History of Steel Beam Design

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This paper is about the history of the design of steel beams. Mainly, it is a brief review of how beams were designed according to the various revisions of the American Institute of Steel Construction's *Specification for the Design, Fabrication and Erection of Structural Steel for Buildings* since this specification was first adopted on June 1, 1923.

The term "beam" as used herein defines structural elements in which axial force is so small as to be negligible. Such beams are relatively long compared to their cross section, and their cross section is made up of plate elements which are relatively thin compared to the depth of the members. Beams are loaded such that they resist the loads, which act transverse to their longitudinal axis, by flexure and shear. Such elements occur in many forms in steel construction, and in any frame they comprise a substantial percentage of all load-carrying members.

Steel beams can be classified in many ways: small ("joists"), medium, and large ("girders"); solid-web and open-web; rolled, cold-formed, and built-up; open, closed (box or multi-cell), and combined open-and-closed cross sections; beams with unstiffened or stiffened webs; uniform, stepped (coverplated), and tapered; hybrid and non-hybrid; composite and noncomposite; simple and continuous; etc., etc. This classification can surely be expanded, but the point has been made; it is possible to design and build a large variety of configurations and combinations to creatively achieve the desired purpose of safe and economical structural elements.

Modern building technology devises ever more new types of configurations; a specification must guide designers to proportion them safely and economically, and it must provide supervising authorities with uniform methods to control construction. Thus, a modern specification is an ever growing and improving instrument. Such is the story of the AISC Specification: it grew from a very modest 15 page pamphlet in 1923 to today's complex document (59 pages of Specification, 54 pages of Appendix, 47 pages of Commentary, 56 pages of Supplements—a total of 216 pages which are augmented by many additional references).

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HISTORY OF BEAM THEORY

The history of beam theory is interwoven with the more general history of applied mechanics, building technology, and industrial development. The history of engineering mechanics is well told by Timoshenko in his *History of the Strength of Materials,*[11] from which the following assessment of the state of beam theory in 1923 is abstracted.

The scientific investigation of beams has its first known origins with Leonardo da Vinci (1452–1519), who first attempted the solution to statics problems of determinate beams, and Galileo (1564–1642) who tried (unsuccessfully) to define their internal resistance. However, engineers were not presented with a usable and correct beam theory until the publication of Navier's (1785–1836) book on the Strength of Materials in 1826. This book contained the basic engineering theory of in-plane bending of beams as we know it today.

The 19th Century saw the Theories of Elasticity and Elastic Stability come into full bloom, and by the first decade of the 20th Century the basic methods of solution for statically indeterminate problems, plate buckling, frame buckling, and lateral-torsional buckling were known. Our century, in addition to bringing theoretical refinements and an almost unmeasurable list of solutions to a variety of practical problems, most recently with the aid of computers and the Finite Element Method, also saw the maturing of the Theory of Plasticity.

The status of beam theory, experiment, and calculation in 1976 is such that most problems of practical and theoretical importance have been solved, or, if they have not been solved, the capabilities of finding a solution exists.

BEAM DESIGN IN THE FIRST AISC SPECIFICATION

Engineers in 1923, to be sure, did not have available to them the knowledge we now have at our fingertips in tables, charts, or computers; nevertheless, they knew the fundamental problems with beams, they knew the theories available to formulate solutions, and they were experienced enough to devise simple means of defining acceptable design criteria.

What were the beam problems that needed to be addressed in a design specification? These were moment and shear capacity, fatigue fracture, lateral-torsional buckling, and plate strength (outstanding elements of flanges and web plates). These limit states are essentially the same ones which are in today's AISC Specification.

In 1923 there was only one grade of constructional steel (ASTM A9 or A7, with a yield stress $F_y = 33$ ksi); there was no extensive welded construction and composite beams were not used, so it was fairly simple to devise beam design criteria. The use of elastic analysis was taken for granted, and the only limit state was yielding. The allowable stress in flexure was 18 ksi (up from 16 ksi in 1918),[6] and it was 12 ksi in shear. Static as well as fatigue stresses (the latter determined for the sum of the maximum stress plus 50% of the opposing minimum stress) were checked against these allowables. So were, in fact, the stresses arising from instability (however, in an indirect manner). Column buckling, lateral-torsional buckling, and web buckling were taken care of by one universal formula: the Rankine-Gordon-Tredgold formula (usually called the Rankine-Gordon formula), which had been in use in the English-speaking world at least since the publication of Rankine's *Manual of Civil Engineering* in 1861[11] and which was still used in the 1950's in many city building codes. This formula is rederived in Appendix II,[4] and it is based on the concept of attaining the yield stress at the center of an imperfect strut. Its general form is

$$F_a = \frac{F_y/(F.S.)}{1 + C\left(\dfrac{L}{r}\right)^2} \qquad (1)$$

for the allowable stress of an axially loaded column. The term C is a constant depending on end conditions, material, and cross-sectional properties and tests, and L/r is the least slenderness ratio.

This formula is used in three forms in the 1923 AISC Specification:

Column axial stress:

$$F_a = \frac{18{,}000 \text{ psi}}{1 + \dfrac{(L/r)^2}{18{,}000}} \qquad (2)$$

Allowable flexural stress:

$$F_b = \frac{20{,}000 \text{ psi}}{1 + \dfrac{(L_b/b_f)^2}{2{,}000}} \leq 18{,}000 \text{ psi} \qquad (3)$$

Allowable web shear stress:

$$F_v = \frac{18{,}000 \text{ psi}}{1 + \dfrac{(h/t)^2}{7{,}200}} \leq 12{,}000 \text{ psi} \qquad (4)$$

In Eqs. (3) and (4) the slenderness ratio L/r is replaced by the unbraced length-to-flange width (L_b/b_f) ratio, and the web height-to-thickness (h/t) ratio, respectively. Physically, the compression flange of the unbraced beam and the web of the unstiffened web (at 45° to the axis) were both considered to be axially loaded struts.

REVISIONS OF THE AISC SPECIFICATION

After 13 years it became apparent that the factor of safety of 33/18 = 1.83 against yield, giving a basic stress of 18 ksi, was too high, and the 1936 revision of the AISC Specification increased the basic allowable stress to 20 ksi, by reducing the factor of safety to 1.65. This is essentially the same value still in use (see Table 1 for data on the evolution of the factor of safety since 1890).

There was need for many other changes also, and as the AISC Specification was periodically revised (see Table 2 for the dates of the revisions), there was a constant improvement, resulting in generally improved design while holding the factor of safety constant (after 1936). Some of the changes pertaining to the important limit states for beams are catalogued in Tables 3 through 7.

What difference did these changes make? Appendix III gives two design examples for a two-span continuous beam. One example is for a braced beam, and the other is for an

Table 1. Factors of Safety Against Yielding Beams

Authority	Basic F_b	F_y (ksi)	F_b (ksi)	*F.S.*
Du Bois, 1890[4]	$0.5F_y$	28.6	14.3	2.00
Ketchum, 1918[6]	$0.58F_y$	27.5*	16	1.72
AISC Specs., 1923, 1928	$0.545F_y$	33	18	1.83
AISC Specs., 1936, 1949	$0.606F_y$	33	20	1.65
AISC Specs., 1963, 1969	$0.6F_y$	36	21.7	1.67

*Based on the ASTM specified value of one-half the tensile strength.

Table 2. Dates of AISC Specification Revisions

First adoption: June 1, 1923	Fifth revision: April 17, 1963
First revision: Nov. 1, 1928	Sixth revision: Feb. 12, 1969
Second adoption: June, 1936	Supplement No. 1: Nov. 1, 1970
Second revision: June, 1937	Supplement No. 2: Dec. 8, 1971
Third revision: June, 1949	Supplement No. 3: June 12, 1974
Fourth revision: Nov. 30, 1961	

unbraced beam. Contemporary section tables were used in these examples. These examples illustrate that, at least for this problem, the required weight of steel in 1976 is about one-half of that required in 1923 to carry the same load. There are many reasons for this: a smaller factor of safety, wider range of available sections, permission to use plastic design and a greatly improved way of handling lateral-torsional buckling.

Some of the significant changes were as follows:

1. The old Rankine-Gordon formula for lateral-torsional buckling was replaced in 1949 by the much improved and more rational de Vries formula,[3] and this was augmented partially in 1961 and fully in 1969 by a double formula with the C_b factor, the latter recognizing the effect of moment gradient.

2. Partial moment redistribution due to plastification at negative moment regions was permitted in 1949, thus making the design of continuous beams more efficient. This was further improved in 1961 when a flexural stress of $F_b = 0.66F_y$ was permitted for "compact" beams, and the original "compactness" requirements (i.e., limiting unbraced lengths and flange and web width-thickness ratio limits) were further relaxed in 1974. Also, plastic design was introduced in Part 2 of the AISC Specification in 1961. In fact, "compact" beam design in the allowable stress method and plastic design are in direct competition, resulting in the unfortunate fact that the more rational plastic design method is not widely used.

3. Fatigue provisions in the 1974 AISC Specification permit a much more creative use of details, providing a great improvement over previous methods.

4. Plate girder design changed little from 1923 to 1961, except for ever changing stiffener spacing formulas, the change from net to gross section moment of inertia in 1936, and the introduction of web crippling rules in 1936. Major changes occurred in 1961, when web buckling in flexure was permitted and shear design was governed by tension field action. Hybrid beam design was introduced in 1969.

Table 3. Lateral Buckling Provisions in AISC Specifications

1923	Rankine-Gordon Formula, $L_b/b_f \leqslant 40$
1936	Same as 1923
1949	De Vries Formula, using $L_b d/A_f$
1961	$F_b = 0.66F_y$ for "compact" shapes Dual formulas (partially) for F_b for unbraced beams Partial recognition of effect of moment gradient (C_b) Plastic design
1969	Dual formulas and moment gradient effect
1974	Liberalization of compactness requirements Box beams

Table 4. Flange and Web Width-Thickness Requirements in AISC Specification

	Non-Compact Shapes		Compact Shapes	
Year	Flange*	Web*	Flange*	Web*
1923	12	60	N.A.	N.A.
1936	12	70	N.A.	N.A.
1949	16**	70	N.A.	N.A.
1961	$95/\sqrt{F_y}$**	None	$50.6/\sqrt{F_y}$	$421/\sqrt{F_y}$
1969	$95/\sqrt{F_y}$***	None	$52.2/\sqrt{F_y}$	$412/\sqrt{F_y}$
1974	$95/\sqrt{F_y}$***	None	$65/\sqrt{F_y}$	$640/\sqrt{F_y}$

* $b_f/2t_f$ for flanges and d/t for webs.

** May be exceeded if stress on section with excess area removed is adequate.

*** May be exceeded if reduction formulas in Appendix C are used

Table 5. Moment Redistribution for Continuous Beams in AISC Specification

1923	Not permitted
1936	Not permitted
1949	20% increase of F_b to 0.72 F_y in all cases
1961, 1969	10% increase of negative moment, $F_b = 0.66F_y$, for "compact" shapes only.

Table 6. Fatigue Provisions in AISC Specifications

1923, 1936	$F_b \geqslant f_{max} - 0.5 f_{min}$
1949	$F_b \leqslant f_{max}$; i.e., no reduction for members, but fatigue considerations required for connections
1961	$n < 10{,}000$ cycles: no effect $10{,}000 < n < 100{,}000$ cycles: $F_b \geqslant f_{max} - \frac{2}{3} f_{min}$ $100{,}000 < n < 2{,}000{,}000$ cycles: F_b for A7 Steel $\geqslant f_{max} - \frac{2}{3} f_{min}$ $n > 2{,}000{,}000$ cycles: $\frac{2}{3} F_b$ for A7 Steel $\geqslant f_{max} - \frac{2}{3} f_{min}$
1969	Stress range, life, stress-category (detail) considerations

Table 7. Plate Girder Provisions in AISC Specifications

1923	Riveted girders only; stiffener spacing based on shear buckling; $h/t \leqslant 160$; I of net section
1936	Riveted girders only; stiffener spacing based on shear buckling; $h/t \leqslant 170$; I of gross section; web crippling rules
1949	Riveted and welded girders, different web stiffener spacing rules, also based on web buckling
1961	Riveted and welded girders, I of gross section except when holes exceed 15% of flange area. Flexure: web buckling permitted. Shear: tension field action permitted.
1969	Same as 1961, but hybrid girders also permitted.

The greatest change in the AISC Specification occurred in 1961. This revision was the culmination of the accelerated postwar research effort which radically changed design procedures in all fields of structural engineering. It was during this time that the concepts of plastic analysis and design, which were theoretically refined after World War II at Brown University, were experimentally tested out and practically implemented by the research at Lehigh University.[14]

For the design of beams, this meant that the conditions under which a plastic mechanism could be achieved were defined: type of steel, lateral bracing spacing, flange and web width-thickness ratios, etc. Another major accomplishment of this period was the research of Basler and Thurlimann at Lehigh on plate girders.[1] The idea of the limit state of elastic plate buckling was abandoned, and it was convincingly demonstrated that one could count on the postbuckling strength of the plate girder web both in flexure and in shear. Three postbuckling strength concepts were in use earlier in aircraft design and they were successfully championed for cold-formed members by Winter, who established well documented formulas for the effective width of buckled plates.[15]

This period also saw increased activity in derivations and computations on many additional lateral buckling problems (expanding on earlier work of Timoshenko[12] and Winter[13]), notably the work of Salvadori,[9] Clark and Hill,[2] and others. The results of this work were summarized in the first edition of the Column Research Council's *Guide to Design Criteria for Metal Compression Members,*[10] which appeared in 1960.

The results of this research work were strongly reflected in the 1961 adoption of the AISC Specification, and in later adoptions of the design specifications for cold-formed members and highway and railroad bridges. Not only were beam design criteria refined and made more rational, but the scope of applicability was expanded greatly.

A CRITIQUE AND A LOOK TO THE FUTURE

The AISC Specification has been greatly expanded in the 43 years of its existence. It has become quite sophisticated, its scope includes rather complex structural systems, and it results in efficient and acceptably safe and economic structures. While some may wish a return to the simple old days, this is not possible without a great sacrifice of material. In the future, human resources are likely to become more plentiful than material resources. There is thus no way back. However, improvements are in order, because there are ways to achieve them, thus making this complex current version of the AISC Specification more understandable.

The following suggestions for improvement and for further research are presented:

1. The present limiting unbraced lengths and the flange and web width-thickness ratio limits for plastic design are conservative, and research should be performed to establish a basis for their liberalization.
2. The present treatment of the lateral-torsional buckling problem through the dual formulas in Sect. 1.5.1.4.6a of the 1969 AISC Specification is inconsistent and needlessly confusing. Using available knowledge,[5,10] a simpler and more rational treatment is possible.
3. Compact beam and composite beam design, while based clearly on the principles of plastic behavior, are currently couched in a pseudo-allowable-stress design. A more consistent, more rational, and more economical design would emerge if these members were explicitly designed for their plastic capacity.

Three possible courses for future development are put forward here as suggestions:

1. The AISC Specification, by its very nature, has to be inclusive of all the types of steel building construction, and thus it is general, extensive, and complex. Yet many designers concern themselves most of the time with simple structures. A simplified and restrictive version of the AISC Specification could be evolved for such uses by abstracting out and simplifying the relevant provisions. This is especially applicable to beam design.
2. The AISC Specification in its present form is a patchwork of many revisions and supplements. As a result, it is difficult for an occasional user to find all relevant provisions for a given design assignment. It would be good if the document could be restructured along a rational decision theory model, so that omissions on the one hand and needless work on the other hand are avoided. Such a restructuring model is in existence and could well be used for rebuilding the AISC Specification.[7]
3. Research on probability-based design criteria has shown that rational and consistent Load and Resistance Factor design criteria can be developed. Such criteria have been developed for steel building structures,[8] but these are keyed to the present AISC Specification and therefore can be used only in tandem. It would be desirable if a complete set of design criteria, fully integrated under the Load and Resistance Factor Design model, could be developed in the future.

CONCLUSION

An attempt has been made in this paper to briefly trace the history of steel beam design and to enumerate some possible future directions. The example used was the AISC Specification, but any other specification could have been utilized as well. The story would have been similar. From simple beginnings, more and more extensive and complex documents resulted. At this time all of the major structural steel specifications suffer from too many not too carefully integrated revisions, and perhaps one should give thought not to new revisions, but to a brand new start.

The road from the beginning of large scale steel construction since the latter part of the 19th Century was challenging and very interesting. The future, while difficult and challenging, appears to be one of excitement, giving opportunity for many creative ventures.

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APPENDIX I—NOTATION

A = Area of cross section
b_f = Flange width
C = Constant in Rankine-Gordon formula
c = Distance from neutral axis to extreme fiber
C_b = Equivalent moment factor
C_c = Column slenderness factor
C_w = Warping constant
d = Beam depth
E = Modulus of elasticity
e = Eccentricity
$F.S.$ = Factor of safety
F_b = Allowable flexural stress
F_{cr} = Critical stress
F_v = Allowable shear stress
F_u = Tensile strength
F_y = Yield stress
f_b = Flexural stress
G = Shear modulus
h = Web depth
I = Moment of inertia
J = Torsion constant
K = Effective length factor
L = Column length
L_b = Unbraced length
M = Moment
M_p = Plastic moment
P = Axial force
r = Radius of gyration
S = Elastic section modulus
t = Web thickness
Z = Plastic section modulus
δ = Deflection
ϕ = Curvature
σ_{cr} = Critical stress

APPENDIX II—DERIVATION OF RANKINE-GORDON FORMULA

Even though the first theoretical solutions for the elastic lateral buckling of beams (Pradtl, Michelle, Timoshenko, all in the first decade of the 20th Century) were not available to them, 19th Century engineers were very much aware of the problem, and they devised an effective way to deal with it: they turned to the Rankine-Gordon formula. Timoshenko in his *History of the Strength of Materials*[11] describes the history of this formula. The name Rankine-Gordon stems from the fact that it was taught by Rankine in his text *Manual of Civil Engineering* (first published in 1861), and Rankine states that is is a formula revived by Gordon from an earlier form suggested by Tredgold. It is essentially a variation of the secant formula. The Rankine-Gordon formula derivation here follows DuBois,[4] who used the approach found in Rankine's book.

Let a column of length $2L$ be subjected at its pinned ends to an eccentric force P, and let that eccentricity be equal to e at both ends. The column will deform in a single curvature mode, and its maximum moment will be at the center.

$$M_{max} = P(\delta + e) = \frac{EI}{\phi} \tag{5}$$

where δ is the center deflection, EI is the flexural stiffness, and ϕ is the curvature at the column center. Assuming that the column is a segment of a circle, $\phi = L^2/2\delta$. Substitution of ϕ into Eq. (5) yields

$$\delta = \frac{Pe}{2EI - PL^2} \tag{6}$$

The assumed limit state is the attainment of the yield stress due to axial stress and bending stress at the center:

$$F_y = \frac{P}{A} + \frac{M_{max}c}{I} \quad (7)$$

$$F_yA = P\left[1 + \frac{c}{I}(\delta + e)\right] \quad (8)$$

Substituting Eq. (6) into Eq. (8) and rearranging:

$$F_yA[2EI - PL^2] = P[2EI + 2AEec - PL^2] \quad (9)$$

DuBois then states that $PL^2 \ll 2EI + 2EAec$ in the right bracket, and so it can be neglected. Thus,

$$2F_yAEI = P[2EI + 2AEec + F_yAL^2] \quad (10)$$

from which, for an axially loaded column ($e = 0$),

$$\frac{P}{A} = F_{cr} = \frac{F_y}{1 + \frac{F_y}{2E}\left(\frac{L}{r}\right)^2} \quad (11)$$

The term $F_y/2E$ was then replaced with a constant which depended on the material properties, the cross section, and the end conditions, and it also contained adjustments from appropriate test results. In terms of allowable stress, the Rankine-Gordon formula can be written in the form

$$F_a = \frac{F_y/F.S.}{1 + C\left(\frac{L}{r}\right)^2} \quad (12)$$

This formula was used from the 1840's on for about one hundred years in many codes and specifications for many applications. It was a very popular equation for axially loaded columns, although it had to share its popularity with various straight-line and parabolic formulas. For example, Ketchum (1918)[6] recommends a straight line column formula, while the 1923 and the 1928 AISC Specifications use the Rankine-Gordon formula. The 1936 and the 1949 adoptions of this specification use both a parabolic formula (for $L/r \leq 120$) and the Rankine-Gordon formula (for $L/r > 120$). It disappears completely only in the next change (1961) of the AISC Specification.

APPENDIX III—DESIGN EXAMPLES

Statement of Problem—Select a uniform two-span non-composite continuous beam with two equal spans of 30 ft each. This beam is subjected to uniformly distributed loading as follows:

1. 1.5 kips/ft service load; beam is fully braced between supports
2. 0.5 kips/ft construction load; beam is unbraced between supports

Design beams for both loading conditions, using the 1923, 1936, 1949, 1963, and 1969 AISC Specifications with contemporary available sections. Consider only flexure. Do not use beams with less than 15 in. depth.

Case 1: Braced Beam Under Service Loading, Full Bracing

Maximum elastic negative moment:

$$M_{max}^{(-)} = \frac{wL^2}{8} = \frac{1.5 \times 30^2 \times 12}{8} = 2025 \text{ kip-in.}$$

Maximum elastic positive moment:

$$M_{max}^{(+)} = \frac{9wL^2}{128} = \frac{9 \times 1.5 \times 30^2 \times 12}{128} = 1139 \text{ kip-in.}$$

1923 AISC Specification:

$F_b = 18$ ksi; $S_{req'd} = 2025/18 = 112.5$ in.^{3}

From *Carnegie Pocket Companion* of 1923:

Select. **20I65.4,** $S = 116.9$ in.^{3}

1923 AISC Specification, 1930 AISC Manual:

$F_b = 18$ ksi; $S_{req'd} = 112.5$ in.^{3}

Select **22B54.5** Bethlehem beam, $S = 113.34$ in.^{3}

1936 AISC Specification, 1937 AISC Manual:

$F_b = 20$ ksi; $S_{req'd} = 101.2$ in.^{3}

Select **21WF59,** $S = 119.3$ in.^{3}

1949 AISC Specification, 1959 AISC Manual:

At support:

$F_b = 1.2 \times 20 = 24$ ksi; $S_{req'd} = 84.38$ in.^{3}

In the span:

$$F_b = 20 \text{ ksi}; \quad S_{req'd} = \frac{1139}{20} = 56.95 \text{ in.}^3$$

Select **18WF50,** $S = 89.0$ in.^{3}

1963 AISC Specification, 1963 AISC Manual:

Allowable Stress Design:

At support:

$F_b = 0.66F_y = 0.66 \times 36 = 23.76$ ksi

$M = 0.9 \times 2025 = 1822$ kip-in.

$$S_{req'd} = \frac{1822}{23.76} = 76.7 \text{ in.}^3$$

In the span:

$$M = 1139 + \frac{2025 + 0}{2 \times 10} = 1240 < 1822 \text{ kip-in.}$$

Select **18WF45,** $S = 78.9$ in.^{3}

Plastic Design:

$$N_{p(req'd)} = \frac{1.7wL^2}{11.66} = \frac{1.7 \times 1.5 \times 30^2 \times 12}{11.66}$$

$$= 262 \text{ kip-in.}$$

$Z_{(req'd)} = 2362/36 = 65.6$ in.^{3}

Select **16WF40**, $Z = 72.7$ in.^{3}

1969 and 1970 AISC Specifications:

Allowable Stress Design:

$S_{req'd} = 76.7$ in.^{3}

Select **W21X44**, $S = 81.6$ in.^{3}

Plastic Design:

$Z_{(req'd)} = 65.6$ in.^{3}

Select **W18X35**, $Z = 66.8$ in.^{3}

Case 2: Unbraced Beam Under Construction Loading

Maximum elastic negative moment: 675 kip-in.
Maximum elastic positive moment: 380 kip-in.
Unbraced length: 30 ft = 360 in.

1923 AISC Specification:

Minimum flange width: $b_f \geq \frac{L_b}{40} = \frac{360}{40} = 9$ in.

From Carnegie Pocket Companion of 1923:

Select **27I90**, $b_f = 9$ in.

$$F_b = \frac{20}{1 + \frac{360^2}{2000 \times 9^2}} = 11.11 \text{ ksi} > 3.08 \text{ ksi}$$

1923 AISC Specification, 1930 AISC Manual:

$b_f \geq 9$ in.; $f_b \geq 11.11$ ksi

Select **24B70**, Bethlehem beam, $b_f = 9.00$ in.

$f_b = 4.12$ ksi < 11.11 ksi

1936 AISC Specification, 1937 AISC Manual:

$b_f \geq 9$ in.

Select **24WF74**, $b_f = 8.975$ in.

$f_b = 3.96$ ksi

$$F_b = \frac{22.5}{1 + \frac{360^2}{1800 \times 9.975^2}} = 11.88 \text{ ksi} > 3.96 \text{ ksi}$$

1949 AISC Specification, 1959 AISC Manual:

Select **W18X50**

$$f_b = \frac{675}{89.0} = 7.58 \text{ ksi}$$

$$F_b = \frac{12{,}000}{L_b d/bt} = \frac{12{,}000}{(360 \times 18)/(7.5 \times 0.57)} = 7.92 \text{ ksi}$$

1963 AISC Specification and Manual:

Select **W18X50**

$f_b = 7.58$ ksi; $F_{b1} = 7.92$ ksi

$$F_{b2} = 0.6F_y \left[1 - \frac{(L_b/r)^2}{2C_c^2 C_b}\right]$$

$r = 1.96$ in.; $C_c = 126.1$; $C_b = 1.0$

$F_{b2} < 0$, will not control.

1969 AISC Specification and Manual:

The same beam will result if the formulas given in the Specification are used directly. However, it is permitted to use a "more precise analysis." This will be tried here, using the Third Edition of the SSRC Guide.[10]

Eq. 6.9a in the SSRC Guide.

$$\sigma_{cr} = \frac{C_b \pi \sqrt{EI_y GJ}}{S_x(KL)} \sqrt{1 + \frac{\pi^2 EC_w}{GJ(KL)^2}}$$

Assume conservatively that the effective length $K = 1.0$ and $C_b = 1.0$.

Check a **W18X45** section:

$E = 29{,}000$ ksi; $I_y = 34.8$ in.^{4}
$G = 0.385\,E$; $C_w = 2620$ in.^{6}
$S = 79.0$ in.^{3}; $J = 0.889$ in.^{4}

$\sigma_{cr} = 13.91$ ksi

$$f_b = \frac{675}{79.0} = 8.54 \text{ ksi}$$

$\sigma_{cr}/f_b = 1.63$, an adequate margin