On Inelastic Column Buckling

Paper by C. V. SMITH, JR. (3rd Quarter, 1976)

Discussion by **Charles A. Matz**

In 1971, Yura[1] presented the concept of the inelastic K-factor. He suggested a procedure such that in the inelastic range of column behavior, the end restraint factor G was defined as:

$$G_{inelastic} = \frac{E_T}{E} \times G_{elastic} \simeq \frac{F_a}{F_e'} \times G_{elastic} \qquad (1)$$

where

E_T = Slope of the stress-strain curve
E = Initial value of E_T; Young's Modulus

In 1973, Disque[2] presented a design procedure for approximating the value of E_T/E ($E_T/E \simeq F_a/F_e'$). He acknowledged that F_a and F_e' have different factors of safety leading to a small error in the approximation.

In 1976, Smith[3] presented a technique for establishing the value of E_T/E which eliminates the above error. The Smith procedure requires the calculation of the value $\alpha = (Kl/r)/C_c$ and the resulting value of

$$\frac{E_T}{E} = \alpha^2(2 - \alpha^2) \qquad (\text{for } 0 < \alpha \leq 1) \qquad (2)$$

As suggested by Smith, tables for the values of E_T/E for the different values of $f = P/A$ are presented for 36 ksi steel and for 50 ksi steel (Tables 1 and 2, respectively).

REFERENCES

1. *Yura, Joseph A.* The Effective Length of Columns in Unbraced Frames *AISC Engineering Journal, April 1971, pp. 37–42.*
2. *Disque, Robert O.* Inelastic K-Factor for Column Design *AISC Engineering Journal, 2nd Quarter, 1973, pp. 33–35.*
3. *Smith, C. V., Jr.* On Inelastic Column Buckling *AISC Engineering Journal, 3rd Quarter, 1976, pp. 86–88.*

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Table 1

$\frac{Kl}{r}$	F_a(ksi)	E_t/E	$\frac{Kl}{r}$	F_a(ksi)	E_t/E
126.	9.41	1.000	63.	17.14	0.437
125.	9.55	1.000	62.	17.24	0.425
124.	9.70	0.999	61.	17.33	0.413
123.	9.85	0.998	60.	17.43	0.402
122.	9.99	0.996	59.	17.53	0.390
121.	10.14	0.994	58.	17.62	0.378
120.	10.28	0.991	57.	17.71	0.367
119.	10.43	0.988	56.	17.81	0.356
118.	10.57	0.985	55.	17.90	0.344
117.	10.71	0.981	54.	17.99	0.333
116.	10.85	0.976	53.	18.08	0.322
115.	10.99	0.972	52.	18.17	0.311
114.	11.13	0.967	51.	18.26	0.300
113.	11.26	0.961	50.	18.35	0.290
112.	11.40	0.955	49.	18.44	0.279
111.	11.54	0.949	48.	18.53	0.269
110.	11.67	0.943	47.	18.61	0.259
109.	11.81	0.936	46.	18.70	0.248
108.	11.94	0.929	45.	18.78	0.238
107.	12.07	0.922	44.	18.86	0.229
106.	12.20	0.914	43.	18.95	0.219
105.	12.33	0.906	42.	19.03	0.210
104.	12.47	0.898	41.	19.11	0.200
103.	12.59	0.889	40.	19.19	0.191
102.	12.72	0.880	39.	19.27	0.182
101.	12.85	0.871	38.	19.35	0.173
100.	12.98	0.862	37.	19.42	0.165
99.	13.10	0.853	36.	19.50	0.156
98.	13.23	0.843	35.	19.58	0.148
97.	13.35	0.833	34.	19.65	0.140
96.	13.48	0.823	33.	19.73	0.132
95.	13.60	0.813	32.	19.80	0.125
94.	13.72	0.803	31.	19.87	0.117
93.	13.84	0.792	30.	19.94	0.110
92.	13.97	0.781	29.	20.01	0.103
91.	14.09	0.770	28.	20.08	0.096
90.	14.20	0.759	27.	20.15	0.090
89.	14.32	0.748	26.	20.22	0.083
88.	14.44	0.737	25.	20.28	0.077
87.	14.56	0.725	24.	20.35	0.071
86.	14.67	0.714	23.	20.41	0.065
85.	14.79	0.702	22.	20.48	0.060
84.	14.90	0.691	21.	20.54	0.055
83.	15.02	0.679	20.	20.60	0.050
82.	15.13	0.667	19.	20.66	0.045
81.	15.24	0.655	18.	20.72	0.040
80.	15.36	0.643	17.	20.78	0.036
79.	15.47	0.631	16.	20.83	0.032
78.	15.58	0.619	15.	20.89	0.028
77.	15.69	0.607	14.	20.95	0.025
76.	15.79	0.595	13.	21.00	0.021
75.	15.90	0.582	12.	21.05	0.018
74.	16.01	0.570	11.	21.10	0.015
73.	16.12	0.558	10.	21.16	0.013
72.	16.22	0.546	9.	21.21	0.010
71.	16.33	0.534	8.	21.25	0.008
70.	16.43	0.521	7.	21.30	0.006
69.	16.53	0.509	6.	21.35	0.005
68.	16.64	0.497	5.	21.39	0.003
67.	16.74	0.485	4.	21.44	0.002
66.	16.84	0.473	3.	21.48	0.001
65.	16.94	0.461	2.	21.52	0.001
64.	17.04	0.449	1.	21.56	0.000^+

F_y = 36 ksi (C_c = 126.1)

Table 2

$\frac{Kl}{r}$	F_a (ksi)	E_t/E	$\frac{Kl}{r}$	F_a (ksi)	E_t/E
106.	13.29	1.000	53.	23.88	0.431
105.	13.53	0.999	52.	24.04	0.417
104.	13.77	0.997	51.	24.19	0.403
103.	14.00	0.995	50.	24.35	0.389
102.	14.24	0.992	49.	24.51	0.375
101.	14.47	0.988	48.	24.66	0.362
100.	14.71	0.984	47.	24.81	0.349
99.	14.94	0.979	46.	24.96	0.335
98.	15.17	0.974	45.	25.11	0.322
97.	15.39	0.968	44.	25.26	0.310
96.	15.62	0.962	43.	25.40	0.297
95.	15.84	0.955	42.	25.55	0.284
94.	16.06	0.948	41.	25.69	0.272
93.	16.29	0.940	40.	25.83	0.260
92.	16.50	0.932	39.	25.97	0.248
91.	16.72	0.923	38.	26.11	0.236
90.	16.94	0.914	37.	26.25	0.225
89.	17.15	0.905	36.	26.38	0.214
88.	17.37	0.895	35.	26.51	0.203
87.	17.58	0.885	34.	26.64	0.192
86.	17.79	0.875	33.	26.77	0.181
85.	17.99	0.864	32.	26.90	0.171
84.	18.20	0.853	31.	27.03	0.161
83.	18.41	0.841	30.	27.15	0.151
82.	18.61	0.830	29.	27.28	0.142
81.	18.81	0.818	28.	27.40	0.132
80.	19.01	0.806	27.	27.52	0.123
79.	19.21	0.793	26.	27.63	0.115
78.	19.41	0.780	25.	27.75	0.106
77.	19.61	0.768	24.	27.86	0.098
76.	19.80	0.754	23.	27.97	0.090
75.	19.99	0.741	22.	28.08	0.083
74.	20.19	0.728	21.	28.19	0.076
73.	20.38	0.714	20.	28.30	0.069
72.	20.56	0.701	19.	28.40	0.062
71.	20.75	0.687	18.	28.51	0.056
70.	20.94	0.673	17.	28.61	0.050
69.	21.12	0.659	16.	28.71	0.044
68.	21.31	0.645	15.	28.80	0.039
67.	21.49	0.630	14.	28.90	0.034
66.	21.67	0.616	13.	28.99	0.029
65.	21.85	0.602	12.	29.08	0.025
64.	22.02	0.588	11.	29.17	0.021
63.	22.20	0.573	10.	29.26	0.017
62.	22.37	0.559	9.	29.34	0.014
61.	22.55	0.544	8.	29.42	0.011
60.	22.72	0.530	7.	29.50	0.009
59.	22.89	0.516	6.	29.58	0.006
58.	23.06	0.501	5.	29.66	0.004
57.	23.22	0.487	4.	29.73	0.003
56.	23.39	0.473	3.	29.80	0.002
55.	23.55	0.459	2.	29.87	0.001
54.	23.72	0.445	1.	29.94	0.000^+

F_y = 50 ksi (C_c = 107.0)