End Restraints on Steel Joist Floor Vibrations

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The problem of perceptible vibrations in steel joist/concrete slab floor systems has been under investigation for nearly 20 years. The vibrations, which can prove to be annoying to occupants, occur as a result of a human footfall. The floors, although structurally safe, lack the flexural rigidity necessary to prevent noticeable oscillations. Two articles by Lenzen[6] and Murray,[10] which have presented detailed methods for attacking the vibrations problem, were the primary sources for the background information presented in this paper. The discussion that follows leads to a new method of controlling the floor vibrations problem by varying the rotational end restraints of the floor.

BACKGROUND

In the dynamic analysis, the floor is considered to be composed of a series of parallel T-beams. The steel joist is considered to be the web of the T-beam, while the slab forms its flange. Complete interaction between the web and flange reduces the solution of the problem to that of a composite, simply-supported beam. The frequency of the floor is that of the transformed single T-beam. The deflection is obtained by considering a simply-supported T-beam with a concentrated load at midspan. The deflection of the floor is resisted by a number of T-beam sections. The number of T-beams that resist the deflection is determined by the ratio of flexural rigidities parallel and perpendicular to the joists.[4,7,9]

In order to predict the reaction of occupants to the vibration chracteristics of the floor, the question of perceptibility was quantified. Figure 1, which was originally developed by Lenzen,[5] relates various combinations of frequency and amplitude to different degrees of occupant discomfort. In general, for the same amplitude of vibration, the vibration is more perceptible in the higher frequency range than in the lower frequency range. The figure is applicable only in cases where the percent of critical damping is less than 5–6%. If the critical damping is greater than 5–6%, the amplitude of vibration will be damped to 20% or less of its initial amplitude within 5 cycles. Experiment has shown that this will result in the occupant experiencing only the initial displacement and not the oscillatory motion.

Traditionally, the control of vibrations has been approached in two ways. First, by varying the design of the floor, its vibration characteristic can be improved. Secondly, additional damping can be added to the floor system.

In general, the former method is restrictive, in that many of the parameters are prescribed by the problem. Variation of the parameters that are not prescribed usually results in negligible results. The exception is an increase in the slab thickness. Such an increase not only improves the vibration characteristics, but also increases the damping in the system. However, the increase of slab-weight could be substantial.

The second method involves increasing the damping either through the construction or by adding artificial forms of damping. Rugs, ceilings, partitions, and mechanical fixtures all add damping to a system. Two mechanical devices have been developed to add extra damping to severely problemed floors. The first, a vibration absorber,

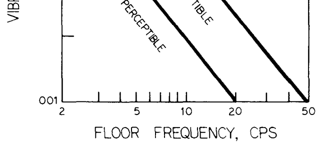

Fig. 1. Lenzen's human response chart[5]

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developed at the University of Kansas,[6,8] is attached to the joists below a problem floor. It consists of a mass, springs, and shock absorber. The unit is tuned to a frequency that will induce the beat effect in the system, thus damping the vibrations quickly. The second device, developed by Allen,[1] consists of a floor-to-ceiling post attached to the problem floor with neoprene pads. When the floor vibrates, the pads are placed in shear, thus dissipating the energy and damping the vibrations quickly.

In this paper, which is extracted from an earlier study[3] at RPI, it is shown that a distinctly perceptible vibration can be reduced to a slightly perceptible level by varying the rotational end restraints. Thus, the variation of end restraints can also be considered as a means of controlling the floor vibrations in the future.

THEORETICAL DEVELOPMENT

The frequency of the floor with variable end restraints will be determined using the Rayleigh-Ritz energy method. This method determines the natural frequency by equating the strain energy of bending to the kinetic energy of vibration. The deflected shape of the floor is approximated by a series of "admissible functions". These functions satisfy the end conditions. Each "admissible function" is the product of two characteristic equations for a vibrating beam. One equation represents the floor as a beam parallel to the joists and is designated as for the beam in the x-direction. The other equation represents the floor as a beam perpendicular to the joists and is designated as for the y-direction. The end restraints will be represented by rotational springs, as shown in Fig. 2.

The development of the equation begins by considering the vertical deflection of the beam at its ends to be zero. Only the rotational restraints at the ends will be considered elastic. The variation of the spring constant will correspond to the stiffening or relaxing of the end restraint.[13] The ideal conditions of completely fixed and simply supported will be represented by spring constants of ∞ and 0, respectively. For mathematical convenience, a dimensionless restraint constant ρ, which will represent the ratio of the rotational spring constant to the flexural stiffness of the beam, KL/EI, will be introduced.

Note that the solutions for the hinged and fixed conditions can be found in Ref. 14, while for elastic rotational end restraints, the solutions are given in Ref. 3.

The second part of the development for elastic end restraints will deal with the calculation of the amplitude of vibration. The condition of symmetry requires that the maximum deflection occurs at midspan, when a load is placed at midspan. As before it will be assumed that there is no vertical deflection at the supports. The problem becomes the calculation of the midspan deflection due to a statistically equivalent concentrated load applied at midspan for the beam. By applying an appropriate dynamic amplification factor, the dynamic deflection is obtained.

A human footfall can be represented as a triangular impulse with an initial magnitude of 606 lb and a duration of 0.05 sec.[11] The dynamic effect of the human footfall requires that a Dynamic Magnification Factor (*DMF*) be included in the determination of the amplitude of vibration. The Dynamic Magnification Factor (*DMF*) can be found in standard text books.[2,12]

VIBRATION CONTROL

The determination of the frequency and amplitude of vibration, of a floor with rotational end restraints, leads to the application to vibration control.

In Ref. 3, five hypothetical floors were used to determine any trends that resulted from various variations of ρ_x and ρ_y , where ρ_x and ρ_y are dimensionless rotational restraint constants in the x- and y-directions, respectively. It was observed from the data[3] that there was a definite trend in the variation of frequency with ρ_x. However, the effect of varying ρ_y amounts to only a 1% variation over the range

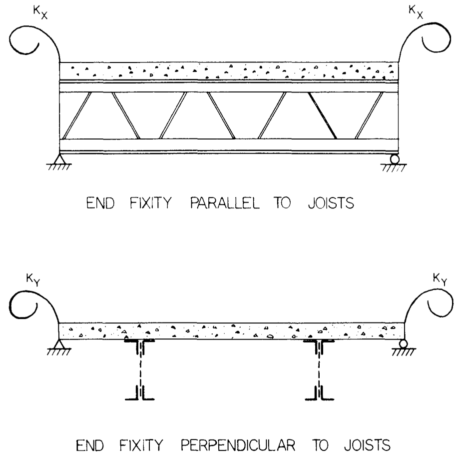

Fig. 2. Cross sections of steel joist floors including end restraints

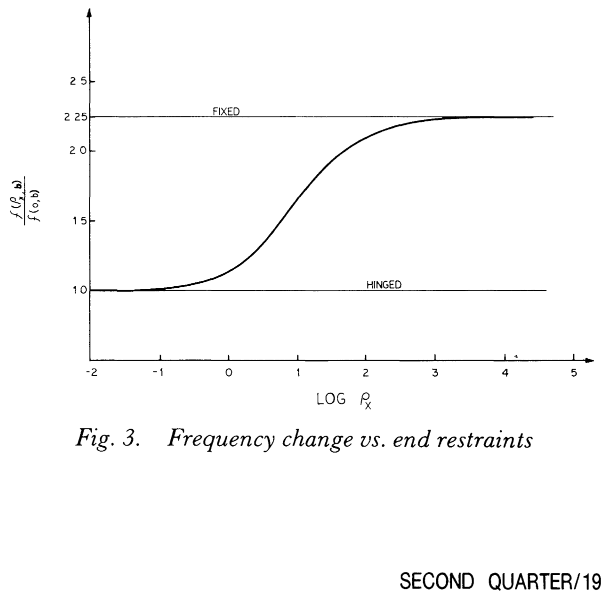

Fig. 3. Frequency change vs. end restraints

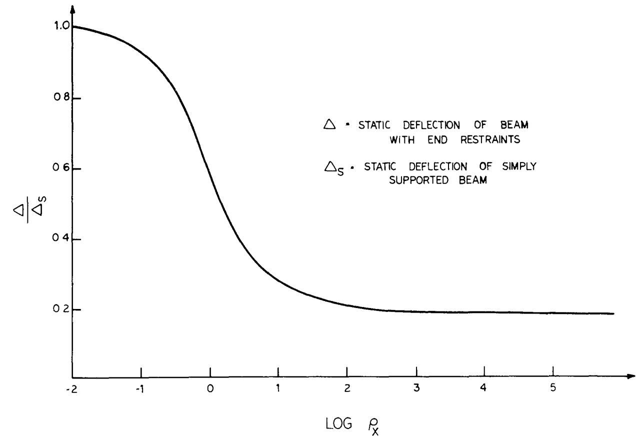

Fig. 4. *Static deflection vs. end restraints*

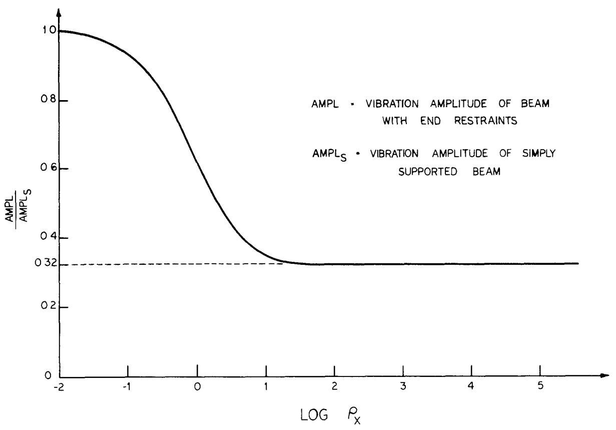

Fig. 5. *Amplitude change vs. end restraints*

of values used (0.01 to 100,000). It was decided, therefore, to neglect the effect of ρ_y in the problem. Figure 3 shows the variation of frequency with ρ_x. This variation is the same at any fixed value of ρ_y, and a $\rho_y = b$ is used in Fig. 3. The ordinate is represented by the ratio of the frequency of the floor at a particular ρ_x to the frequency of the simply-supported floor. The trend is a "growth curve" that builds up gradually, then rises quickly, and finally levels off slowly. An increase of 125% has been observed. In regard to the perceptibility of vibration, the increase of frequency due to the increase of rotational end restraints is not desirable, as shown in Fig. 1.

In contrast, Fig. 4 shows that the deflection drops off slowly at first, then rapidly, with a leveling off at about $\rho_x = 100$ or $\log \rho_x = 2$. The ordinate is represented as a ratio of the deflection at a particular value of ρ_x to the simply-supported deflection. This trend is encouraging. If the amplitude decreases at a faster rate than the frequency increases, then the floor may have a more desirable combination of amplitude and frequency, i.e., a less perceptible level. Since it is the amplitude of vibration and not the static deflection of the floor that is involved, the variation of *DMF* with ρ_x must be observed. Note that the variation of the *DMF* with ρ_x is dependent on the period (frequency) of the floor under consideration. It is therefore necessary to investigate a specific problem, in order to observe the effect of the variation of ρ_x on the perceptibility of the floor.

A simply-supported floor which has a natural frequency of 12 cps and an amplitude of 0.0065 in. (and as a result is located in the lower part of the "distinctly perceptible" region of the Lenzen graph, as shown in Fig. 6) is modified by adding rotational end restraints. The sample calculations with several variations of ρ_x are shown in Table 1. The values shown in Table 1 are explained in the Appendix. The amplitude (Column 5) is the product of initial static deflection, Δ_s, under simply-supported conditions (assuming $\rho_x = 0.01$) and deflection ratio for varying end condition (Column 4) and the dynamic amplification factor (Column 3). In this example, $\Delta_s = 0.005$ in. is used. For example, when $\rho_x = 1$, amplitude $AMPL = 0.005 \times 0.583 \times 1.39 = 0.0041$ in.

Figure 5 presents the variation of the amplitude normalized with that of the simply-supported condition for varying ρ_x (Column 6). The results are promising; the amplitude seems to decrease at a faster rate than the frequency increases, for the lower values of ρ_x. As final proof, the vibration characteristics of the floor are plotted on the Lenzen graph (see Fig. 6) for various values of ρ_x. This graph confirms the proposed trend, i.e., the increasing ρ_x moves the vibration characteristics of the floor in the direction of lower perceptibility.

DISCUSSION

There are three points which must be considered to complete the topic under discussion. The first is the effect, on the number of joists resisting deflection, of elastic end restraints. The work done in this area by Lenzen[5] considered the floor as a simply-supported plate. If the change of deflection profile, caused by introducing elastic end restraints, reduces the number of resisting joints, then the problem should be re-evaluated. More experimental work should be done in this area.

Table 1. Sample Problem

Pt.	(1) ρ_x	(2) *FREQ*	(3) *DMF*	(4) Δ/Δ_s	(5) *AMPL*	(6) $AMPL/AMPL_S$
1	10^{-2}*	12.00	1.30	1.00	0.0065	1.00
2	10^{-1}	12.33	1.31	0.93	0.0061	0.94
3	10^{0}	14.04	1.39	0.583	0.0041	0.63
4	10^{1}	20.92	1.56	0.271	0.0021	0.32
5	10^{2}	26.02	1.65	0.25	0.0021	0.32
6	10^{3}	26.91	1.66	0.25	0.0021	0.32
7	10^{4}	27.01	1.66	0.25	0.0021	0.32
8	10^{5}	27.02	1.66	0.25	0.0021	0.32

*Assumed value for hinged support.

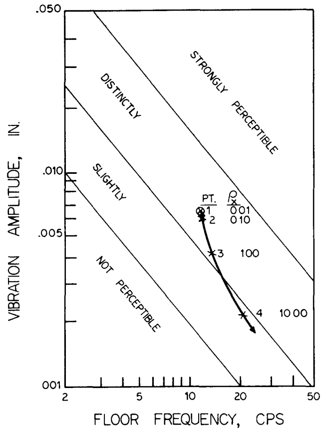

Fig. 6. Change in perceptibility with increasing end restraints

The second point to be considered is the method by which the end restraints can be varied. No actual work has been done; however, several suggestions can be made. The overlapping and rigid connection of joist extenders between bays of a floor system may be a way of increasing rotational rigidity. Reinforcement in the slab at points joining two floor systems, or some form of bracing, could also be used to make the end supports more rotationally rigid. The actual methods can be refined and elaborated upon through experiment and actual construction.

The third point to be stressed is that the bottom chord of the joist near the ends must be braced and capable of taking the compression caused by end restraint.

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APPENDIX
EXPLANATION OF TABLE 1 VALUES

Column 1 gives the variation of the dimensionless rotational restraint parameter ρ_x (defined as K_xL/EI) from a very small value ($\rho_x = 0.01$), representing the simply supported condition, to a very large value ($\rho_x = 10^5$), representing the fixed end condition. For simplicity of calculations, values of $\rho_x = 0.01, 0.1, 1$, etc., were used in the example.

Column 2 gives the frequency of the slab-joist beam system. Initially, the frequency of the simply supported system ($\rho_x = 0.01$) was taken as 12 cycles/sec. As ρ_x changes, the frequency of the system also changes. For

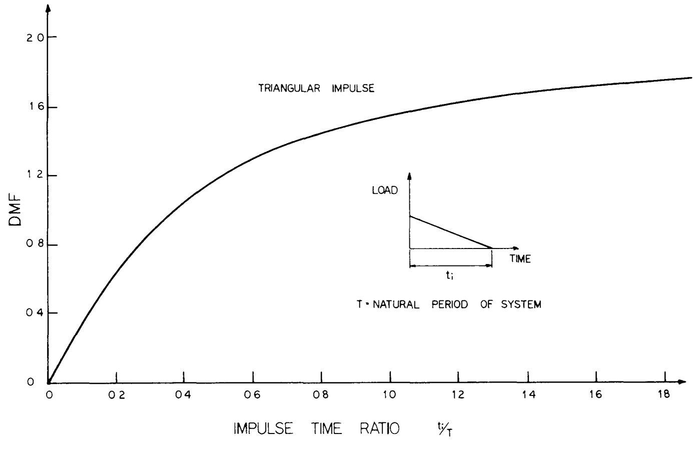

Fig. 7. Dynamic Magnification Factor vs time duration

example, when $\rho_x = 1$ ($\log \rho_x = 0$), one can find the dimensionless frequency from Fig. 3 to be 1.17. Thus, the frequency for the $\rho_x = 1$ system is:

$$f(\rho_x = 1) = 1.17 \times 12 = 14.04 \text{ cycles/sec}$$

Column 3 gives the values of the dynamic magnification factor (*DMF*) for the human footfall impact, which may be represented by a triangular impulse with an initial magnitude of 606 lbs and a duration $t_i = 0.05$ second.[11] Figure 7 displays the *DMF* for a triangular impulse[2,12] with t_i/T as the abscissa. Initially, the system has a period $T = 1/f = 1/12$ sec. Thus, from $t_i/T = 0.05 \times 12 = 0.6$, one can find that $DMF = 1.30$ from Fig. 7. Similarly, when $\rho_x = 1$, $T = 1/f = 1/14.04$ sec., $t_i/T = 0.05 \times 14.04 = 0.7$, and one finds from Fig. 7 that $DMF = 1.39$.

Column 4 gives the ratio of static midspan deflection for various end restraints to the initial simply-supported deflection, which can be found in Fig. 4.

Column 5 and Column 6 have been explained in the text and thus will not be repeated.