Discussion

Drift Reduction Factors for Belted High-Rise Structures

Paper by J. W. McNABB and B B. MUVDI (3rd Quarter, 1975)

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In this discussion, the one belt truss system, as a means of controlling drift in high-rise structures, is considered further and the analysis extended to two and many belt-truss systems. Using the same basic assumptions as well as the same loading used in the paper under discussion, the authors developed the equation for the deflection y at the top of an idealized high-rise structure, shown in Fig. 1. The deflection y was then transformed into a dimensionless drift R, given by the equation

$$R = 1 - \frac{2}{3}\alpha[1 + \gamma_2 - \gamma_1{}^3\gamma_2 - \gamma_2{}^3 + \gamma_1\gamma_2{}^3 - \gamma_1{}^4] \quad (1)$$

where $\gamma_1 = x_1/l$, $\gamma_2 = x_2/l$ and the dimensionless drift is given by $R = y/(Wl^4/8E_{CR}I)$.

For a given value of the parameter α, Eq. (1) gives the dimensionless drift R as a function of the dimensionless coordinates γ_1 and γ_2, which locate the two belt trusses with respect to the top of the structure. This is a problem of classical indirect optimization, whose purpose would be to choose γ_1 and γ_2 to minimize R. Necessary, but not sufficient, conditions for a minimum value of R are given by:

$$\frac{\partial R}{\partial \gamma_1} = 0 \qquad \frac{\partial R}{\partial \gamma_2} = 0 \quad (2)$$

Applying these conditions to Eq. (1), one obtains:

$$4\gamma_1{}^3 + 3\gamma_1{}^2\gamma_2 - \gamma_2{}^3 = 0 \quad (3)$$

and

$$\gamma_1{}^3 - 3\gamma_1\gamma_2{}^2 + 3\gamma_2{}^3 - 1 = 0 \quad (4)$$

Equations (3) and (4) are simultaneous, algebraic, non-linear equations in γ_1 and γ_2. In general, non-linear algebraic equations are usually solvable only by extensive

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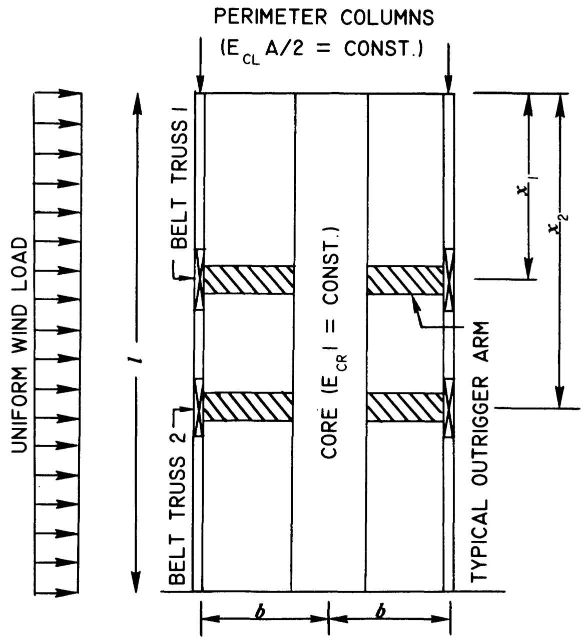


Fig. 1. Schematic diagram of simplified structure

numerical iterative procedures, but an elegant and relatively rapid technique has been discovered for solving the two equations arising in this optimization problem. Divide Eq. (3) by $\gamma_2{}^3$ and let $\eta = \gamma_1/\gamma_2$. This leads to the following equation:

$$4\eta^3 + 3\eta^2 - 1 = 0 \quad (5)$$

Equation (5) has one real, positive root: $\eta = 0.455$. Use the same technique to write Eq. (4) in the form:

$$(\eta^3 - 3\eta)\gamma_2{}^3 + 3\gamma_2{}^2 - 1 = 0 \quad (6)$$

For $\eta = 0.455$, γ_2 has only one real positive value within its admissible domain (i.e., $0 \leq \gamma_2 \leq 1$) which satisfies Eq. (6). This root of Eq. (6) is $\gamma_2 = 0.685$. Since $\eta = \gamma_1/\gamma_2 = 0.455$, γ_1 corresponding to $\gamma_2 = 0.685$ is $\gamma_1 = 0.312$. The sufficient condition for establishing that R takes on a relative minimum value at these values of γ_1 and γ_2 is given by:

$$\frac{\partial^2 R}{\partial \gamma_1{}^2}\frac{\partial^2 R}{\partial \gamma_2{}^2} - \frac{\partial^2 R}{\partial \gamma_1 \partial \gamma_2} > 0 \quad (7)$$

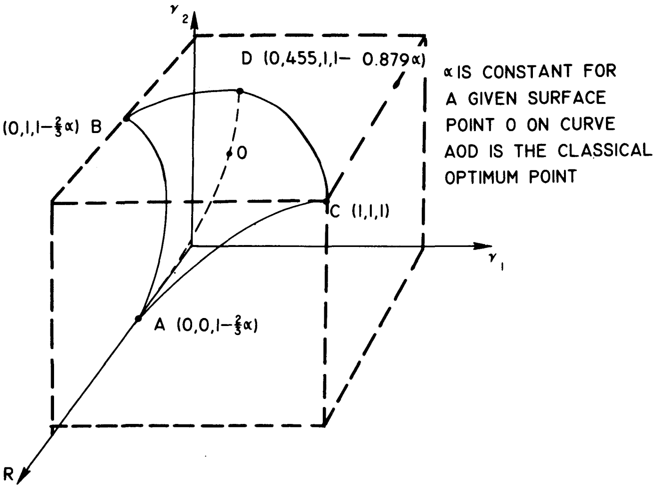
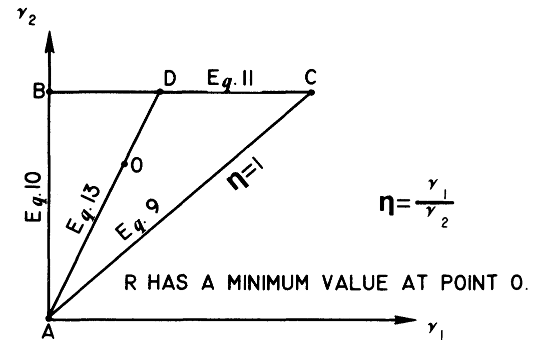


Fig. 2. Qualitative representation of a typical dimensionless drift surface for a given value α



POINT ON SURFACE $R=f(\alpha,\gamma_1,\gamma_2)$	COORDINATES (γ_1,γ_2,R)
A	0,0,1–⅔α
B	0,1,1–⅔α
C	1,1,1
O	0.312,0.685,1–0.956α
D	0.455,1.000,1–0.879α

Fig. 3. Plan view of dimensionless drift surface showing special cases of $R = f(\alpha, \gamma_1, \gamma_2)$

provided

$$\frac{\partial^2 R}{\partial \gamma_1{}^2} > 0$$

Performing the appropriate differentiations of Eq. (1) and substituting the critical values of γ_1 and γ_2 reveals that R is a minimum at this classical optimum point, which is denoted by the letter O in Fig. 2. In this figure, **AOD** is the curve in which the plane $\eta = 0.455$ intersects the dimensionless drift surface. The tangent plane at point 0 is parallel to the (γ_1-γ_2) plane and point 0 has the coordinates:

$$\gamma_1 = 0.312; \; \gamma_2 = 0.685; \; R = 1 - 0.956\alpha \qquad (8)$$

Equation (8) provides the solution to the problem of deciding where to locate the two belt trusses in order to minimize the drift of this idealized high rise structure.

Figure 3 provides a plan view of a dimensionless drift surface for which α is constant. Values of γ_1 and γ_2 that are the coordinates of a point lying either inside or on the boundaries of triangle **ABC** are admissible values of γ_1 and γ_2.

If one specializes Eq. (1) by letting $\gamma_1 = \gamma_2 = \gamma$, then:

$$R = 1 - \frac{2}{3}\alpha(1 + \gamma - \gamma^3 - \gamma^4) \qquad (9)$$

This corresponds to locating both belt trusses at the same level and is of the same form given for a single belt truss in the paper under discussion. The function R of Eq. (9) is valid along line **AC** of Fig. 3.

If γ_1 is equated to zero in Eq. (1), this corresponds to locating Belt Truss 1 at the top of the structure, and R is given by:

$$R = 1 - \frac{2}{3}\alpha(1 + \gamma_2 - \gamma_2{}^3) \qquad (10)$$

The function R of Eq. (10) is valid along line **AB.**

$$\frac{\partial R}{\partial \gamma_2} = -\frac{2}{3}\alpha(1 - 3\gamma_2) = 0$$

$$\gamma_2 = \pm\frac{\sqrt{3}}{3} = \pm 0.577$$

Rejecting the negative root, $\gamma_2 = +0.577$, provided $\alpha \neq 0$. The second derivative of R with respect to γ, evaluated at $\gamma_2 = 0.577$, is greater than zero. Hence, R is a minimum at $\gamma_2 = +0.577$.

Next, let $\gamma_2 = 1$ in Eq. (1), which means that Belt Truss 2 is moved to the base of the structure and is no longer effective in reducing the drift. In this case, R is given by:

$$R = 1 - \frac{2}{3}\alpha(1 + \gamma_1 - \gamma_1{}^3 - \gamma_1{}^4) \qquad (11)$$

The function R of Eq. (11) is valid along line **BC** of Fig. 3. Equation (11) is identical to the single belt truss equation given in the paper under discussion. It can readily be shown that R reaches a minimum at $\gamma_1 = 0.455$.

Finally, let $\gamma_1 = \eta\gamma_2$ in Eq. (1) to obtain:

$$R = 1 - \frac{2}{3}\alpha[1 + \gamma_2 - \gamma_2{}^3 + (\eta - \eta^3 - \eta^4)\gamma_2{}^4] \quad (12)$$

Equation (12) expresses R as a function of α, η, and γ_2 and is simply an alternate way to write Eq. (1). If one further lets η assume the special value $\eta = 0.455$, then R is given by:

$$R = 1 - \frac{2}{3}\alpha[1 + \gamma_2 - \gamma_2{}^3 + 0.318\,\gamma_2{}^4] \qquad (13)$$

The function R of Eq. (13) is valid along line **AD** of Fig. 3. R reaches a minimum at the classical optimum point 0 which has coordinates given by Eq. (8).

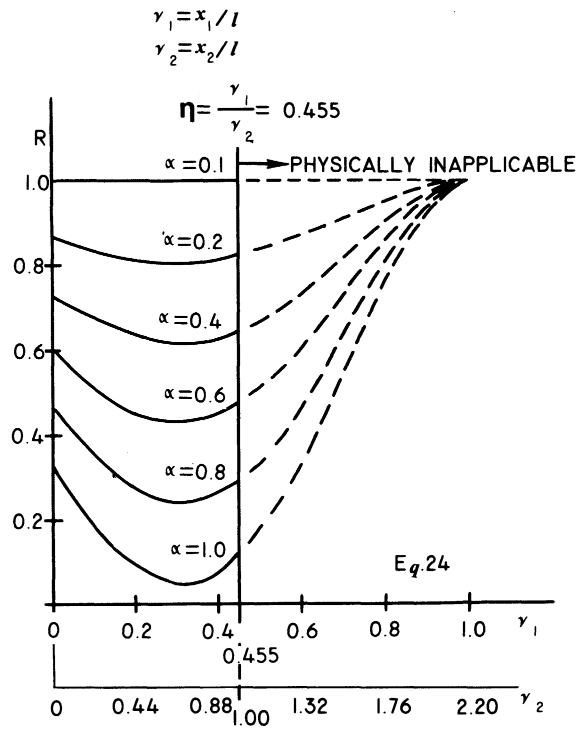


Fig. 4. Dimensionless drift vs. γ_1 and/or γ_2, for various values of α and for η = 0.455

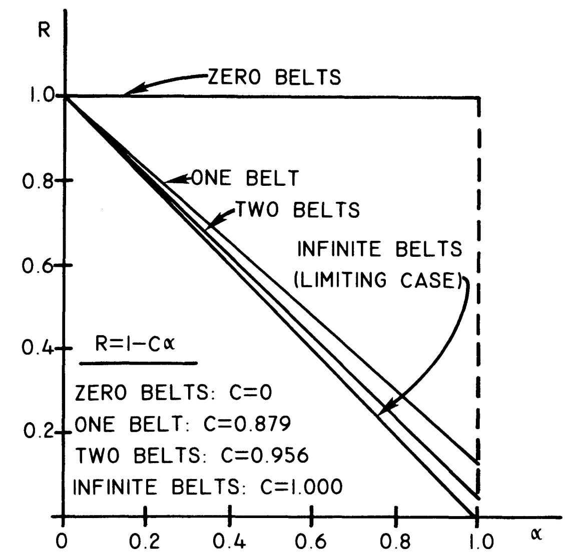


Fig. 5. Comparison of minimum dimensionless drift at various values of α for several belt systems

Equation (13) was obtained as a result of minimizing Eq. (1), and thus represents the absolute minimum drift of a structure with two belt trusses. Under such conditions, it is found that the minimum drift occurs when the ratio of the two belt truss locations γ_1/γ_2 assumes a value of 0.455. Figure 4 is a plot of Eq. (13), and shows the variation of the dimensionless drift R as a function of γ_1 and/or γ_2 for several arbitrary values of the drift reduction factor α, resulting in a family of curves that shows the pronounced effect of the parameter α. The dimensionless drift decreases considerably as the value of α is increased. Note also that regardless of the value of α, minimum drift occurs at γ_1 = 0.312 and γ_2 = 0.685. Furthermore, the plot in Fig. 4 depicts a range ($\gamma_1 > 0.455$ and $\gamma_2 > 1$) which is physically inapplicable, since under these conditions the second belt truss would be under ground level.

The effect on the dimensionless drift in a simplified high-rise structure of more than two belt trusses can be deduced from comparing Eqs. (1) and (11). Equation (11) gives the dimensionless drift R when only one belt truss is placed on the structure. For constant values of γ_1, Eq. (11) may be expressed in the form

$$R = 1 - C\alpha \tag{14}$$

in which C assumes the value of 0.879 for the optimum location of the belt truss, i.e., for γ_1 = 0.455. Similarly, Eq. (1), which gives the dimensionless drift R when two belt trusses are placed on the structure, can be expressed in the same form as Eq. (14), in which C becomes equal to 0.956 for the optimum location of the two belt trusses, i.e., for $\eta = \gamma_1/\gamma_2 = 0.455$.

A review of the development of Eq. (11) from basic principles, as was done in the paper under discussion for one belt truss, and that of Eq. (1) for two belt trusses, reveals that in each case the drift reduction factor α is factorable from the relations expressing the resisting moments at the belt truss locations. While the mathematics involved in examining the effects of more than two belt trusses would be cumbersome, it can be concluded that, even in such a case, the drift reduction factor α is factorable from the equations expressing the resisting moments at the various belt truss locations. One can extrapolate this conclusion to the case of a very large number of belt trusses (approaching infinity) and conclude, as before, that the dimensionless deflection would be given by Eq. (14), in which C assumes the limiting value of 1.000 in order to avoid negative values of the dimensionless drift R. These various relations are illustrated in Fig. 5.

The conclusions reached on the basis of this investigation are summarized as follows:

1. If only one belt truss is considered, it should be placed such that γ_1 = 0.455 to produce minimum drift at the top of the ideal structure. This minimum drift depends dramatically upon the drift reduction factor, decreasing as the drift reduction factor increases.
2. When two belt trusses are considered, they should be placed such that γ_1 = 0.312 and γ_2 = 0.685 to produce minimum drift at the top of the ideal structure. As in the case of the one belt truss, the minimum drift depends dramatically on the drift reduction factor, decreasing as the drift reduction factor increases.

3. As shown in Fig. 5, the one belt-truss system is the most effective in reducing the drift of a simplified high-rise structure. The two-belt truss system is seen to further reduce the drift, but the benefit of the second belt truss is considerably less than that of the first. From the straight line representing a very large number of belt trusses (approaching infinity), it can also be stated that the benefit of the third, fourth, etc. belt trusses becomes progressively less as the number of belt trusses is increased. Thus, it can be concluded that, for the ideal high-rise structure, the decision to add more than one belt truss would have to be made in light of economic and other considerations.