Column Stability under Elastic Support

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PROFESSOR GALAMBOS has shown[1] that the amount of diagonal bracing or masonry wall needed to prevent sidesway of a steel building frame under gravity loading is relatively small. The approach which he used can provide useful stability solutions for (a) frames that depend entirely upon their own bending stiffness for lateral support yet include columns without rigid connections and (b) columns with flexible lateral support.

It has been shown[2, 3] that, in elastic frames, the value of the critical gravity load producing incipient sidesway in a rectangular frame is not greatly influenced by concurrent bending stress resulting from moment at the ends of the columns. Hence presence of such stresses will be ignored in the following procedures for designing to prevent premature joint translation.

CASE I: FRAME SIDESWAY

Consider the case where one or more of the columns in a given tier of an otherwise rigidly-connected, laterally-unbraced, rectangular frame is not rigidly connected. Such a situation is indicated in Fig. 1a.

In order to realize their full capacity as axially loaded compression members of unbraced length L_c each "pin-connected" column requires a stabilizing force capable of limiting the lateral movement of the top of the column, with respect to its bottom end, to some very small displacement δ.

The total force thus required for all of the columns is represented in Fig. 1b as being furnished by a spring. The force is equal to $k\delta$, where k is the required spring constant or *stiffness factor* and δ is a differential displacement which would take place as the axial stress P_{cr}/A in all of the columns slightly exceeds $\pi^2 \; E_t/(L_c/r)^2$. Considering any one of the columns as a free body subject to the critical vertical load P_n and lateral stabilizing force $k_n\delta$

$$P_n\delta = k_n\delta L_c$$

$$k_n = P_n/L_c$$

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As indicated in Fig. 1b, since the lateral displacement for all of the columns is the same

$$k = \Sigma k_n = \Sigma P_{cr}/L_c$$

Let it be required that the bending stiffness of the beams rigidly framed to one of the columns provide the stabilizing force for all of the columns. This condition is shown in Fig. 2a. That portion of the framing which is assumed to provide the needed lateral stability is shown separately in Fig. 2b. It has conservatively been assumed that the far end of the beams are only pin-connected to the adjacent columns.

P_o, the critical value of load P_6, is smaller than it would be if all of the columns were rigidly connected to the in-plane beams and able to provide their own lateral support. The sum of the critical loads on the "pinned" columns, determined by their unbraced length, is equal to $\Sigma P_{cr} - P_o$, where ΣP_{cr} is the total capacity of the frame to support vertical loading.

$$\text{Req'd } k = \frac{\Sigma P_{cr} - P_o}{L_c} \tag{1}$$

Considering equilibrium

$$k\delta L_c + (P_o + R_1 - R_2)\delta = R_1 L_1 + R_2 L_2 \tag{2}$$

Considering compatibility:

$$\frac{\Delta_1}{\Delta_2} = \frac{L_1}{L_2} = \frac{R_1 L_1^3 I_2}{R_2 L_2^3 I_1} \tag{3}$$

and

$$\theta_3 = \theta_1 - \theta_2$$

or

$$\frac{R_2 L_2^2}{3EI_2} = \frac{\delta}{L_c} - \psi\left[\frac{k\delta L_c^2 + (P_o + R_1 - R_2)\delta L_c}{3\,E_t I_c}\right]$$

$$= \frac{\delta}{L_c} - \frac{\psi}{\tau}\left[\frac{k\delta L_c^2 + (P_o + R_1 - R_2)\delta L_c}{3\,EI_c}\right] \tag{4}$$

where ψ is a *stability* factor measuring the influence of the load P_o upon the angle θ_2 and $\tau = E_t/E$.

Solving these simultaneous equations, the *stiffness factor* provided by the partial framing shown in Fig. 2b can be expressed as

$$k = \frac{3\,E\gamma_B\gamma_c - P_oL_c\left(\gamma_c + \frac{\psi}{\tau}\gamma_B\right)}{L_c^2\left(\gamma_c + \frac{\psi}{\tau}\gamma_B\right)} \tag{5}$$

where

$$\gamma_B = \frac{I_1}{L_1} + \frac{I_2}{L_2} \text{ and } \gamma_c = \frac{I_c}{L_c}$$

Equating (1) and (5)

$$\text{Req'd } I_c = \frac{\psi}{\tau}\cdot\frac{AL_c\gamma_B}{\gamma_B - A} \tag{6}$$

or

$$\text{Req'd } \gamma_B = \frac{A\gamma_c}{\gamma_c - \frac{\psi}{\tau}A} \tag{7}$$

where

$$A = \frac{\Sigma P_{cr}L_c}{3\,E}$$

Replacing ΣP_{cr} with the equivalent design values, P, in kips, multiplied by a factor of safety of, say, 1.85

$$A = \frac{\Sigma PL_c}{47{,}000} \tag{8}$$

The value of ψ is a function of $L_c\sqrt{P_o/E_tI_c}$ and τ can be expressed as $\tau = 4(P_o/P_y)(1 - P_o/P_y)$. Using the computed values of f_a/F_a and L_c/r, the corresponding ψ/τ-value can be obtained directly from Chart A. This chart is based upon a 36 ksi yield point. However, within the limits of practical design, it is suitable for steels having a yield point up to 50 ksi.

It has already been noted that concurrent bending moments do not greatly affect the value of the gravity loading producing incipient sidesway. However, if the column whose bending stiffness is counted upon to provide frame stability is also required to resist bending moments caused by lateral loads applied to the frame, its axial load P_o, included in ΣP in Equation (8), should be that which the column could support in the absence of the bending moment, rather than the smaller axial load for which it is designed. A conservative estimate of the equivalent axial load can be computed as

$$P_o = \frac{P + MB_xF_a}{(1 - f_a/F'_e)\,F_b}$$

where P and M are, respectively, the actual axial load and concurrent bending moment considered in the design of the column and B_x (or B_y if appropriate) is given in the column load tables commencing on page 3–13 of the AISC Manual.

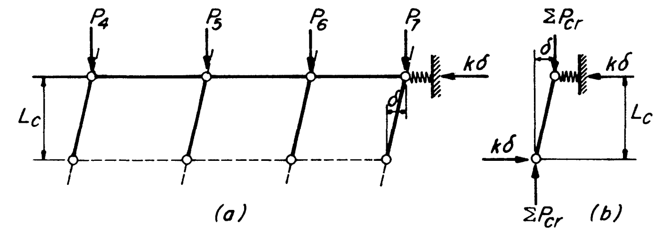

Figure 1

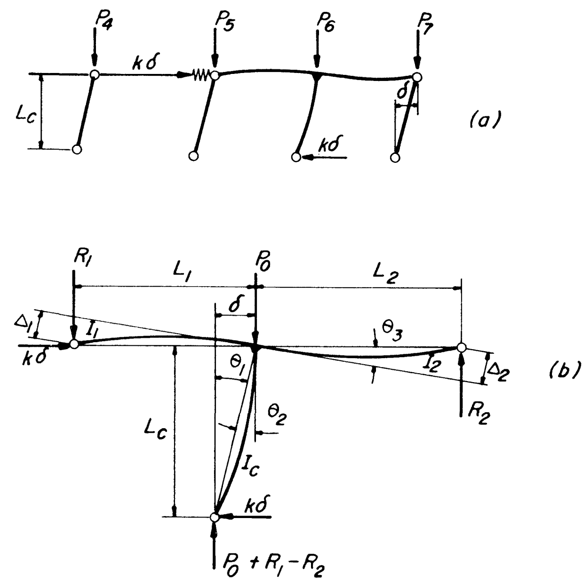

Figure 2

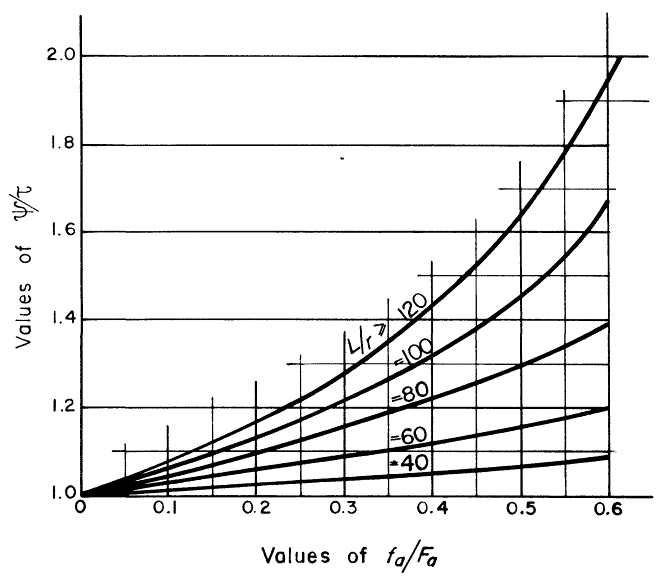

Chart A

Example 1—The roof beams shown in Fig. 3 will have ordinary framed connections at all interior columns. Assuming the absence of other means of lateral support and using rigid beam-to-column connections at the eaves, determine the exterior columns needed to prevent sidesway. Assume A36 steel.

The bending stiffness of each 16WF45 beam, together with its adjacent exterior column, must provide lateral support for one exterior column and 1½ interior columns as well.

$$\gamma_B = \frac{583.3}{35 \times 12} = 1.39$$

$$\Sigma P = 40 + (1.5 \times 70) = 145 \text{ kips}$$

$$A = \frac{145 \times 192}{47{,}000} = 0.60$$

Neglecting ψ/τ,

$$\text{Req'd } I_c = \frac{0.60 \times 192 \times 1.39}{1.39 - 0.60} = 204 \text{ in.}^4$$

Try 10WF39 (I = 209.7 in.^{4})

Considering ψ/τ,

$$f_a = \frac{40}{11.48} = 3.49 \text{ ksi}$$

$$\frac{L_c}{r_x} = \frac{192}{4.27} = 45$$

From AISC Manual, p. 5-68,

$$F_a = 18.78 \text{ ksi}$$

$$\frac{f_a}{F_a} = \frac{3.49}{18.78} = 0.185$$

From Chart A, when $f_a/F_a = 0.185$ and $L/r = 45$,

$$\psi/\tau \cong 1.04$$

Req'd I_c = 1.04 × 204 = 212 in.^{4} ≅ 209.5 in.^{4}
10WF39 is inadequate.

CASE II: ELASTIC LATERAL SUPPORT INDEPENDENT OF COLUMN STIFFNESS

Situations arise where the only lateral support pro-provided a multi-story column, with respect to one of its axes, is dependent upon the lateral bending stiffness of beams framing into it.

The exterior wall column shown in Fig. 4 adjacent to a stair well is such a case. A beam framing to the web of the column normal to the exterior wall would interfere with headroom. Assuming that the column is braced laterally in the plane of the wall it is necessary to determine the stiffness of Beams A and B needed to stabilize the column in a direction normal to the wall.

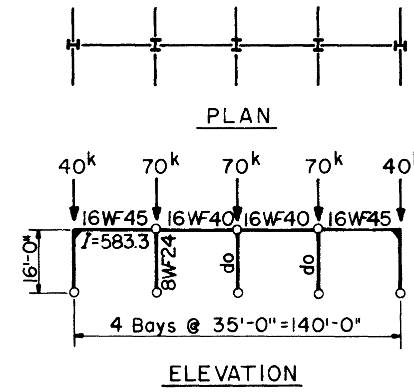

Figure 3

A conservative solution of the problem lies in neglecting any lateral support due to column bending stiffness and matching the lateral displacement Δ of the beams, at successive floor levels, with a virtual column displacement δ, so as to maintain the capacity of the column as that of a pin-ended member of unbraced story-height length. The basic assumptions leading to the solution are shown in Fig. 5. Note that, to allow for possible reversed beam deflections at adjacent floors, the virtual column displacement is taken as twice the beam displacement. The beam-to-column connections conservatively are assumed to be non-moment-resisting in the plane of the floor.

The column stabilizing force F has two components, F_1 and F_2, each of which is related to the same horizontal deflection by the expression[5]

$$\Delta = \frac{F_a^2\,(b + a)}{3\,EI} \quad (9)$$

Therefore

$$F = F_1 + F_2 = k\delta = \frac{3\,EI_1\Delta}{a_1^2(b_1 + a_1)} + \frac{3\,EI_2\Delta}{a_2^2\,(b_2 + a_2)}$$

Substituting $\delta/2$ for Δ

$$k = \frac{3E}{2}\left[\frac{I_1}{a_1^2(b_1 + a_1)} + \frac{I_2}{a_2^2(b_2 + a_2)}\right] = \frac{P_{cr}}{L_c}$$

Then, taking P_{cr} approximately equal to 1.85 times the column design load P,

$$\text{Req'd } \frac{I_1}{a_1^2(b_1 + a_1)} + \frac{I_2}{a_2^2(b_2 + a_2)} = \frac{P}{24{,}000L_c} \quad (10)$$

Depending upon the details of floor construction, other deflection formulas than Equation (9) may be more appropriate. Its use in the above derivation is sufficient to illustrate the method of analysis. It should be noted that the integrity of the analysis is dependent upon the presence of a floor slab stiff enough to prevent movement of Beams C, D, E and F normal to Beams A and B.

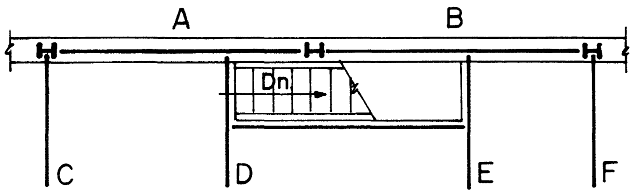

Figure 4

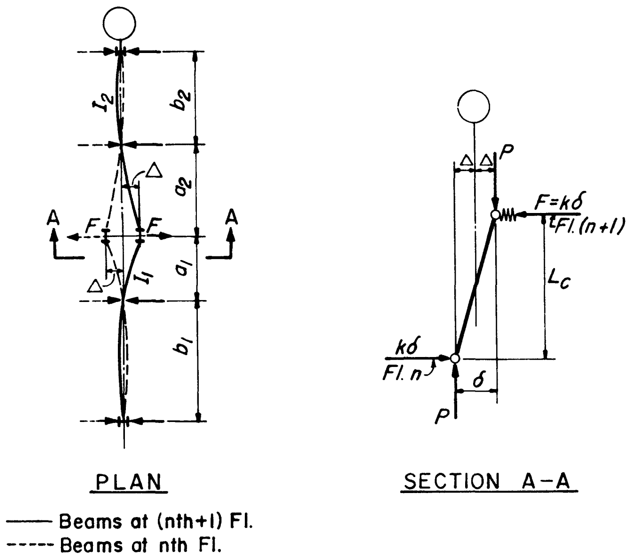

Figure 5

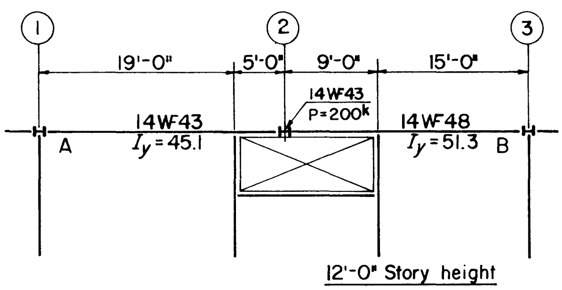

Figure 6

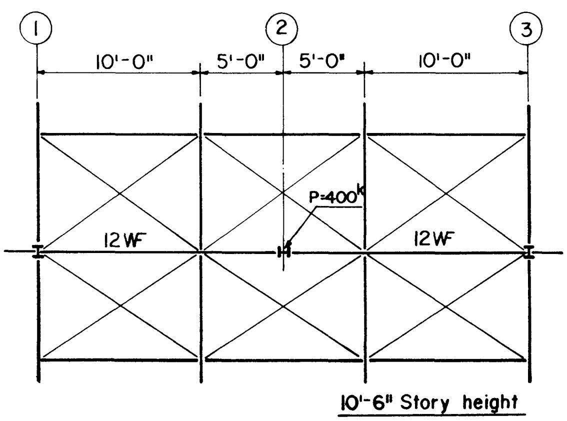

Figure 7

Example 2—Determine whether the combined lateral stiffness of Beams A and B in Fig. 6 is sufficient to stabilize Col. 2. The column, a 14WF43, must support a 200 kip load on an unbraced length of 12 ft. From Equation 10,

$$\frac{45.1}{60^2(228 + 60)} + \frac{51.3}{108^2(180 + 108)} = \frac{P}{24{,}000 \times 144}$$

$$P = 203 \text{ kips} > 200 \text{ kips (OK)}$$

Example 3—Determine the size of the 12WF beams in Fig. 7 needed in order to brace Col. 2 when the load on this column is 400 kips.

Since

$$a_1 = a_2 \quad \text{and} \quad b_1 = b_2$$

$$\text{Req'd } I = \frac{Pa^2(b + a)}{48{,}000\ L_c} \tag{10}$$

$$= \frac{400 \times 60^2 \times 180}{48{,}000 \times 126} = 42.8 \text{ in.}^4$$

Use 12WF40 (I_y = 44.1 in.^{4})

Example 4—Determine the size of beams that would be required had Col. 2 in the previous example been a 14WF158 turned 90° and supporting a 900 kip load.

$$r_x/r_y = 1.60 \text{ (\textit{AISC Manual} p. 3-18)}$$

Since the column load is limited by its y-axis slenderness ratio it is necessary to develop but 1/1.60 times its x-axis buckling load. Therefore the value of P can be taken as 900 kips/1.60 in Equation (10), or 563 kips.

$$\text{Req'd } I = \frac{563 \times 60^2 \times 180}{48{,}000 \times 126} = 60.0 \text{ in.}^4$$

Use 10WF49 (I_y = 93.0 in.^{4})

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5. Manual of Steel Construction *American Institute of Steel Construction, New York, N. Y., pp. 2–129.*