An Introduction to Load and Resistance Factor Design for Steel Buildings

C. W. PINKHAM AND W. C. HANSELL

Load and Resistance Factor Design (LRFD) is a major advance toward a simple, rational design of steel-framed buildings. It combines limit states of strength and serviceability with a modern probability-based approach to structural reliability. The essence of LRFD is the assignment of resistance and load factors on the basis of consistent reliability. Developed by T. V. Galambos and his associates at Washington University, St. Louis, LRFD is based on a project[1] sponsored by the American Iron and Steel Institute Committees of Structural Steel Producers and Steel Plate Producers.

This paper introduces LRFD, illustrating the treatment of loads and resistance and describing how the numerical values of load and resistance factors were established. Two companion papers also appear in this issue of the Enginnering Journal: one[2] describes the proposed LRFD Criteria and Commentary[1] for steel buildings, which are the end product of the research project; the other summarizes design office studies conducted by two consultants during the project's evaluation phase.[3]

DEFINITION OF LRFD

Load and Resistance Factor Design is a method of proportioning structural members and connections so that strength and serviceability limit states exceed factored load combinations. To illustrate this definition, consider the limit state of yielding at a tension member's gross section. For this limit state, the LRFD design equation may be expressed as follows:

C. W. Pinkham is President, S. B. Barnes and Associates, Los Angeles, Calif.

W. C. Hansell is Structural Consultant, Bethlehem Steel Corporation, Bethlehem, Pa.

Ed. Note: This paper is based on a talk given at the 1977 AISC National Engineering Conference in Washington, D.C. and is the first of three papers on LRFD that appear in this issue.

$$0.9T_y \geq 1.1[1.1(DL) + 1.4(LL)] \tag{1}$$

where

T_y = yield strength of the gross section, equal to the product of gross area, A_g, and yield strength of steel, F_y

(DL) = mean dead load

(LL) = mean live load

0.9 = resistance factor, ϕ

$1.1, 1.1, 1.4$ = load factors γ_o, γ_{DL}, γ_{LL}, referred to as analysis factor, dead load factor, and live load factor, respectively

Equation (1) illustrates the LRFD format as used by the designer. The remainder of this paper provides background on numerical factors and their origin in structural reliability, and discusses the terms representing member resistance and loads. It is aimed at familiarizing the reader with the origin of individual terms in the basic LRFD format represented by Eq. (1).

Resistance factor ϕ, a number less than 1.0, accounts for resistance uncertainties for a particular limit state. It depends upon variability in material properties like yield stress or ultimate tensile strength; geometric deviations in depth, thickness, and straightness as the result of mill, fabrication, and erection practice; and statistical measures of agreement between design models predicting resistance for a particular limit state versus experimental data for that same limit state. Limit states with more variable resistance have lower resistance factors.

Load factors γ_{DL} and γ_{LL}, numbers larger than 1.0, provide for dead and live load variations with time, as well as for uncertainty about their location on the structure. Design idealizations, such as assumption of uniform load distribution, are recognized in these load factors. Because dead load magnitude and location usually involve less uncertainty than most live loads, the dead load factor is the smaller of the two.

There is an exception to the rule that load factors are larger than unity. This occurs when a load type, such as dead load, increases resistance for limit states like uplift or stability against overturning. In this case the dead load factor must be less than 1.0.

Analysis factor γ_o , also larger than 1.0, accounts for uncertainties in structural analysis. For example, normal three-dimensional behavior of real buildings is usually idealized for two-dimensional analysis. Connections are assumed either simple or rigid, but real connections lie somewhere between these bounds.

The purpose of citing these approximations is not to label them as errors or deficiencies in design, for they each represent accepted and proven practice. Nor does LRFD require changes in that practice. However, it is an advantage of LRFD that these uncertainites in structural analysis are recognized and considered in a simple manner, using an analysis factor.

In a more general form, the LRFD format is:

$$\phi R_n \geq \gamma_o \Sigma \gamma_i Q_i \quad i = (DL), (LL), W, S, \ldots \qquad (2)$$

where the left side is nominal resistance, R_n , times a resistance factor and the right side sums the products of mean load effects, Q_i , and load factors γ_i and γ_o. Nominal resistance and mean load effect both have the same dimensions, depending on the limit state. For example, they are both an axial force, a shear force, or a bending moment. The subscript i represents load types such as dead load, live load, wind load, and snow load, which are summed for any particular load combination.

Several current structural codes, including the Canadian standard for limit states design,[4] use an LRFD-type format. The resistance factor ϕ can be lumped together with the load factors, as in either Part 2 of the current AISC Specification[5] or the load factor design provisions of the AASHTO specifications for highway bridges.[6] The resistance factor can also be retained on the nominal resistance side of the equation, as in the ACI building code.[7] The proposed criteria[1] prescribe separate numerical values for all resistance and load factors, as illustrated in Eq. (1).

LOADS AND LOAD COMBINATIONS

The load side of the LRFD equation introduces some new, but simple, concepts for structural design. First, all loads are mean values. The best or most likely load estimate, rather than an extreme, is used. Extreme loads are not appropriate, because the load factors of LRFD allow for load variability.

In cases such as dead load, the product of nominal density and original material volume gives a good estimate of mean load. Allowance for future dead loads, such as reroofing or a computer sub-floor, remains a designer-owner decision. LRFD *does not* alter the decision process aimed at providing a building with levels of maintenance and operational flexibility required by the owner-user. But, since dead and live load have different load factors, LRFD requires the designer to be more specific in identifying load sources and assigning load factors.

In fact, it has been suggested that one advantage of LRFD is to make the designer more "loads conscious". For example, consider load duration. Loads that remain in place for long periods, such as dead load, tend to be less variable. On the other hand, loads applied for brief periods, such as wind load, tend to have larger variations.

This leads to an important feature of LRFD. Load sources are subdivided according to load duration and frequency, items that govern the likelihood of load combinations.

For wind loads, LRFD uses the mean value of both maximum annual wind, W_A , and maximum lifetime wind, W. Basic information, typical of current code values, is available in the 1972 American National Standards Institute requirements for minimum design loads.[8] Wind maps in that publication were developed by fitting an extreme value probability distribution to wind velocity data for many stations. Using the same probability distribution, mean values of maximum annual wind and maximum lifetime wind can be related to the wind loads, q_{ANSI} , in the form:[9]

$$W_A = 0.5 q_{ANSI} \qquad (3)^*$$

$$W = 1.2 q_{ANSI} \qquad (4)^*$$

where the lifetime wind is based on a 50-year life. This illustrates the ease of translating from ANSI or any other code loads to the mean maximum loads used in LRFD.

Live loads depend on type of occupancy. Consider, for example, office loading. LRFD uses the mean value of instantaneous live load, $(LL)_I$, also called sustained live load, and the mean value of maximum lifetime live load, (LL). Instantaneous live load is that normally present and observed in live load surveys. Maximum lifetime live load represents both sustained loads and extraordinary loads. The latter are high density loads of short duration and infrequent application. An example would be furniture stacked during remodeling.

Office live load surveys give as mean value of instantaneous live load:

$$(LL)_I = 12 \text{ psf} \qquad (5)$$

or about 1/4 of the ANSI code specified office load. Based on an extreme value probability model developed at MIT,[10] the mean value of maximum lifetime live load in offices is:

$$(LL) = 15 + 760/\sqrt{A_I} \leq 60 \text{ psf} \qquad (6)^{**}$$

* *Values of 0.5 and 1.2 have been rounded off from calculated factors 0.49 and 1.17 in Ref. 9.*

** *Values of 15 and 760 have been rounded off from calculated factors 14.9 and 763 in Ref. 10.*

The influence area, A_I, is taken as twice the tributary area for beams and four times the tributary area for columns.

One advantage of LRFD is the rational, but simple, treatment of load combinations. Many maximum lifetime loads are of short duration and rare frequency. The probability of an event combining maximum lifetime loads from two different sources is extremely small; to combine loads in this way is to adopt an unrealistic design decision. LRFD gives the designer flexibility to select load combinations on the basis of conservative, but realistic, expectations.

Dead load is always present, except for members that do not support gravity loads. Hence, dead load combines with any maximum lifetime load that can be applied to a member. If a third load source is present, its level is taken at the annual or sustained load level, rather than its maximum lifetime level. For example, if W is the mean value of the maximum lifetime wind load and $(LL)_I$ is the mean value of the sustained live load, the tension member considered in Eq. (1) would be designed also for the dead plus live plus wind load combination, using

$$0.9T_y \geq 1.1[1.1(DL) + 2.0(LL)_I + 1.6W]$$

where 2.0 is the load factor on sustained live load and 1.6 is the load factor on maximum lifetime wind.

The topic of loads and load combinations is currently undergoing active study. LRFD is consistent with the present state-of-the-art, but the future is certain to provide more comprehensive data. As better information on loads and load combinations becomes available, the LRFD model provides a ready means for incorporating such information into design practice while a consistent approach to structural reliability is maintained.

STRENGTH AND SERVICEABILITY LIMIT STATES

Another feature of LRFD is consistent application of strength and serviceability limit states. Strength limit states deal with capacity to survive extreme loads; the main issue is structural safety. Serviceability limit states aim to avoid malfunctions during expected service of the structure. The main objective is for the structure to serve its intended functions without distress to either occupants or nonstructural elements.

The LRFD Criteria[1] include comprehensive strength limit state provisions for all major types of structural steel members and connections. Some examples include: (1) for a tension member, yield in the gross section and fracture in the net section; (2) for a column with axial load, column buckling and local buckling; (3) for a beam-column, inplane bending capacity and lateral torsion buckling; and (4) for a composite beam, maximum plastic strength of steel, concrete, or shear studs.

For some strength limit states, allowable stress design models from Part 1 of the AISC Specification[5] have been reworked to an ultimate strength form for use in LRFD. For other limit states, the best available information on steel or composite elements has been incorporated. For instance, maximum plastic strength provisions for composite beams are included in the LRFD Criteria.

Serviceability limit states deal with mean loads and structural response during normal use. Included are permanent deformation due to gross yielding, excessive elastic sag or drift, major slip in high-strength bolted joints not erected in bearing, and unacceptable vibrations. Many serviceability criteria rely on empirical, but accepted, indexes like span-to-deflection, span-to-depth, or height-to-drift. LRFD does not change these indexes, although it does modify the manner of accounting for uncertainty in load and stiffness evaluations. LRFD serviceability criteria are included in the Commentary.[1]

A brief conceptual sketch of strength limit states for flexure of rolled beams and plate girders in Fig. 1 illustrates some features of LRFD. The principal limit states are flange local buckling (FLB), web local buckling (WLB), and lateral-torsional buckling (LTB) between laterally braced points. Depending on slenderness, each limit state has three strength zones ranging from plastic, through inelastic, into elastic buckling.

This illustrates another advantage of LRFD for the designer. LRFD uses a comprehensive set of limit state provisions that apply to the full range of slenderness. In contrast, previous limit state type code provisions for steel design limit slenderness to the plastic range. From this it can be seen that plastic design is a subset of LRFD, with LRFD providing a unity of approach to structural steel design. Instead of vacillating between historical approaches of allowable stress design and modern but restricted approaches of plastic design, the future lies in the unified approach of Load and Resistance Factor Design.

STRUCTURAL RELIABILITY

The analysis of structural reliability starts by recognizing that the principal variables of design—strength or resistance, R, and load effect, Q—cannot be precisely determined in advance. They vary above or below some mean values, and the statistical measure of this variation is the "standard deviation". The basic idea is illustrated in Fig. 2, which compares probability distributions for two loads, L_1 and L_2. Both loads have the same mean value, but L_1,

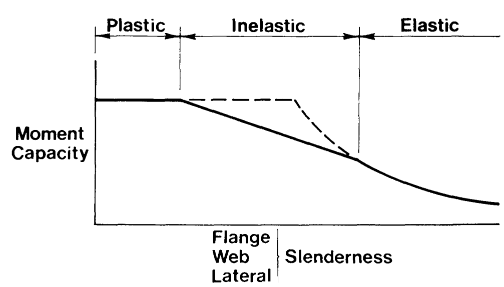

Fig. 1. Bending strength zones for FLB, WLB, LTB

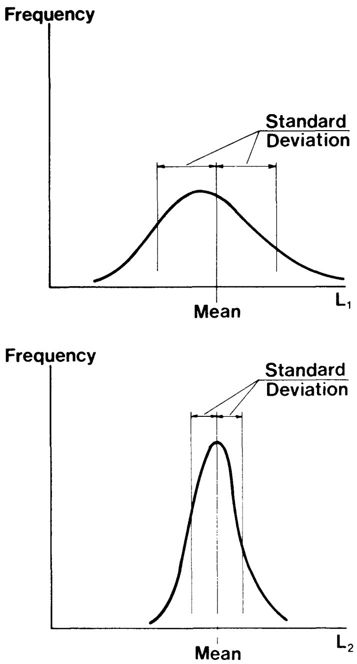

Fig. 2. Typical probability distributions

with larger standard deviation, is more variable. In approximate terms, resistance and load effects will be within two standard deviations of the mean about 95% of the time.

For convenience, variability can also be measured as a ratio of standard deviation to the mean. This ratio is called the "coefficient of variation", or COV. Thus, a simple way to describe random variables of resistance and load effect is to use mean values, R_m and Q_m, together with corresponding coefficients of variation, V_R and V_Q.

Using only these simple terms, LRFD reliability analysis[11,13] applies the safety index β in the form:

$$\beta = \frac{\ln(R_m/Q_m)}{\sqrt{V_R{}^2 + V_Q{}^2}} \tag{7}$$

Figure 3, a hypothetical probability distribution for $\ln(R/Q)$, helps to explain the safety index. The horizontal axis is the natural logarithm of R/Q. The vertical axis is relative frequency. The bell shaped curve can be thought of as a smoothed-out bar chart or histogram of $\ln(R/Q)$. If resistance is just equal to load effect, we have $R = Q$ or $R/Q = 1.0$.

Note that when $R/Q = 1.0$, the logarithm is zero and that negative values of $\ln(R/Q)$ correspond to R/Q between 1.0 and 0. The vertical axis through 0 forms the boundary between failure condition $R < Q$ and survival condition $R > Q$. The risk of failure is represented by the hatched area under the curve in the failure zone.

Rather than attempt to evaluate failure risk directly as the area under the curve, the LRFD reliability analysis uses the mean value and standard deviation of $\ln(R/Q)$. In these terms, safety index β is simply the ratio of mean value $\ln(R/Q)$ to the standard deviation. In graphical terms, shown in Fig. 3, the mean is β standard deviations to the right of 0. A safety index increase corresponds to shifting the probability distribution to the right—a reduction in risk of failure. The converse is also true. As the safety index is reduced, failure risk increases. This illustrates that the safety index is a valid measure of structural reliability.

Note that the form of probability distribution remains unknown and is of little practical interest. The safety index is described as a distribution-free measure of structural reliability based on modern probability theory.[12]

CALIBRATION

The next step in reliability analysis is to establish a value for the safety index. The best place to turn for guidance is current structural design practice, as codified in the AISC Specification[5] and the ANSI code for loads.[8] The process of determining safety index values from design codes is called *calibration*, and can be thought of as the bridge between LRFD and prior experience.

Calibration[13] proceeds in a straightforward manner, although algebraic details can become intricate. The first step, using AISC and ANSI codes plus available statistics, is to establish equations for the mean and COV of resistance and load effect as functions of load, span, and tributary area. This step must be repeated for various structural forms and a range of load combinations.

The second step is to study how safety index varies with loads and area. A sample safety index calibration for beams, Fig. 4, illustrates the result. The curves indicate how the safety index varies with tributary area for simple beams designed to the AISC Specification. Uniform dead and live loads are applied with live load reductions from the ANSI code.

In this sample, the safety index varies from approximately 2.6 to 3.4. The trend of increasing safety index with increasing dead load is typical.

The third step is to select a representative value of safety index that characterizes the level of safety inherent in

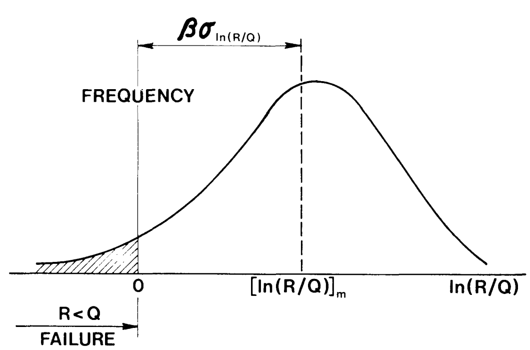

Fig. 3. LRFD safety index

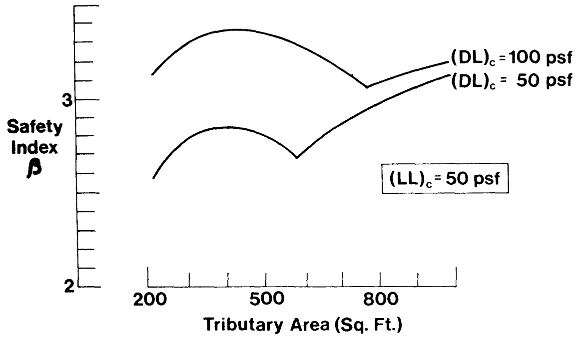

Fig. 4. LRFD calibration—simple beams[13]

current design practice. In effect, this section establishes the overall level of safety in LRFD to be consistent with the current AISC Specification. Based on calibrations for simple beams and centrally loaded columns, and on comparative designs,[3] the Washington University studies[13] recommend a safety index of $\beta = 3.0$ for members and 4.5 for connections. Figure 4 shows that even lower values of the safety index are common in existing structures.

Sample calibration curves such as Fig. 4 illustrate another advantage of LRFD. Structural reliability underlying current design practice is inherently safe—but surprisingly variable. LRFD opens the way for better control of structural reliability. Overdesign that results from the application of historical patterns of structural codes can be largely erased. With a proper choice of the safety index, LRFD will result in new economies, while maintaining levels of safety proven adequate by existing structures.

SEPARATION

There is one final link in the sequence between safety index and LRFD format. Termed *separation,* its basic objective is to separate terms describing loads from those relating to resistance. It will be sufficient to describe the beginning of the process and then indicate the final result.

Equation 7, defining the safety index, can be solved for mean resistance:

$$R_m = Q_m e^{\beta\sqrt{V_R{}^2 + V_Q{}^2}} \qquad (8)$$

The right side of Eq. (8) includes terms dependent on both loads and member resistance. To simplify the design process, it is desirable to have all resistance terms on one side of the equation and all load terms on the other side. This is accomplished by observing that:

$$\sqrt{V_R{}^2 + V_Q{}^2} \cong \overline{\alpha}(V_R + V_Q) \qquad (9)$$

where $\overline{\alpha}$ is the separation constant. Separation of V_R from V_Q gives:

$$R_m e^{-\overline{\alpha}\beta V_R} = Q_m e^{\overline{\alpha}\beta V_Q} \qquad (10)$$

All resistance terms are now on the left; all load related terms are on the right. Further steps in the separation process involve decomposing the load term into separate terms for load sources such as dead, live, wind, and snow. Omitting intermediate algebra, the result leads directly to the LRFD format:

$$\phi R_n = \gamma_o \Sigma \gamma_i Q_i \quad i = (DL), (LL), W, S, \ldots \qquad (2)$$

where the resistance and analysis factors are:

$$\phi = \frac{R_m}{R_n} e^{-\alpha\beta V_R} \quad \text{and} \quad \gamma_o = e^{\alpha\beta V_o} \qquad (11)$$

and the load factors have the form:

$$\gamma_i = 1 + \alpha\beta V_i \qquad (12)$$

In this completely separated form, each factor contains the separation constant α, the safety index β, and an individual COV (V_R or V_o or V_i) associated with that term. Numerical studies at Washington University[13] concluded that $\alpha = 0.55$ is appropriate for the separation factor.

The proposed criteria[1] prescribe numerical values for all resistance and load factors. Accordingly, Eqs. (11) and (12) are needed only for solution of non-routine problems outside the scope of the proposed criteria.

CONCLUDING REMARKS

The LRFD format used by the designer is illustrated by Eq. (1). The remainder of this paper discusses loads and resistance as applied in LRFD, and describes how the safety index was used to establish numerical values of load and resistance factors.

LRFD, with roots in a safety index, provides the basis for assigning load and resistance factors with consistent reliability. This is a powerful new tool for the structural engineering profession. It will permit new economies, while maintaining levels of safety proven adequate by existing structures.

Codes dealing with resistance and standards defining loads can now have a common reliability base for their separate tasks. LRFD identifies required information and the means for maintaining consistent structural reliability within a code and between codes.

Structural engineers will find the LRFD format and reliability model useful for dealing with non-routine problems. For example, load magnitude and location are often vaguely defined during the preliminary design phase. At this stage, the engineer can adjust load factors to be consistent with his assessment of likely variations and uncertainties. Later, when reviewing his design with more definite information on loads and their placement, he can adjust load factors to reflect reduced load uncertainty.

It is true that the experienced engineer, confronted with such problems, will routinely adjust loads according to available information. In this circumstance, the LRFD format is simply an additional tool to assist him in applying and sharpening his judgment.

LRFD provides much needed guidance to the structural research community. In the past, well aware that random factors had influenced his data, the researcher would choose some arbitrary "lower bound" estimate for design recommendations. LRFD suggests a more consistent approach. Information needed from structural research is the best estimate of the mean, together with the COV, obtained from statistical analysis of the data. The COV can then be adjusted to reflect sources of uncertainty beyond the experimental program. Research recommendations for design, presented in this fashion, will provide the code body and ultimate user with information needed for applying the results with consistent reliability.

Several other LRFD features of value to the engineer were previously mentioned. They included:

1. Rational but simple treatment of load combinations.
2. A framework for progress that can easily adjust to new information on loads and resistance.
3. A unity in approach to structural steel design due to consistent application of limit states across the full range of slenderness.

One additional feature deserves mention. LRFD offers the opportunity to unify structural design. It appears possible, and highly desirable, to move toward common load factors that would apply to all major structural materials and systems. Each material discipline would be responsible for its own limit states and associated strength factors, but all disciplines would use a common reliability analysis and safety index. This is yet another illustration of the proposition that LRFD is the framework for progress in the art of structural design.

ACKNOWLEDGMENTS

The authors have enjoyed the privilege of many fruitful discussions on this topic within the AISI Advisory Task Force on LRFD, and especially with Professors T. V. Galambos and C. A. Cornell.

REFERENCES

1. Proposed Criteria for Load and Resistance Factor Design of Steel Building Structures *Research Report 45, C.E. Dept., Washington University, May 1976.*
2. *Galambos, T. V. and M. K. Ravindra* Proposed Criteria for Load and Resistance Factor Design *AISC Engineering Journal, Jan. 1978.*
3. *Wiesner, K. B.* LRFD Design Office Study *AISC Engineering Journal, Jan. 1978.*
4. Steel Structures for Buildings—Limit States Design *CSA Standard S16.1—1974 Canadian Standards Association, Rexdale, Ontario.*
5. Specification for the Design, Fabrication and Erection of Structural Steel for Buildings *American Institute of Steel Construction, New York, N.Y. 1969, plus Supplements 1, 2, and 3.*
6. Standard Specifications for Highway Bridges *American Association of State Highway and Transportation Officials, Washington, D.C., 1973.*
7. Building Code Requirements for Reinforced Concrete *ACI 318-71, American Concrete Institute, Detroit, Mich., 1971.*
8. Building Code Requirements for Minimum Design Loads in Buildings and Other Structures *ANSI A58.1-1972, American National Standards Institute, New York, N.Y.*
9. *Ravindra, M. K. and T. V. Galambos* Load Factors for Wind and Snow Loads for Use in Load and Resistance Factor Design Criteria *Research Report No. 34, C.E. Dept., Washington University, Jan. 1976.*
10. *McGuire, R. K. and C. A. Cornell* Live Load Effects in Office Buildings *ASCE Journal of the Structural Division, Vol. 100, No. ST7, July 1974.*
11. *Cornell, C. A.* A Probability Based Structural Code *Journal, American Concrete Institute, Vol. 66, No. 12, Dec. 1969.*
12. *Benjamin, J. R. and C. A. Cornell* Probability, Statistics and Decisions for Civil Engineers *McGraw-Hill Company, New York, N.Y., 1970.*
13. *Galambos, T. V. and M. K. Ravindra* Tentative Load and Resistance Factor Design Criteria for Steel Buildings *Research Report 18, C.E. Dept., Washington University, Sept. 1973.*