section modulus of beam
t_f = Thickness of beam flange
t_p = Thickness of tee flange or end plate in prying force method
t_s = End-plate thickness by author's proposed method
t_w = Thickness of beam web

T = Bolt force
w = Tributary width of tee flange or end plate per bolt
w_s = Size of fillet weld (taken as zero for unreinforced groove welds)
w_t = Throat size of fillet weld (= $0.707w_s$ if 45-degree weld)
α_m = Modification factor for plate moment (= M_d/M_t)
β_i = Beam coefficients in prediction equations ($i = d, m, t$)
μ_i = Material coefficients in prediction equations ($i = d, m, t$)

APPENDIX B

DEVELOPMENT OF PREDICTION EQUATION FOR PLATE MOMENT

By regression analysis of finite element results from 168 end-plate connections for 559 load cases, the prediction equation for plate moment M_d was found to be:

$$M_d = 0.124\beta_m\mu_m t_s^{0.671} p_e^{0.829} f_b^{1.040}/a_b^{0.406} \quad \text{(A.1)}$$

in which

$$\beta_m = \frac{b_f^{1.198} t_f^{0.673} A^{0.406}}{d^{0.183} t_w^{0.188}}$$

$$\mu_m = F_y^{0.216}/F_{bu}^{0.256}$$

Also, for design,

$$M_d = F_p(b_s t_s^2/6) \quad \text{(A.2)}$$

Substituting for t_s from Eq. (A.2) into Eq. (A.1), and collecting terms in M_d:

$$M_d = 0.106\beta_d\mu_d p_e^{1.247} f_b^{1.565}/(a_b^{0.611} b_s^{0.505}) \quad \text{(A.3)}$$

in which

$$\beta_d = \frac{b_f^{1.803} t_f^{1.012} A^{0.611}}{d^{0.275} t_w^{0.282}}$$

and

$$\mu_d = \frac{F_y^{0.325}}{F_{bu}^{0.385} F_p^{0.505}}$$

Let the moment modification factor α_m be defined as follows:

$$M_d = \alpha_m M_t \quad \text{(A.4)}$$

in which M_t is the split-tee moment given by

$$M_t = 0.25F_f p_e \quad \text{(6)}$$

as explained in the text.

From Eqs. (A.3), (A.4), and (6),

$$\alpha_m = M_d/M_t$$
$$= 0.426\beta_d\mu_d p_e^{0.247} f_b^{0.565}/(a_b^{0.611} b_s^{0.505} F_f) \quad \text{(A.5)}$$

From other finite element and regression analyses, it was determined that (p_e/d_b) was a dominant parameter. To reduce all the variables to dimensionless parameters, the bolt area term was selectively substituted as follows:

To satisfy the allowable stress requirement,

$$a_b = 0.5F_f/F_{bt} \quad \text{(A.6)}$$

Also, for two bolts per row,

$$a_b = 2(\pi d_b^2/4) = 0.5\pi d_b^2 \quad \text{(A.7)}$$

Thus, from Eqs. (A.6) and (A.7),

$$a_b^{0.611} = a_b^{0.1235} a_b^{0.4865}$$
$$= (0.5\pi d_b^2)^{0.1235}(0.5F_f/F_{bt})^{0.4865}$$
$$= 0.755 d_b^{0.247} F_f^{0.487}/F_{bt}^{0.487} \quad \text{(A.8)}$$

In Eqs. (A.5) and (A.8), the term F_f may be written as

$$F_f = f_b S/(d - t_f) \quad \text{(A.9)}$$

Substituting Eq. (A.8) into Eq. (A.5), and Eq. (A.9) into the result, and grouping the non-dimensional terms,

$$\alpha_m = 0.564\beta_t\mu_t(f_b/F_y)^{0.078}(p_e/d_b)^{0.247}(b_f/b_s)^{0.505} \quad \text{(A.10)}$$

in which

$$\beta_t = \frac{b_f^{1.298} t_f^{1.102} A^{0.611}(d - t_f)^{1.487}}{d^{0.275} t_w^{0.282} S^{1.487}} \quad \text{(A.10a)}$$

and

$$\mu_t = \frac{F_y^{0.403} F_{bt}^{0.487}}{F_{bu}^{0.385} F_p^{0.505}} \quad \text{(A.10b)}$$

By regression analysis of 192 standard rolled wide-flange sections and 44 typical built-up I sections, it was determined that the beam coefficient Eq. (A.10a), reduced to:

$$\beta_t = 2.387(A_f/A_w)^{0.322} \quad \text{(A.11)}$$

Further, with a maximum extreme fiber stress about 0.6 times the yield stress, in Eq. (A.10),

$$(f_b/F_y)^{0.078} = 0.961 \quad \text{(A.12)}$$

By rounding exponents in Eq. (A.10b) and combining terms, the following simplified expressions were adopted:

$$\alpha_m = C_a C_b (A_f/A_w)^{0.32}(p_e/d_b)^{0.25} \quad \text{(13)}$$

in which

$$C_a = 1.29(F_y/F_{bu})^{0.4}(F_{bt}/F_p)^{0.5} \quad \text{(13a)}$$

and

$$C_b = (b_f/b_s)^{0.5} \quad \text{(13b)}$$

APPENDIX C—MOMENT MODIFICATION FACTOR

Moment Modification Factor (α_m)

A_f/A_w \ p_e/d_b	$C_aC_b = 0.95$							
	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50
0.25	0.57	0.61	0.64	0.67	0.70	0.72	0.75	0.77
0.50	0.71	0.76	0.80	0.84	0.88	0.91	0.93	0.96
0.75	0.81	0.87	0.92	0.96	1.00	1.03	1.06	1.09
1.00	0.88	0.95	1.00	1.05	1.09	1.13	1.16	1.19
1.25	0.95	1.02	1.08	1.13	1.17	1.21	1.25	1.28
1.50	1.01	1.08	1.14	1.20	1.24	1.29	1.32	1.36
1.75	1.06	1.14	1.20	1.26	1.31	1.35	1.39	1.43
2.00	1.10	1.19	1.25	1.31	1.36	1.41	1.45	1.49
2.25	1.15	1.23	1.30	1.36	1.42	1.46	1.51	1.55
2.50	1.19	1.27	1.35	1.41	1.46	1.51	1.56	1.60

A_f/A_w \ p_e/d_b	$C_aC_b = 1.00$							
	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50
0.25	0.60	0.64	0.68	0.71	0.74	0.76	0.79	0.81
0.50	0.75	0.80	0.85	0.89	0.92	0.95	0.98	1.01
0.75	0.85	0.91	0.96	1.01	1.05	1.08	1.12	1.15
1.00	0.93	1.00	1.06	1.11	1.15	1.19	1.22	1.26
1.25	1.00	1.07	1.14	1.19	1.24	1.28	1.32	1.35
1.50	1.06	1.14	1.20	1.26	1.31	1.35	1.39	1.43
1.75	1.11	1.20	1.26	1.32	1.38	1.42	1.46	1.50
2.00	1.16	1.25	1.32	1.38	1.44	1.48	1.53	1.57
2.25	1.21	1.30	1.37	1.43	1.49	1.54	1.59	1.63
2.50	1.25	1.34	1.42	1.48	1.54	1.59	1.64	1.69

A_f/A_w \ p_e/d_b	$C_aC_b = 1.05$							
	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50
0.25	0.63	0.67	0.71	0.75	0.77	0.80	0.83	0.85
0.50	0.78	0.84	0.89	0.93	0.97	1.00	1.03	1.06
0.75	0.89	0.96	1.01	1.06	1.10	1.14	1.17	1.20
1.00	0.98	1.05	1.11	1.16	1.21	1.25	1.29	1.32
1.25	1.05	1.13	1.19	1.25	1.30	1.34	1.38	1.42
1.50	1.11	1.20	1.26	1.32	1.37	1.42	1.46	1.50
1.75	1.17	1.26	1.33	1.39	1.44	1.49	1.54	1.58
2.00	1.22	1.31	1.39	1.45	1.51	1.56	1.61	1.65
2.25	1.27	1.36	1.44	1.51	1.57	1.62	1.67	1.71
2.50	1.31	1.41	1.49	1.56	1.62	1.67	1.72	1.77

A_f/A_w \ p_e/d_b	$C_aC_b = 1.10$							
	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50
0.25	0.66	0.71	0.75	0.78	0.81	0.84	0.86	0.89
0.50	0.82	0.88	0.93	0.98	1.01	1.05	1.08	1.11
0.75	0.93	1.00	1.06	1.11	1.15	1.19	1.23	1.26
1.00	1.02	1.10	1.16	1.22	1.27	1.31	1.35	1.38
1.25	1.10	1.18	1.25	1.31	1.36	1.40	1.45	1.49
1.50	1.17	1.25	1.32	1.39	1.44	1.49	1.53	1.57
1.75	1.22	1.32	1.39	1.46	1.51	1.56	1.61	1.65
2.00	1.28	1.37	1.45	1.52	1.58	1.63	1.68	1.73
2.25	1.33	1.43	1.51	1.58	1.64	1.70	1.75	1.79
2.50	1.37	1.47	1.56	1.63	1.70	1.75	1.81	1.85

APPENDIX D—MATERIAL COEFFICIENTS FOR COMMON COMBINATIONS

The general expression for C_a (Eq. 13a) is:

$$C_a = 1.29\,(F_y/F_{bu})^{0.4}(F_{bt}/F_p)^{0.5}$$

For $F_p = 0.75F_y$:

$$C_a = 1.49F_{bt}^{0.5}/(F_{bu}^{0.4}F_y^{0.1})$$

For A325 bolts:

$$F_{bt} = 44.0 \text{ ksi and } F_{bu} = 93.0 \text{ ksi}$$

For A490 bolts:

$$F_{bt} = 54.0 \text{ ksi and } F_{bu} = 116.0 \text{ ksi}$$

F_y (ksi)	A325	A490
36.0	1.13	1.14
42.0	1.11	1.13
45.0	1.10	1.12
50.0	1.09	1.11
55.0	1.08	1.10
60.0	1.07	1.09
65.0	1.06	1.08
90.0	1.03	1.04

APPENDIX E—EXAMPLE OF END-PLATE CONNECTION DESIGN

Given:

W16 × 45 section:
$d = 16.12$ in., $b_f = 7.039$ in., $t_f = 0.563$ in., $t_w = 0.346$ in., $S = 72.5$ in.^{3}

A36 steel: $F_y = 36.0$ ksi and $F_p = 27.0$ ksi

A325 bolts: $F_{bt} = 44.0$ ksi and $F_{bu} = 93.0$ ksi

Maximum bending moment for maximum bending stress of 0.66 F_y:

$$M_b = (72.5)(0.66)(36.0) = 1722.6 \text{ kip-in.}$$

Design:

1. Nominal flange force, by Eq. (4):
   $F_f = 1722.6/(16.12 - 0.563) = 110.7$ kips
2. Bolt area per row, by Eq. (14):
   $a_t = (0.5)\,(110.7)/44.0 = 1.26$ sq. in.

   Use two 1-in. diameter bolts, to provide 1.57 sq. in.
3. Set edge distance:
   $d_e = (1.75)\,(1.0) = 1.75$ in.

   Set bolt distance at (say) 1.5 diameters:
   $p_f = (1.5)\,(1.0) = 1.5$ in.

   Set weld size w_s to transfer F_f to the end plate:
   Use ½-in. fillet welds with E70 electrodes.

   Effective bolt distance, by Eq. (5):
   $p_e = 1.5 - (0.25)(1.0) - (0.707)\,(0.5) = 0.897$ in.
4. Split-tee moment, by Eq. (6):
   $M_t = (0.25)(110.7)(0.897) = 24.83$ kip-in.
5. Material coefficient, by Eq. (13a):
   $C_a = (1.29)\,(3.60/93.0)^{0.4}(44.0/27.0)^{0.5} = 1.127$

   Minimum plate width:
   $b_f + (2 \times \text{weld size}) = 7.039 + (2 \times 0.5) = 8.039$ in.

   Set plate width b_s at 8.5 in.

   Width correction factor, by Eq. (13b):
   $C_b = (7.039/8.5)^{0.5} = 0.910$

   Area of beam tension flange:
   $A_f = (7.039)(0.563) = 3.963$ sq. in.

   Area of beam web (between the two flanges):
   $A_w = (0.346)\,[16.12 - (2 \times 0.563)] = 5.188$ sq. in.

   Hence, $A_f/A_w = 3.963/5.188 = 0.764$

   $(p_e/d_b) = 0.897/1.0 = 0.897$

   Moment modification factor, by Eq. (13):
   $\alpha_m = (1.127)(0.910)(0.764)^{0.32}(0.897)^{0.25} = 0.916$

   (Note: From Appendix D, C_a could have been read off as 1.13. From Appendix C, for C_aC_b of (1.127) (0.910) or 1.03, α_m could have been estimated at about 0.92.)
6. Design moment, by Eq. (11):
   $M_d = (0.916)\,(24.83) = 22.74$ kip-in.
7. End-plate thickness, by Eq. (16):

   $$t_s = \sqrt{\frac{(6)\,(22.74)}{(8.5)\,(27.0)}} = 0.771 \text{ in.}$$

   Try 13⁄16-in. plate ($t_s = 0.8125$ in.)
8. Check effective plate width, by Eq. (15):
   $b_e = 7.039 + (2 \times 0.5) + 0.8125 = 8.852$ in. $> b_s = 8.5$ in. **o.k.**
9. Check maximum shear stress, by Eq. (17):
   $f_s = 110.7/[(2)(8.5)(0.8125)] = 8.02$ ksi

   Allowable shear stress $= 0.4F_y = (0.4)\,(36.0) = 14.4$ ksi $> f_s = 8.02$ ksi **o.k.**

Use 13/16-in. thick plate, 8.5 in. wide, and 22.5 in. deep.

Design for the same situation by the prying force method requires 1-in. diameter bolts, but a $1\frac{7}{16}$-in. thick plate.

Note: Figure 5 happens to be based on the same materials and p_f/d_b ratio as used in this problem, and hence its use as a design chart may be demonstrated as follows:

Thickness of end plate for the split-tee moment, by the simple bending theory, is:

$$\sqrt{\frac{(6)(24.83)}{(8.5)\,(27.0)}} = 0.806 \text{ in.}$$

The value of the thickness ratio for the modified split-tee method, corresponding to A_f/A_w of 0.764, as read off from Fig. 5, is 0.97.

Hence, the thickness by the proposed procedure is

$$t_s = (0.806)(0.97) = 0.78 \text{ in.}$$

(In Figs. 5 and 6, the plate widths were set to the whole inch above the beam flange width; in this problem, b_s would be 8.0 in.)

APPENDIX F—ADDITIONAL TEST RESULTS

After the rest of the paper had been set up for publication, a tenth specimen was tested in April 1978, with results that reinforce the statements made in the paper. The beam was of W16 × 45 section, with d of 16.12 in., b_f of 7.039 in., t_f of 0.563 in., and t_w of 0.346 in.; d_f was 1.0 in. and p_f was 1.75 in.; the plate had b_s of 8.75 in., t_s of 0.875 in., and d_s of 20.0 in., projecting beyond the beam tension flange only and cut off nearly flush on the compression flange side. The weld size w_s was 0.5 in. and the average flange yield stress F_y was 39.54 ksi. The (M_m/M_p) ratio at failure was 1.04, the specimen failing by torsional twisting. The brittle coating indicated considerable yielding of the beam flanges and web, but no yielding of the plate. The strain gages on the end plate, next to the weld attaching the plate to the junction of beam tension flange and web, indicated very localized yielding, but strain gages elsewhere read low strains. The specimen was supplied by Butler Manufacturing Company of Grandview, Missouri.