Floor Vibrations and Cantilevered Construction

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A previous paper by the senior author presented a design method to estimate the acceptability of a proposed floor system from the standpoint of occupant induced floor vibrations.[1] The procedures developed therein are applicable only to steel beam-concrete slab floor systems where the beams can be considered simply supported at each end. However, some of the most severe vibration problems have occurred in construction involving free cantilevers, to which the proposed method of Ref. 1 is not applicable. The purpose of this paper is to present modifications to the method suggested in Ref. 1, so that cantilever floor systems and systems with overhanging beams can be analyzed for annoying floor vibrations. The modifications were verified by tests on seven floor systems reported in Ref. 2. One of the tested systems is used to demonstrate the suggested analysis procedure.

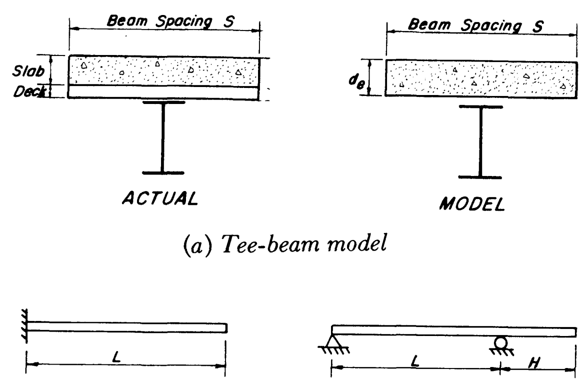
DESIGN PROCEDURE

Human sensitivity to vibration has been shown to depend on three parameters: frequency, amplitude, and damping. Scales relating these parameters to human reaction have been developed. The modified Reiher-Meister and the Wiss-Parmelee scales are probably the most suitable for occupant induced floor vibration analysis (see Ref. 1 for a complete discussion). The use of either scale requires an estimate of the frequency and amplitude for a specified impact. In addition, an estimate of the critical damping is required. The following sections describe methods that can be used to estimate the three parameters for cantilever and overhanging floor systems. For brevity, both types of systems will be referred to as "cantilevered" floor systems in the following discussion.

Damping—The damping in a cantilevered floor system can be estimated as the sum of the damping of the separate elements in the system. From the guidelines suggested for floor systems supported by simply supported beams: bare floor, 1%–3% (lower limit for thin slab of lightweight concrete, upper limit for thick slab of normal weight concrete); ceiling, 1%–3% (lower limit for hung ceiling, upper limit for sheetrock or furring attached to beams); ductwork and mechanical, 1%–10%, depending on amount; partitions, 10%–20%, if attached to the floor and spaced not more than every five floor beams. These values were originally suggested in Ref. 1 and are based on observation only, not on the results of a systematic study.

Frequency—From test results presented in Ref. 2, the frequency of a cantilevered system can be estimated using a single tee-beam, if the transformed moment of inertia is computed assuming:

(a) Composite action, regardless of the method of construction.
(b) An effective slab width, S, equal to the sum of half the distance to adjacent beams.
(c) An effective slab depth, d_e, based on an equivalent slab of rectangular cross section, is equal in weight to the actual slab including concrete in the valleys of decking and the weight of decking.



(a) *Tee-beam model*

(b) *Cantilever beam*

(c) *Overhanging beam*

Fig. 1. Analytical models

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Figure 1(a) shows the tee-beam model for computing the transformed moment of inertia.

The first natural frequency of a cantilevered tee-beam, Fig. 1(b), is given by:

$$f_b = 1.875 \left[\frac{gEI_t}{WL^3} \right]^{1/2} \tag{1}$$

where

g = 386 in./sec^2
E = modulus of elasticity, psi
I_t = transformed moment of inertia, in.^{4}
W = total weight supported by the tee-beam, lbs
L = length of cantilever, in.

The first natural frequency of a simply supported beam with one overhanging end, Fig. 1(c), is given by:

$$f_b = K \left[\frac{gEI_t}{WL^3} \right]^{1/2} \tag{2}$$

where g, E, I_t, and W are as defined previously, L = backspan length, in., and K is a coefficient which depends on the overhanging length to backspan ratio, H/L. The coefficient K is determined by setting the determinant of the coefficient matrix of the boundary condition equations equal to zero. A closed form solution for K is not possible and values for specific H/L ratios were obtained numerically. The value of the coefficient K can be determined from Fig. 2.

Equations (1) and (2) were derived for free lateral vibration of prismatic, straight, elastic beams considering bending deformations only.

In practice, overhanging beams are usually supported by flexural members rather than rigid supports. The flexibility of these members can significantly affect the frequency of the floor system. In Ref. 2, it is shown that the system frequency for such cases can be approximated by:

$$\frac{1}{f_s^2} = \frac{1}{f_b^2} + \frac{1}{f_g^2} \tag{3}$$

where

f_s = the system frequency, Hz
f_b = overhanging beam frequency, Hz

$$f_g = 1.57 \left[\frac{gEI_g}{W_g L_g^3} \right]^{1/2} \tag{4}$$

where

W_g = total supported weight, lbs
I_g = girder moment of inertia, in.^{4}
L_g = girder span, in.

In the computation of I_g, composite action should not be assumed unless the slab or deck rests directly on the girder flange. The effective slab width should be estimated as for normal composite construction, even if shear connections are not used.

Table 1. Dynamic Load Factors for Heel-Drop Impact

f, Hz	*DLF*	*f*, Hz	*DLF*	*f*, Hz	*DLF*
1.00	0.1541	5.50	0.7819	10.00	1.1770
1.10	0.1695	5.60	0.7937	10.10	1.1831
1.20	0.1847	5.70	0.8053	10.20	1.1891
1.30	0.2000	5.80	0.8168	10.30	1.1949
1.40	0.2152	5.90	0.8282	10.40	1.2007
1.50	0.2304	6.00	0.8394	10.50	1.2065
1.60	0.2456	6.10	0.8505	10.60	1.2121
1.70	0.2607	6.20	0.8615	10.70	1.2177
1.80	0.2758	6.30	0.8723	10.80	1.2231
1.90	0.2908	6.40	0.8830	10.90	1.2285
2.00	0.3058	6.50	0.8936	11.00	1.2339
2.10	0.3207	6.60	0.9040	11.10	1.2391
2.20	0.3356	6.70	0.9143	11.20	1.2443
2.30	0.3504	6.80	0.9244	11.30	1.2494
2.40	0.3651	6.90	0.9344	11.40	1.2545
2.50	0.3798	7.00	0.9443	11.50	1.2594
2.60	0.3945	7.10	0.9540	11.60	1.2643
2.70	0.4091	7.20	0.9635	11.70	1.2692
2.80	0.4236	7.30	0.9729	11.80	1.2740
2.90	0.4380	7.40	0.9821	11.90	1.2787
3.00	0.4524	7.50	0.9912	12.00	1.2834
3.10	0.4667	7.60	1.0002	12.10	1.2879
3.20	0.4809	7.70	1.0090	12.20	1.2925
3.30	0.4950	7.80	1.0176	12.30	1.2970
3.40	0.5091	7.90	1.0261	12.40	1.3014
3.50	0.5231	8.00	1.0345	12.50	1.3058
3.60	0.5369	8.10	1.0428	12.60	1.3101
3.70	0.5507	8.20	1.0509	12.70	1.3143
3.80	0.5645	8.30	1.0588	12.80	1.3185
3.90	0.5781	8.40	1.0667	12.90	1.3227
4.00	0.5916	8.50	1.0744	13.00	1.3268
4.10	0.6050	8.60	1.0820	13.10	1.3308
4.20	0.6184	8.70	1.0895	13.20	1.3348
4.30	0.6316	8.80	1.0969	13.30	1.3388
4.40	0.6448	8.90	1.1041	13.40	1.3427
4.50	0.6578	9.00	1.1113	13.50	1.3466
4.60	0.6707	9.10	1.1183	13.60	1.3504
4.70	0.6835	9.20	1.1252	13.70	1.3541
4.80	0.6962	9.30	1.1321	13.80	1.3579
4.90	0.7088	9.40	1.1388	13.90	1.3615
5.00	0.7213	9.50	1.1454	14.00	1.3652
5.10	0.7337	9.60	1.1519	14.10	1.3688
5.20	0.7459	9.70	1.1583	14.20	1.3723
5.30	0.7580	9.80	1.1647	14.30	1.3758
5.40	0.7700	9.90	1.1709	14.40	1.3793

Amplitude—The "heel-drop" impact has been used to develop acceptability criteria when the modified Reiher-Meister scale is used. The amplitude of a single tee-beam subjected to a heel-drop impact can be computed from:

$$A_{ot} = (DLF)_{max} \, \Delta_s \tag{5}$$

where

A_{ot} = amplitude
$(DLF)_{max}$ = maximum dynamic load factor
Δ_s = static deflection caused by a 600-lb force

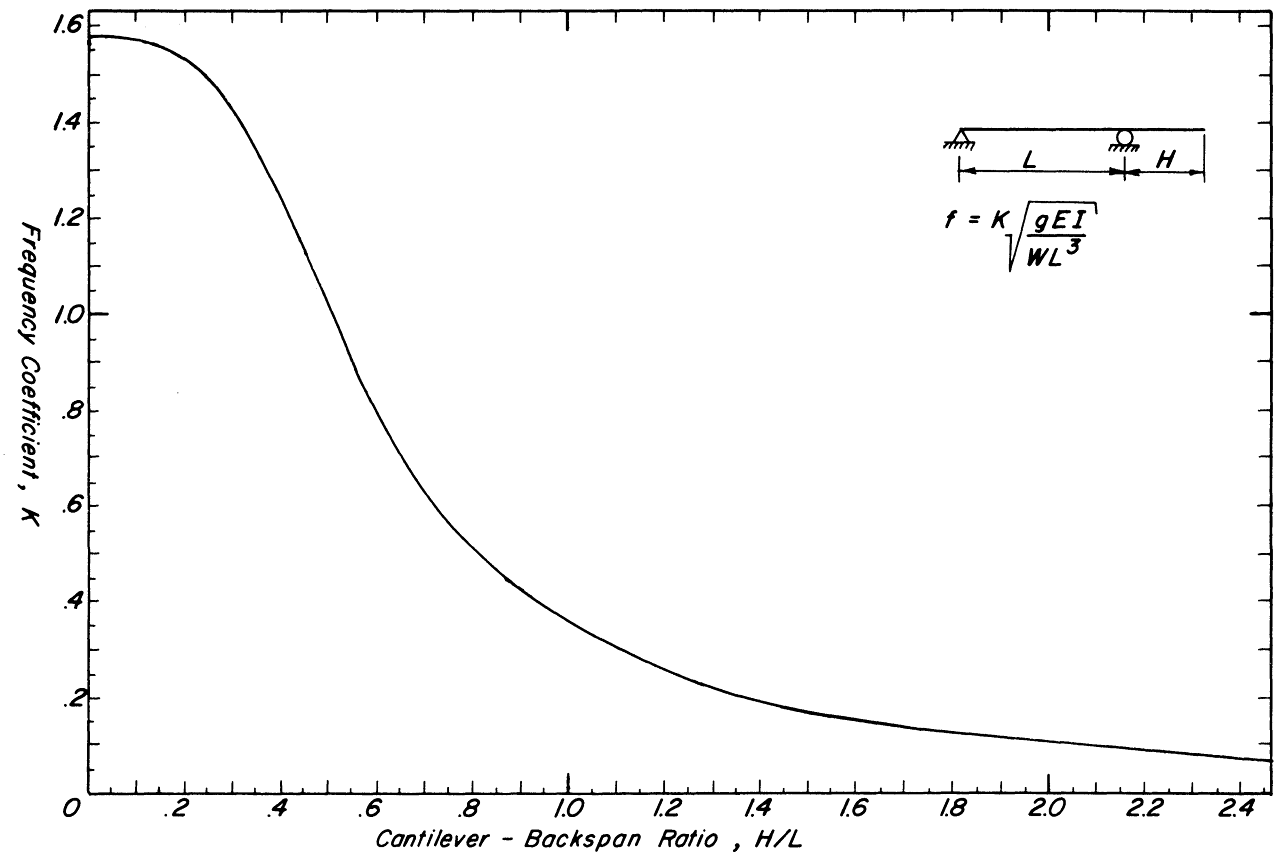


Fig. 2. Frequency coefficients for overhanging beams

Therefore, for a cantilever tee-beam,

$$A_{ot} = (DLF)_{max} \times \frac{600L^3}{3EI_t} \tag{6}$$

and for an overhanging tee-beam,

$$A_{ot} = (DLF)_{max} \times \frac{600H^2(L + H)}{3EI_t} \tag{7}$$

Equations for $(DLF)_{max}$ are given in Ref. 1 and values of $(DLF)_{max}$ are given in Table 1.

Usually more than one tee-beam is effective in resisting an impact. The first maximum amplitude of a floor system can be estimated from:

$$A_o = A_{ot}/N_{eff} \tag{8}$$

where N_{eff} = number of effective tee-beams. For a series of tee-beams with equal effective flange width and simple supports, it was shown in Ref. 3 that:

$$N_{eff} = 2.967 - 0.05776(S/d_e) + 2.556 \times 10^{-8}(L^4/I_t) + 0.00010(L/S)^3 \tag{9}$$

Unless L/S is very large, i.e., greater than 10, N_{eff} can be approximated from:

$$N_{eff} = 2.97 - \frac{S}{17.3d_e} + \frac{L^4}{1.35EI_t} \tag{10}$$

where $E = 29 \times 10^6$ psi and S, d_e, L, I_t are in inch units. As a design approximation, it is assumed here that the number of effective tee-beams for cantilevered construction is the same as for simple supports and no overhang.

Equation (9) was developed assuming at least five identical tee-beams exist and the impact location is at the center of the five beams. Frequently, the framing for cantilevered balconies is irregular and Eq. (9) cannot be used. For such cases, it is suggested that a static finite element analysis be used to determine A_o. A computer program such as STRUDL[4] can be used by dividing the slab into a mesh and treating the beams as line elements with a moment of inertia equal to the transformed moment of inertia, and determining the maximum static deflection caused by a 600-lb concentrated load. In lieu of a finite element analysis, the designer may conservatively take $N_{eff} = 1$.

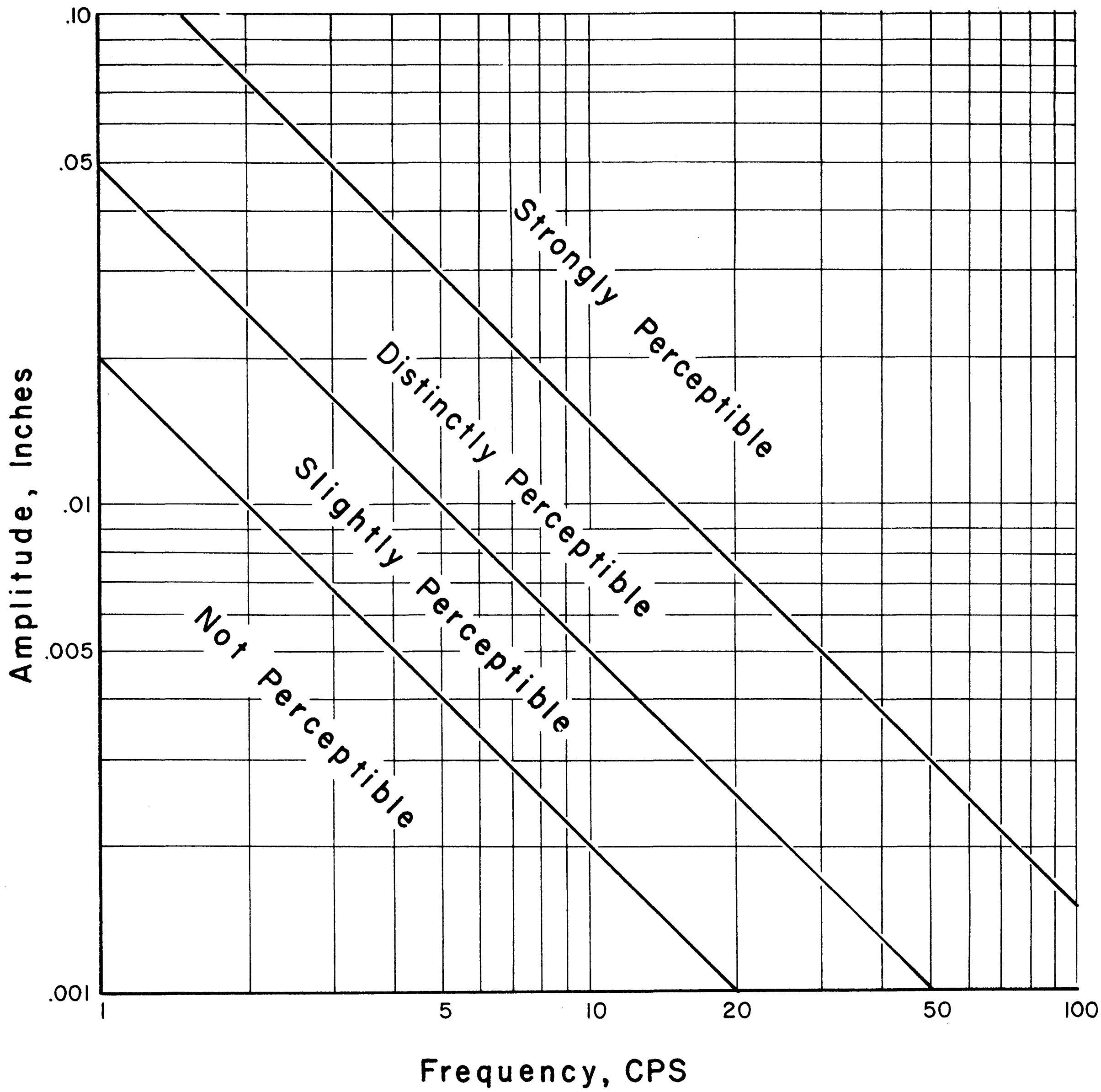
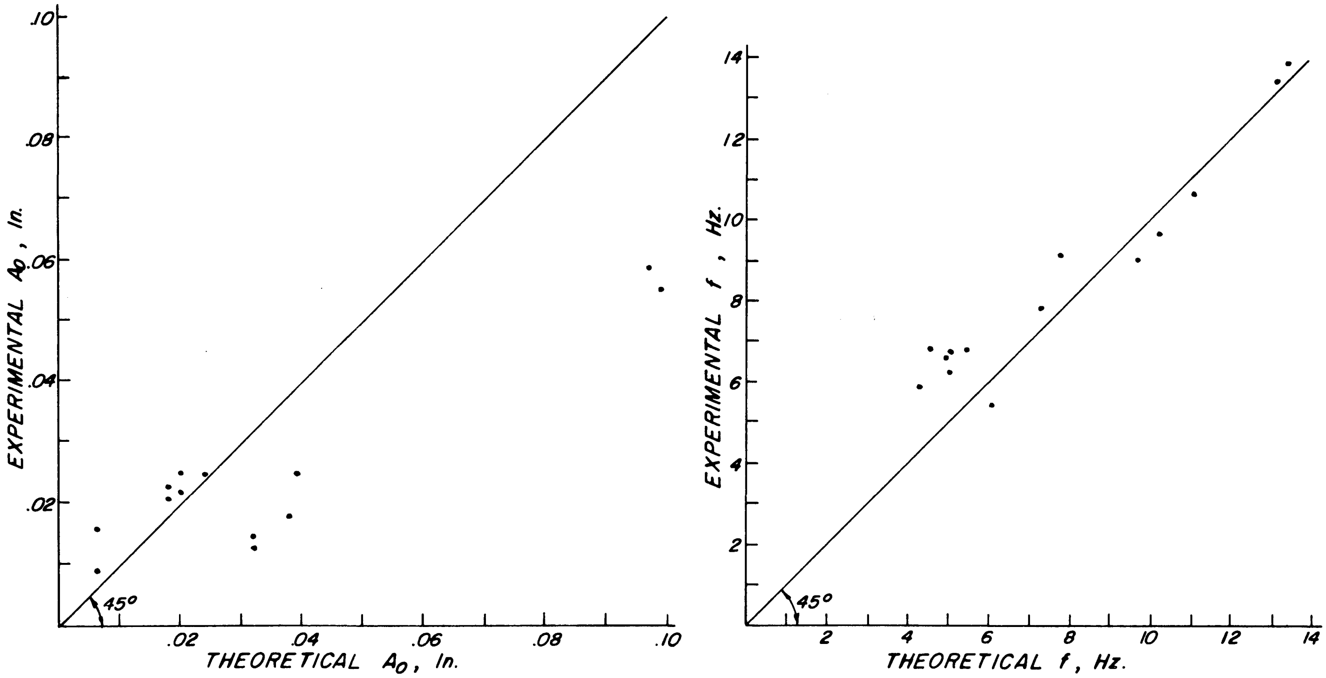


Fig. 3. Modified Reiher-Meister scale

Proposed Design Method

1. Estimate the damping in the finished floor system; if greater than 8%–10% there is no need for a vibration analysis.
2. Compute the transformed moment of inertia of a single tee-beam, I_t, using the guidelines presented.
3. Compute the frequency from Eqs. (1) or (2) and (3), as applicable.
4. Compute the heel-drop amplitude of a single tee-beam, A_{ot}, using Eq. (6) or (7) and Table 1.
5. If the effective slab widths are equal, estimate the number of effective tee-beams, N_{eff}, using Eq. (10); otherwise, perform a static finite element analysis or conservatively use $N_{eff} = 1$.
6. Compute the amplitude of the floor system, using $A_o = A_{ot}/N_{eff}$.
7. Estimate perceptibility, using the modified Reiher-Meister scale, Fig. 3.
8. If the system plots below the lower half of the distinctly perceptible range, the system is satisfactory if the damping is less than 3%–4%. If the system plots in the upper half of the distinctly perceptible range and the damping is relatively low, less than 6%–8%, complaints from occupants may occur. If the system plots above the distinctly perceptible range, the system will be unacceptable if the damping is less than 10%–12%.



(*a*) *Initial amplitude* (*b*) *Frequency*

Fig. 4. Comparison of theoretical and experimental results

EXPERIMENTAL VERIFICATION

Tests were conducted in seven buildings at a total of 15 locations to verify the proposed design method.[2] Nine of the locations were on church balconies, three were on the upper level of a shopping mall, two were on the second floor exterior walkway of a motel, and one location was on the second floor exterior balcony of an office building. At each location the floor was impacted by an approximately 190-lb man executing a heel-drop. The resulting floor motion was recorded, together with timing lines, on light sensitive paper using an engineering seismograph. From the record, it was possible to determine the initial amplitude, frequency, and damping of the floor system.

Comparisons of predicted and measured frequencies and amplitudes are shown in Figs. 4(a) and 4(b), respectively. Considering there was no laboratory-type control of the floor system construction or tolerances, the results are considered to be excellent. Furthermore, some of the beams supporting the church balconies were not prismatic, varying in both section and slope to meet pew location and walkway requirements, and engineering judgment was used to obtain equivalent stiffness for use in the proposed design formulas.

EXAMPLE

Figure 5 shows structural details of a test location on the upper level of the shopping mall.[2] The floor is 1½-in. clay tile laid over a 2½-in. concrete slab on a steel deck. The slab is supported by W8 × 15 beams at 30 in. on center, which rest on the top flange of a W27 × 94. The beams are assumed to act compositely with the concrete slab and the clay tile for transformed moment of inertia calculations. Composite action is not assumed for the girder.

Damping:	Slab and beam	2%
	Soffit	3
		5% < 8%

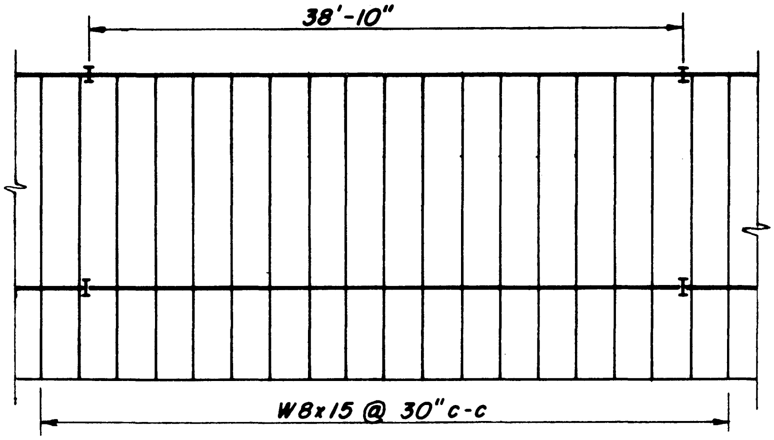
∴ Need to investigate floor system.

Beam Transformed Section Properties (see Fig. 5):

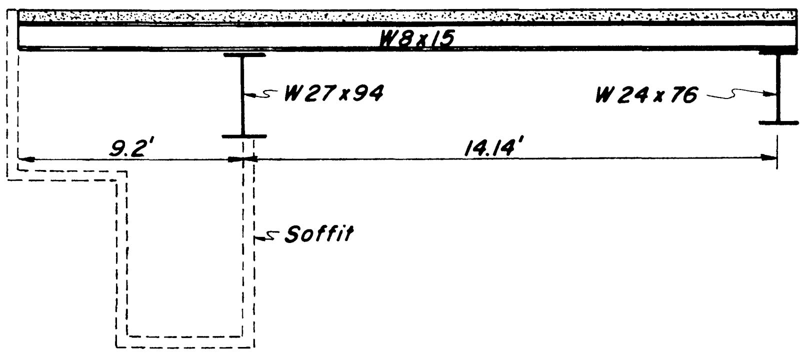
$d_e = 3.5$ in.; $n = 7.6$

$$\frac{Ac}{n} = \frac{30(3.5)}{7.6} = 13.82 \text{ in.}^2$$

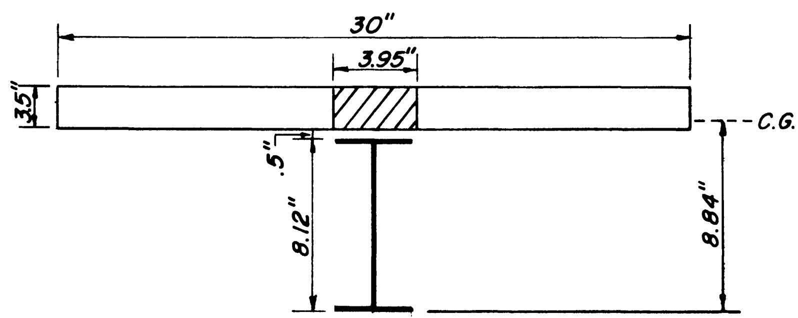
W8 × 15: $A = 4.43$ in.^{2}; $I = 48.1$ in.^{4}; $d = 8.12$ in.



(*a*) *Framing plan*



(*b*) *Section*



(*c*) *Tee-beam model*

Fig. 5. Framing system for example

$$Y_b = \frac{4.43(8.12/2) + 13.82(8.12 + 0.5 + 1.75)}{4.43 + 13.82}$$

$$= 8.84 \text{ in.}$$

$$I_t = \frac{13.82(3.5)^2}{12} + 13.82(1.53)^2 + 48.1$$

$$+ 4.43\left(8.84 - \frac{8.12}{2}\right)^2$$

$$= 195.8 \text{ in.}^4$$

Beam Frequency:

Floor Weight:

Clay tile = 20 psf
Concrete = 150 pcf

$$W = \left[150\left(\frac{2}{12}\right) + 20\right](23.34)(2.5) + 23.34(15)$$

$$= 2976 \text{ lbs}$$

$$\frac{H}{L} = \frac{9.20}{14.14} = 0.65$$

From Fig. 2: $K = 0.72$

$$f_b = 0.72 \left[\frac{386(29 \times 10^6)(195.8)}{(2976)[14.14(12)]^3} \right]^{1/2}$$

$$= 8.84 \text{ Hz}$$

Girder Frequency:

W27 × 94: $I = 3270$ in.^{4}

$$W = \left[\frac{(23.34)^2}{2(14.14)} \left(\frac{2(150)}{12} + 20 + \frac{15}{2.5} \right) + 94 \right] 38.83$$

$$= 41{,}796 \text{ lbs}$$

$$f_g = 1.57 \left[\frac{386(29 \times 10^6)(3270)}{(41796)[38.83(12)]^3} \right]^{1/2}$$

$$= 4.62 \text{ Hz}$$

System Frequency:

$$\frac{1}{f_s^2} = \frac{1}{(4.62)^2} + \frac{1}{(8.84)^2} = 0.0596$$

$$f_s = 4.09 \text{ Hz}$$

Amplitude:

Neglect influence of girder.

$$\Delta_s = \frac{600[9.2(12)]^2(9.2 + 14.14)(12)}{3(29 \times 10^6)(195.8)} = 0.120 \text{ in.}$$

From Table 1, with $f_s = 4.09$ Hz:

$$(DLF)_{max} = 0.604$$

$$A_{ot} = 0.604(0.12) = 0.072 \text{ in.}$$

$$N_{eff} = 2.97 - \frac{30}{(17.3)(3.5)} + \frac{[14.14(12)]^4}{1.35(29 \times 10^6)(195.8)}$$

$$= 2.58$$

$$A_o = 0.072/2.58 = 0.028 \text{ in.}$$

Perceptibility:

With a frequency of 4.09 Hz and an initial amplitude of 0.028 in., the system plots in the upper third of the "distinctly perceptible" range on the modified Reiher-Meister scale, Fig. 3.

Field Measurements:

The system was measured before the soffit was completed:

Damping = 1.39 %

f = 4.72 Hz

A_o = 0.029 in.

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