Direct Feasible and Optimal Design of Laterally Unsupported Beams*

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WHEN A BEAM is laterally supported throughout its length, the axis of bending is prescribed, and most efficient use of material results if the moment of inertia around that axis is made as large as possible. The only structural restriction is that width-thickness ratios must not become so large as to cause local buckling; of course, architectural and clearance considerations may impose other limitations. Such beams may be rapidly and optimally designed in steel, using either the Section Modulus Tables in the AISC Manual[1] (pgs. 2-7 to 2-20) or the Allowable Uniform Load Tables (pgs. 2-28 to 2-81).

When a beam is laterally unsupported over some of its length further investigation is required, since the beam tends to become unstable and twist or buckle about its weak axis. If the unsupported length is small enough (L less than L_u), design proceeds using the same tables referred to above, although the allowable bending stress may need to be decreased from that allowed for compact sections to $0.60F_y$. Selecting the optimum is still relatively easy.

When the unsupported length exceeds L_u, however, the problem becomes more difficult. Theoretical solutions are much too involved for ordinary design office use, so the current AISC Specification[2] provides for this rather complex phenomenon by the provisions of Sect. 1.5.1.4.6a. In essence, these provisions reduce the allowable primary bending stress as the laterally unsupported length increases. Examination of the Specification reveals that an allowable stress must be computed from a column-like slenderness ratio (l/r_T) using two different formulas [(1.5-6a) and (1.5-6b)] in applicable ranges, a second allowable stress must be computed from a second ratio (Ld/A_f), and the larger of these stresses chosen, with an upper limit of $0.60F_y$. In all the formulas, a coefficient (C_b) is used, whose value depends upon the variation in compressive stress along the unbraced length caused by corresponding variation in the bending moment.

To assist the designer in working with the formulas, Stockwell[3] has shown that the formulas which contain l/r_T can be simplified in terms of a factor, considerably reducing the amount of calculation. Brandt[4] has produced a one-page chart that enables approximate solutions to all the formulas to be obtained with no calculations. Both these procedures, however, are useful only after a section has been chosen, or in what is usually termed "review".

When the problem addressed is "design", i.e., the selection of a satisfactory section, trial and error procedures are usually used, unless $C_b = 1$ and F_y is either 36 ksi or 50 ksi, in which case the Allowable Moment Charts in the AISC Manual (pgs. 2-87 to 2-105) can be used. Except in this latter case, optimization is largely a matter of perseverence and/or luck.

Table 1 (see Appendix) can be used for all values of C_b and all values of F_y directly to select feasible sections which are guaranteed to satisfy the Specification. The Table 1 also makes it possible to check whether or not the feasible section chosen is optimal and, if it is not, to replace it with the optimal one. Calculations are not extensive and the procedure is illustrated with examples.

DERIVATION OF EQUATIONS AND DESIGN PROCEDURE

A quick look at Table 1 reveals that it is a list in order of decreasing weight of all the sections commonly used as beams: W, S, HP, M, C, and MC. For each section, its customary designation (including its weight) is given, along with parameters r_T and S_x from the AISC Manual, and a derived parameter, X, which is the section modulus, S_x, divided by d/A_f.

All the parameters follow the weights only in a very general way, being largest for the heaviest sections, smallest for the lightest. An asterisk next to any parameter signifies that there is no larger value for this parameter below it in the list.

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* *In this paper the words "feasible" and "optimal" are used with their meanings from Operations Research. Thus a "feasible" choice of beam section satisfies all the constraints (allowable stress, depth limitation, etc.), and an "optimal" choice, in addition to satisfying all the constraints, also maximizes or minimizes an objective function, in this case achieving least weight.*

Stage I of the design operation is the selection of feasible sections, and consists of finding sections having two parameters that exceed minimum values. The first of these parameters is the section modulus S_x, which, if the bending stress is not to exceed $0.60F_y$, must not exceed the value

$$S_{min} = \frac{M}{0.60F_y} \tag{1}$$

The second parameter is based upon AISC Formula (1.5-7), $F_b = 12{,}000\ C_b/ld/A_f$. If this is used as the allowable stress, the resisting moment capacity can be written as

$$M = F_b S_x = \frac{12{,}000\ C_b A_f S_x}{ld} = \frac{12{,}000\ C_b X}{l}$$

which can be solved for

$$X = \frac{Ml}{12{,}000\ C_b}$$

Any section having X at least equal to this value can support M, and therefore:

$$X_{min} = \frac{Ml}{12{,}000\ C_b} \tag{2}$$

It is clear, then, that sections, selected from the Table 1, having both S_x greater than S_{min} and X greater than X_{min}, must be feasible, since the bending stress is less than $0.60F_y$ and also less than that allowed by Formula (1.5-7).

The selection of such sections from Table 1 is very simple. Since the values of S_x and X having asterisks are in numerical order, it is easy to find a suitable starting point in each of these columns and to scan upward for possible beam sections.

In many designs, the process may be stopped here, since the savings achieved by going to the next stage are not usually great.

Stage II, the check for optimality, is based on Eq. (1) and AISC Formulas (1.5-6a) and (1.5-6b).

Stockwell's observations concerning these formulas may be represented by the graph of Fig. 1. In this graph, it is obvious that the point of demarcation between the formulas may be expressed either by the abscissa (which yields $l/r_T = \sqrt{510 \times 10^3 C_b/F_y}$) or by the ordinate (which yields $F_b/F_y = 0.333$). In this paper, the ordinate representation is used.

Solving Formulas (1.5-6a) and (1.5-6b) for r_T yields the following:

When

$$\frac{F_b}{F_y} \geq \frac{1}{3}: \quad r_T = l\sqrt{\frac{F_y}{\left(\frac{2}{3} - \frac{F_b}{F_y}\right) 1530 \times 10^3 C_b}} \tag{3}$$

When

$$\frac{F_b}{F_y} < \frac{1}{3}: \quad r_T = l\sqrt{\frac{F_b}{170 \times 10^3 C_b}} \tag{4}$$

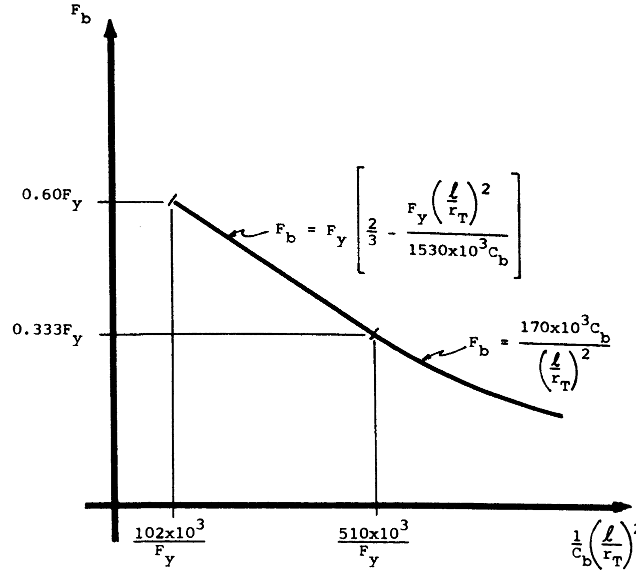

Figure 1

These values of r_T are the minimum values which a given section can have if it is to have a given allowable stress, F_b, when l, F_y, and C_b are also given. Call the appropriate one of these r_{TCR}.

Picking up the design procedure from Stage I, the designer selects a feasible section having section modulus S_1, and calculates $F_b = M/S_1$, then F_b/F_y and r_{TCR}. Returning to Table 1, any section above the feasible one is a heavier section and is rejected. Sections below the feasible one fall into four categories:

1. $r_T < r_{TCR}$ and $S_x < S_1$. These cannot work and are rejected. (Most sections fall into this category. When asterisked values of r_T and S_x in this category are reached, the downward scan can be terminated.)
2. $r_T > r_{TCR}$ and $S_x > S_1$. These are definitely satisfactory, and can replace the previous feasible selection. (Sections rarely fall into this category.)
3. $r_T > r_{TCR}$ and $S_x < S_1$.
4. $r_T < r_{TCR}$ and $S_x > S_1$.

Sections in categories 3 and 4 will not work if $S_x < S_{min}$. If $S_x > S_{min}$, these sections may or may not work and require calculation. Fortunately, there are not usually very many sections in these final categories, and if the calculations proceed from S_x to F_b to F_b/F_y to r_T, one calculation can frequently serve to test several sections.

If any satisfactory section is found from category 2, 3, or 4, this section can replace the feasible one. If not, the starting feasible one is the optimal beam.

It is simple enough for the designer to restrict his selections to include any other requirement, in addition to lowest weight consistent with that requirement. For instance, clearance problems may impose maximum depths; deflection considerations may impose minimum depths; sections not readily available may be excluded.

The following examples illustrate the procedure.

EXAMPLE 1

Given: A simple span of 40 ft, with lateral braces at ends and center, supporting uniform load of 1 kip/ft.

Find: The lightest satisfactory beam of A572 Grade 65 (F_y = 65 ksi) steel.

Solution:

Stage I:

Since $M_1/M_2 = 0$, $C_b = 1.75$

$M_{max} = wL^2/8 = 1 \times (40)^2/8$

$= 200$ kip-ft $= 2400$ kip-in.

$$S_{min} = \frac{M}{0.60\,F_y} = \frac{2400}{39} = 61.53 \text{ in.}^3$$

$$X_{min} = \frac{Ml}{12000\,C_b} = \frac{2400 \times 240}{12{,}000 \times 1.75} = 27.43$$

A quick scan down the asterisked values in the X column in Table 1 arrives at 24.2 (for a W14X48) as the value next below the required X_{min} of 27.43. Feasible sections must be above this point in the table. Similarly, scanning down the asterisked S_x values, 57.9 is found as the value next below the required S_{min} of 61.53. Feasible sections must also be above this point, so the upward search can start at the higher of the two.

W12X50 ($S_x = 64.7, X = 27.5$) is found as the lightest feasible section, but additional moment is produced by the weight of the beam.

$M_{DL} = 0.050 \times (40)^2/8 = 10.0$ kip-ft

Revised:

$M_{max} = 210$ kip-ft, $S_{min} = 64.6$ in.^{3}, $X_{min} = 28.8$

W14X53 ($S_x = 77.8, X = 29.6$) is now the lightest feasible section, and because it is a Group I section, it is available with F_y = 65 ksi. (See AISC Manual, pgs. 1-6, 1-7.)

Stage II:

For W14X53, $F_b = 210 \times 12/77.8 = 32.39$ ksi

$F_b/F_y = 32.39/65 = 0.498 > 0.333$

$\therefore$ use Eq. (3) for r_{TCR}:

$$r_{TCR} = l\sqrt{\frac{F_y}{(2/3 - F_b/F_y)1{,}530{,}000C_b}}$$

$$= 240 \times \sqrt{\frac{65}{(0.667 - 0.498) \times 1{,}530{,}000 \times 1.75}}$$

$= 2.88$ in.

In order to be satisfactory, any section below W14X53 must have S_x greater than S_{min} (64.6) and either r_T greater than r_{TCR} (2.88) or S_x greater than S_1 (77.8).

The asterisks again aid in limiting the search. In the column of r_T values, searching down from W14X53, no value of r_T greater than r_{TCR} is found before coming to the first marked value (2.77 for the W10X49. Since the asterisk signifies no larger values of r_T below this point, this search is terminated.

Similarly, in the S_x column, searching down from W14X53, the values of S_x are examined until the asterisked value 68.4 (for the W18X40), which is less than 77.8, stops this search. In this examination, the following sections are noted:

W16X50 ($S_x = 80.8, r_T = 1.87$)

W18X50 ($S_x = 89.1, r_T = 1.96$)

W21X49 ($S_x = 93.3, r_T = 1.63$)

W18X45 ($S_x = 79.0, r_T = 1.94$)

W18X44 ($S_x = 81.6, r_T = 1.59$).

None looks promising, so investigate the one with the largest S_x value, i.e., W21X49. For this section, $F_b = 210 \times 12/93.30 = 27.01$ and $F_b/F_y = 0.416$, so Eq. (3) is still used for calculating r_{TCR} and this can be done by ratio:

$$r_{TCR} = 2.88\sqrt{\frac{0.667 - 0.498}{0.667 - 0.416}} = 2.36 \text{ in.}$$

Since this value of r_T required a larger than r_T furnished by any of the five sections, none of these sections will work, and the feasible section, W14X53, is optimal.

EXAMPLE 2

One of the commonest uses of laterally unsupported beams is as the bridge beam of a crane girder. S sections rather than W sections are often used for these beams, since the trolley wheels which roll on the lower flange are made to fit the standard 1:6 slopes of the flange of an S section. As an illustration of a problem with an additional constraint, then, let the problem be:

Given: A simple span of 23.17 feet. Maximum moment due to all causes (including a reasonable estimate for the weight of the beam) is 33.2 kip-ft (398.4 kip-in.). Lateral braces are at ends only (C_b=1).

Find: The lightest satisfactory S section of A36 steel (F_y = 36 ksi).

Solution:

Stage I:

$$S_{min} = \frac{M}{0.60\,F_y} = \frac{398.4}{22} = 18.1 \text{ in.}^3$$

$$X_{min} = \frac{Ml}{12000\,C_b} = \frac{398.4 \times (23.17 \times 12)}{12000 \times 1}$$

$= 9.23$

The lightest feasible S section is S12X40.8 ($S_x = 45.4$, $X = 13.1$)

Stage II:

For S12X40.8: $F_b = \dfrac{398.4}{45.4} = 8.78$ ksi

$F_b/F_y = 8.78/36 = 0.243 < 0.333$

$\therefore$ Use Eq. (4) for r_{TCR}:

$$r_{TCR} = l\sqrt{\frac{F_b}{170{,}000\ C_b}}$$

$$= 23.17 \times 12\sqrt{\frac{8.78}{170{,}000 \times 1}}$$

$$= 2.00 \text{ in.}$$

In order to be satisfactory, any S section below S12×40.8 must have S_x greater than S_{min} (18.1) and either r_T greater than r_{TCR} (2.00) or S_x greater than S_1 (45.40).

No such S section exists. Therefore, the S12×40.8 is optimal.

EXAMPLE 3

To establish confidence in the method, and also to illustrate a case in which the feasible solution of Stage I is *not* the optimal solution of Stage II, solve the problem presented on pg. 2-86 of the AISC Manual.

In its final form, this problem is:

Given:

Total moment: $M = 231$ kip-ft $= 2772$ kip-in.
Unbraced length: $L = 15 \times 12 = 180$ in.
$C_b = 1$; $F_y = 36$ ksi

Find:

The lightest satisfactory section.

Solution:

Stage I:

$$S_{min} = \frac{M}{0.60\ F_y} = \frac{2772}{22} = 126 \text{ in.}^3$$

$$X_{min} = \frac{Ml}{12{,}000\ C_b} = \frac{2772 \times 180}{12{,}000 \times 1} = 41.58$$

Lightest feasible sect.: W18×70 ($S_x = 129$, $X = 47.1$)

Stage II:

For W18×70: $$F_b = \frac{M}{S_1} = \frac{2772}{129} = 21.49 \text{ ksi}$$

$$\frac{F_b}{F_y} = \frac{21.49}{36} = 0.597$$

$\therefore$ Eq. (3) applies:

$$r_{TCR} = l\sqrt{\frac{F_y}{(2/3 - F_b/F_y)\ 1{,}530{,}000\ C_b}}$$

$$= 180\sqrt{\frac{36}{(0.667 - 0.597)\ 1{,}530{,}000 \times 1}}$$

$$= 3.30 \text{ in.}$$

To be considered, any lighter section must have $S_x > 126$, and either $r_T > 3.30$ or $S_x > 129$. The only possible sections are:

W21×68 ($S_x = 140$, $r_T = 2.15$)
W24×68 ($S_x = 153$, $r_T = 2.28$)
W24×61 ($S_x = 130$, $r_T = 1.74$)

By inspection, W24×61 is rejected, and it is clear that W24×68 is the best candidate for further investigation.

For 24×68: $$F_b = \frac{M}{S_x} = \frac{2772}{153} = 18.12 \text{ ksi}$$

$$\frac{F_b}{F_y} = \frac{18.12}{36} = 0.503$$

$\therefore$ Eq. (3) applies:

$$r_{TCR} = 3.30\sqrt{\frac{0.667 - 0.597}{0.667 - 0.503}} = 2.15 \text{ in.}$$

Since W24×68 has $r_T = 2.28$, which is greater than the value of r_{TCR} required to make its allowable stress 18.12 ksi, this section is satisfactory and optimal. Use W24×68.

This is the same solution as in the AISC Manual and, of course, can be more rapidly obtained using the graphs therein. The method of this article, however, is good for all grades of steel, and all values of C_b.

NOMENCLATURE

A_f = area of compression flange, in.^{2}

C_b = bending coefficient depending upon the moment gradient, equal to

$$1.75 + 1.05\left(\frac{M_1}{M_2}\right) + 0.3\left(\frac{M_1}{M_2}\right)^2$$

F_b = bending stress permitted, ksi

F_y = specified minimum yield stress of the type of steel being used, ksi

L = overall span length, ft

L_u = maximum unbraced length of the compression flange at which the allowable bending stress may be taken at 0.6 F_y, ft

M = moment, kip-in.

M_1 = smaller moment at end of unbraced length, kip-in.

M_2 = larger moment at end of unbraced length, kip-in.

M_{DL} = moment produced by the weight of the beam, kip-in.

S_1 = the section modulus of the feasible section selected in Stage I, in.^{3}

S_{min} = that value of S_x which will make the computed bending stress equal to 0.60 F_y, in.^{3}

S_x = elastic section modulus about the x-axis, in.^{3}

X = $S_x A_f/d$, in.^{4}

X_{min} = that value of X which will make the computed bending stress equal to 12,000 $C_b/(ld/A_f)$, in.^{4}

d = depth of beam, in.
l = actual unbraced length
r_T = radius of gyration of a section comprising the compression flange plus one-third of the compression web area, taken about an axis in the plane of the web, in.
r_{TCR} = that value of r_T which will make the allowable bending stress calculated from l/r_T equal to the computed bending stress calculated from M/S_x, in.
l = actual unbraced length

REFERENCES

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3. *Stockwell, Frank W., Jr.* Simplified Approach to AISC Bending Formulas *Engineering Journal, Third Quarter, 1974, American Institute of Steel Construction, New York, N.Y.*
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APPENDIX

Table I

R_T (in.)	Section	S_x (in.^{3})	X (in.^{4})	R_T (in.)	Section	S_x (in.^{3})	X (in.^{4})
5.27*	W14X730	1280.*	5010.*	3.82	W24X160	414.	268.
5.17*	W14X665	1150.*	4230.*	3.75	W27X160	446.	248.
5.08*	W14X605	1040.	3600.*	3.06	W36X160	542.*	184.
5.00*	W14X550	933.	3030.*	4.36*	W14X158	253.	312.*
4.92*	W14X500	840.	2550.*	2.98	W33X152	487.	177.
4.85*	W14X455	758.	2150.*	4.35*	W14X150	240.	282.*
4.81*	W14X426	707.	1920.*	3.03	W36X150	504.*	158.
4.76*	W14X398	657.	1690.*	3.79	W24X145	373.	218.
4.72	W14X370	608.	1480.*	3.72	W27X145	404.	205.
4.68	W14X342	559.	1290.*	4.33*	W14X142	227.	254.*
4.73*	W14X320	493.	1030.	3.58	W21X142	317.	212.
4.63*	W14X314	512.	1100.*	2.96	W33X141	448.*	149.
4.46	W36X300	1110.*	846.	4.12*	W14X136	216.	229.*
4.58*	W14X287	465.	934.*	2.97	W36X135	440.*	117.
4.43	W36X280	1030.*	735.	3.47	W12X133	183.	209.*
4.54*	W14X264	427.	804.*	2.72	W30X132	380.	132.
4.40	W36X260	952.*	626.	3.76	W24X130	332.	173.
4.51*	W14X246	397.	706.*	2.92	W33X130	406.*	121.
4.38	W36X245	894.*	553.	4.10*	W14X127	202.	203.*
4.23	W33X240	813.	539.	3.55	W21X127	284.	172.
4.50*	W14X237	382.	659.*	2.70	W30X124	355.	115.
4.36	W36X230	837.*	484.	1.93	S24X120	252.	93.1
4.48*	W14X228	368.	616.*	3.44	W12X120	163.	169.
4.20	W33X220	742.*	450.	3.22	W24X120	300.	139.
4.47*	W14X219	353.	571.*	4.08*	W14X119	189.	179.*
4.46*	W14X211	339.	532.*	2.87	W33X118	359.*	92.6
4.05	W30X210	651.	426.	4.06	HP14X117	173.	146.
4.44*	W14X202	325.	492.*	2.68	W30X116	329.*	97.9
4.17	W33X200	671.*	368.	3.23	W18X114	220.	140.
3.12	W36X194	665.*	278.	2.62	W27X114	300.	103.
4.42*	W14X193	310.	452.*	2.94	W10X112	126.	144.
3.59	W12X190	263.	402.	3.52	W21X112	250.	134.
4.01	W30X190	587.	347.	4.07*	W14X111	176.	156.*
4.41*	W14X184	296.	415.*	3.20	W24X110	276.	118.
3.10	W36X182	622.*	244.	2.64	W30X108	300.*	80.2
3.77	W27X177	494.	303.	3.41	W12X106	145.	136.
4.39*	W14X176	282.	380.*	1.93	S24X105	236.	85.3
3.98	W30X172	530.	283.	3.21	W18X105	202.	118.
3.08	W36X170	580.*	212.	4.05*	W14X103	164.	136.*
4.38*	W14X167	267.	344.*	4.02	HP14X102	150.	111.
3.53	W12X161	222.	297.	2.59	W27X102	267.	81.7

(Cont'd next page)

Table I (*cont'd*)

R_T (in.)	Section	S_x (in.³)	X (in.⁴)	R_T (in.)	Section	S_x (in.³)	X (in.⁴)
1.65	S24×100	199.	52.3	1.98	W18×55	98.4	25.8
2.91	W10×100	112.	116.	2.10	W21×55	110.	22.7
3.18	W24×100	250.	96.9	1.70	W24×55	114.*	17.0
3.39	W12×99	135.	119.*	1.41	S18×54	89.4	20.6
2.61	W30×99	270.*	63.8	2.78	W10×54	60.4	37.0*
3.16	W16×96	166.	103.	3.23*	HP12×53	66.9	29.8
3.19	W18×96	185.	99.5	2.74	W12×53	70.7	33.8*
2.39	W21×96	198.	79.1	2.18	W14×53	77.8	29.6
1.70	S20×95	161.	53.1	0.00	MC18×51.9	69.7	9.92
4.04*	W14×95	151.	116.*	0.00	MC12×50	44.9	10.8
2.37	W24×94	221.	71.9	0.00	MC13×50	48.4	10.0
2.56	W27×94	243.*	67.4	0.00	C15×50	53.8	8.66
3.38	W12×92	125.	103.*	1.31	S12×50	50.8	15.3
1.65	S24×90	187.	48.3	1.31	S15×50	64.8	15.2
3.99	HP14×89	131.	85.6	2.19	W12×50	64.7	27.5
2.88	W10×89	99.7	94.0	1.87	W16×50	80.8	22.1
3.14	W16×88	151.	85.4	1.96	W18×50	89.1	21.2
4.02*	W14×87	138.	98.3*	2.77*	W10×49	54.6	30.5*
1.69	S20×85	152.	49.1	1.63	W21×49	93.3*	15.5
3.36	W12×85	116.	89.4*	2.27	W 8×48	43.2	28.2*
2.37	W18×85	157.	69.0	2.16	W14×48	70.2	24.2*
3.32	W14×84	131.	86.4*	0.00	MC18×45.8	64.3	8.93
2.34	W24×84	197.	56.9	0.00	MC12×45	42.0	9.83
2.52	W27×84	212.*	50.3	2.21	W10×45	49.1	24.1*
2.35	W21×82	169.	57.7	2.18	W12×45	58.2	22.4*
1.66	S24×79	175.	44.5	1.85	W16×45	72.5	17.8
3.34	W12×79	107.	76.8*	1.94	W18×45	79.0	16.5
3.31	W14×78	121.	74.1*	1.59	W21×44	81.6*	11.6
2.32	W16×78	128.	58.9	2.14	W14×43	62.7	19.4
2.85	W10×77	86.1	71.7*	1.31	S15×42.9	59.6	13.6
2.36	W18×77	142.	57.1	0.00	MC18×42.7	61.6	8.45
2.32	W24×76	176.*	45.1	2.72*	HP10×42	43.4	18.8
1.48	S20×75	128.	32.3	0.00	MC10×41.1	31.5	7.83
3.31	HP12×74	93.4	57.1	1.28	S12×40.8	45.4	13.1
2.76	W14×74	112.	62.2	0.00	MC12×40	39.0	8.85
3.94*	HP14×73	108.	58.4	0.00	MC13×40	42.0	8.25
2.16	W21×73	151.	43.6	0.00	C15×40	46.5	7.09
2.84	W10×72	80.1	62.7	2.24*	W 8×40	35.5	19.4*
3.33*	W12×72	97.5	64.3*	2.16	W12×40	51.9	17.9*
2.30	W16×71	116.	48.8	1.84	W16×40	64.6	14.2
1.41	S18×70	103.	24.7	1.54	W18×40	68.4*	12.1
2.34	W18×70	129.	47.1	2.19	W10×39	42.2	17.9*
2.74	W14×68	103.	52.8	1.80	W14×38	54.7	13.5
2.15	W21×68	140.	37.5	2.11	M 8×37.7	32.6	16.7*
2.28	W24×68	153.*	33.7	0.00	MC12×37	34.2	6.16
2.33	W 8×67	60.	51.9	2.22	HP 8×36	29.9	13.5
2.82	W10×66	73.7	53.7*	1.77	W12×36	46.0	13.3
1.48	S20×65	118.	29.1	1.81	W16×36	56.5	10.7
3.31*	W12×65	88.0	52.8*	0.00	MC12×35	36.1	7.93
2.28	W16×64	104.	39.5	0.00	MC13×35	38.8	7.41
2.32	W18×64	118.	39.5	1.15	S10×35	29.4	7.14
2.13	W21×62	127.	30.7	1.20	S12×35	38.2	8.79
2.73	W14×61	92.2	42.6	2.22*	W 8×35	31.1	15.2*
1.74	W24×61	130.*	22.7	1.51	W18×35	57.9*	8.42
2.80	W10×60	67.1	45.0*	2.08	M 8×34.3	29.1	13.4*
1.99	W18×60	108.	31.1	1.78	W14×34	48.6*	10.6
0.00	MC18×58	75.1	11.0	0.00	C15×33.9	42.0	6.19
2.31	W 8×58	52.0	39.5	1.64	M 6×33.7	20.7	12.3
2.75	W12×58	78.1	41.1*	0.00	MC10×33.6	27.8	6.55
2.26	W16×58	94.4	32.5	2.16	W10×33	35.0	12.4
2.78	HP10×57	58.8	33.9	0.00	MC12×32.9	31.8	5.57

(Cont'd next page)

Table I (*cont'd*)

R_T (in.)	Section	S_x (in.^{3})	X (in.^{4})	R_T (in.)	Section	S_x (in.^{3})	X (in.^{4})
2.08	M 8×32.6	28.4	12.9*	1.40	W 8×17	14.1	2.85
0.00	MC13×31.8	36.8	6.91	1.03	W10×17	16.2	2.11
1.20	S12×31.8	36.4	8.25	0.98	W12×16.5	17.6*	1.58
2.21*	W 8×31	27.4	11.9*	0.00	MC 6×16.3	8.68	2.06
1.75	W12×31	39.5	9.91*	1.06	M 4×16.3	6.67	2.95
1.41	W16×31	47.2*	7.28	1.39	W 5×16	8.53	3.07*
0.00	MC12×30.9	30.6	5.28	1.10	W 6×16	10.2	2.66
0.00	C10×30	20.7	2.74	1.63*	W 6×15.5	10.0	2.69*
0.00	C12×30	27.0	3.57	0.00	MC 6×15.3	8.47	1.90
1.75	W14×30	41.9*	7.80	0.00	C10×15.3	13.5	1.53
1.40	M10×29.1	26.6	6.22	0.89	S 7×15.3	10.5	2.15*
1.57	W10×29	30.8	8.74	0.00	MC 6×15.1	8.32	1.94
0.00	MC10×28.5	25.3	5.75	0.00	C 9×15	11.3	1.29
0.00	MC10×28.3	23.6	4.75	1.04	W 8×15	11.8	1.83
1.80*	W 8×28	24.3	9.13*	1.00	W10×15	13.8	1.49
1.74	W12×27	34.2	7.43*	0.00	C 7×14.7	7.78	0.935
1.29	W14×26	35.1	5.31	0.78	S 5×14.7	6.09	1.30
1.38	W16×26	38.3*	4.64	0.96	W12×14	14.8*	1.11
0.00	MC 9×25.4	19.6	4.19	1.05	M 4×13.8	5.42	2.01*
1.13	S10×25.4	24.7	5.65	0.00	C 8×13.7	9.03	1.03
0.00	MC10×25.3	21.4	3.80	0.00	C 9×13.4	10.6	1.18
0.00	C10×25	18.2	2.29	0.00	C 6×13	5.80	0.715
0.00	C12×25	24.1	3.07	1.04	M 4×13	5.24	1.92*
1.69	W 6×25	16.7	7.27*	1.11*	W 4×13	5.45	1.84*
1.56	W10×25	26.5	6.51	1.02	W 8×13	9.90	1.26
0.00	MC10×24.9	22.0	4.30	0.82	S 6×12.5	7.37	1.47*
1.78*	W 8×24	20.8	6.79*	0.00	C 7×12.2	6.93	0.795
0.00	MC 9×23.9	18.9	3.99	0.00	MC 6×12	6.24	0.974
0.99	S 8×23	16.2	3.59	1.07*	W 6×12	7.25	1.35*
1.40	M10×22.9	23.6	5.35*	0.69	M12×11.8	12.0*	0.690
0.00	MC 8×22.8	16.0	3.68	0.00	C 8×11.5	8.14	0.897
0.00	MC 7×22.7	13.6	3.50	0.98	W10×11.5	10.5*	0.857
1.55	M 6×22.5	13.7	5.24*	0.00	MC12×10.6	9.23*	0.357
1.28	M 8×22.5	17.1	4.07	0.00	C 6×10.5	5.06	0.588
1.03	W12×22	25.3	3.51	0.74	S 5×10	4.92	0.964*
1.26	W14×22	28.9	3.53	1.00	W 8×10	7.80*	0.794*
0.00	MC10×21.9	19.7	3.40	0.00	C 7×9.8	6.08	0.664
0.00	MC 8×21.4	15.4	3.49	0.68	S 4×9.5	3.39	0.694*
1.53	W10×21	21.5	4.25	0.00	MC 3×9	2.10	0.521
0.00	C12×20.7	21.5*	2.64	0.00	C 5×9	3.56	0.429
0.00	MC 8×20	13.6	2.57	0.62	M10×9	7.76*	0.430
0.00	C 9×20	13.5	1.64	0.00	MC 8×8.5	5.83	0.425
0.00	C10×20	15.8	1.89	1.04*	W 6×8.5	5.08	0.666*
0.91	S 7×20	12.1	2.62	0.00	MC10×8.4	6.40*	0.269
1.54	M 6×20	13.0	4.88	0.00	C 6×8.2	4.38	0.481
1.66*	W 6×20	13.4	4.77*	0.66*	S 4×7.7	3.04	0.593*
1.42	W 8×20	17.0	4.16*	0.62*	S 3×7.5	1.95	0.424*
0.00	MC 7×19.1	12.3	3.03	0.00	C 4×7.2	2.29	0.292
1.05	W10×19	18.8	2.91	0.00	MC 3×7.1	1.82	0.413*
1.01	W12×19	21.3*	2.45	0.00	C 5×6.7	3.00	0.336
1.32	M 5×18.9	9.63	4.01	0.00	MC10×6.5	4.42	0.101
0.00	MC 8×18.7	13.1	2.44	0.54	M 8×6.5	4.62*	0.249
0.00	C 8×18.7	11.0	1.36	0.00	C 3×6	1.38	0.200
1.28	M 8×18.5	15.5	3.59	0.58*	S 3×5.7	1.68	0.339*
1.40	W 5×18.5	9.94	4.10*	0.49*	M 7×5.5	3.44*	0.184
0.97	S 8×18.4	14.4	3.06	0.00	C 4×5.4	1.93	0.226*
0.00	MC 6×18	9.91	2.75	0.00	C 3×5	1.24	0.169*
0.00	MC 7×17.6	10.8	2.20	0.44*	M 6×4.4	2.40*	0.126
0.85	S 6×17.2	8.77	1.87	0.00	C 3×4.1	1.10*	0.141*
0.93	M14×17.2	21.1*	1.64				