Failure of Simply-Supported Flat Roofs by Ponding of Rain

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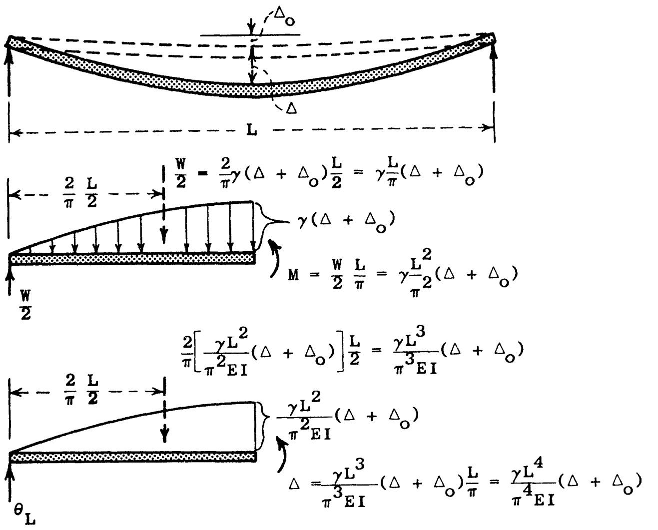
A FEW STRUCTURES supporting flat roofs have collapsed during rain storms even though they were apparently adequately designed and constructed. Such collapses have occurred in structures constructed of all three of the major building materials.

The mechanism of failure is as follows: The structure has an initial deflection due to loads or initial downward camber or both, which creates a depression. Rain water ponds in the depression, causing additional deflection. This collects more water causing still more deflection. The cycle continues until the water spills over the edges or collapse occurs.

The action can be analyzed in a step-by-step fashion as just described. Mathematically, this corresponds to an infinite series. If the series converges, the structure will assume a final deflected position, and the water will spill over the edges. If the series diverges, the structure will, mathematically, continue to deflect as long as the rain continues.



γ = unit weight of liquid times beam spacing
Δ_o = initial centerline deflection
Δ = additional centerline deflection due to ponding
E = modulus of elasticity of material
I = moment of inertia of beam

Fig. 1. Maximum moment and deflection due to ponding

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MATHEMATICAL ANALYSIS

The simply-supported roof beam can be analyzed exactly as long as it remains elastic. Consider the simply-supported beam of Fig. 1, initially deflected in a half sine wave. Water filling this depression gives a load having a half-sine-wave distribution. Remembering that a half-sine-wave loading results in half-sine-wave moment and deflection diagrams, the maximum moment can be found from the appropriate free body diagram, and the maximum deflection caused by the ponded water can be found by the conjugate beam method.

The expression for the maximum deflection obtained in Fig. 1 contains maximum deflection on the right-hand side of the equation as well as on the left. Manipulating to remove it from the right-hand side:

$$\Delta = \frac{\gamma L^4}{\pi^4 EI}(\Delta + \Delta_0) = \frac{\gamma L^4}{\pi^4 EI}\Delta + \frac{\gamma L^4}{\pi^4 EI}\Delta_0$$

$$\Delta = \Delta_0 \frac{\dfrac{\gamma L^4}{\pi^4 EI}}{1 - \dfrac{\gamma L^4}{\pi^4 EI}} \qquad (1)$$

The total deflection at the centerline, $\Delta + \Delta_0$, is then:

$$\Delta + \Delta_0 = \Delta_0 \left[\frac{1}{1 - \dfrac{\gamma L^4}{\pi^4 EI}}\right] \qquad (2)$$

If the term, $\gamma L^4/\pi^4 EI$ becomes unity, the fraction and, consequently, the deflection, become infinite. This is true regardless of the value of Δ_0, as long as Δ_0 is greater than zero. This, then, predicts a collapse even though the structure remains completely elastic. This is similar to the elastic buckling of a column.

What causes Δ_0 is immaterial. It can be load, downward camber of the beam, or a combination of both.

The initial deflection need not be in the form of a half sine wave for the above conclusions to be essentially correct. It is easily shown that any simple-beam deflection equation can be expressed in a Fourier sine series. The half-sine-wave term of the series is the dominant one in nearly every practical case.

If $\gamma L^4/\pi^4 EI$ is equal to or greater than unity, collapse is inevitable in a sustained storm. If $\gamma L^4/\pi^4 EI$ is less than unity, collapse can still occur from overstress. The maximum moment in the beam when the depression is filled with water is:

$$M = M_0 + \frac{\gamma L^2}{\pi^2}(\Delta + \Delta_0)$$

$$= M_0 + \frac{\gamma L^2}{\pi^2}\left[\frac{\Delta_0}{1 - \dfrac{\gamma L^4}{\pi^4 EI}}\right]$$

where M_0 is the initial moment due to all loads on the structure before ponding. The maximum stress is:

$$f_{\max} = \frac{Mc}{I} f_0 + \frac{\gamma L^2 c}{\pi^2 I}\left[\frac{\Delta_0 \pi^4 EI}{\pi^4 EI - \gamma L^4}\right]$$

$$= f_0 + \frac{\gamma L^2 \pi^2 E c\, \Delta_0}{\pi^4 EI - \gamma L^4} \qquad (3)$$

where $f_0 = M_0 c/I$.

If the calculated maximum stress exceeds the yield point, then there is a good probability that collapse will occur. When yielding occurs, only the portions of the cross-section of the beam which are still elastic contribute to the moment of inertia resisting the next increment of moment. With this decrease in stiffness, deflection increases at an increasing rate.

Equation (3) can be rearranged to determine the moment of inertia required to provide a given factor of safety against yield. Let n = the desired factor of safety; then,

$$f_{\max} = \frac{f_y}{n} = f_0 + \frac{\gamma L^2 \pi^2 E c \Delta_0}{\pi^4 EI - \gamma L^4}$$

$$\frac{f_y}{n} - f_0 = \frac{\gamma L^2 \pi^2 E c \Delta_0}{\pi^4 EI - \gamma L^4}$$

$$\pi^4 EI - \gamma L^4 = \frac{\gamma L^2 \pi^2 E c \Delta_0}{\dfrac{f_y}{n} - f_0} \qquad (4)$$

$$I = \frac{\gamma L^4}{\pi^4 E} + \frac{\gamma L^2}{\pi^2}\left[\frac{\Delta_0 c}{\dfrac{f_y}{n} - f_0}\right]$$

Δ_0 is made up of load effect and camber. The deflection due to load, Δ_{0-1}, can be calculated using readily available formulas. For the most common case, though, that of uniform load, an alternate form may be preferable.

$$\Delta_{0-1} = \frac{5}{384}\left[\frac{wL^4}{EI}\right] = \frac{wL^2}{8}\left[\frac{d}{2I}\right]\left[\frac{5L^2}{24Ed}\right]$$

$$= M_{\max}\left[\frac{c}{I}\right]\left[\frac{5L^2}{24Ed}\right]$$

If Δ is in inches, L is in feet, d is in inches, E = 30,000 ksi, and $f_{\max} = M_{\max}\, c/I$ is in ksi,

$$\Delta = f_{\max}\frac{5L^2(144)}{24(30{,}000)d} = f_{\max}\left[\frac{L^2}{1{,}000d}\right]$$

For any other value of E,

$$\Delta = \frac{30{,}000}{E \text{ in ksi}}\left[\frac{fL^2}{1{,}000d}\right] \qquad (5)$$

For many purposes, $f_{\max}$ can be taken as the allowable flexural stress, F_b.

The designer may wish to consider the camber equal to the maximum permissible by ASTM A6 or to a fraction of the span, such as $L/360$. He may also wish to consider a minimum value of $\Delta_0 = \Delta_{0-1} + \Delta_{0-2}$ such as 1 in. or $L/180$.*

The use of Equations (2), (3), (4), and (5) is illustrated in the following example:

ILLUSTRATIVE EXAMPLE

Steel beams at 20 ft centers will support a roof on a 50 ft simple span. Dead load is 10 psf and live load is 20 psf. Beam weight is estimated at 55 plf. Design the beam of A36 steel.

Design load $w = (10 + 20)(20) + 55 = 655$ plf

$$M_0 = \frac{wL^2}{8} = 0.655\left(\frac{50^2}{8}\right) = 204.7 \text{ kip-ft}$$

$$S_{\text{req}} = \frac{M}{F_b} = \frac{204.7(12)}{24} = 102.4 \text{ in.}^3$$

Use 21WF 55 with S = 109.7 in.^{3} and I = 1,140.7 in.^{4}.

Check for effect of ponding:

Deflection due to load is

$$\Delta_{0-1} = \frac{5}{384}\frac{wL^4}{EI} = \frac{5}{384}\frac{(0.655)(50)^4(1{,}728)}{(29{,}000)(1{,}140.7)} = 2.78 \text{ in.}$$

This is less than $L/180$ = 3.33 in. and will be considered satisfactory.

$$\frac{\gamma L^4}{\pi^4 EI} = \frac{(62.4)(20)(50)^4(144)}{\pi^4(29{,}000)(1{,}140.7)} = 0.344 < 1$$

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* *In the interest of simplification Sect. 1.13 of the AISC Specification provides a simple safeguard against ponding by arbitrarily limiting the depth-to-span ratio to $F_b/600{,}000$. This is equivalent to $L/240$ for the given case. This requirement can at times be overly conservative. The above analysis affords a more precise solution.*

The total deflection after ponding, $\Delta_t = \Delta + \Delta_0$, is found from Equation (2).

$$\Delta_t = \Delta_0 \left[\frac{1}{1 - \dfrac{\gamma L^4}{\pi^4 EI}} \right] = 2.78\left(\frac{1}{1 - 0.344}\right)$$

$$= \frac{2.78}{0.656} = 4.24 \text{ in.}$$

The additional moment due to the water load, M_w, is

$$M_w = \gamma \Delta_t \frac{L^2}{\pi^2} = 1.248\left(\frac{4.24}{12}\right)\left(\frac{50^2}{\pi^2}\right) = 111.7 \text{ kip-ft}$$

The total moment, M, is

$$M = M_0 + M_w = 204.7 + 111.7 = 316.4 \text{ kip-ft}$$

and the maximum stress is

$$f_{\max} = \frac{M}{S} = \frac{316.4(12)}{109.6} = 34.6 \text{ ksi} < (F_y = 36 \text{ ksi})$$

No collapse will occur under this condition. If the beam has a downward camber, Δ_{0-2}, equal to the maximum permissible by ASTM A6,

$$\Delta_{0-2} = \frac{1}{8} \text{ in.} \left(\frac{L \text{ in ft.}}{10}\right) = \frac{1}{8}\left(\frac{50}{10}\right) = \frac{5}{8} \text{ in.}$$

$$\Delta_0 = \Delta_{0-1} + \Delta_{0-2} = 2.78 + 0.63 = 3.41 \text{ in.}$$

$$\Delta_t = \Delta_0 \left[\frac{1}{1 - \dfrac{\gamma L^4}{\pi^4 EI}} = \frac{3.41}{0.656} \right] = 5.20 \text{ in.}$$

The additional moment due to water load is now

$$M_w = \gamma \Delta_t \frac{L^2}{\pi^2} = 1.248\left(\frac{5.20}{12}\right)\left(\frac{50^2}{\pi^2}\right) = 137 \text{ kip-ft}$$

and the total moment becomes

$$M = M_0 + M_w = 204.7 + 137 = 342 \text{ kip-ft}$$

and the maximum stress is

$$f_{\max} = \frac{M}{S} = \frac{342(12)}{109.7} = 37.4 \text{ ksi} > (F_y = 36\text{ksi})$$

The beam will yield and could collapse.

An estimate can be made of the deflection at collapse by assuming that the deflection curve is still a half sine wave (Fig. 2) and that the maximum moment equals the plastic moment. (This assumes that the secondary effects of this deflection do not produce prior failure of supporting beams, columns or connections.)

$$M_p = F_y Z = \frac{36(125.4)}{12} = 376.2 \text{ kip-ft}$$

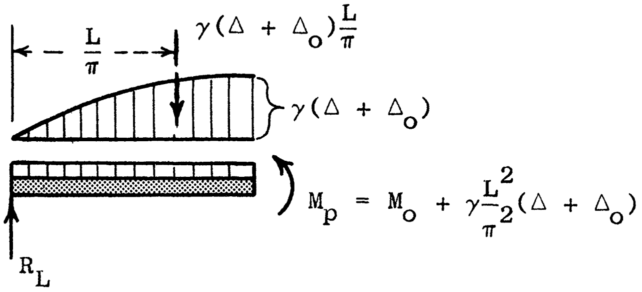

Fig. 2. Moment in beam at collapse

$$M_p = M_0 + M_w = M_0 + \frac{\gamma L^2}{\pi^2}(\Delta + \Delta_0)$$

$$376.2 = 204.7 + 1.248\left(\frac{50^2}{\pi^2}\right)(\Delta_t)$$

$$316 \, \Delta_t = 171.5$$

$$\Delta_t = 0.542 \text{ ft} = 6.51 \text{ in.}$$

The moment of inertia required of a 21 in. WF to provide a factor of safety of one against yielding under this loading condition is found from Equation (4) assuming that $f_0 = F_b = 24$ ksi.

$$I = \frac{\gamma L^2}{\pi^2}\left[\frac{L^2}{\pi^2 E} + \frac{\Delta_0 c}{\dfrac{f_y}{n} - f_0}\right]$$

$$\Delta_{0-1} = \left[\frac{30}{291{,}000}\frac{fL^2}{d}\right] = \frac{30}{29}\left(\frac{24(50)^2}{1{,}000(21)}\right) = 2.96 \text{ in.}$$

$$\Delta_{0-2} = \frac{5}{8} \text{ in.}$$

$$I = \frac{1.248(50)^2}{\pi^2}\left(\frac{(50 \times 12)^2}{\pi^2(29{,}000)} + \frac{(2.96 + 0.63)(10.5)}{36 - 24}\right)$$

$$= 316(1.26 + 3.14) = 1{,}390 \text{ in.}^4$$

A 21 WF 68 provides $I = 1{,}578.3$ in.^{4} and is adequate.

The preceding example is subject to criticism since the full live load, if a snow load, would probably not be on the roof during a rain. Another factor which should be considered, however, is the presence of parapet walls or gravel stops.

If only one half of the live load is acting on the beam having maximum downward camber, and a 2 in. gravel stop allows water to accumulate before spilling over, the problem takes the following form:

$$\text{Weight of 2 in. of water} = 62.4(2/12)$$
$$= 10.4 \text{ psf} = 208 \text{ plf}$$

This, added to the dead load and half the live load gives the load to be considered in determining Δ_{0-1}:

$$w = 255 + 200 + 208 = 663 \text{ plf}$$

This would be a worse condition after ponding than dead load plus full live load of 655 plf, and yielding would occur.

The moment of inertia required of a 21 in. WF beam to provide a factor of safety of one against yielding under this loading condition is found by applying Equation (4). The value of f_0 is assumed to be F_b times w for this condition divided by w due to dead and live load.

$$f_0 = \frac{w_d + \frac{1}{2} w_l + w_w}{w_d + w_l} = \frac{663}{655}(24) = 24.5 \text{ ksi}$$

$$\Delta_{0-1} = \frac{30}{29}\,\frac{fL^2}{(1{,}000\,d)} = \frac{30}{29}\,\frac{(24.5)(50)^2}{(1{,}000(21))} = 3.02 \text{ in.}$$

$$\Delta_{0-2} = 5/8 \text{ in.}$$

$$I = 316\left(1.26 + \frac{(3.02)(10.5)}{36 - 24.5}\right) = 316(1.26 + 3.33)$$

$$= 1{,}450 \text{ in.}^4$$

The 21 WF 68 will still be satisfactory.

SUMMARY

Beams supporting flat roofs sometimes collapse in rain storms due to ponding of water. If the flexural stiffness of a beam is less than a certain critical stiffness, collapse is inevitable in a sustained storm and is independent of the initial deflection. If the flexural stiffness is greater than the critical stiffness, collapse may or may not occur and is a function of stiffness and initial deflection of the beam and of the yield point of the steel.