Girder Stiffness Distribution for Unbraced Columns

FRANK W. STOCKWELL, JR.

IN THE GRAVITY design of unbraced columns, the Alignment Chart (Fig. C1.8.2) in the AISC Commentary[1] is very useful. However, this chart is conservative due to several assumptions made in its preparation. One of these assumptions is that the column above and the column below a particular joint share equally in the girder stiffness available at that joint. In reality this is rarely how it works. What happens is that each column takes stiffness in proportion to its particular need for restraint.* This paper discusses a rational method for distributing girder stiffness to each column tier at a given joint, and the significance of this distribution in actual design.

PROCEDURE

Assume the girder stiffness taken by one column at a joint is $X(\Sigma I_g/L_g)$ and the column on the other side of the same joint takes $(1 - X)(\Sigma I_g/L_g)$, where $\Sigma I_g/L_g$ is the total rotational restraint at the joint and X is the stiffness distribution factor. The corresponding G-terms are given by Eqs. (1) and (2).

$$G_1 = \frac{I_{c1}/L_{c1}}{X(\Sigma I_g/L_g)} \tag{1}$$

$$G_2 = \frac{I_{c2}/L_{c2}}{(1 - X)(\Sigma I_g/L_g)} \tag{2}$$

It can be shown that even if the column sections are different on either side of the joint, an equal assignment of girder stiffness ($X = 1 - X = 0.5$) will result in Eq. (3), which is as defined in the Commentary.

$$G = \frac{\Sigma I_c/L_c}{\Sigma I_g/L_g}$$

Prior to the assignment of X-values, consider the typical 2-tier column shown in Fig. 1, where the upper tier (**AB**) is loaded less than the lower tier (**BC**). However, at joint

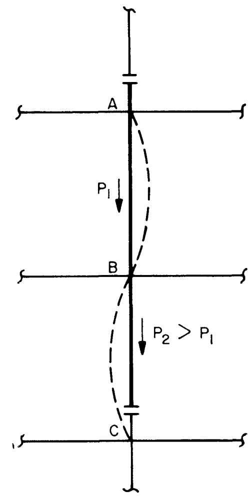

Figure 1

B, both tiers must rotate through the same angle. Recognizing that this rotation is restrained by the girder stiffnesses, and further recognizing that the greater the load, the greater the restraint required, it is obvious that tier **BC** will require more restraint than tier **AB** because load P_2 is greater than load P_1.

The same logic holds true at joints **A** and **C**. (Even though the column below joint **C** is more heavily loaded, it too is an upper tier requiring proportionally less girder stiffness than the column above joint **C**.)

Arbitrarily assign a 60-40 distribution of the girder stiffness:

$$\text{Column } \mathbf{AB}: \quad G_A = G_B = \frac{I_c/L_c}{0.4\Sigma I_g/L_g} \tag{4}$$

$$\text{Column } \mathbf{BC}: \quad G_B = G_C = \frac{I_c/L_c}{0.6\Sigma I_g/L_g} \tag{5}$$

Next find the value of K for each section from the alignment chart and find $P_{allowable}$ from the AISC Manual[2] Column Tables for the particular column (y-y axis assumed braced gives initial choice of column). If $P_{allowable}$ for each tier is greater than P_{actual}, the arbitrary assignment is acceptable. This does *not* mean that the 60-40 distribu-

Frank Stockwell, Jr., is Asst. Chief Engineer—Technical Projects, American Institute of Steel Construction, New York, N.Y.

* *The concept demonstrated in this paper was first presented by Joseph A. Yura, University of Texas, Austin, in his 1975 T. R. Higgins lecture at the AISC National Engineering Conference in St. Louis.*

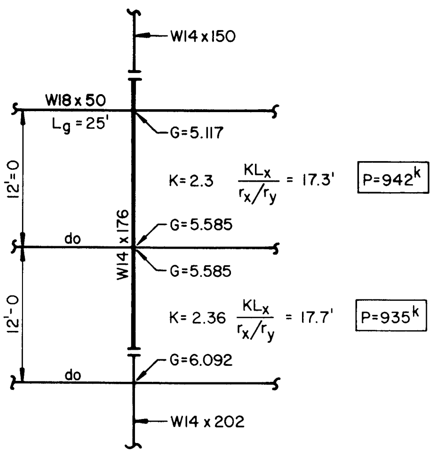

Figure 2

tion represents how each joint will behave. It does mean that a 60-40 distribution gives each column tier enough stiffness to satisfy stability for the applied loads.

If, with the above distribution, one of the columns tiers (say **BC**) was not satisfied but the other (**AB**) was, a reassignment can be made (say 70-30), and each tier checked a second time. It is interesting to note that with this procedure *both* tiers of every column must be checked as opposed to checking only the lower tier with the conventional 50-50 girder stiffness distribution. This procedure can be repeated until either both tiers check (problem is solved) or until neither tier checks (must go to a larger column).

The advantage of a more rational girder stiffness distribution can be further demonstrated by comparing Fig. 2 with Fig. 3. Figure 2 represents a 50-50 girder stiffness distribution [Eq. (3)]. Note that here the upper column tier has the greater capacity, although in a real building this tier would not be loaded as heavily as the lower tier. Thus, there is obviously some wasted column capacity. Figure 3 shows the same column tier, but with the girder stiffness distributed 80-20. The column capacities are more logical than in Fig. 2, since the lower tier has the largest capacity.

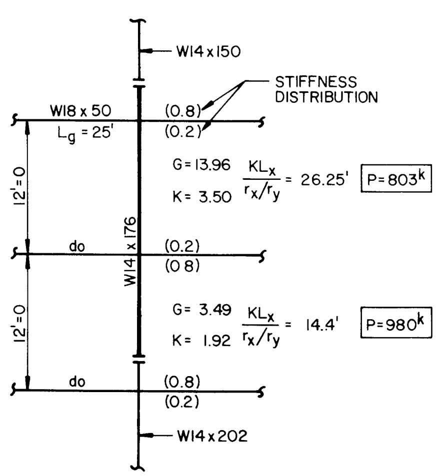

Figure 3

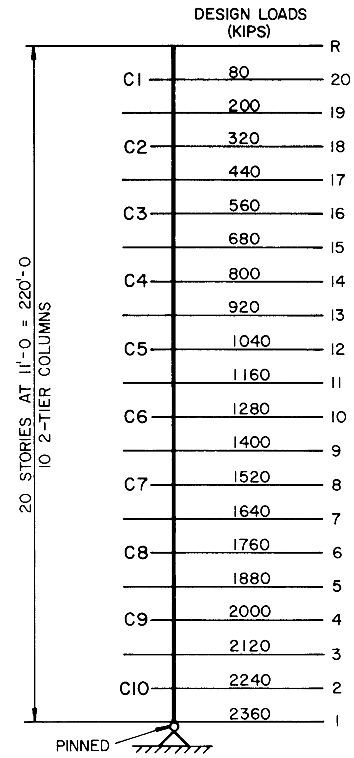

Figure 4

DESIGN EXAMPLE

Determine the column sizes required for the 20-story frame shown in Fig. 4, using 10 2-tier columns, y-axis braced.

Given:

F_y = 36 ksi
$(KL)_y$ = 11.0 ft
Girders: W21 × 55 (I_x = 1140 in.^{4})
L_g = 25.0 ft
Roof load: 80 kips/interior bay
Floor load: 120 kips/interior bay

Solution:

A. Conventional Design:

Assume a trial column section. Using Eq. (3), determine G-values assuming that y-axis effective length $(KL)_y$ controls; then check x-axis effective length and find the allowable column load from the AISC Manual Column Tables. Compare the allowable load with the actual load. If n.g., repeat with new trial section.

Column C1 (Levels 19–20):

Try W14 × 43:

P = 207 kips @ $(KL)_y$ = 11.0 ft
I_x = 429 in.^{4}; r_x/r_y = 3.08

Assume column below is W14 × 84:
I_x = 928 in.^{4}

$$G_T = \frac{2(429)/11}{2(1140)/25} = 0.86$$

$$G_B = \frac{(429 + 928)/11}{2(1140)/25} = 1.35$$

$K = 1.35$ (from alignment chart)

$$\frac{(KL)_x}{r_x/r_y} = \frac{(1.35)(11)}{3.08} = 4.82 \text{ ft} < 11 \text{ ft}$$

$\therefore$ y-y axis controls

Use: W14 × 43

Columns C2 and C3 are also found to be y-axis controlled. Columns C4 through C10 are x-axis controlled; the design of Column C4 is shown below.

Column C4 (Levels 13–14):

Try W14 × 158:

$P = 917$ kips @ $(KL)_y = 11.0$ ft

$I_x = 1900$ in.^{4}; $r_x/r_y = 1.6$

Assume column below is W14 × 202:

$I_x = 2540$ in.^{4}

$$G_T = \frac{2(1900)/11}{2(1140)/25} = 3.79$$

$$G_B = \frac{(1900 + 2540)/11}{2(1140)/25} = 4.43$$

$K = 2.08$

$$\frac{(KL)_x}{r_x/r_y} = 14.3 \text{ ft}$$

$P_{all} = 881$ kips < 920 kips **n.g.**

Use: W14 × 167

Technically, the above procedure should be repeated for the new column selected, W14 × 167, but by inspection it is unnecessary. Note that I_x of the column increases by about 5% (1900 to 2020) and r_x/r_y stays the same. This indicates a slight increase in the G-values and a corresponding increase in K. However, the capacity of 920 kips corresponds to an x-axis effective length of $(KL)_x/(r_x/r_y)$ = 15.2 ft, or a 7% increase (14.3 to 15.2), which is unlikely (actual value = 14.5 ft with G_T increased by 5% and G_B increased by 10%).

Table 1 lists the column sizes determined for all tiers by the conventional method.

B. Girder Stiffness "As Needed" Design:

Columns C1 through C3 are y-axis controlled; therefore, they are designed the same as by the conventional method.

Column C4:

(As previously discussed, *both* tiers must be checked in this method.)

TABLE I

		Conventional Design		Stiffnesses "as needed"	
Column	Level	Column Size	Weight (lbs)	Column Size	Weight (lbs)
C1	19–R	W14 × 43	946	W14 × 43	946
C2	17–19	W14 × 84	1,848	W14 × 84	1,848
C3	15–17	W14 × 119	2,618	W14 × 119	2,618
C4	13–15	W14 × 167	3,674	W14 × 158[a]	3,476
C5	11–13	W14 × 211	4,642	W14 × 202[b]	4,444
C6	9–11	W14 × 264	5,808	W14 × 246[c]	5,412
C7	7–9	W14 × 314	6,908	W14 × 287[c]	6,314
C8	5–7	W14 × 370	8,140	W14 × 320[c]	7,040
C9	3–5	W14 × 426	9,372	W14 × 370[c]	8,140
C10	1–3	W14 × 500	11,000	W14 × 426[c]	9,372
		Total	54,956	Total	49,610

[a] Stiffness distribution design.
[b] Stiffness distribution top tier, stiffness distribution plus inelastic K-factor bottom tier.
[c] Stiffness distribution plus inelastic K-factor design.

Level 14–15:

Column above is W14 × 119 (see Table 1.)

$I_x = 1370$ in.^{4}

Try W14 × 158:

$I_x = 1900$ in.^{4}; $r_x/r_y = 1.60$

$$G_T = \frac{(1900 + 1300)}{2(1140)/25}$$

$= 3.26$ (50-50 stiffness distribution)

From Manual Column Tables ($P = 800$ kips):

Req'd $(KL)_x/(r_x/r_y) = 20$ ft

Req'd $K = (20/11)1.60 = 2.90$

From the Alignment Chart:

Req'd $G_B > 100$

Arbitrarily set an upper limit of 50 on Req'd G_B, because values above 50 cannot be read accurately from the Alignment Chart (this is conservative):

$$50 = \frac{1900/11}{X(2)(1140)/(25)} \text{ or } X = 0.04$$

This means that at Level 14, the column above requires only 4% of the girder stiffness available; thus, 96% of that stiffness may be used for the column below.

Level 13–14:

$$G_T = \frac{1900/11}{(0.96)2(1140)/25} = 1.97$$

From Manual Column Tables ($P = 920$ kips):

Req'd $(KL)_x/(r_x/r_y) = 11.0$ ft

Req'd $K = 1.60$

From the Alignment Chart:

Req'd $G_B = 2.30$

$$2.3 = \frac{1900/11}{X(2)(1140)/25} \text{ or } X = 0.82$$

From the above we can conclude that of the girder stiffness available at Level 13, 82% is needed for the column above and 18% is left for the column below. Continue this procedure with Column C5.

Column C5:

Level 12–13 ($P = 1040$ kips):

Try W14 × 202:

$I_x = 2540$; $r_x/r_y = 1.61$

$G_T = 14.07$

Req'd $(KL)_x/(r_x/r_y) = 20$ ft

Req'd $K = 2.93$

Req'd $G_B = 6.6$

$$6.6 = \frac{2540/11}{X(2)(1140)/25} \text{ or } X = 0.38$$

At Level 12, girder stiffness distribution is 38% to column above and 62% to column below.

Level 11–12 ($P = 1160$ kips):

$$G_T = \frac{2540/11}{(0.62)(2)(1140)/25} = 4.08$$

Req'd $(KL)_x/(r_x/r_y) = 12.0$ ft

Req'd $K = 1.76$

Req'd $G_B = 1.51$

$$1.51 = \frac{2540/11}{X(2)(1140)/25} \text{ or } X = 1.68\ (?)$$

A value of X greater than 1.0 means that at Level 11, the column above needs 168% of the girder stiffness available, which is of course impossible. The design can proceed in either of two ways. First, the column section can be increased and both Levels 12–13 and 11–12 can be rechecked. Alternatively, advantage can be taken of another of the conservative assumptions used in preparing the alignment chart; i.e., the assumption of elastic column behavior. As Yura[3] pointed out, most columns are designed not in the elastic range, but in the inelastic range. If this is the case here, the design can be based on the inelastic approach, as follows:

$f_a = 1160/59.4 = 19.5$ ksi

$\therefore$ design is in the inelastic range

G_T (elastic) $= 4.08$

From Ref. 4, Table 1, stiffness reduction factor for $f_a = 19.5$ ksi and $F_y = 36$ ksi is 0.187.

G_T (inelastic) $= 4.08\ (0.187) = 0.76$

Req'd $K = 1.76$

Req'd G_B (inelastic) $= 7.0$

$$7.0 = \frac{2540/11}{X(2)(1140)/25} \text{ or } X = 0.07$$

Thus, at Level 11, considering a rational girder stiffness distribution plus inelastic K-factors, the column above needs only 7% of the girder stiffness and 93% of that stiffness is available to the column below for its restraint.

Table 1 lists the sizes determined for all columns in this example, both by the "as-needed" method and by the conventional method.

SUMMARY

An "as-needed" girder stiffness distribution can be economical, is logical, and more truly represents how columns and girders interact in reality. However, noting the results listed in Table 1, the most significant departure from conventional analysis is not the concept of "as-needed" girder stiffness distribution, but rather the concept of inelastic K-factors.

Finally, this paper was prepared for the purpose of demonstrating rational stiffness distribution and logical column and girder interaction in unbraced frames. It was written with only gravity loads in mind to, hopefully, avoid clouding the issue. In a real design the procedure shown in the design example is merely a start; the final analysis must be completed with due regard to wind, seismic, drift, etc. as the particular design criteria might require.

REFERENCES

1. Commentary on the Specification for the Design, Fabrication and Erection of Structural Steel for Buildings *American Institute of Steel Construction, February 12, 1969.*
2. Manual of Steel Construction *7th Edition, First Revised Printing, American Institute of Steel Construction, 1973.*
3. *Yura, Joseph A.* The Effective Length of Columns in Unbraced Frames *AISC Engineering Journal, Vol. 8, No. 2, 1971.*
4. *Disque, Robert O.* Inelastic K-factor for Column Design *AISC Engineering Journal, Vol. 10, No. 2, 1973.*